Should We Discount the Welfare of Future Generations?

Peter J. Hammond,
University of Warwick and Stanford University (Emeritus)
Based on joint work with
Graciela Chichilnisky, Columbia University
and Nicholas Stern, London School of Economics

2019 June 25th; typeset from `WBSsummerSchool2019-06.tex`
Online working paper accessible
from `http://wrap.warwick.ac.uk/107726/`
Or search the web for
"Ramsey and Suppes versus Koopmans and Arrow".

Outline

Kenneth J. Arrow (1921–2017)
Holding "The Core of the Arrow–Debreu Economy"

Two Particular Contributions by Arrow

- (1951, 1963) *Social Choice and Individual Values*
- (1982) [with 8 co-authors, including Sen and Stiglitz] *Discounting for Time and Risk in Energy Policy*

Other classic works on inventory theory, decision theory, welfare economics, general equilibrium, health economics, innovation, learning by doing, production functions, etc.

Patrick C. Suppes (1922–2014), Scientific Philosopher

Some Major Contributions by Suppes

- (1957) *Introduction to Logic* (republished by Dover)
- (with David Krantz, Duncan Luce, and Amos Tversky)
  Foundations of Measurement (3 volumes)
  republished by Dover, 2006.
- (edited, 1976) *Logic and Probability in Quantum Mechanics*
- Articles with Bing Han and Zhong-Lin Lu

  (1997) "Brain-wave recognition of words"
  PNAS 94 (26): 14965–14969

  (1998) "Brain-wave recognition of sentences"
  PNAS 95 (26): 15861–15866.

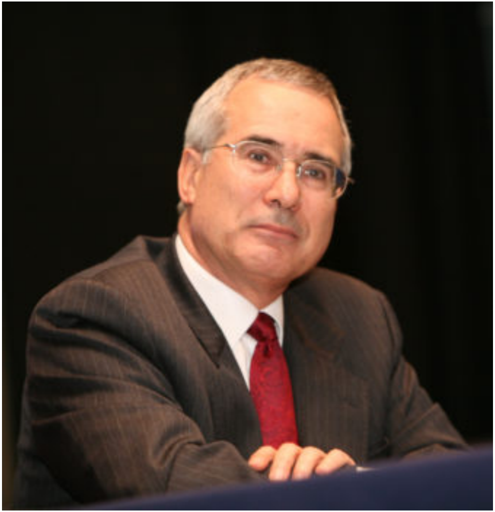
  Three other similar papers in PNAS.

Outline

Coauthor: Graciela Chichilnisky

Graciela in Beijing with Al Gore

Coauthor: Nick Stern



Outline

Graciela's Prior Articles and Book

- (1996) "An Axiomatic Approach to Sustainable Development" *Social Choice and Welfare* 13: 231–257.
- (1997) "What is Sustainable Development?" *Land Economics* 73 (4): 467–491.
- (2009) "Avoiding Extinction: Equal Treatment of the Present and the Future" *Economics: The Open-Access, Open-Assessment E-Journal* `dx.doi.org/10.5018/economics-ejournal.ja.2009-32`.
- (with Kristen A. Sheeran) (2009) *Saving Kyoto: An Insider's Guide to What it is, How it Works and What it Means for the Future* (New Holland Publishers Ltd)

Graciela's Company: Global Thermostat LLC

https://globalthermostat.com/

A carbon negative solution

Uses direct air capture of carbon dioxide from the ambient air.

Intends to sell concentrated/compressed carbon dioxide
for commercial purposes.

Nick's Books

- Stern, Nicholas H. (2006) *The Economics of Climate Change: The Stern Review* (Cambridge University Press).
- Stern, Nicholas H. (2015) *Why Are We Waiting? The Logic, Urgency, and Promise of Tackling Climate Change* (MIT Press).

Four of Nick's Articles

- (2008) "The Economics of Climate Change"
  American Economic Review 98 (2): 1–37.
- (2013) "The Structure of Economic Modeling
  of the Potential Impacts of Climate Change:
  Grafting Gross Underestimation of Risk
  onto Already Narrow Science Models"
  Journal of Economic Literature 51(3): 838–859.
- (2014a) "Ethics, Equity and the Economics
  of Climate Change. Paper 1: Science and Philosophy"
  Economics and Philosophy 30 (3): 397–444.
- (2014b) — —. Paper 2: Economics and Politics"
  Economics and Philosophy 30 (3): 445–501.

Outline

Toward Future Values

See chapter 10, "Topics in Financial Mathematics"
in *Essential Mathematics for Economic Analysis*
coauthored with with Knut Sydsæter, Andrés Carvajal,
and Arne Strøm (Pearson Education).

In a world where savings and investment can earn interest,
each £ of money in a savings account now is worth more
than a £ in the same savings account at any later time t.

That is, the future value increases with t.

Powers of Compound Interest

Indeed, suppose that $r \geq 0$
is the constant rate of interest each period.

Suppose too that S_0 is the amount of money in the account right now, which we take to be date 0.

If this money is left in the savings account,
without any further deposits or withdrawals,
the ensuing future value will be:

- $S_1 = (1 + r)\, S_0$ at date 1,
  after earning interest at rate r for one period;
- $S_2 = (1 + r)^2\, S_0$ at date 2,
  after earning interest at rate r for two periods;
- ...
- $S_t = (1 + r)^t\, S_0$ at date t,
  after earning interest at rate r for t periods.

Present Values and Discount Factors

Conversely, the associated present values of wealth W_t at time t are the respective inverses:

- $P_1 = (1+r)^{-1} W_1$ when $t = 1$;
- $P_2 = (1+r)^{-2} W_2$ when $t = 2$;;
- $\ldots$
- $P_t = (1+r)^{-t} W_t$ for general t.

For each time $t \in T$, the discount factor d_t indicates the present value at time 0 of 1 unit of money received at date t.

Present Discounted Values

A cash flow in discrete time is a function $T \ni t \mapsto C_t \in \mathbb{R}$ which specifies, for each future time $t \in T = \{0, 1, 2, \ldots\}$, what amount of money C_t is expected at time t.

Given the discounting function $T \ni t \mapsto d_t$, the total present discounted value of this cash flow is

$$\text{PDV} = \sum\nolimits_{t \in T} d_t \, C_t$$

Undiscounted Total Utilitarianism

1. Let G_0 denote the finite set of individuals who are alive now;
2. For each time $t \in T$,
   let G_t denote the finite set of individuals who will be born at date t.
3. For each individual $i \in G_0 \cup (\cup_{t \in T} G_t)$,
   let u_i denote the utility that will accrue to i.
   (The meaning of this will be discussed below).

The undiscounted total utilitarian social welfare functional is

$$W = \sum_{i \in G_0} u_i + \sum_{t \in T} \sum_{i \in G_t} u_i$$

This will typically diverge when T and $\cup_{t \in T} G_t$ are infinite.

This is what Arrow calls
"Koopmans' strong argument" for discounting.

Total Utilitarianism with Extinction Discounting

Given the discounting function $T \ni t \mapsto d_t$,
the discounted total utilitarian social welfare functional is

$$W = \sum_{i \in G_0} u_i + \sum_{t \in T} d_t \sum_{i \in G_t} u_i$$

This will typically converge absolutely provided that:

1. there exists a uniform bound b such that $|u_i| \leq b$ everwhere;
2. the sum $\sum_{t \in T} d_t \# G_t$ converges.

We postulate that, due to astrophysics, for each time $t \in T$,
there is an exogenous minimum probability of extinction by date t.

There is an associated maximum probability
that humanity will survive up to date t.

With extinction discounting,
we define d_t as this maximum survival probability.

Outline

The Case Against Discounting: Sidgwick and Ramsey

Sidgwick, Henry (1907) *The Methods of Ethics* (Macmillan).

> *"...the interests of posterity must concern a Utilitarian as much as those of his contemporaries, ..." (p. 414)*

Ramsey, Frank P. (1928) "A Mathematical Theory of Saving" *Economic Journal* 38 (4): 543–549.

> *"...we do not discount later enjoyments*
> *in comparison with earlier ones,*
> *a practice which is ethically indefensible*
> *and arises merely from the weakness of the imagination."*

The Case Against Discounting: Suppes

Suppes, Patrick C. (1966)
"Some Formal Models of Grading Principles"
Synthese 6: 284–306.

Suppose two profiles of individual utility levels differ
only because two individuals' levels have been interchanged.

Then equity demands that these two profiles
should be regarded as ethically indifferent.

This remains true even if the two individuals
belong to two different generations.

The Case for Discounting: Koopmans

Koopmans, Tjalling C. (1960)
"Stationary Ordinal Utility and Impatience"
Econometrica 28 (2): 287–309.

Koopmans, Tjalling C., Peter A. Diamond
and Richard E. Williamson (1964)
"Stationary Utility and Time Perspective"
Econometrica 32 (1–2): 82–100.

Without some discounting, it is logically impossible
to construct a preference ordering
that is both monotonic and continuous
over the entire domain $\mathbb{R}^{\mathbb{N}} = \mathbb{R}^{\infty}$
of all logically possible infinite utility streams.

Peter Diamond's Caveat

Tjalling Koopmans always thought this result should apply only to an individual consumer or household.

In his pioneering work on optimal growth theory,
he explored *inter alia*
the Ramsey framework without any discounting.

(1965) "On the Concept of Optimal Economic Growth" *Academiae Scientiarum Scripta Varia* 28 (1): 225–287; reprinted as Cowles Foundation Paper 238.

(1967) "Objectives, Constraints, and Outcomes in Optimal Growth Models" *Econometrica* 35 (1): 1–15.

The Case for Discounting: Arrow

Significant parts of the following two articles overlap:

Arrow, Kenneth J. (1999a) "Inter-Generational Equity and the Rate of Discount in Long-Term Social Investment" In: Sertel, Murat R. (ed.) *Contemporary Economic Issues. International Economic Association Series*. (Palgrave), pp. 89–102.

— — (1999b) "Discounting, Morality, and Gaming" in Portney, Paul R. and John P. Weyant (eds.) *Discounting and Intergenerational Effects* (Resources for the Future Press: Washington DC) pp. 13–21.

Also

— — (2007) "Global Climate Change: A Challenge to Policy" *The Economists' Voice*. DOI: `https://doi.org/10.2202/1553-3832.1270`.

Arrow on "Koopmans' Weak Argument"

- The weak argument is that, without sufficient discounting, current generations will be asked to make excessive sacrifices.
- Too much empirical content for this theoretical paper ! ...
- I simply note that Graciela's company Global Thermostat®, amongst others, may be reducing the sacrifices needed to avoid excessive risk of catastrophic climate change.

  The company's product is highly purified carbon dioxide captured directly from ambient air.

Arrow on "Koopmans' Strong Argument"

The strong argument is that logic requires discounting.

We show that this does not apply if:

1. each individual's utility function is bounded,
   both above and below,
   as Arrow claimed it should be;
2. as time $t \to \infty$, the likelihood of extinction by time t
   approaches 1 sufficiently fast
   to make the expected total population finite.

Discounting Justified by Unavoidable Extinction

A Cambridge View before the *Stern Review*:

Sidgwick (1907) *The Methods of Ethics*

> *...the effect of his actions on posterity*
> *and even the existence of human beings to be affected*
> *must necessarily be more uncertain. (p. 414)*

Ramsey, Frank P. (1931) "Epilogue" in R.B. Braithwaite (ed.) *The Foundations of Mathematics and Other Logical Essays*.

> *In time the world will cool and everything will die;*
> *but that is a long way off still, and its present value*
> *at compound interest is almost nothing." (p. 291)*

Mirrlees, James A. (1967) "Optimum Growth when Technology is Changing" *Review of Economic Studies* 34 (1): 95–124.

> *I admit the possibility of discounting at a rate r,*
> *to allow for the likelihood of extinction.*

Outline

Discrete Lotteries

Definition

Let Z be any abstract non-empty set.

- A discrete lottery ζ on Z is a mapping

$$Z \ni z \mapsto \zeta(z) \in [0,1] \subset \mathbb{R}$$

  for which there exists a countable support $S_\zeta \subseteq Z$ such that
  1. $\zeta(z) > 0 \Longleftrightarrow z \in S_\zeta$;
  2. $\sum_{z \in Z} \zeta(z) = \sum_{z \in S_\zeta} \zeta(z) = 1$.
- The lottery $z \mapsto \zeta(z)$ is simple just in case its support S_ζ is a finite subset of Z.
- Let
  1. $\Delta^*(Z)$ denote the set of all discrete lotteries on Z;
  2. $\Delta(Z) \subsetneq \Delta^*(Z)$ denote the set of all simple lotteries on Z.

Ethical Decisions are Rational

Assumption

For an appropriate consequence domain Z,
we postulate that a social benefactor should maximize
a (complete and transitive) preference ordering $\succsim$ on $\Delta^*(Z)$,
with strict preference and indifference relations $\succ$ and $\sim$.

Moreover, for all $\lambda, \mu, \nu \in \Delta(Y^n)$, the ordering $\succsim$ should satisfy:

- *independence* (*I*): *whenever $\alpha \in (0, 1]$, then*

$$\lambda \succsim \mu \iff \alpha\lambda + (1-\alpha)\nu \succsim \alpha\mu + (1-\alpha)\nu$$

- *continuity* (*C*): *whenever $\lambda \succ \mu \succ \nu$, the two sets*

$$\begin{array}{rl} & \{\, \alpha \in [0,1] \quad | \quad \alpha\lambda + (1-\alpha)\nu \succsim \mu \,\} \\ \textit{and} & \{\, \alpha \in [0,1] \quad | \quad \mu \succsim \alpha\lambda + (1-\alpha)\nu \,\} \end{array}$$

are both closed.

Dominance

David H. Blackwell and Meyer A. Girshick (1954)
Theory of Games and Statistical Decisions (John Wiley).

Both these statisticians were collaborators of Kenneth Arrow.

Definition

The preference ordering $\succsim$ on $\Delta^*(Y)$ satisfies:

1. dominance (D) just in case,
   whenever the countable collection of lotteries $\lambda_i, \mu_i \in \Delta^*(Y)$
   and convex weights $\alpha_i > 0$ satisfy both:
   (i) $\lambda_i \succsim \mu_i$ (all $i = 1, 2, \ldots$); and (ii) $\sum_{i=1}^{\infty} \alpha_i = 1$;
   then $\sum_{i=1}^{\infty} \alpha_i \lambda_i \succsim \sum_{i=1}^{\infty} \alpha_i \mu_i$;
2. strict dominance (D*) (B & G, page 105, H_1)
   just in case, if in addition $\lambda_i \succ \mu_i$ for some i,
   then $\sum_{i=1}^{\infty} \alpha_i \lambda_i \succ \sum_{i=1}^{\infty} \alpha_i \mu_i$. □

When the collection of lotteries is finite,
condition (D*) and, *a fortiori* (D), is implied by (I).

Expected Utility Representation

Definition

The preference ordering $\succsim$ on $\Delta^*(Z)$
has an expected utility representation just in case
there exists a von Neumann–Morgenstern utility function (NMUF)

$$Z \ni z \mapsto v(z) \in \mathbb{R}$$

such that, for all $\lambda, \mu \in \Delta^*(Z)$, one has

$$\lambda \succsim \mu \Longleftrightarrow \mathbb{E}_\lambda v \geq \mathbb{E}_\mu v$$

where $\mathbb{E}_\lambda v := \sum_{z \in Z} \lambda(z)\, v(z)$
denotes the expectation of v w.r.t. $\lambda \in \Delta^*(Z)$.

Unbounded Utility and the St. Petersburg Paradox

Christian Seidl (2013) "The St. Petersburg Paradox at 300" *Journal of Risk and Uncertainty* 46 (3): 247–264.

Karl Menger (1934) "Das Unsicherheitsmoment in der Wertlehre, Betrachtungen im Anschluss an das sogenannte Petersburger Spiel" *Zeitschrift für Nationalökonomie* 5: 459–485.

Translated under the title "The Role of Uncertainty in Economics" in M. Shubik (ed.) (1967) *Essays in Mathematical Economics in Honor of Oskar Morgenstern* (Princeton University Press), ch. 16, pp. 211–231.

Kenneth J. Arrow (1965) *Aspects of the Theory of Risk-Bearing* (North-Holland).

— — (1974) "The Use of Unbounded Utility Functions in Expected-Utility Maximization: Response" *Quarterly Journal of Economics* 88: 136–138.

Dominance and Bounded Expected Utility

Definition

An NMUF $Z \ni z \mapsto v(z) \in \mathbb{R}$ is bounded just in case
there exist $\underline{v}, \bar{v} \in \mathbb{R}$ such that $\underline{v} \leq v(z) \leq \bar{v}$ for all $z \in Z$.

Peter J. Hammond (1998)
"Objective Expected Utility: A Consequentialist Perspective"
in Salvador Barberà, Peter J. Hammond and Christian Seidl (eds.)
Handbook of Utility Theory, Vol. 1: Principles
(Kluwer Academic Publishers) ch. 5, pp. 145–211.

Inspired by Blackwell & Girshick and by Arrow,
the results in Section 8 of this survey chapter imply:

Theorem

*Provided that $\#Z \geq 3$, the preference ordering $\succsim$ on $\Delta^*(Z)$
has a bounded expected utility representation
if and only if it satisfies conditions (I), (C) and (D) — or (D*).*

Of course (I) $\Longrightarrow$ (D) when Z is finite.

Outline

Defining Consequentialism

Arrow, Kenneth J. (1951) "Alternative Approaches to the Theory of Choice in Risk-Taking Situations" *Econometrica* 19 (4) : 404–437.

> *Among the actions actually available, then, that action is chosen whose consequences are preferred to those of any other available action. (p. 404)*

Aquinas, St. Thomas (1265–1274, 1947) *Summa Theologiae* (Benziger Bros. edition)

First Part of the Second Part > Question 20, Article 5, Objection

> *Sed contra, eventus sequens non facit actum malum qui erat bonus, nec bonum qui erat malus.*

Translation:

> *On the Contrary, The consequences do not make an action evil that was good; nor good one that was evil.*

A Universal Domain of Personal Consequences

We will apply prescriptive social choice theory
with an individualistic theory of social consequences.

We start with a "universal" personal consequence domain,
taken to be a non-empty set Y in some high-dimensional space.

We postulate that the typical member $y \in Y$
includes everything that should be ethically relevant
to any decision, whether individual or social,
that concerns the entire experience of a person's life history.

It even includes a special non-existence consequence y_0
to recognize the possibility
that a person may not come into existence.

It may also include individuals' rights to choose for themselves.

And equality in the society to which one belongs.

Outline

Fundamental Utility: References

Tinbergen, Jan (1957)
"Welfare Economics and Income Distribution"
AER, Papers and Proceedings 47 (2): 490–503.

Kolm, Serge-Christophe (1994)
"The Meaning of 'Fundamental Preferences'"
Social Choice and Welfare 11 (3): 193–198.

Cites Tinbergen, and criticizes:
Broome, John (1993)
"A Cause of Preference Is Not an Object of Preference"
Social Choice and Welfare 10 (1): 57–68.

Fleurbaey, Marc, and Peter J. Hammond (2004)
"Interpersonally Comparable Utility"
in *Handbook of Utility Theory, Vol. 2: Extensions*
(Kluwer Academic Publishers) ch. 21, pp. 1181–1285.

Expected Utility for Adam Smith's Impartial Spectator

What if Adam Smith had understood modern decision theory?

What if he had considered what consequence lottery
would be best for an individual
from the point of view of his impartial spectator?

Assumption

*On the domain Y of personal consequences,
the impartial spectator has a preference ordering $\succsim$
over the domain $\Delta^*(Y)$
of discrete personal consequence lotteries λ.*

*Moreover, the ordering $\succsim$ can be represented
by the expected value $\Delta^*(Y) \ni \lambda \mapsto \mathbb{E}_\lambda v := \sum_{y \in Y} \lambda(y), v(y) \in \mathbb{R}$
of a fundamental bounded utility function $Y \ni y \mapsto u(y) \in \mathbb{R}$.*

Interpersonal Comparisons of Utility

Remark

With this device, "interpersonal" comparisons of utility get replaced by comparisons of different personal consequences based on the fundamental interpersonal utility function.

Indeed, the fundamental utility function $Y \ni y \mapsto u(y) \in \mathbb{R}$ *allows, for any pair of individuals* h, i *and their personal consequences* $y_h, \tilde{y}_h, y_i, \tilde{y}_i \in Y$*, both:*

- *ordinal interpersonal comparisons of the form* $u(y_h) \gtreqless u(y_i)$*;*
- *cardinal interpersonal comparisons of the form* $u(y_h) - u(\tilde{y}_h) \gtreqless u(y_i) - u(\tilde{y}_i)$*.*

Outline

Social Consequences as Personal Consequence Profiles

Assume a finite fixed potential population of size $n \in \mathbb{N}$.

Thus, the set of personal numerical labels is taken to be

$$\mathbb{N}_n := \{1, 2, \ldots, n\} = \{i \in \mathbb{N} \mid i \leq n\}$$

Later, we will analyse potentially infinite populations
— the case of most concern to Ramsey, Koopmans and Arrow.

For now, with population fixed at size n,
take the social consequence domain
to be the n-fold Cartesian product Y^n
of the personal consequence domain Y.

So each social consequence $y^n \in Y^n$ is an n-dimensional profile

$$y^n = \langle y_i \rangle_{i=1}^n = \langle y_i \rangle_{i \in \mathbb{N}_n}$$

of personal consequences for the n individuals $i \in \mathbb{N}_n \subset \mathbb{N}$.

Social and Marginal Personal Consequence Lotteries

Recall that, for a society with n individuals,
we defined a social consequence y^n
as a member of the Cartesian product Y^n
of n copies of the personal consequence domain Y.

We will consider decisions that yield outcomes
described by a social consequence lottery $\lambda \in \Delta(Y^n)$.

Given any such social consequence lottery,
each person numbered $i \in \mathbb{N}_n$ experiences the relevant
marginal personal consequence lottery $\lambda_i = \text{marg}_i \, \lambda \in \Delta(Y)$
over i's personal copy of the personal consequence domain Y.

Given any $\lambda \in \Delta(Y^n)$,
this marginal lottery has probabilities given by

$$\lambda_i(y) = \underset{i}{\text{marg}} \, \lambda(y) := \lambda(\{y^n \in Y^n \mid y_i = y\})$$

Outline

Original Positions before Rawls's *Theory of Justice*

- Hume, David (1739) *A Treatise on Human Nature*.
- Rousseau, Jean-Jacques (1762) *The Social Contract*.
- Vickrey, William S. (1945) "Measuring Marginal Utility by Reactions to Risk" *Econometrica* 13 (4): 319–333.
- Harsanyi, John C. (1953) "Cardinal Utility in Welfare Economics and in the Theory of Risk-Taking" *Journal of Political Economy* 61: 434–435.
- Harsanyi, John C. (1955) "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility" *Journal of Political Economy* 63 (4): 309–321.
- Vickrey, William S. (1960) "Utility, Strategy and Social Decision Rules" *Quarterly Journal of Economics* 74 (4): 507–535.

From Original Positions to Personal Lotteries

Definition

An original position with n persons is a lottery $\mu \in \Delta(\mathbb{N}_n)$.

The original position lottery μ is unbiased or impartial just in case it satisfies $\mu(i) = \frac{1}{n}$ for all $i \in \mathbb{N}_n$; otherwise it is biased.

Definition

Given any original position $\mu \in \Delta(\mathbb{N}_n)$
and any social consequence lottery $\lambda \in \Delta^*(Y^n)$,
there is an equivalent personal consequence lottery
defined as the compound lottery $\mu \circ \lambda \in \Delta^*(Y)$
that gives every personal consequence $y \in Y$ the probability

$$(\mu \circ \lambda)(y) = \sum\nolimits_{i \in \mathbb{N}_n} \mu(i)\, \lambda_i(y)$$

where λ_i denotes the marginal lottery $\text{marg}_i\, \lambda$.

Reduced Consequentialism

Assumption

Individualistic ethics requires rational behaviour in the (almost) unrestricted domain of finite risky decision trees whose consequences $(\mu, \lambda) \in \Delta(\mathbb{N}_n) \times \Delta^*(Y^n)$ *combine:*

1. *an original position lottery* $\mu \in \Delta(\mathbb{N}_n)$*;*
2. *a social consequence lottery* $\lambda \in \Delta^*(Y^n)$*.*

Moreover, such rational behaviour must be explained by the reduced consequences $\mu \circ \lambda \in \Delta^*(Y)$ *of each pair* $(\mu, \lambda) \in \Delta(\mathbb{N}_n) \times \Delta^*(Y^n)$*.*

Fundamental Cardinal Utility and Its Expectation

Theorem

Suppose that for each $n \in \mathbb{N}$, ethical behaviour in decision trees with consequences $(\mu, \lambda) \in \Delta(\mathbb{N}_n) \times \Delta^*(Y^n)$ maximizes a complete and transitive preference ordering on $\Delta^*(Y)$ that satisfies independence, continuity, and strict dominance.

Then there exists a unique cardinal equivalence class of fundamental bounded utility functions $Y \ni y \mapsto u(y) \in \mathbb{R}$ whose expected values

$$\mathbb{E}_{\mu \circ \lambda} u := \sum_{y \in Y} (\mu \circ \lambda)(y)\ u(y) = \sum_{i \in \mathbb{N}_n} \mu(i) \sum_{y \in Y} \lambda_i(y)\, u(y)$$

represent the preference ordering on $\Delta(\mathbb{N}_n) \times \Delta(Y^n)$ or, equivalently, on $\Delta(Y)$.

Critique of Prioritarianism

Derek Parfit (1997) "Equality and Priority" *Ratio* 10 (3): 202–221.

> *A utilitarian (or radical egalitarian) might also claim that their theory is more parsimonious than prioritarianism (which values well-being and priority).*
>
> — *from the Wikipedia entry on "prioritarianism"*

The (online) *Stanford Encyclopedia of Philosophy* has no entry on "prioritarianism"!

If priority should matter — for example, because of a concern for poverty — then it should be included in the definition of personal consequence.

So the impartial spectator's utility function should take it into account.

In this way, priority is subsumed into utility.

The Consequentialist Pre-Axiom . . .

Consider finite decision trees, with a hierarchy of decision nodes where the decision maker has a decision to make.

Also continuation subtrees emanating from any decision node.

Attached to each terminal node is a consequence (or personal consequence profile).

We postulate that behaviour in any decision tree,
including any continuation subtree,
should be explicable as the choice of consequence.

This postulate implies that:

1. behaviour should maximize a (complete and transitive) preference ordering over the domain of consequences;
2. preferences satisfy the independence axiom that is violated in the Allais paradox;
3. preferences satisfy Savage's sure-thing principle that is violated in the Allais paradox.

…Implies Subjective Expected Utility Maximization

Supplementary condition: the set of chosen consequences varies continuously with respect to positive hypothetical probabilities at any chance node of a decision tree.

Together with the consequentialist pre-axiom,
this continuity condition implies that chosen consequences should maximize subjective expected utility (SEU).

If planned behaviour satisfies the consequentialist pre-axiom,
planned behaviour in any continuation subtree
continues planned behaviour in the full decision tree.

But if the consequentialist pre-axiom is not satisfied,
a decision theory may have to specify
what is chosen in continuation subtrees
as well as in whole decision trees.

The Consequentialist Pre-Axiom: Bibliography I

- (1977) "Dynamic Restrictions on Metastatic Choice" *Economica* 44, 337–350.
- (1988) "Consequentialism and the Independence Axiom" in B.R. Munier (ed.) *Risk, Decision and Rationality* (Dordrecht: D. Reidel), pp. 503–516.
- (1988) "Consequentialist Foundations for Expected Utility" *Theory and Decision* 25: 25–78.
- (1996) "Consequentialism, Structural Rationality and Game Theory" in K.J. Arrow, E. Colombatto, M. Perlman, and C. Schmidt (eds.) *The Rational Foundations of Economic Behaviour* (IEA Conference Volume) ch. 2, pp. 25–42.

The Consequentialist Pre-Axiom: Bibliography II

- (1998) in the *Handbook of Utility Theory, Vol. 1: Principles* (co-edited with Salvador Barberà and Christian Seidl):
  - "Objective Expected Utility: A Consequentialist Perspective" ch. 5, pp. 145–211;
  - "Subjective Expected Utility" ch. 6, pp. 213–271.
- (2007) "Schumpeterian Innovation in Modelling Decisions, Games, and Economic Behaviour" *History of Economic Ideas* XV, 179–195.
- (2014) (with Horst Zank) "Rationality and Dynamic Consistency under Risk and Uncertainty" in Mark J. Machina and W. Kip Viscusi (eds.) *Handbook of the Economics of Risk and Uncertainty, Vol 1* ch. 2, pp. 41–97.

Suppes Equity: Origins

Suppes, Patrick C. (1966)
"Some Formal Models of Grading Principles"
Synthese 6: 284–306.

Sen, Amartya K. (1970, 2017)
Collective Choice and Social Welfare: Expanded Edition
(Penguin Books).

Hammond, Peter J. (1979)
"Equity in Two-Person Situations: Some Consequences"
Econometrica 47: 1127–1135.

Suppes Equity Implies Unbiasedness

Definition

The social preference satisfies Suppes equity just in case,
in any two-person decision situation $\nu, \rho \in \Delta^*(Y^n)$
where $\nu_i = \rho_i$ for all $i \in \mathbb{N}_n \setminus \{j, k\}$,
interchanging the pairs of marginal lotteries ν_j, ν_k and ρ_j, ρ_k
for these two individuals has no effect
on the social preference between ν and ρ in $\Delta^*(Y^n)$.

Theorem

The original position welfare function $\Delta^*(Y^n) \ni \lambda \mapsto \mathbb{E}_{\bar{\mu} \circ \lambda} u \in \mathbb{R}$ satisfies Suppes equity if and only if $\bar{\mu}(i) = \frac{1}{n}$ for all $i \in \mathbb{N}_n$.

Outline

The Exogenous Hazard of Unavoidable Extinction

Astrophysics dictates that meaningful life
within many light years of the Earth
will eventually become impossible
at some finite extinction date $e \in \mathbb{N}$ that is unavoidable.

We regard e as the exogenous background date of extinction;
human folly may well advance the date e endogenously.

The current Extinction Rebellion in London is against such folly.

For each $t \in \mathbb{N}$, let σ_t denote the survival probability that $e > t$,
implying that generation t can live (possibly a very short life)
before extinction; to avoid triviality, assume that $\sigma_1 = 1$.

Thus $\eta_t := \sigma_t - \sigma_{t+1}$ is the probability that $e = t$.

Also, the conditional probability that $e = t$ given that $e \geq t$
is the hazard rate $\eta_t / \sigma_t = (\sigma_t - \sigma_{t+1}) / \sigma_t = 1 - \sigma_{t+1} / \sigma_t$.

An Extinction Biased Original Position

For simplicity, suppose that for each $t \in \mathbb{N}$
generation t consists entirely of the lone individual t.

Definition

In the extinction biased original position with n individuals:

1. For each $t \in \mathbb{N}$ the conditional probability
that individual t comes into existence
soon enough to avoid unavoidable extinction
is proportional to the survival probability σ_t.
2. The conditional probability
that person t avoids exogenous extinction
is therefore equal to $\sigma_t / \sum_{k=1}^n \sigma_k$.
3. The conditional expected utility of person t is therefore

$$\sum_{t=1}^n \sigma_t \, u_t / \sum_{k=1}^n \sigma_k$$

An Original Position with Infinitely Many Individuals

For simplicity, suppose that:

1. the set of potential individuals is $\mathbb{N} := \{1, 2, \ldots, \}$;
2. generation t consists entirely of individual t, for all $t \in \mathbb{N}$.

The probability that t is small enough to exist is σ_t.

We consider an extended original position in which:

1. there is an arbitrary but fixed unconditional positive probability P of coming into existence;
2. for each $t \in \mathbb{N}$, given that person t does come into existence, there is a conditional probability of becoming individual t given by $k\,\sigma_t$, for a suitable constant $k > 0$.

For these to be conditional probabilities,
we require that the infinite sum $\sum_{t \in \mathbb{N}} k\,\sigma_t = 1$,
implying that $k = 1 / \sum_{t \in \mathbb{N}} \sigma_t$.

When Do Conditional Probabilities Exist?

Lemma

Given the extinction process with survival probabilities σ_t, the expected number of individuals is $\sum_{t\in\mathbb{N}} \sigma_t$.

Proof.

If the extinction date is t, there will be t individuals.

So the expected number of individuals is

$$\sum_{t\in\mathbb{N}} \eta_t \, t = \sum_{t\in\mathbb{N}} (\sigma_t - \sigma_{t+1}) t$$
$$= \sum_{t\in\mathbb{N}} \sigma_t [t - (t-1)] = \sum_{t\in\mathbb{N}} \sigma_t \quad \square$$

Theorem

There exists an original position $\mu \in \Delta^*(\mathbb{N})$ with conditional existence probabilities $\mu_t := \sigma_t / \sum_{s\in\mathbb{N}} \sigma_s$ if and only if the expected number of individuals is finite.

Conditional Expected Utility Given Existence

Recall that P denotes the unconditional probability of existence, whereas 0 is the utility of non-existence.

Note that the unconditional EU in the original position is given by

$$P \times \text{conditional EU} \mid \text{existence} + (1 - P) \times 0$$

Because P is fixed, the following are equivalent:

1. maximizing unconditional EU in the original position;
2. maximizing conditional EU given existence.

But the second maximand is $\sum_{t \in \mathbb{N}} \mu_t \, u_t$.

After multiplying by the finite positive constant $\sum_{t \in \mathbb{N}} \sigma_t$,
the expected number of individuals,
the maximand becomes $\sum_{t \in \mathbb{N}} \sigma_t \, u_t$.

But this is expected discounted future utility,
with the rates of discount
equal to exogenous maximum possible survival probabilities.

Main Result in This Presentation

We consider an exogenous background stochastic process of unavoidable extinction.

Assumption (Finite Expected Population)

Consider the total population of all individuals who, at or before the randomly determined extinction date, may live at any time in history, and so could have non-zero utility.

We consider the restricted domain of infinite utility streams for which the expectation of this population is finite.

Under this assumption, along with bounded utility, extinction discounting gives a well-defined and equitable intergenerational welfare criterion.

So extinction discounting avoids the strong Koopmans argument that was cited by Arrow.

Two More Results in the Full Paper

Even under this assumption of a finite expected population:

1. Extinction discounting fails to avoid what Chichilnisky calls a “dictatorship of the present”.
2. Combining extinction discounting with what Chichilnisky calls a “dictatorship of the future” can give what she calls a “sustainable” welfare criterion that avoids dictatorship of either the present or the future.

Envoi

Many thanks:

- to Kenneth Arrow, for inspiring not only several generations of people interested in these topics, but also myself;
- to Patrick Suppes, who never ceased treating me as if I might eventually become some sort of philosopher;
- to co-authors Graciela and Nick not only for their past work, as well as numerous improvements that they have suggested, but also for their patience while I have been distracted by other obligations from pursuing work on our paper as intensively as we would desire;
- to all of you for your attention!