Sentiment and Beta Herding[1]

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Abstract

We propose a non-parametric measure of beta herding based on linear factor models and apply it to investigate the nature of (slowing moving) herd behaviour in the US, UK, and South Korean stock markets. We find clear evidence of beta herding when investors believe that they know where market is heading rather than when the market is in crises.

Keyword Herding, Sentiment, Non-central Chi Square Distribution, Market Crises.

JEL Code C12,C31,G12,G14

1 Introduction

Herding is widely believed to be an important element of behaviour in financial markets and yet the empirical evidence is not conclusive.[1] Most studies have failed to find strong evidence of herding except in a few particular cases, for example herding by market experts such as analysts and forecasters (see Hirshleifer and Teoh, 2001). One difficulty lies in the failure of methods to differentiate between a rational reaction to changes in fundamentals and irrational herding behaviour. It is critical to discriminate empirically between the two, since the former simply reflects an efficient reallocation of assets whereas the latter potentially leads to market inefficiency.

In this study we define herding in a slightly different way from the conventional definition and then propose an empirical method that makes this critical distinction between fundamental adjustment and herding. We define herding as the behavior of investors who simply follow the performance of specific factors such as the market portfolio itself or particular sectors, styles, or macroeconomic signals, and hence buy or sell individual assets at the same time disregarding the long-run risk-return relationship. Although our measure can be easily applied to these other specific factors, say herding towards the new technology sector, we will only focus here on herding towards the market portfolio which we call 'beta herding' in our study. Our definition of herding is therefore at the market-wide level similar to Christie and Huang (1995) and Hwang and Salmon (2004).[2] The existence of this type of herding suggests that individual

[1]See Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), and Welch (1992) for information-based herding, Scharfstein and Stein (1990) reputation-based herding, and for compensation-based herding Brennan (1993), and Roll (1992). These studies of herd behaviour are closely related to the study of contagion, see Eichengreen, Mathieson, Chadha, Jansen, Kodres and Sharma (1998) for example.

[2]Throughout this paper we implicitly assume that herding should naturally be viewed in a relative sense rather than as an absolute and that no market can ever be completely free of some element of herding. Thus we argue that there is either more or less herding (including 'adverse' herding) in a market at some particular time and herding is a matter of degree. It seems to us conceptually difficult if not impossible to rigorously define a statistic which could measure an absolute level of herding. However, most herd measures that have been proposed, such as Lakonishok, Shleifer, and Vishny (1992), Wermers (1995), and Chang, Cheng and Khorana (2000), have apparently tried to identify herding in absolute terms.

assets will be mispriced as equilibrium beliefs are suppressed. For practitioners, using styles to form hedge portfolios may not be effective when there is herding towards the styles. Hedging strategies could work well when there is 'adverse' herding where factor sensitivities (betas) are widely dispersed.[3] Based on this definition, Hwang and Salmon (2004) (from now on HS) propose an approach based on a disequilibrium CAPM which leads to a measure that can empirically capture the extent of herding in a market. This measure can be estimated using the cross-sectional variance of the factor sensitivities of the individual assets in the market.

In this paper, we extend the previous study of HS in several significant ways. First, we investigate herding in the presence of market-wide sentiment. While sentiment may affect the entire return distribution and hence all moments we will follow the majority of the literature and define sentiment with reference to the mean of noise traders' subjective returns. If it is too high or too low, optimistic or pessimistic sentiment exists.[4] We find that herding activity increases with market-wide sentiment. When there is market-wide positive sentiment so that individual asset returns are expected to increase regardless of their systematic risks, herding increases. On the other hand negative sentiment is found to reduce herding level. Empirical evidence by Brown and Cliff (2004) of high contemporaneous relationship between market returns and sentiment suggests that herding activity increases in bull markets while it decreases during bear markets. Our measure of beta herding therefore is driven by two forces; one from cross-sectional herding within the market towards the market portfolio, and the other from market-wide sentiment that evolves over time.

Second, we study the dynamic nature of herd behavior over an extended time horizon. Herding is generally perceived to be a phenomenon that arises rapidly and thus most studies of herding explicitly or implicitly examine herding over very short time intervals. However, Summers and Porterba (1988) and Fama and French (1988) show that noise may be highly persistent and slow-moving over time.[5] Shiller (2000)

[3] Our term adverse herding is consistent with the use of "disperse" in Hirshleifer and Teoh (2001).

[4] Several different approaches to defining sentiment have been proposed; see De Long, Shleifer, Summers, and Waldmann (1990), Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, Subrahmanyan (1998). Essentially we follow Shefrin (2005) and regard sentiment as an aggregate belief which affects the market as whole.

[5] Noise here is referred to any factors that make asset prices deviated from fundamentals, and thus

argues that if markets are not efficient at both the macro and micro levels and if the "conventional wisdom" given by experts only changes very slowly, then the short-run relationship may provide us with biased information about the level of stock prices. A dramatic case of slow-moving noise is a 'bubble' where the cycle (of a bubble) may not be completed within days, weeks or even months. For example, bubbles such as the Tulip Bubble in seventeenth-century Holland, the real estate bubble in the late 1980s Japan, and the recent dot-com bubble were not formed within short time periods. It took years for these bubbles to develop and finally make their impact on the market.[6] If this argument is correct, then we should find evidence for slow moving herd behavior, and for this reason we use monthly data rather than higher frequency data.

Third, our measure of herding can also explain in which periods betas are relatively more dispersed and in which periods they are not. Fama and French (1992, 1993) show that the average returns of portfolios formed on beta are not cross-sectionally different, and that portfolios formed on size and book-to-market do not show significant difference in their betas. There is clear difference before and after 1963; before 1963 CAPM works well and beta is priced while after 1963 CAPM does not appear to work. Many explanations have been proposed; among others, time varying betas by Jagannathan and Wang (1996) and Ang and Chen (2005), discount rate and cash flow betas by Campbell and Vuolteenaho (2004). Most studies investigate the anomaly in a cross-sectional world. Our approach allows us to investigate in what time periods "herding in betas" causes failure of CAPM.

Finally a non-parametric method is proposed to measure herding. This approach is more flexible than the parametric model of HS as no specific parametric model for herding needs to be specified and hence assumed. In addition, we develop a statistical framework within which the significance of the estimated herd measure can be formally assessed. The measure we derive follows a non-central Chi-square distribution, which enables us to investigate if there is any significant difference in the estimated levels of herd behavior between any two periods, or if the estimated level of herding changes over time significantly.

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includes 'anomalies' such as sentiment and herding. See Black (1986) and DeLong et al (1990) for discussion on noise and asset pricing.

[6] See Shiller (2003) for example.

We apply our non-parametric procedure to the US, UK, and South Korean stock markets and find that beta herding does indeed move slowly, but is heavily affected by the advent of crises. Contrary to the common belief that herding is only significant when the market is in stress, we find that herding can be much more apparent when market continues to rise slowly or when it becomes apparent that market is falling. Once a crisis appears herding toward the market portfolio becomes much weaker, as individuals become more concerned with fundamentals rather than overall market movements. We also show that the herd measure we propose is robust to business cycle and stock market movements.

Our results confirm that herding occurs more readily when investors' expectations regarding the market are more homogeneous, or in other words when the direction towards which the market is heading is relatively clear - whether it be a bull or a bear market. The results suggest that herding is persistent and moves slowly over time like stock prices around the intrinsic values as discussed in Shiller (1981, 2000, 2003), but critically showing a different dynamic to stock prices.

In the next section we briefly explain the concept of the herding and its measurement in the presence of market sentiment; we then develop a non-parametric method to estimate herd behavior based on the cross-sectional variance of the t-statistics of estimates of the betas in a linear factor model. In section 3, we apply this new method to the US, UK, and South Korean stock markets, and provide empirical evidence of the relationship between herding and sentiment in section 4. Finally we draw some conclusions in section 5.

2 Market Sentiment and Measure of Herding

In this section we present a model that can be used to estimate herding in the presence of market sentiment. The type of herding we investigate is the behaviour of investors who follow the performance of factors such as the market index (or market-wide movements) to buy or sell individual assets at the same time disregarding the equilibrium risk-return relationship. Herding is then said to be towards the market (return). To investigate this type of herding we first discuss how individual betas in the CAPM are affected by herding and sentiment. Under certain conditions sentiment has similar effects to herding; cross-sectional variance of the betas decreases as herding and market-wide sentiment increases.

2.1 Herding, Sentiment, and Betas

Consider the following CAPM in equilibrium,

$$E_t(r_{it}) = \beta_{imt} E_t(r_{mt}), \tag{1}$$

where r_{it} and r_{mt} are the excess returns on asset i and the market at time t, respectively, β_{imt} is the systematic risk, and $E_t(.)$ is conditional expectation at time t. In equilibrium, given the view of the market ($E_t(r_{mt})$), we only need β_{imt} in order to price an asset i. When there is beta herding, an individual asset's expected return, $E_t(r_{it})$, is affected by the expected market movement $E_t(r_{mt})$ more than the CAPM suggests and thus β_{imt} is biased towards 1. Conditional on the expected market return HS suggest the following simple model to explain herd behaviour;

$$\frac{E_t^b(r_{it})}{E_t(r_{mt})} = \beta_{imt}^b = \beta_{imt} - h_{mt}(\beta_{imt} - 1), \tag{2}$$

where $E_t^b(r_{it})$ and β_{imt}^b are the market's biased conditional expectation of excess returns on asset i and its beta at time t, and h_{mt} is a parameter that captures herding and changes over time, $h_{mt} \leq 1$. This is a generalized model that encompasses the equilibrium CAPM with $h_{mt} = 0$, but allows for temporary disequilibrium.

Let us consider several cases in order to see how herding affects individual asset prices *given* the evolution of the expected market return, $E_t(r_{mt})$. First of all, when $h_{mt} = 1$, $\beta_{imt}^b = 1$ for all i and the expected excess returns on the individual assets will be the same as that on the market portfolio regardless of their systematic risks. Thus $h_{mt} = 1$ can be interpreted as perfect herding toward the market portfolio. In general, when $0 < h_{mt} < 1$, herding exists in the market, and the degree of herding depends on the magnitude of h_{mt}. In terms of betas, when $0 < h_{mt} < 1$, we have $\beta_{imt} > \beta_{imt}^b > 1$ for an equity with $\beta_{imt} > 1$, while $\beta_{imt} < \beta_{imt}^b < 1$ for an equity with $\beta_{imt} < 1$. Therefore when there is beta herding, the individual betas are biased towards 1. Given that the model is designed to return towards equilibrium betas over time and hence behaviour fluctuates around the equilibrium CAPM, we also explain 'adverse beta herding' when $h_{mt} < 0$. In this case an equity with $\beta_{imt} < 1$ will be less sensitive to movements in the market portfolio (i.e., $\beta^b_{imt} < \beta_{imt} < 1$), while an equity with $\beta_{imt} > 1$ will be more sensitive to movements in the market portfolio (i.e., $\beta^b_{imt} > \beta_{imt} > 1$).

It is worth emphasizing that $E_t(r_{mt})$ is treated as given in this framework and thus h_{mt} is conditional on market fundamentals. Therefore, the herd measure is not assumed to be affected by *market-wide mispricing* like bubbles, but is designed to capture cross-sectional herd behavior *within* the market. Clearly however there is a link between the two and we extend the model by allowing the expected returns of the market portfolio and individual assets to be biased by investor sentiment.

The model in (2) is generalised as follows. Let δ_{mt} and δ_{it} represent sentiment on the market portfolio and asset i respectively.[7] Then the investors' biased expectation in the presence of sentiment is sum of fundamentals and sentiment,

$$\begin{aligned} E^s_t(r_{it}) &= E_t(r_{it}) + \delta_{it}, \text{ and} \\ E^s_t(r_{mt}) &= E_t(r_{mt}) + \delta_{mt}, \end{aligned}$$

where for consistency $\delta_{mt} = E_c(\delta_{it})$ and $E_c(.)$ represents cross-sectional expectation, and the superscript s represents bias due to the sentiment. Then we have

$$\begin{aligned} \beta^s_{imt} &= \frac{E^s_t(r_{it})}{E^s_t(r_{mt})} \\ &= \frac{E_t(r_{it}) + \delta_{it}}{E_t(r_{mt}) + \delta_{mt}} \\ &= \frac{\beta_{imt} + s_{it}}{1 + s_{mt}}, \end{aligned} \tag{3}$$

where $s_{mt} = \frac{\delta_{mt}}{E_t(r_{mt})}$ and $s_{it} = \frac{\delta_{it}}{E_t(r_{mt})}$ represent sentiment in the market portfolio and asset i relative to the expected market return. Positive values of s_{mt} and s_{it} are usually expected in bull markets, while negative market sentiment during bear markets. For example, the Japanese stock market bubble in the late 1980's happened after a long bull market, and the US dot-com bubble happened during the bull market of the 1990's. It is hard to consider any example where positive sentiment has existed during bear

[7]Essentially sentiment reflects a belief and hence probability distribution for returns. We simply take the first moment to characterise the impact as is common elsewhere in the literature.

markets or negative sentiment has existed during bull markets. This is supported by several previous studies such as Neal and Wheatley (1998), Wang (2001), and Brown and Cliff (2004) who report empirical evidence that asset returns are positively related with sentiment.

There are several cases of this structure that show how beta is biased in the presence of sentiment in individual assets and/or the market;

$$\beta^s_{imt} = \begin{cases} \beta_{imt} + s_{it} \text{ when } \delta_{it} \neq 0 \text{ and } \delta_{mt} = 0, \\ \frac{\beta_{imt}}{1+s_{mt}} \text{ when } \delta_{it} = 0 \text{ and } \delta_{mt} \neq 0, \\ \frac{\beta_{imt}+s_{it}}{1+s_{mt}} \text{ when } \delta_{it} \neq 0 \text{ and } \delta_{mt} \neq 0. \end{cases}$$

In the first case, $\delta_{mt} = 0$, assumes that there is no market-wide sentiment though non-zero sentiment could exist for individual assets. Since $\delta_{mt} = E_c(\delta_{it}) = 0$ (or $s_{mt} = E_c(s_{it}) = 0$), a special case arises by assuming s_{it}. Herding towards the market in equation (2) can be obtained with $s_{it} = -h_{mt}(\beta_{imt} - 1)$ conditional on $E_t(r_{mt})$. For a given equilibrium β_{imt}, it is a positive value of h_{mt} that creates herding, but the positive h_{mt} is not necessarily related with positive sentiment of that asset. For example, for an asset with $\beta_{imt} > 1$, herding ($h_{mt} > 0$) is related with negative sentiment ($s_{it} < 0$). Herding can be observed when sentiment in individual assets appears in a systematic way as in (2).

The second case is when there is a market-wide sentiment but no sentiment effect for the specific asset. Even if there is no sentiment for the specific asset its beta is biased because of the market-wide sentiment. For a positive market-wide sentiment the beta is biased downward and vice versa. However, for the market as a whole there should be other individual assets whose sentiment contributes to the non-zero market-wide sentiment.

The final case is when individual and market sentiments are both non-zero. Equation (3) suggests that when $\delta_{it} = \beta_{imt}\delta_{mt}$, we have $\beta^s_{imt} = \beta_{imt}$. That is, when the market-wide sentiment only affects the expected return of the individual asset through the equilibrium relationship will the beta in the presence of sentiment be equal to the equilibrium beta. However, it is hard to expect that the market-wide sentiment affects individual assets via the equilibrium relationship. When investors are overconfident (have positive sentiment), a similar level of sentiment is likely to be expected for individual assets regardless of the equilibrium relationship. In an extreme case, when sentiment is the same for all assets in the market, $s_{mt} = s_{it} > 0$ for all i, β^s_{imt} moves towards 1; $1 > \beta^s_{imt} > \beta_{imt}$ for assets with $\beta_{imt} < 1$ and $1 < \beta^s_{imt} < \beta_{imt}$ for $\beta_{imt} > 1$. Similarly when $s_{mt} = s_{it} < 0$, $1 < \beta_{imt} < \beta^s_{imt}$ for assets with $\beta_{imt} > 1$ and $1 > \beta_{imt} > \beta^s_{imt}$ for $\beta_{imt} < 1$. All three cases suggest that the equilibrium beta would not be observable when there is sentiment in the individual assets or at the market level.

In order to investigate beta herding in the presence of sentiment we assume that the sentiment impact on an individual asset is decomposed into three components, a common market-wide effect, herding, and an idiosyncratic sentiment, such that;

$$s_{it} = s_{mt} - h_{mt}(\beta_{imt} - 1) + \omega_{it}, \tag{4}$$

where ω_{it} is an idiosyncratic sentiment of asset i. There could be other alternative processes for the sentiment, but equation (4) is both simple and general since all three - herding, market and idiosyncratic sentiment components are included. The proposed model implies that herding is effectively treated as one of the driving forces contributing to the sentiment impact on an individual assets price in our study. Note that equation (4) satisfies the constraint that the cross-sectional expectation of all the individual assets' sentiments is in fact the market-wide sentiment;

$$\begin{aligned} E_c(s_{it}) &= E_c(s_{mt} - h_{mt}(\beta_{imt} - 1) + \omega_{it}) \\ &= s_{mt}, \end{aligned}$$

since $E_c(\beta_{imt} - 1) = E_c(\omega_{it}) = 0$. By substituting s_{it} into equation (3), we have beta in the presence of herding and sentiment;

$$\beta^s_{imt} = 1 + \frac{1}{1 + s_{mt}} \left[(1 - h_{mt})(\beta_{imt} - 1) + \omega_{it}\right]. \tag{5}$$

Only when all three components - herding, market-wide and idiosyncratic sentiments - are zero, does (5) deliver the equilibrium beta, $\beta^s_{imt} = \beta_{imt}$. For given s_{mt} a positive h_{mt} (herding) make β^s_{imt} move towards 1 while a negative h_{mt} (adverse herding) make β^s_{imt} move away from 1. On the other hand, when s_{mt} increases for given h_{mt}, β^s_{imt} moves toward 1 and vice versa.

When β_{imt} is not related with ω_{it}, we have

$$\begin{aligned} Var_c(\beta^s_{imt}) &= E_c\left[\left(\frac{1}{1+s_{mt}}\left[(1-h_{mt})(\beta_{imt}-1)+\omega_{it}\right]\right)^2\right] \\ &= \frac{1}{(1+s_{mt})^2}\left[(1-h_{mt})^2 Var_c(\beta_{imt})+Var_c(\omega_{it})\right]. \end{aligned} \tag{6}$$

We assume that $Var_c(\beta_{imt})$ is constant; $Var_c(\beta_{imt})$ is not expected to change significantly during a short time period though individual β_{imt}'s may change over time very slowly. With a large number of stocks, idiosyncratic movements in β_{imt}'s are expected to be cancelled out. HS show no evidence that the cross-sectional variance of betas is explained by macroeconomic variables or firm characteristic based variables. Likewise there is no strong reason not to assume the cross-sectional variance of the idiosyncratic sentiment ω_{it} could be constant.

Therefore for given $Var_c(\beta_{imt})$ and $Var_c(\omega_{it})$ the left hand side of equation (6) decreases, *ceteris paribus*, when h_{mt} and s_{mt} increase. That is, we observe a reduction in $Var_c(\beta^s_{imt})$ when there is herding towards the market and positive market-wide sentiment. When there is no herding but market-wide sentiment exists, i.e., $h_{mt}=0$ and $s_{mt}\neq 0$, changes in $Var_c(\beta^s_{imt})$ are due to market wide movements in sentiment. For positive sentiment $Var_c(\beta^s_{imt})$ decreases suggesting that a bubble could reduce $Var_c(\beta^s_{imt})$. This has similar effects to herding, $h_{mt}>0$. However negative sentiment increases $Var_c(\beta^s_{imt})$ and thus during bear markets we could observe larger $Var_c(\beta^s_{imt})$, but the increase could become even higher when there is adverse herding, $h_{mt}<0$. It is possible that a rise in s_{mt} could cancel out a fall in h_{mt} so that $Var_c(\beta^s_{imt})$ does not change. However, it is quite well documented that sentiment is positively contemporaneously correlated with market returns and lagged to market returns. Thus a fall in $Var_c(\beta^s_{imt})$ from increase in s_{mt} is more likely during bull markets rather than bear markets. On the other hand, a fall in $Var_c(\beta^s_{imt})$ through a rise in h_{mt} is possible any time.

We can also derive a similar equation for portfolios while equation (6) is useful for the investigation of herding in individual stocks. There are several benefits from using portfolios. First, for a well diversified portfolio the idiosyncratic sentiment of the portfolio s_{pt} is zero. i.e., $\omega_{pt}=0$. Then the sentiment of the portfolio is decomposed into two components, market-wide sentiment and herding, such that;

$$s_{pt} = s_{mt} - h_{mt}(\beta_{pmt} - 1). \tag{7}$$

Then we have

$$\begin{aligned} Var_c(\beta^s_{pmt}) &= E_c\left[\left(\frac{1}{1+s_{mt}}\left[(1-h_{mt})(\beta_{pmt}-1)\right]\right)^2\right] \\ &= \frac{(1-h_{mt})^2}{(1+s_{mt})^2} Var_c(\beta_{pmt}). \end{aligned} \tag{8}$$

Therefore under the assumption that $Var_c(\beta_{pmt})$ is invariant over time, we could observe herding by measuring $Var_c(\beta^s_{pmt})$. Second, using portfolio betas has an important empirical advantage in that the estimation error would be reduced. That is as the number of equities increases we have $p\lim \widehat{\beta}^s_{pmt} = \beta^s_{pmt}$. However in practice the number of equities is limited and thus we could still have estimation error but it would be much smaller than that from using individual equities.

When the impacts of the idiosyncratic sentiment (i.e., $Var_c(\omega_{it})$) and the estimation error are disregarded, herding measured with individual stocks (6) is equivalent to that measured with portfolios (8) only when β_{imt}'s are not correlated with each other;

$$Var_c(\beta_{pmt}) = \frac{1}{N_p} Var_c(\beta_{imt}) + \frac{(N_p - 1)}{N_p} Cov_c(\beta_{imt}, \beta_{jmt}),$$

where N_p is the number of stocks in a portfolio and $Cov_c(\beta_{imt}, \beta_{jmt})$ is covariance between β_{imt} and β_{jmt}, $i \neq j$. However in general betas are correlated and thus the results obtained with individual stocks and portfolios are expected to be different.

2.2 A Herd Measure and Non-parametric Test Methods

In this section we use a simple market model to develop a test of herding.[8] We also explain why the t-statistics of the estimated betas are a better way of measuring herd behaviour than estimates of the betas themselves.

Following the discussion in the previous section our definition of beta herding is as follows. This definition of beta herding represents changes in the cross-sectional variance of the betas that originate from both herding and sentiment.

[8] A similar explanation is possible in multifactor models such as Fama and French (1993), but is more complicated.

Definition 1 *The degree of beta herding (or herding towards the market portfolio) is given by*

$$H_{mt} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left(\beta_{imt}^s - 1\right)^2, \tag{9}$$

where N_t is the number of stocks at time t. Herding towards the market portfolio therefore decreases with H_{mt}.

One major obstacle in calculating the herd measure is that β_{imt}^s is unknown and needs to be estimated. It is well documented that betas are not constant but time-varying. (See Harvey (1989), Ferson and Harvey (1991, 1993), and Ferson and Korajczyk (1995) for example.) Several methods have been proposed to estimate time-varying betas by Gomes, Kogan and Zhang (2003), Santos and Veronesi (2004), and Ang and Chen (2005).

In what follows, we use rolling windows to capture the time variation in betas for the following reasons. First, as in Jagannathan and Wang (1996) multi-factor unconditional models can capture the same effects as a single factor conditional model. We estimate the betas using multi-factor models in our empirical tests below and as argued by Fama and French (1996) if size, book-to-market and momentum factors are used, then this can minimise the problems in estimating betas with the OLS. Second we avoid the potentially spurious parametric restrictions used in Ang and Chen (2005), where the latent processes - the betas, market risk premia, and volatility are assumed to follow first order autoregressive processes. Ghysels (1998) points out that it is difficult to obtain time-varying betas unless the true model for the betas is known. Third, betas are extremely persistent. For example the monthly autocorrelations of conditional betas reported by Gomes, Kogan and Zhang (2003) and Ang and Chen (2005) are 0.98 and 0.99 respectively. Therefore for the highly persistent process choosing shorter windows (i.e., 24 months rather than 60 months) could minimise the problems that come from time-variation in betas. In addition we can easily monitor any unfavourable effects of using long windows on the herd measure. Finally using the OLS estimates of betas we could investigate how estimation error affects the herd measure. The state space model estimated using Gibbs sampling and Markov Chain Monte Carlo methods used by Ang and Chen (2005) is not very helpful in evaluating how estimation errors affect the herd measure (H_{mt}).

A simple market model is used as an example to show the difficulty in using OLS estimates of betas and why using the t-statistics of the OLS estimates of the betas is a better way to measure herding than directly using the estimates of betas themselves. The same argument applies in multi-factor models if the factors are orthogonal to each other. Given τ (window size) observations, the market model is represented as

$$r_{it} = \alpha^s_{it} + \beta^s_{imt} r_{mt} + \varepsilon_{it}, \ t = 1, 2, ..., \tau, \tag{10}$$

where ε_{it} is the idiosyncratic error which we assume $\varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon it})$. The OLS estimator of β^s_{imt} for asset i at time t, b^s_{imt}, is then simply

$$b^s_{imt} = \widehat{\sigma}^2_{imt} / \widehat{\sigma}^2_{mt}, \tag{11}$$

$$Var(b^s_{imt}) = \widehat{\sigma}^2_{\varepsilon it} / \widehat{\sigma}^2_{mt}, \tag{12}$$

where $\widehat{\sigma}^2_{imt}$ is the sample covariance between r_{it} and r_{mt}, $\widehat{\sigma}^2_{mt}$ is the sample variance of r_{mt}, and $\widehat{\sigma}^2_{\varepsilon it}$ is the sample variance of the OLS residuals. Using the OLS betas, we could then estimate the measure of herding as

$$H^O_{mt} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left(b^s_{imt} - 1\right)^2. \tag{13}$$

However, H^O_{mt} will also be numerically affected by statistically insignificant estimates of β^s_{imt}'s. The significance of the OLS estimates of the betas could change over time, affecting H^O_{mt} even though β^s_{imt} was constant. In addition, the OLS estimates in equations (11) and (12) have several undesirable properties. Suppose that the market model in (10) were multiplied by a non-zero κ. Then we would have

$$r^*_{it} = \alpha^*_{it} + \beta^s_{imt} r^*_{mt} + \varepsilon^*_{it}, \tag{14}$$

where $r^*_{it} = \kappa r_{it}$, $r^*_{mt} = \kappa r_{mt}$, $\alpha^*_{it} = \kappa \alpha^s_{it}$, and $\varepsilon^*_{it} = \kappa \varepsilon_{it}$, leaving b^s_{imt} unchanged. Only when r_{it}, r_{mt}, and ε_{it} move at the same rate, would the market model hold with the same beta and the OLS estimator will not be affected; however, in general this is unlikely. A similar econometric problem has been discussed when measuring contagion during market crises. When the volatility in one country increases dramatically during international financial crises, the volatility of the returns in its neighbour country may not move in proportion, and the correlation coefficient between the two countries may not reflect the true relationship.[9]

When r_{it}, r_{mt}, and ε_{it} do not move at the same rate, $Var(b^s_{imt})$ is affected by heteroskedasticity in ε_{it} or r_{mt}. To see the impact of the heteroskedasticity on our herd measure, we first note that

$$E_c[b^s_{imt}] = E_c[\beta^s_{imt} + \eta_{imt}] = 1,$$

where η_{imt} is the OLS estimation error, $\eta_{imt}\tilde{}N\left(0, \sigma^2_{\varepsilon it}/\sigma^2_{mt}\right)$. So using the estimated parameters in the herd measure, H^O_{mt}, is given by

$$\begin{aligned} E_c[H^O_{mt}] &= E\left[\frac{1}{N_t}\sum_{i=1}^{N_t}(b^s_{imt}-1)^2\right] \\ &= E\left[\frac{1}{N_t}\sum_{i=1}^{N_t}(\beta^s_{imt}+\eta_{imt}-1)^2\right] \\ &= E\left[\frac{1}{N_t}\sum_{i=1}^{N_t}(\beta^s_{imt}-1)^2 + \frac{1}{N_t}\sum_{i=1}^{N_t}\eta^2_{imt}\right] \\ &= H_{mt} + \frac{1}{N_t}\sum_{i=1}^{N_t}\frac{\sigma^2_{\varepsilon it}}{\sigma^2_{mt}}, \end{aligned} \tag{15}$$

since $E\left[\frac{1}{N_t}\sum_{i=1}^{N_t}(\beta^s_{imt}-1)\eta_{imt}\right] = 0$. When r_{it}, r_{mt}, and ε_{it} all move in unison, $\frac{1}{N_t}\sum_{i=1}^{N_t}\sigma^2_{\varepsilon it}/\sigma^2_{mt}$ (the cross-sectional average of estimation errors, from now on we call it CAEE) is constant over time and any movement in H_{mt} can be captured by H^O_{mt}. However, if either the cross-sectional average of idiosyncratic variances (i.e., $\frac{1}{N_t}\sum_{i=1}^{N_t}\sigma^2_{\varepsilon it}$) or the market variance (i.e., σ^2_{mt}) is heteroskedastic, then changes in H^O_{mt} do not necessarily only arise from herd behavior, but also come from the changes in the ratio of firm level variance against market variance.

[9] For example Forbes and Rigobon (2002) show that the correlation coefficient between the two countries increases during market crises when the volatility of idiosyncratic errors remains unchanged. Therefore they conclude that increased correlations between two countries may not necessarily be the evidence of contagion. However, many studies point out that the assumption is not appropriate. See Corsetti, Pericoli, and Sbracia (2003), Bae, Karolyi and Stulz (2003), Pesaran and Pick (2004), Dungey, Fry, Gonzalez-Hermosillo, and Martin (2003, 2004) among others.

To avoid this unpleasant property of H^{O}_{mt}, we standardize b^{s}_{imt} with its standard deviation; in other words we used the t statistic which will have a homoskedastic distribution and thus will not be affected by any heteroskedastic behaviour in CAEE. Using t statistics will also reduce the influence of the impact of changes in market volatility in particular during market crises. The t statistic in the measure of herding based on t statistics is;

$$\frac{b^{s}_{imt}-1}{\widehat{\sigma}_{\varepsilon it}/\widehat{\sigma}_{mt}} \sim t\left(DF; \frac{\beta^{s}_{imt}-1}{\sigma_{\varepsilon it}/\sigma_{mt}}\right), \tag{16}$$

where DF is the degrees of freedom and $\frac{\beta^{s}_{imt}-1}{\sigma_{\varepsilon it}/\sigma_{mt}}$ is a non-centrality parameter.

Definition 2 *Standardised beta herding is defined using*

$$H^{*}_{mt} = \frac{1}{N_t}\sum_{i=1}^{N_t}\left(\frac{b^{s}_{imt}-1}{\widehat{\sigma}_{\varepsilon it}/\widehat{\sigma}_{mt}}\right)^{2}, \tag{17}$$

where b^{s}_{imt} are the observed estimates of betas for the market portfolio for stock i at time t, and $\widehat{\sigma}_{\varepsilon it}$ and $\widehat{\sigma}_{mt}$ are as defined in equations (11) and (12). Standardised beta herding increases with decreasing H^{*}_{mt}.

We call expression (9) as the beta-based herd measure while H^{*}_{mt} in (17) is the standardised herd measure. The following distributional result applies to (17).[10]

Theorem 1 *Let $\mathbf{B}^{*}_{mt} = \begin{pmatrix} B^{*}_{1mt} & B^{*}_{2mt} & . & B^{*}_{N_t mt}\end{pmatrix}'$, where $B^{*}_{imt} = \frac{b^{s}_{imt}-1}{\widehat{\sigma}_{\varepsilon it}/\widehat{\sigma}_{mt}}$. Then with the classical OLS assumptions,*

$$\underset{N_t\times N_t}{\mathbf{B}^{*}_{mt}} \sim N\left(\underset{N_t\times 1}{\boldsymbol{\delta}^{*}_{mt}}, \underset{N_t\times N_t}{\mathbf{V}^{*}_{mt}}\right),$$

where $\boldsymbol{\delta}^{*}_{mt} = \begin{pmatrix} \delta^{*}_{1mt} & \delta^{*}_{2mt} & . & \delta^{*}_{N_t mt}\end{pmatrix}'$, $\delta^{*}_{imt} = \frac{\beta^{s}_{imt}-1}{\sigma_{\varepsilon it}/\sigma_{mt}}$, and $\mathbf{V}^{*}_{mt}$ is covariance matrix of $\mathbf{B}^{*}_{mt}$. Then

$$\begin{aligned} H^{*}_{mt} &= \frac{1}{N_t}\mathbf{B}^{*\prime}_{mt}\mathbf{B}^{*}_{mt} \\ &\sim \frac{1}{N_t}\left[\chi^{2}(R;\delta^{*R}_{k}) + c^{*}\right], \end{aligned} \tag{18}$$

[10] A similar result can be obtained for the beta-based herd measure in (9).

where R is the rank of $\mathbf{V}^*_{mt}$, $\delta^{*R}_m = \sum_{j=1}^{R}(\delta^{A*}_j)^2/\lambda^*_j$, and $c^* = \sum_{j=R+1}^{N}(\delta^{A*}_j)^2$, where δ^{A*}_j is the jth element of the vector $\mathbf{C}^{*\prime}_{mt}\mathbf{B}^*_{mt}$, where $\mathbf{C}^*_{mt}$ is the $(N_t \times N_t)$ matrix of eigenvectors of $\mathbf{V}^*_{mt}$, i.e., $\mathbf{V}^*_{mt} = \mathbf{C}^*_{mt}\mathbf{\Lambda}^*_{mt}\mathbf{C}^{*\prime}_{mt}$, where $\mathbf{\Lambda}^*_{mt}$ is the $(N_t \times N_t)$ diagonal matrix of eigenvalues. The eigenvalues are sorted in descending order.

Proof. See the Appendix. ■

This measure can be calculated easily using any standard estimation program since it is based on the cross-sectional variance of the t statistics of the estimated coefficient on the market portfolio. Theorem 1 shows that this new measure of herding is distributed as $1/N_t$ times the sum of non-central χ^2 distributions with degrees of freedom R and with non-centrality parameters δ^{*R}_m and a constant. Therefore the variance of H^*_{mt} is given by;

$$Var[H^*_{mt}] = \frac{2}{N_t^2}\left[R + 2\delta^{*R}_m\right]. \tag{19}$$

It is important to note that this distributional result depends on the assumption that the number of observations to estimate β^s_{imt} is sufficiently large and $\mathbf{B}^*_{kt}$ is multivariate normal. With too few observations, the confidence level implied in the theorem above would be smaller than it would be asymptotically and we will reject the null hypothesis too frequently. In practice, the non-centrality parameter would be replaced with its sample estimate.

3 Empirical Tests

One straight forward approach to testing herding towards the market is then to calculate the measure given in (18) and its confidence level as given in Theorem 1 for particular periods of interest. If there is any significant difference between two periods, we may conclude that one period shows relatively more herding than the other. Many studies on herding and contagion take this view, especially when examining behavior around and during market crises. See Bikhchandani and Sharma (2000) for a survey of empirical studies.

An alternative approach that we adopted below is to calculate the statistics recursively and in this way we can investigate in a more detailed manner whether the degree of herding has increased or decreased significantly over time. That is, using equation

(17), we calculate the herd measure at time t given an appropriate window of data (τ), and obtain confidence intervals from equation (19). The same procedure is then repeated over time by rolling windows (advancing the start date by one period, i.e., t, $t+1$, ...). The test statistics provide us effectively with a sequence of hypothesis tests. That is, we can use the confidence level calculated at time t to test if the value of the test statistic at $t+1$ is changed significantly. In this way we can determine if the level of herding is significantly different over time.

In the empirical study, we use two different datasets; individual stocks and portfolios. We first present our results using individual stocks in the US, UK and South Korean markets, and then compare herd behaviour across these different markets. We then apply the method for the Fama-French 25 and 100 portfolios formed on size and book-to-market from January 1927 to December 2003.

3.1 Herding in the US Market

3.1.1 Data

We use monthly data from the Center for Research in Security Prices (CRSP) to investigate herding in the US stock market. Ordinary common stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ markets are included, and thus ADRs, REITSs, closed-end-funds, units of beneficial interest, and foreign stocks are excluded from our sample. Our sample period consists of 488 monthly observations from July 1963 to December 2003. For excess market returns we use the CRSP value weighted market portfolio returns and 1 month treasury bills. For the other factors we use Fama-French's (1993) size (Small minus Big, SMB) and book-to-market (High Minus Low, HML), and momentum from Kenneth French's data library in addition to the excess market returns.

As explained above, an appropriate number of monthly observations, i.e., τ, needs to be chosen to obtain the OLS estimate. We have chosen a shorter window, i.e., τ=24, but we have also tried a range of values, i.e., τ=36, 48, and 60, and found that the results are effectively not different from one other. (See the results in the Appendix.) The procedure by which we calculated each herd measure is as follows. We use the first 24 observations up to June 1965 to obtain the OLS estimates of betas and their t

statistics for each stock (or portfolio) and then calculate H^*_{mt} and its test statistic for June 1965. We then add one observation at the end of the sample and drop the first and so use the next 24 observations up to July 1965 to calculate the herd measure and its statistic for July 1965, and so on.

An important issue in the accurate estimation of the betas arises from the lack of liquidity in particular assets. When prices do not reflect investors expectations because of illiquidity, our measure using observed prices may not fully reflect what it intends to show. One problem is nonsynchronous trading problem for illiquid stocks, which was first recognized by Fisher (1966). Scholes and Williams (1977) show that the OLS estimates of betas of infrequently traded stocks are negatively biased while those of frequently traded stocks are positively biased. They derive consistent estimators of the market model in (10). However, the estimators are consistent only for the returns that suffer the nonsynchronous trading. In general the effects of illiquidity on asset returns are multifaceted and difficult to summarise in a single explanatory variable and so we filter out small illiquid stocks in our empirical work by controlling for the following three liquidity proxies; volatility, size, and turnover rate.[11]

- (Volatility) When the true betas are not known, the market-adjust-return model of Campbell et al (2001) is useful;

  $$r_{it} = r_{mt} + \varepsilon_{it}, \tag{20}$$

  which is a restricted version of the market model in (10) with $\alpha^s_{it} = 0$ and $\beta^s_{imt} = 1$. Here it is easy to show that $\sigma_{it} \geq \sigma_{mt}$. We remove any stocks whose volatility is less than half of the market volatility, i.e., $\sigma_{it} < 0.5\sigma_{mt}$.[12] We find that there are less than 5 percent of stocks (19 percent in market value) whose volatility is less than half of the market volatility.

- (Turnover Rate) Annual turnover rates on the NYSE from 1980 to 2000 range from 33 to 88 percents (Swan, 2002). In our study we remove stocks from our

[11] There are many studies on liquidity. Liquidity is a function of 1) the cost of liquidating a portfolio quickly, 2) the ability to sell without affecting prices, 3) the ability of prices to recover from shocks, and 4) costs associated with selling now, not waiting. See Kyle (1981) and Grossman and Miller (1998) for example.

[12] At each time t we estimate σ_{it} and σ_{mt} using past τ observations

sample whose average monthly turnover rates (average over τ period) are less than 0.5 percent (6 percent a year). The proportion of stocks removed by this process is less than 45 percent in both numbers and capitalisation values.

- (Size) We also remove small stocks whose market values are less than 0.01 percent of the total market capitalisation. The proportion of stocks removed by this process is less than 14 percent in values, but as large as 88 percent in numbers.

Applying these filters leaves the number of stocks ranging from 570 to 1185 for our sample period.[13] Compared with the total number of stocks, the method seems to be strict and looks arbitrary. However the filtering should not matter for discovering whether the measure changes over time significantly in order to detect herding empirically. In the Appendix we try several different values of cut-off size to see if our choice affects the results significantly. Controlling firm sizes is also important in our study for another purpose. The statistics proposed in Theorem 1 are for equally weighted herd measure rather than value weighted measure since value weights are hard to include in the principal component analysis. By controlling for firm size we could also investigate if there is any difference in herding between big and small firms.

3.1.2 Herd Behaviour

The Fama-French three factor model with momentum we use to estimate the beta is

$$r_{it} = \alpha_i^s + \beta_{im}^s r_{mt} + \beta_{is}^s r_{smbt} + \beta_{ih}^s r_{hmlt} + \beta_{imm}^s r_{mmt} + \varepsilon_{it}, \tag{21}$$

where r_{it} and r_{mt} are excess returns of asset i and the market portfolio, and r_{smbt}, r_{hmlt}, and r_{mmt} are Fama-French's SMB, HML, and momentum factor returns. The estimated betas and t-statistics are used to calculate the herd measure as in (17). We also calculate the herd measures with the market model or based on OLS estimates of betas for comparison purpose.

[13]We find a small number of stocks that have very small standard deviations of regression residuals, i.e., less than 0.1 percent a month. The low standard deviations make the standard errors of estimated betas very small, and thus their t-statistics becomes unusually large. These outliers are excluded from our sample. The number of these stocks is less than 10.

Table 1 reports some basic statistical properties of the herd measures. The four herd measures we calculate are not normally distributed, in particular when calculated with the estimated betas they are positively skewed and leptokurtic. On the other hand the non-normality of the herd measure calculated with t-statistics is much less pronounced. Because of the non-normality rank correlations are calculated to investigate the relationship between the four measures.

There are noticeable differences in herding towards the market portfolio between the market model and the four factor model, in particular with the standardised measure. The difference between the market model and the four factor model in the beta-based measure (first two columns of Table 1) suggests that beta needs to be estimated with the other factors. As in Fama and French (1992, 1993, 1996) if SMB and HML are important factors to explain returns, betas estimated without these factors (and momentum) would not represent systematic risk appropriately. Another issue is that the standardised measure is more sensitive to these additional factors, suggesting that standard errors of the estimated betas are also affected significantly by the additional factors. The last row in Table 1 reports a considerable difference between the beta-based and standardised herd measures; the rank correlations between H^O_{mt} and H^*_{mt} are significantly negative. As explained earlier, the CAEE in equation (15) (i.e., $\frac{1}{N_t}\sum_{i=1}^{N_t}\eta^2_{imt}$) could affect these herd measures in an opposite direction.

To examine the effects of heteroskedasticity of the estimation errors, we regress the beta-based and standardised herd measures on the CAEE. The herd measures are calculated using rolling windows (overlapping samples) and are highly persistent. Although overlapping information provides efficiency, it causes moving average effects. Therefore we report the Newey and West (1987) heteroskedasticity consistent standard errors for the regressions.

Table 2 shows that the coefficients on the CAEE are large and positive for the betas-based herd measures and the R^2 values are 0.836 for the four factor model. On the other hand the coefficients for the standardised herd measures are negative significant, but the R^2 values are only 0.329. Therefore the dynamics of the beta-based herd measures are dominated by the heteroskedasticity in the estimation errors, while the dynamics of the standardised herd measures are only marginally explained by the heteroskedasticity of the estimation errors. These positive and negative coefficients on the CAEE for the beta-based and t-statistics based herd measures respectively explain the negative correlation between the standardised and beta-based herd measures in Table 1. Both measures have the undesirable property that the estimation errors affect the dynamics of the herd measures, but the t-statistic based herd measures are far less affected by the errors.

We conclude that the results in Tables 1 and 2 support the standardised herd measure with the four factors. Several other cases are reported in the Appendix to examine the robustness of the measure. We next turn to test if the behaviour in the herd measure can be explained by market or macroeconomic variables, and then the observed herding will be related to historical events in the US.

3.1.3 Macroeconomic Variables

We test if the proposed measure can be explained by stock market movements or macroeconomic activity which are known to affect stock returns. When a herd measure is explained by these variables, the dynamics of herding merely reflects changes in fundamentals and thus efficient reallocations of assets in the stock market. Irrational herding would not be explained by changes in these fundamentals.

The herd measures are regressed on a number of market and macroeconomic variables. Four macroeconomic variables are used; the dividend-price ratio (DP_t), the relative treasury bill rate (RTB_t), the term spread (TS_t), and the default spread (CS_t). The choice of these four macroeconomic variables follows from previous studies such as those of Chen, Roll, Ross (1986), Fama and French (1988, 1989), Ferson and Harvey (1991), and Goyal and Santa-Clara (2003). We also add market returns and market volatility to investigate how our herd measure is related to the movements in the mean and variance of the market portfolio.[14] We use the log-dividend-price ratio of S&P500 index for DP_t, the difference between the US 3 month treasury bill rate and its 12 month moving average for RTB_t, the difference between the US 10 year treasury bond rate and the US 3 month treasury bill rate for TS_t, and the difference between Moody's AAA and BAA rated corporate bonds for CS_t. The dividend price ratio of S&P500 index is obtained from Robert Shiller's homepage and the other data are from the

[14]Market volatility is calcualted by summing squared daily returns as in Schwert (1989).

Federal Reserve Board of the US.

The results of the linear regression for the standardised and beta-based herd measures are reported in Table 3. Panel A shows that four variables - market volatility, dividend-price ratio, term spread, and credit spread, explain the standardised herd measure calculated with the four factor model. However the value of R^2 is only 0.09. The explanatory power of these variables become much weaker when the CAEE is added in the regression. For the beta-based herd measure the dividend-price ratio is the only significant variable, but it only explains 25% of the herding. These results suggest that although several market related variables appear to be significant in explaining the standardised herd measure, the proportion that these variables explain is very limited and vast majority of the dynamics in the standardised herd measure arise from herding.[15]

3.1.4 Herding and Economic Events

Figure 1 shows the evolution of our herd measure towards the market portfolio. With hundreds or thousands of stocks, the confidence level calculated by equation (19) becomes very small indeed and thus is not clearly visible in the figure. As expected, the dynamic behaviour of H^O_{mt} is almost the same as that of the CAEE in equation (15). This confirms our preference towards measuring herding based on t-statistics (or standardised betas). In the following we focus on our standardised herd measure.

Let us investigate the relationship between herding towards the market portfolio with economic events. There are a number of significant sudden changes in H^*_{mt} in the sense that these changes are far above or below the previous upper and lower boundaries at the 95% confidence level. Several sharp positive jumps can be found in 1970 (Recession), 1973 (Oil Shock), 1982 (Mexican Crisis), 1987 (Market Crash), 1991 (Desolution of USSR), 1998 (Russian Crisis), 2001 (September the Eleventh). When these shocks happened, the herding level decreased significantly. On the other hand there are three sharp declines in the measure during 1980, 1989, and 1993, of which the

[15] We also calculated rank correlation between the t-statistic-based herd measure and the regression residuals from panel A in in Table 3, and found that it was 0.94. When the residuals were plotted in Figure 1 together with the t-statistic-based herd measure there was little difference between them.

last two simply reflect the reversals from the positive increase of 1987 and 1991 after 24 months (τ). The large drop in the herd measure during 1980 comes from sudden increase in the interest rate at the beginning of the recession of 1980. We might infer that the sudden increase in the interest rate increased herding through an increased expectation of a future bear market.

Herding is also clear before the Russian crisis in 1998 and between late 1999 and August 2001. The first herding period could be characterized by a bull market, and the second period by a bear market. When there were shocks such as Russian crisis or 'September the Eleventh', herding disappears. Interestingly the US market does not seem to be affected significantly by the Asian crisis despite the sudden jump in market volatility. This could be interpreted as despite the crisis, the US market was dominated by a strong positive herding, and the Asian crisis in 1997 was not strong enough to remove the positive mood.

Two implications can be drawn from the results. The first is that herding happens in both bull or bear markets. When the economy is in a recession like the early 1980's or during 2001, we observe a high level of herding and on the other hand we can also observe herding when economy is booming, for example in the late 1990's or mid-1980's. The second result is that when there are crises or unexpected shocks, herding disappears. See for example 1973 Oil Shock, 1987 crash, 1988 Russian crisis, and 'September the Eleventh'.

Our findings are not necessarily inconsistent with previous studies. We note that many empirical studies on herding in advanced markets find little concrete evidence of herd behavior, see Bikhchandani and Sharma (2000). However, in the South Korean case, Kim and Wei (1999) and Choe, Kho, and Stulz (1999) study herd behavior around the Asian Crisis in 1997 and find some evidence during the Crisis. These studies use the Lakonishok, Shleifer, and Vishny (1992) measure which focuses a subset of market participants. Therefore, we cannot conclude that their results are inconsistent with ours since our measure considers beta herding in the whole market rather than a subset of participants. Chang, Cheng and Khorana (2000), using a variant of the method of Christie and Huang (1995), suggest the presence of herding in emerging markets such as South Korea and Taiwan, but failed to find evidence in the US, Hong Kong and Japanese markets.

However, our evidence *is not consistent* with the view that herding happens when financial markets are in stress (or in crisis). On the contrary, the figures show that herd behavior can be clearly detected when the markets are not in stress and thus investors are confident on the outlook of the future stock market. If the direction towards which the market is heading is assured, herding begins to occur regardless of whether it is a bull or a bear market; these periods are the late 1960's, before the Oil Shock in 1973, early 1980's and before the 1987 crash, the 1990's and early 2000's. We could interpret this as suggesting that it is the investors' over-confidence or consensus that induces herding. When market is in stress, however, investors lose confidence and begin to focus more on fundamentals. In this sense market stress is beneficial to the market rather than harmful, although it may create stress for market participants.

From this point of view, we do not agree with the view that herding only arises when financial markets are in stress. When a market is in crisis, we can observe large negative returns in the market index and the majority of the individual assets will also show negative returns, which could be interpreted as herding in whole market. However, as far as individual asset returns move following their systematic risks, the argument may be different. Instead in popular linear factor models we could claim herding only arises when the factor loadings of individual assets are systematically biased by the crisis and thus the long-run relationship between individual asset returns and factor returns no longer holds. So the fact that the majority of assets show negative returns during a market crisis is not sufficient evidence of herding itself.

3.2 International Comparison with UK and South Korean Stock Markets

3.2.1 Data

We use monthly data from January 1993 to November 2002 to compare slowly moving herding in the UK and South Korean stock markets. The inclusion of the South Korean market is useful as we can compare herding in both advanced markets and an emerging market and also to examine how the South Korean market reacted during the 1997 Asian crisis. In general investigating herding in emerging markets is interesting given their structural and institutional differences, see Bekaert, Erb, Harvey, and Viskanta (1997) for example, for some key discussions of what are the important factors in emerging markets. Our sample period covers several important crises, the 1997 Asian crisis and the 1998 Russian crisis, as well as the bull market during the 1990's and the bear market in the early 2000's.

The herd measures are calculated using the constituents of the FTSE350 index for the UK market (247 stocks), and the KOSPI index for the South Korean market (454 ordinary stocks). We use FTSE350 index and KOSPI index monthly returns as our market portfolio returns. To calculate the excess returns, 3 month treasury bills are used for the UK market, whereas for the South Korean market, the 1 year Korea Industrial Financial Debentures are used.[16] For these two markets SMB and HML using the same methods as described in Fama and French (1993) are used with the constituents in the FTSE350 index and the KOSPI index.

3.2.2 Herding toward the Market Portfolio

Figures 2 and 3 show the evolution of our herd measures towards the market portfolio for the two countries. As in the US market, herding disappeared in the UK during the Russian crisis. However, the UK market does not show any significant movement in herding around 'September the Eleventh'. Certainly this event had a huge impact on the US investors, but apparently not much impact on UK investors although volatility in the UK market increased sharply. Another difference between the US and UK markets is that in the UK we do not observe herding during the bear market. As in the US the UK market does not seem to be affected significantly by the Asian crisis. On the other hand the South Korean market which directly suffered from the Asian crisis shows that the high level of herding disappeared during the crisis. In the late 1999 and during the 2000, when investors began to regain confidence in the market, the level of herding in the South Korean market returned to the pre-Asian crisis level, but then began to decrease with global economic uncertainty.

As in the US case, market volatility does not explain herd behaviour. Market

[16] Becuase of the underdevelopment of the fixed income market in South Korea, there is no treasury bill available during our sample period.

volatilities have increased in the UK market since 1997, and peaked several times since then until the end of our sample period. However, in the South Korean market, volatility picked up during the Asian crisis of 1997, and then tended to decrease slowly afterwards. We find several examples of when volatility is not positively related with herd behaviour. Volatility peaked at December 1997 and July 2002 in the South Korean and the UK markets respectively. However our herd measure based on t-statistics is relatively smaller compared with neighbouring periods of herding activity. We also plot the CAEE in Figures 2 and 3. As in the US, these are highly correlated with the herd measures based on the estimated betas (not reported).

3.3 Herding in Portfolios

An application of our herd measure to portfolios provides several important insights, both in terms of methodology and the implications. First, we could minimise the estimation error of beta, and thus the beta-based and standardised herd measures should become similar for portfolios. Note that it is the CAEE that makes the two herd measures negatively correlated, and the beta-based herd measure is seriously biased by the estimation error. Second, herding in portfolios may not be the same as herding in individual stocks since it is less affected by the CAEE and the idiosyncratic sentiments of individual stocks. In addition herding in portfolios could be influenced by changes in the cross-correlation of β_{imt}'s. Therefore the dynamics of herding measured by portfolios and individual stocks should not be significantly different from each other.

When too many portfolios are formed from a given number of stocks, the idiosyncratic sentiment and the estimation error may not be minimised because of the small number of stocks that are included in each portfolio. On the other hand when the number of portfolios is too small, the herd measure has large standard errors because of the small number of portfolios (See equation (19)). In our study we use Fama-French 25 and 100 equally weighted portfolios formed on size and book-to-market, which are widely used in finance literature.[17] A total number of 924 monthly observations from January 1927 to December 2003 is used. Betas are estimated in the Fama-French three

[17] The minimum numbers of portfolios used for the calculation of herd measures are 24 and 74 for the 25 and 100 portfolios respectively because of missing dada in the early sample period.

factor model with momentum by rolling windows of 60 monthly observations. As explained earlier using 60 monthly observations increases the moving average effects, but the effects are not significant (see the Appendix). In addition the results are comparable with other previous studies. These monthly factors are obtained from Kenneth French's data library.

The herd measures are plotted in Figure 4. First, the beta-based and standardised herd measures appear to have similar dynamics. The rank correlations between the beta-based and standardised herd measures are 0.54 and 0.26 for the 25 and 100 portfolios respectively. Note that the rank correlations between the two measures were negative for individual stocks because of the estimation error (see Table 1). Forming a small number of portfolios for given number of stocks (or a larger number of stocks in each portfolio) reduces the estimation error and thus we have large positive correlations, in particular for the 25 portfolio case. In order to investigate if the increase in the correlations is attributed to the reduction in the estimation error in beta, we calculate the rank correlation of 0.67 between the 25 and 100 portfolios for the beta-based measure and the rank correlation of 0.94 between the 25 and 100 portfolios for the standardised herd measure. Using a larger number of stocks in each portfolio has little impact on the standardised herd measure while it creates a large difference in the beta-based herd measure, most of which comes from the reduction in estimation error. Again the standardised herd measure is more robust to estimation error. Second, as expected when the number of portfolios is small, the confidence level in the herd measures increases dramatically. For example we could not plot the 95% confidence level for the beta-based herd measure since it is too large to be plotted on the same scale. The confidence level for the standardised herd measure is relatively small, but still much larger than that we obtained for the individual stocks.

Since the dynamics of the standardised herd measures from the 25 and 100 portfolios are not different, we explain herding with the results of the 100 portfolios given their tight confidence bands. Some interesting patterns in herding emerge. There was herding in the early 1930's and before the beginning of World War II. At the outset of the war herding decreased but it started to increase as the war continued. This supports the view that regardless of bull or bear markets, a clear homogeneity of view in the direction in which the market is likely to move creates herding. In the early 1930's the US economy was obviously in deep trouble and we empirically observe herding with our measure. The outbreak of the Second World War however and the resultant uncertainty brought about adverse herding. During the 1950's and 1960's there was herding except for a few years after 1957 when a sharp recession began.

Herding is clearly observed between 1974 and 1978 (after the first Oil Shock in 1973) which was not clear when we considered individual stocks above. Therefore during this period the Oil shock in 1973 affected individual stocks differently and thus there were significant differences in the idiosyncratic elements of sentiment in individual stocks. Figure 4 reports that during the 1980's herding was either at an average level or low in particular just before the 1987 Crash. The herding measure calculated with individual stocks on the other hand showed strong herding during the same period. In the early 1980's the US economy was dominated by high interest rates and thus market-wide sentiment became much larger than the idiosyncratic sentiments. Finally we still observe strong herding during the late 1990's, but there was no sudden decrease or increase in herding from the recent crises. Therefore herding calculated with portfolios does not necessarily give the same pattern as that calculated with individual assets.

Recall that we interpret the level of dispersion in the betas as a measure of herding when the market-wide dispersion changes without changes in fundamentals. So the measure gives an insight into why CAPM does not work after 1963. Since Fama and French (1992, 1993) showed beta does not explain cross-sectional average returns, the failure of CAPM has been investigated by many academics. Our beta-based herd measure provides some basic insight into this question. As we can see before 1963 there was a relatively large dispersion in the betas, but after 1963 the level of dispersion was generally low except for the recent decade. For CAPM to work a necessary condition needs to be satisfied; the betas of portfolios should be sufficiently different from each other regardless of whether the dispersion in the betas reflects fundamentals or herding. Otherwise even if the equity premium is large, cross-sectional average returns are not differentiable. Although the purpose of our measure is to find herding in the market, the measure also provides an explanation as to why and when CAPM works; it is mainly between 1935 to 1945 and between 1954 to 1963 that it works.

4 Market Sentiment and Herding

Our model proposes that herding increases with sentiment *ceteris paribus*. The relationship between herding and sentiment is more clear in (8) with portfolios. To investigate the relationship between the two, we take log in equation (8) to give

$$\ln Var_c(\beta^s_{pmt}) = \ln\left[(1-h_{mt})^2 Var_c(\beta_{pmt})\right] - 2\ln(1+s_{mt}), \tag{22}$$

suggesting a negative relationship between $\ln Var_c(\beta^s_{imt})$ and $\ln(1+s_{mt})$. Thus the first obvious hypothesis is to test if the market sentiment is negatively related with the cross-sectional variance of estimated betas. We run the following regression

$$\ln Var_c(\beta^s_{pmt}) = \alpha + \beta \ln S_{mt} + \eta_t, \tag{23}$$

where $\ln\left[(1-h_{mt})Var_c(\beta_{pmt})\right] = \alpha + \eta_t$ and $\beta \ln S_{mt} = -2\ln(1+s_{mt})$ and S_{mt} is a sentiment index. The simple regression allows us to decompose $\ln Var_c(\beta^s_{pmt})$ into two components; herding and sentiment. This could provide further information on how cross-sectional herding evolves with sentiment.

There are more than a dozen sentiment measures proposed by many authors. In the recent study on sentiment Brown and Cliff (2004) and Baker and Wurgler (2004) investigate various sentiment indices and conclude that direct sentiment measures (surveys) are closely related to indirect measures. In this study we have taken a direct sentiment measure, i.e., the market sentiment index constructed by Investors Intelligence for the period of December 1963 to December 2003.[18] Each week newsletter opinions on the future market movements are grouped as bullish, bearish, or neutral and we use the bull-bear ratio as a proxy of sentiment. Since these newsletters reflect opinions of professionals in financial markets, the index constructed can be regarded as a proxy for institutional sentiment.

The cross-sectional variance of betas in the left hand side of equation (23) is a moving average of the period of τ (e.g., 24 or 60 months) because of the rolling windows we have adopted. In order to match the dependent and independent variables, we apply

[18] By counting the last week's sentiment index in each month we construct monthly sentiment index. Thus the monthly sentiment index could suffer measurement error. However because of the nature of the sentiment index, in particular smoothness, the impact should not seriously devalue our results.

the same rolling windows method for the sentiment index to calculate a moving average sentiment index. As in the previous section Newey and West (1987) heteroskedasticity consistent standard errors are reported. We tested various cases; beta-based and standardised measures, various values of τ, herd measures from individual stocks and portfolios, etc. The results with the Fama-French 100 portfolios are not different from those of the Fama-French 25 portfolios, and to save space we report herd measures calculated with the Fama-French 25 portfolios.

Table 4 supports that sentiment index is negatively related with the herd measures. All cases show significance at least at the 5 percent level even in the presence of the market and macroeconomic variables and the CAEE. The values of R^2 however indicate that sentiment explains only 4 to 6 percent of the dynamics of the herd measure. Although our beta herd measure is explained by both herding and sentiment, the main force of herding comes from herding. For herding measured with individual stocks we obtained similar results but the negative relationship is weaker than that with portfolios because of the presence of idiosyncratic sentiments and estimation error. Thus a fall in $Var_c(\beta^s_{pmt})$ from an increase in sentiment is more likely during bull markets rather than bear markets because of the contemporaneous relationship between returns and sentiment. On the other hand, a decrease in $Var_c(\beta^s_{pmt})$ by increased h_{mt} is possible at any time.

The relationship between herding and sentiment is plotted in Figure 5. In most periods sentiment and beta herding move in opposite directions. However there are some periods that the two move in the same direction; from 1980 to 1982 and from 1998 to 2003. During 1980 to 1982 we expect increase in $Var_c(\beta^s_{pmt})$ because of decreasing sentiment, but the result shows that both decrease. This suggests that during this time h_{mt} increases far more than sentiment decreases, and thus $Var_c(\beta^s_{pmt})$ decreases. We find the opposite case during 1998 to 2003. Market sentiment increases, but herding (h_{mt}) begins to decrease more than the increase in sentiment such that $Var_c(\beta^s_{pmt})$ increases. However although sentiment contributes to the dynamics of $Var_c(\beta^s_{pmt})$ the proportion of $Var_c(\beta^s_{pmt})$ that is explained by sentiment is far less than h_{mt}.

5 Conclusions

Herding is widely believed to be an important element of behavior in financial markets and particularly when the market is in stress, such as during the Asian and Russian Crises of 1997 and 1998. In this study, we have proposed an alternative method of measuring and testing for slow moving herd behavior in the market. We have applied our measure to the US, UK, and South Korean stock market and found that beta herding disappeared during the Russian crisis in 1998 in the US and UK markets while herding in the South Korean market disappeared during the Asian crisis in 1997. Contrary to a common belief that beta herding is significant when the market is in stress, we find that beta herding can be more apparent when investors feel confident regarding the future direction of the market. Once a crisis appears beta herding becomes much weaker as a concern for fundamentals takes over.

We also find that the proposed herd measure is robust to business cycle and stock market movements. This is consistent with our underlying assumption on market wide herding. That is, herding occurs when investors' expectations on the market is homogeneous, or in other words when the direction towards which the market is heading is clear regardless of whether it is a bull or a bear market. Then investors are obsessed by the prospects or the market outlook rather than the equilibrium relationship between individual asset returns and factors.

The herd measure calculated with Fama-French portfolios supports our main findings. However there were several time periods in which this measure provides a different explanation from the herd measure calculated with individual stocks. We argue that the difference can be explained by changes in the idiosyncratic sentiments of individual stocks. Methodologically it has been shown that the standardised herd measure is more robust than the beta-based herd measure.

Clearly our empirical work has just scratched the surface of the potential applications of the approach we have developed here and more detailed analyses of herding attractors in different phases of market development now seem possible. This study has applied the new measure of herding to the market as a whole. However, the approach can also be applied at a sector (industry) level and different herding behavior may well be found in different sectors such as IT or old economy stocks or on a geographical basis.

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Appendix

Proof of Theorem 1

With the assumption of $b^s_{imt} \sim N(\beta^s_{imt}, \sigma^2_{it}/\sigma^2_{mt})$ and τ observations, we obtain the following non-central t distribution with the degrees of freedom $\tau - K - 1$;

$$\frac{b^s_{imt} - E_c(\beta^s_{imt})}{\widehat{\sigma}_{it}/\widehat{\sigma}_{mt}} \sim t\left(\tau - K - 1; \delta^*_{imt}\right), \tag{24}$$

where δ^*_{imt} is a non-centrality parameter, i.e., $\delta^*_{imt} = (\beta^s_{imt} - E_c(\beta^s_{imt}))/\sqrt{\sigma^2_{it}/\sigma^2_{mt}}$. Let $B^*_{imt} \equiv (b^s_{imt} - E_c(\beta^s_{imt}))/\sqrt{\widehat{\sigma}^2_{it}/\widehat{\sigma}^2_{mt}}$, then for a large $\tau - K - 1$,

$$B^*_{imt} \sim N\left(\delta^*_{imt}, 1\right). \tag{25}$$

Let $\mathbf{B}^*_{mt} = \begin{pmatrix} B^*_{1mt} & B^*_{2mt} & . & B^*_{Nmt} \end{pmatrix}'$. Then with the classical OLS assumption, for a large $\tau - K - 1$,

$$\underset{N\times N}{\mathbf{B}^*_{mt}} \sim N\left(\underset{N\times 1}{\boldsymbol{\delta}^*_{mt}}, \underset{N\times N}{\mathbf{V}^*_{mt}}\right),$$

where $\boldsymbol{\delta}^*_{mt} = \begin{pmatrix} \delta^*_{1mt} & \delta^*_{2mt} & . & \delta^*_{Nmt} \end{pmatrix}'$, and $\mathbf{V}^*_{mt}$ is covariance matrix of $\mathbf{B}^*_{mt}$.

In general, we may not assume that the matrix $\mathbf{V}^*_{mt}$ is fully ranked, since a large number of equities could mean $\tau - K - 1 < N$, suggesting the $(N \times N)$ variance-covariance matrix $\mathbf{V}^*_{mt}$ being singular. Let $\mathbf{Z} = \mathbf{C}'\mathbf{B}^*_{mt}$, where $\mathbf{C}$ is the $(N \times N)$ matrix of eigenvectors of the symmetric matrix of $\mathbf{V}^*_{mt}$, i.e., $\mathbf{V}^*_{mt} = \mathbf{C}\boldsymbol{\Lambda}\mathbf{C}'$ and $\boldsymbol{\Lambda}$ is the $(N\times N)$ diagonal matrix of eigenvalues. Note that the eigenvalues are sorted in descending order and the eigenvectors are also sorted according to the sorted eigenvalues. Then using $\mathbf{C}'\mathbf{C} = \mathbf{I}$ and

$$\begin{aligned} E(\mathbf{Z}) &= \mathbf{C}'E(\mathbf{B}^*_{mt}) = \mathbf{C}'\boldsymbol{\delta}^*_{mt} \\ Var(\mathbf{Z}) &= E\left[\left(\mathbf{C}'\mathbf{B}^*_{mt} - \mathbf{C}'\boldsymbol{\delta}^*_{mt}\right)\left(\mathbf{C}'\mathbf{B}^*_{mt} - \mathbf{C}'\boldsymbol{\delta}^*_{mt}\right)'\right] \\ &= \mathbf{C}'\mathbf{V}^*_{mt}\mathbf{C} \\ &= \boldsymbol{\Lambda}, \end{aligned}$$

we have $\mathbf{Z} \sim (\boldsymbol{\delta}^{*A}_{mt}, \boldsymbol{\Lambda})$,where $\boldsymbol{\delta}^{*A}_{mt} = \mathbf{C}'\boldsymbol{\delta}^*_{mt}$. When the rank (R) of the matrix $\mathbf{V}$ is less than N, i.e., $R \leq N$, the first R variables in the vector $\mathbf{Z}$ are normally distributed,

$z_i \sim N(\delta^{*A}_{imt}, \lambda_i)$, where z_i is the ith variable of $\mathbf{Z}$, δ^{*A}_{imt} is the ith element of vector $\boldsymbol{\delta}^{*A}_{mt}$, and λ_i is the ith eigenvalue of the diagonal matrix $\mathbf{\Lambda}$. On the other hand, the remaining $N-R$ variables of z_i, $i = R+1, ..., N$, are just constants since $\lambda_i = 0$ for $i = R+1, ..., N$. Thus we have

$$\begin{aligned}
\mathbf{B}^{*\prime}_{mt}\mathbf{B}^{*}_{mt} &= (\mathbf{CZ})'\mathbf{CZ} \\
&= \mathbf{Z}'\mathbf{C}'\mathbf{CZ} \\
&= \mathbf{Z}'\mathbf{Z} \\
&= \sum_{i=1}^{R} z_i^2 + \sum_{i=R+1}^{N} z_i^2.
\end{aligned}$$

Since $z_i \sim N(\delta^{*A}_{imt}, \lambda_i)$ is independent (orthogonal) of z_j for all $i \neq j$ for $i, j \leq R$, the first component is

$$\sum_{i=1}^{R} z_i^2 \sim \chi^2(R; \delta^R_k),$$

where δ^R_k is the non-centrality parameter, i.e., $\delta^R_k = \sum_{i=1}^{R} (\delta^{*A}_{imt})^2/\lambda_i$. The second component is a constant, i.e., $c \equiv \sum_{i=R+1}^{N} z_i^2 = \sum_{i=R+1}^{N} (\delta^{*A}_{imt})^2$. Thus

$$\mathbf{B}^{*\prime}_{mt}\mathbf{B}^{*}_{mt} \sim \chi^2(R; \delta^R_k) + c.$$

Therefore, our herd measure follows

$$\begin{aligned}
h_{kt} &= \frac{1}{N_t}\mathbf{B}^{*\prime}_{mt}\mathbf{B}^{*}_{mt} \\
&\sim \frac{1}{N_t}\left[\chi^2(R; \delta^R_k) + c\right]. \; QED.
\end{aligned}$$

Robustness of the Herd Measure

We have proposed a measure of herding and argued that the measure is designed to capture investor sentiment and biased pricing. The empirical results we reported are obtained with some assumptions which could be relaxed to see how robust are our arguments. Using the US data, we investigate several extensions.

The Effects of Small Stocks

By removing small stocks whose market values are less than 0.01 percent of the total market capitalisation, the analysis above focused on herding among larger stocks. To investigate the effects of small stocks on our herd measure we calculate herd measure with stocks whose market values are 0.1, 0.01, 0.001, 0.0001 percents of the total market capitalisation, and stocks whose returns are available for the entire sample period (243 stocks). In particular in the last case we could expect survivourship bias in the data.

Figure A1 shows that the herd measures calculated with these four different sets of stocks are close to each other. There is little noticeable difference in our herd measure from including small stocks whose market values are less than 0.01 percent of the total market capitalisation. In particular the result we obtain with 243 stocks that are available for the entire sample period indicates that our measure is robust against survivourship bias. This result is important in our study since we use some sets of stocks that may suffer survivourship bias in the UK and South Korean stock markets.

The Effects of Betas

Since our sample is a subgroup, our results may also be exposed to selection bias. For example we have remove certain stocks which we believe are less liquid and thus may cause bias in the t-statistic. Since our measure is based on the estimates of betas and their standard deviations, we need to investigate how robust is the herd measure for different subsets of betas. Using the estimated betas to rank the stocks, we make four subgroups; large beta stocks (top 70%), small beta stocks (bottom 70%), middle beta stocks (middle 70%) and high-low beta stocks (except middle 30%). Then for each of these sub-samples we apply the same procedure outlined above. The calculated herd measures for the four subgroups are plotted in Figure A2. The figure shows that there little difference in the herd measures between these groups.

Factors in Linear Factor Models

We use factor mimicking portfolios such as SMB, HML and momentum as control variables. Some correlation between the factors within the sample is inevitable given that firm specific characteristics are used to construct the factors. During our sample period positive correlations are found between the excess market return, SMB, and HML, whereas SMB is negatively correlated with momentum (not reported). Another impact from using additional factors could come from changes in the standard deviation of estimated beta. Adding factors can change the idiosyncratic errors and thus the standard deviation of the estimated beta even if there is no multicollinearity problem.

To evaluate the effects of using additional factors, we calculate the herd measures using the simple market model, the Fama-French three factor model, and the four factor model. We find that there is no significant difference in herding towards the market portfolio between the Fama-French three factor model and the four factor model except for the early 1980's and 2003. However, the herd measure based on the simple market model is generally larger than the two models, and is more volatile than the two. For example the large increase of the herd measure with the market model during the late 1990's and 2000's is attributed to the SMB, HML and momentum. Therefore without considering these factors, our measure may be biased.

Number of Observations Used When Estimating the Betas

As mentioned earlier, when the number of observations for the estimation of beta (i.e., τ) is small, we could capture the time variation in betas, but small sample effects may be significant. For example in many academic studies 60 months is common, and in practice 5 to 7 years are used. To evaluate the effects of different periods of τ on herd measure, we also use 36, 48, and 60 monthly observations to estimate betas.

Figure A4 shows that the standardised herd measures move in similar directions. As expected the longer the time period to calculate betas, the smoother the herd measure becomes. An unfavourable moving average effect increases when τ becomes larger, but all cases we report in Figure A4 keep similar dynamics. These results suggest that the negative effects of using 24 months are not serious, and we could get more dynamics from using a short window.

Table 1 Properties of Beta Herd Measure in the US Market

The Beta-based herd measure is calculated with the cross-sectional variance of OLS estimates of betas while the standardised herd measure is calculated with the cross sectional variance of t-statistics of OLS estimates of betas. We use 24 past monthly returns to estimate betas in the market model and in the Fama-French three factor model with momentum. Using 486 monthly observations from July 1963 to December 2003 and rolling windows of 24 months, we obtain 463 monthly herd measures from June 1965 to December 2003. In order to reduce possible bias from illiquid stocks we choose ordinary stocks whose market values are larger than 0.01% of the total market capitalisation, turnovers are larger than 6% a year, and volatilities are larger than half of the market portfolio's volatility. ** represents significance at the 1% level.

	Beta-Based Herd Measure		Standardised Herd Measure	
	Market Model (A)	Fama-French Three Factor with Momentum (B)	Market Model (C)	Fama-French Three Factor with Momentum (D)
Mean	0.322	0.416	3.518	1.896
Standard Deviation	0.216	0.188	1.171	0.530
Skewness	2.744	1.084	0.862	0.569
Excess Kurtosis	9.154	1.062	0.988	-0.470
Jarque-Bera Statistics	2197.756 **	112.484 **	76.153 **	29.280 **
Spearman Rank Correlations between A and B, and C and D		0.856 **		0.454 **
Spearman Rank Correlation between Cross-sectional Standard Deviations of Betas and t-Statistics	Between A and C	Between B and D		
	-0.200 **	-0.361 **		

Table 2 Regression of Herd Measures on the Cross-sectional Average of the Variances of Estimation Errors of Betas

The herd measures are regressed on the cross-sectional average of variances of the estimation errors of betas (CAEE). A total number of 463 monthly observations from June 1965 to December 2003 is used. The numbers in brackets are Newey and West (1987) heteroskedasticity consistent standard errors. ** represents significance at the 1% level.

		Constant	Average of the Variances of Estimation Errors of Betas	R^2
Cross-sectional Standard Deviation of OLS Betas	Market Model	0.096** (0.025)	1.554** (0.228)	0.577
	Fama-French Three Factor with Momentum	0.105** (0.014)	0.973** (0.050)	0.836
Cross-sectional Standard Deviation of OLS t-Statistics	Market Model	4.290** (0.189)	-5.313** (1.045)	0.230
	Fama-French Three Factor with Momentum	2.444** (0.079)	-1.717** (0.172)	0.329

Table 3 Regression of Beta Herding in Individual Stocks on Various Variables

Herd measures calculated with Fama-French three factor model with momentum are regressed on various explanatory variables using 463 monthly observations from June 1965 to December 2003. Rm and Vm represent market return and volatility. DP and RTB represent the dividend price ratio and the relative treasury bill rate, while TS and CS show the term spread and the default spread. The numbers in the brackets are Newey-West heteroskedasticity robust standard errors. ** represents significance at the 1% level and * represents significance at the 5% level.

A. standardised herd Measure

Constant	Rm	Vm	DP	RTB	TS	DS	CAEE	R^2
1.596**	-0.002	0.176*	12.242**	-0.010	0.112*	-0.390**		0.092
(0.172)	(0.006)	(0.075)	(4.439)	(0.056)	(0.052)	(0.142)		
2.919**	-0.001	0.090	-1.394	0.002	0.088*	-0.474**	-2.138**	0.479
(0.176)	(0.004)	(0.057)	(3.907)	(0.038)	(0.039)	(0.102)	(0.198)	

B. Beta-Based Herd Measure

Constant	Rm	Vm	DP	RTB	TS	DS	CAEE	R^2
0.685**	0.000	-0.015	-7.100**	0.003	0.008	-0.033		0.250
(0.069)	(0.002)	(0.030)	(1.844)	(0.018)	(0.016)	(0.045)		
0.095**	-0.001	0.023	-1.011	-0.002	0.018	0.004	0.955**	0.860
(0.034)	(0.001)	(0.018)	(0.971)	(0.008)	(0.007)	(0.024)	(0.048)	

Table 4 Regression of Beta Herding in Portfolios on Various Variables

The herd measures calculated with the Fama-French three factor model with momentum are regressed on log sentiment index and other control variables. For the sentiment index we use bull-bear ratio of Investors Intelligence, where bull and bear are calculated by the proportions of weekly newsletters for the future market movements. Rm and Vm represent market return and volatility. DP and RTB represent the dividend price ratio and the relative treasury bill rate, while TS and CS show the term spread and the default spread. The numbers in the brackets are Newey-West heteroskedasticity robust standard errors. The bold numbers represent significance at the 5% level.

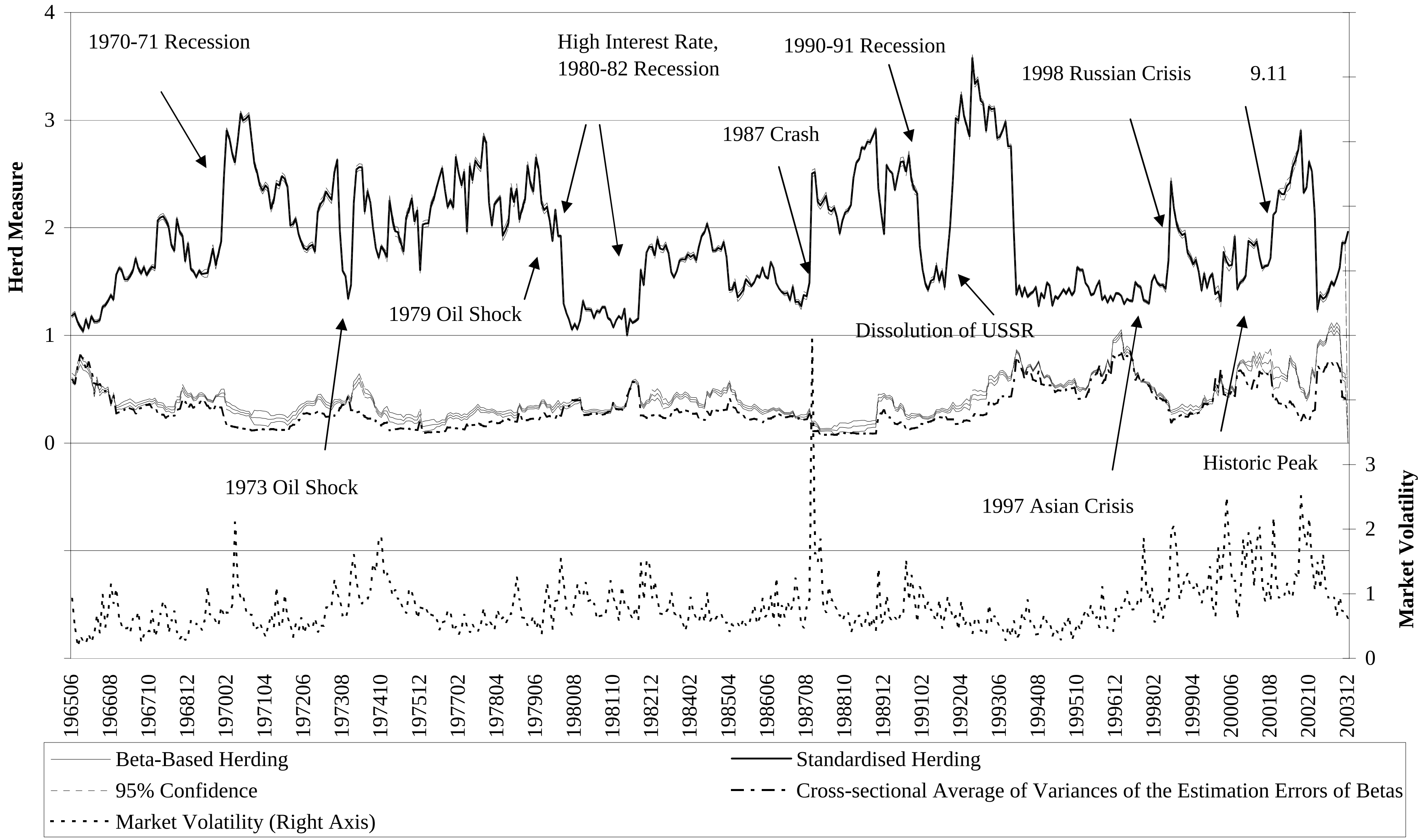
A. t-statistics-Based herd Measure Calculated with Fama-French 25 Portfolios Based on Size and Book-to-Market

Constant	$\log(S_{mt})$	Rm	Vm	DP	RTB	TS	DS	CAEE	R^2
1.427	**-0.346**								0.040
(0.060)	(0.152)								
1.219	**-0.357**	0.000	0.026	**7.795**	-0.040	0.048	-0.130		0.116
(0.163)	(0.178)	(0.003)	(0.057)	(2.871)	(0.038)	(0.035)	(0.085)		
1.542	**-0.396**	0.000	0.041	1.733	-0.039	0.047	-0.082	-56.791	0.145
(0.219)	(0.183)	(0.003)	(0.063)	(4.332)	(0.038)	(0.035)	(0.093)	(32.432)	

B. Beta-Based herd Measure Calculated with Fama-French 25 Portfolios Based on Size and Book-to-Market

Constant	$\log(S_{mt})$	Rm	Vm	DP	RTB	TS	DS	CAEE	R^2
-4.396	**-0.651**								0.061
(0.073)	(0.286)								
-3.979	**-0.848**	0.003	0.098	**-22.185**	-0.076	0.034	0.212		0.303
(0.207)	(0.254)	(0.004)	(0.103)	(4.612)	(0.059)	(0.051)	(0.125)		
-5.279	**-0.691**	0.001	0.038	2.239	-0.077	0.038	0.018	**228.811**	0.504
(0.267)	(0.202)	(0.003)	(0.068)	(4.862)	(0.050)	(0.043)	(0.106)	(36.541)	

Figure 1 Beta Herding in the US Market



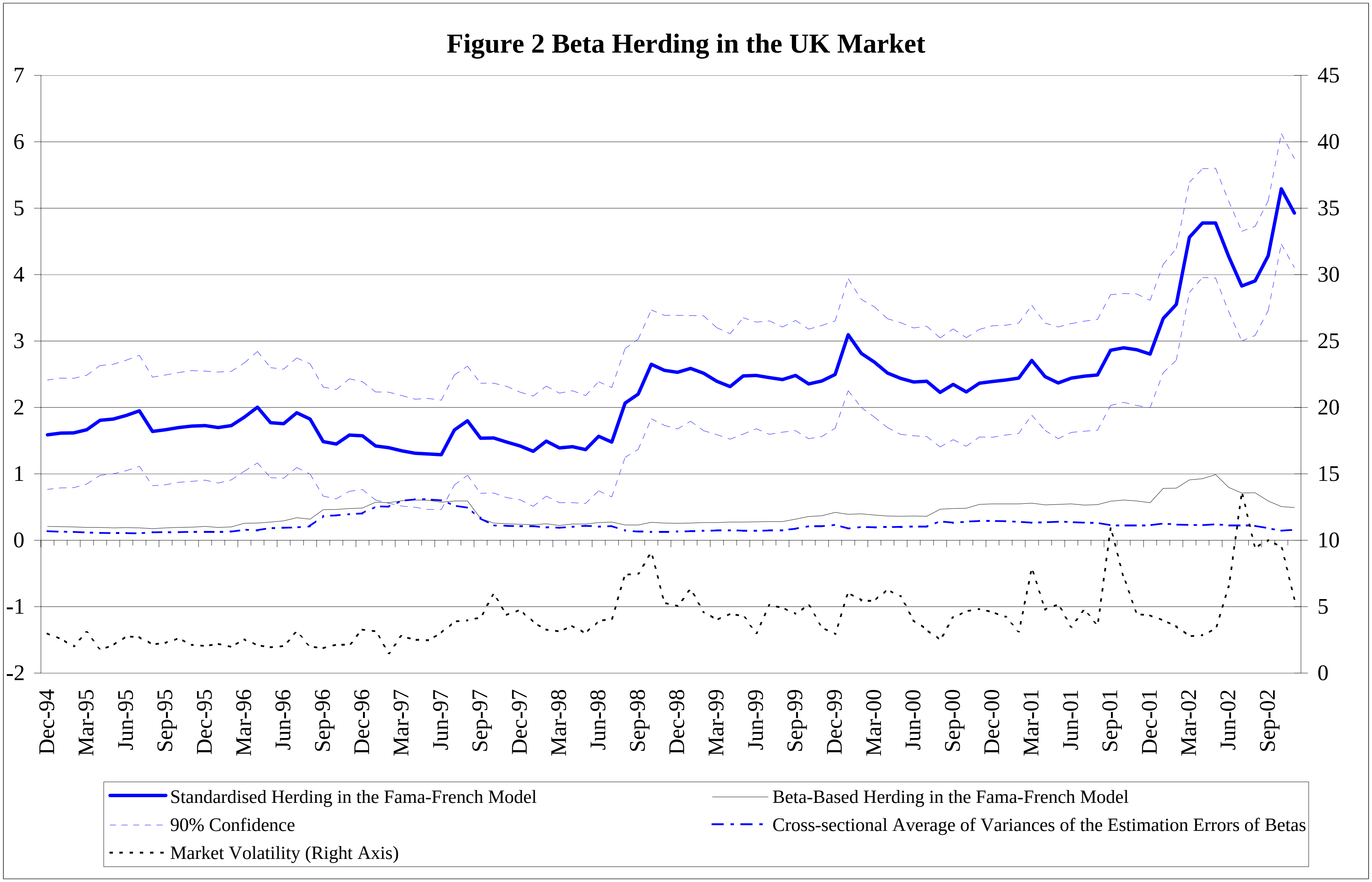


Figure 2 Beta Herding in the UK Market
7
6
5
4
3
2
1
0
-1
-2
45
40
35
30
25
20
15
10
5
0
Dec-94
Mar-95
Jun-95
Sep-95
Dec-95
Mar-96
Jun-96
Sep-96
Dec-96
Mar-97
Jun-97
Sep-97
Dec-97
Mar-98
Jun-98
Sep-98
Dec-98
Mar-99
Jun-99
Sep-99
Dec-99
Mar-00
Jun-00
Sep-00
Dec-00
Mar-01
Jun-01
Sep-01
Dec-01
Mar-02
Jun-02
Sep-02
Standardised Herding in the Fama-French Model
Beta-Based Herding in the Fama-French Model
90% Confidence
Cross-sectional Average of Variances of the Estimation Errors of Betas
Market Volatility (Right Axis)

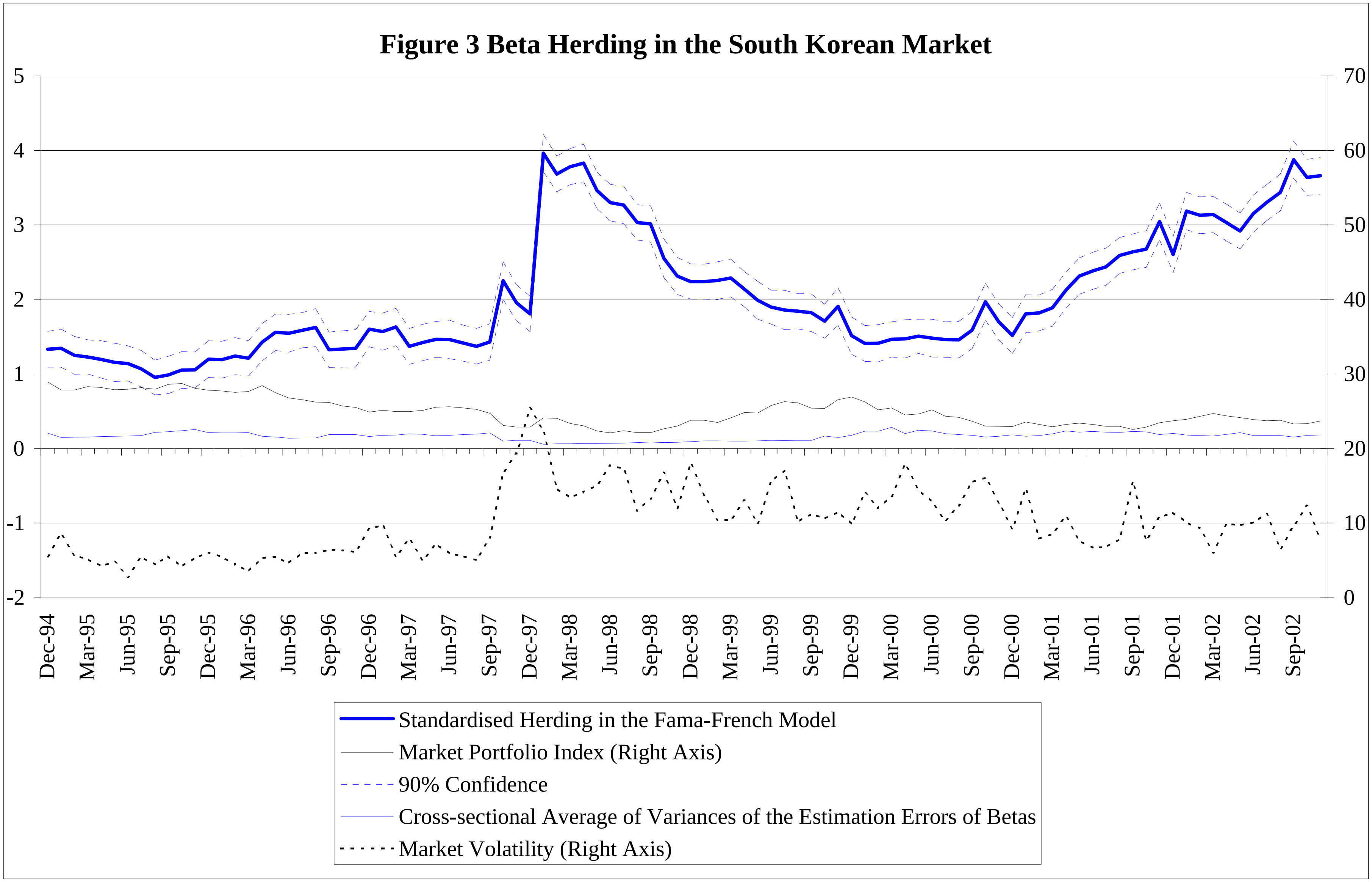
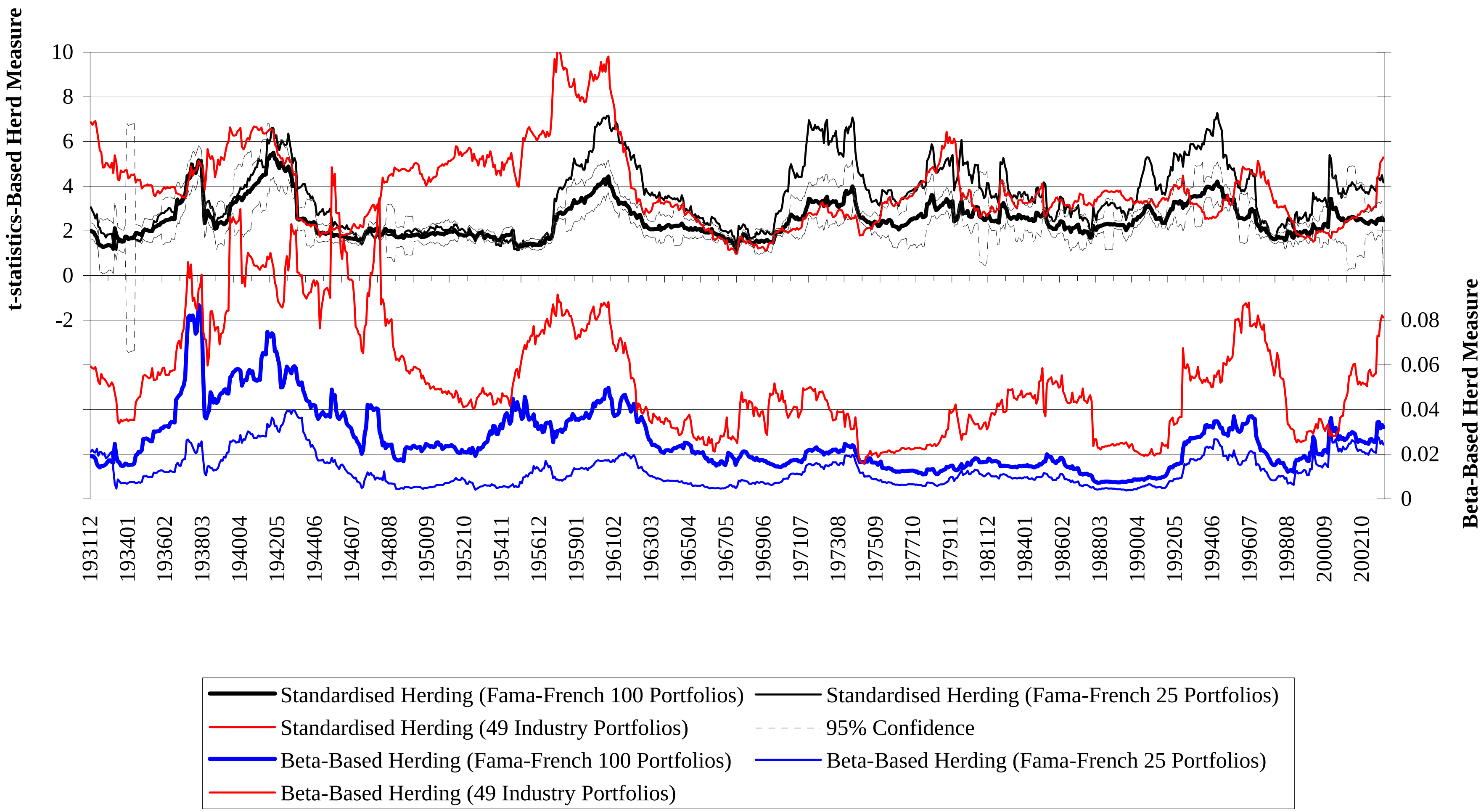
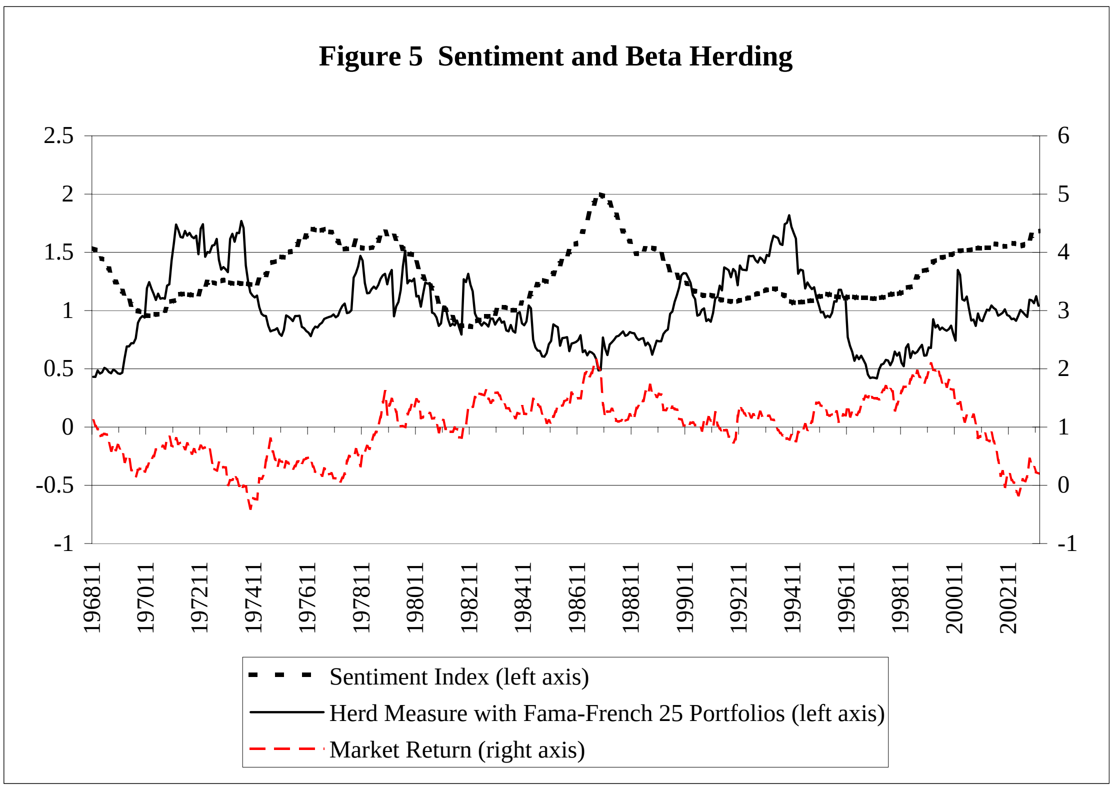


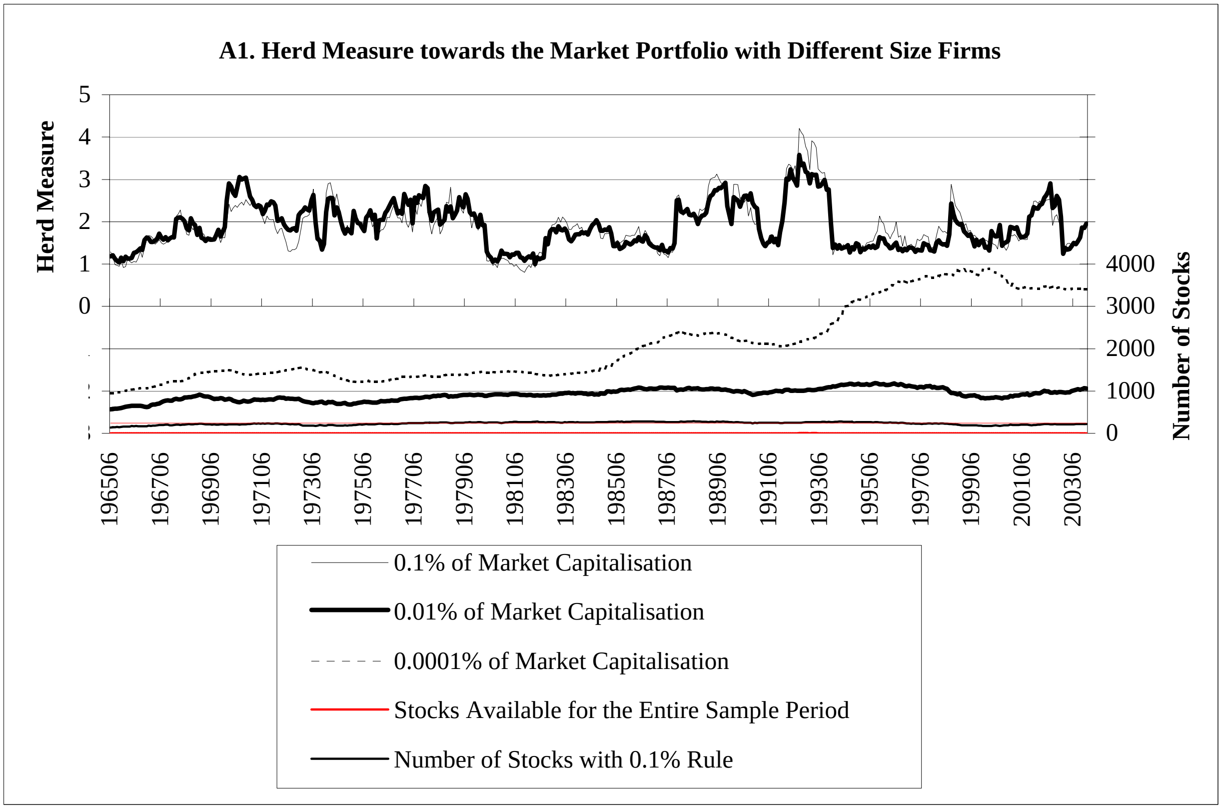
Figure 3 Beta Herding in the South Korean Market
5
4
3
2
1
0
-1
-2
70
60
50
40
30
20
10
0
Dec-94
Mar-95
Jun-95
Sep-95
Dec-95
Mar-96
Jun-96
Sep-96
Dec-96
Mar-97
Jun-97
Sep-97
Dec-97
Mar-98
Jun-98
Sep-98
Dec-98
Mar-99
Jun-99
Sep-99
Dec-99
Mar-00
Jun-00
Sep-00
Dec-00
Mar-01
Jun-01
Sep-01
Dec-01
Mar-02
Jun-02
Sep-02
Standardised Herding in the Fama-French Model
Market Portfolio Index (Right Axis)
90% Confidence
Cross-sectional Average of Variances of the Estimation Errors of Betas
Market Volatility (Right Axis)

Figure 4 Beta Herding Calculated with Fama-French 25 Portfolios Formed on Size and Book-to-Market

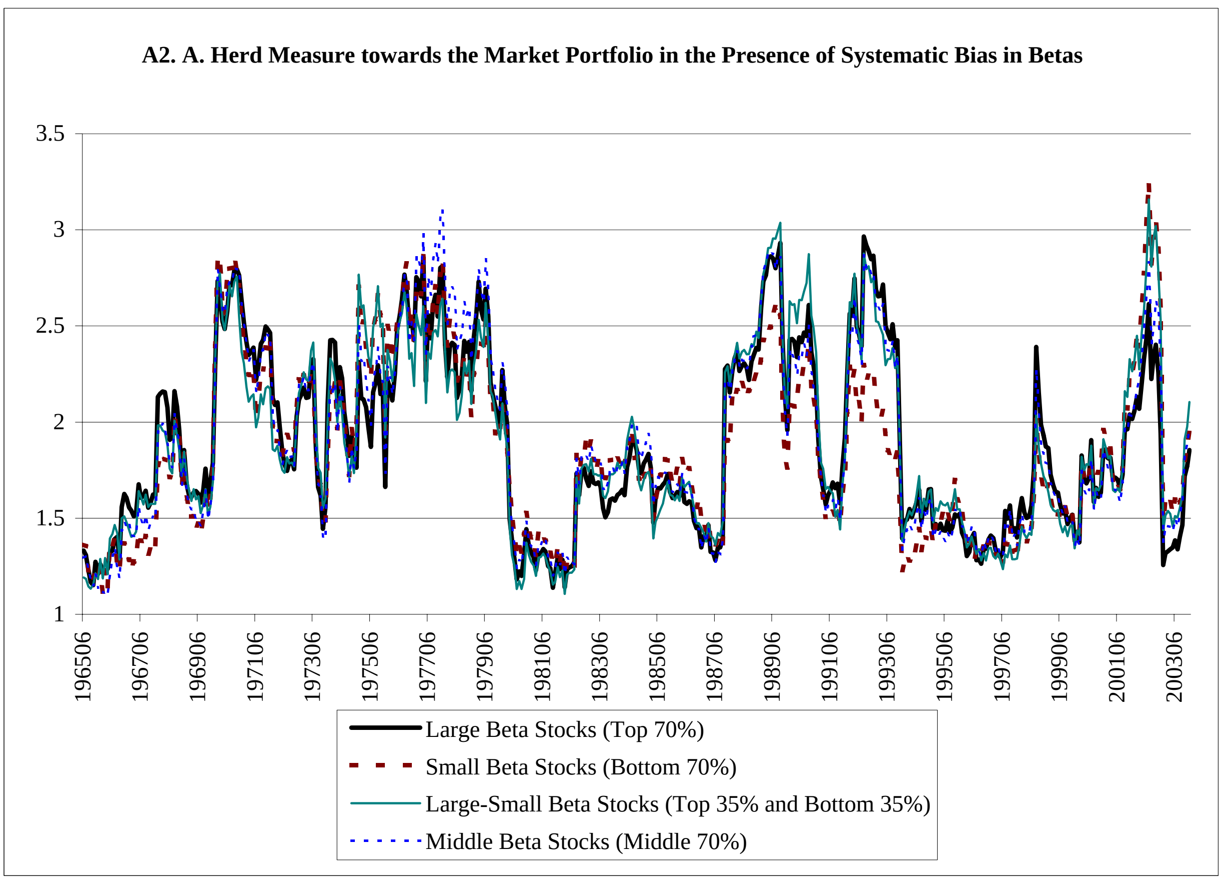




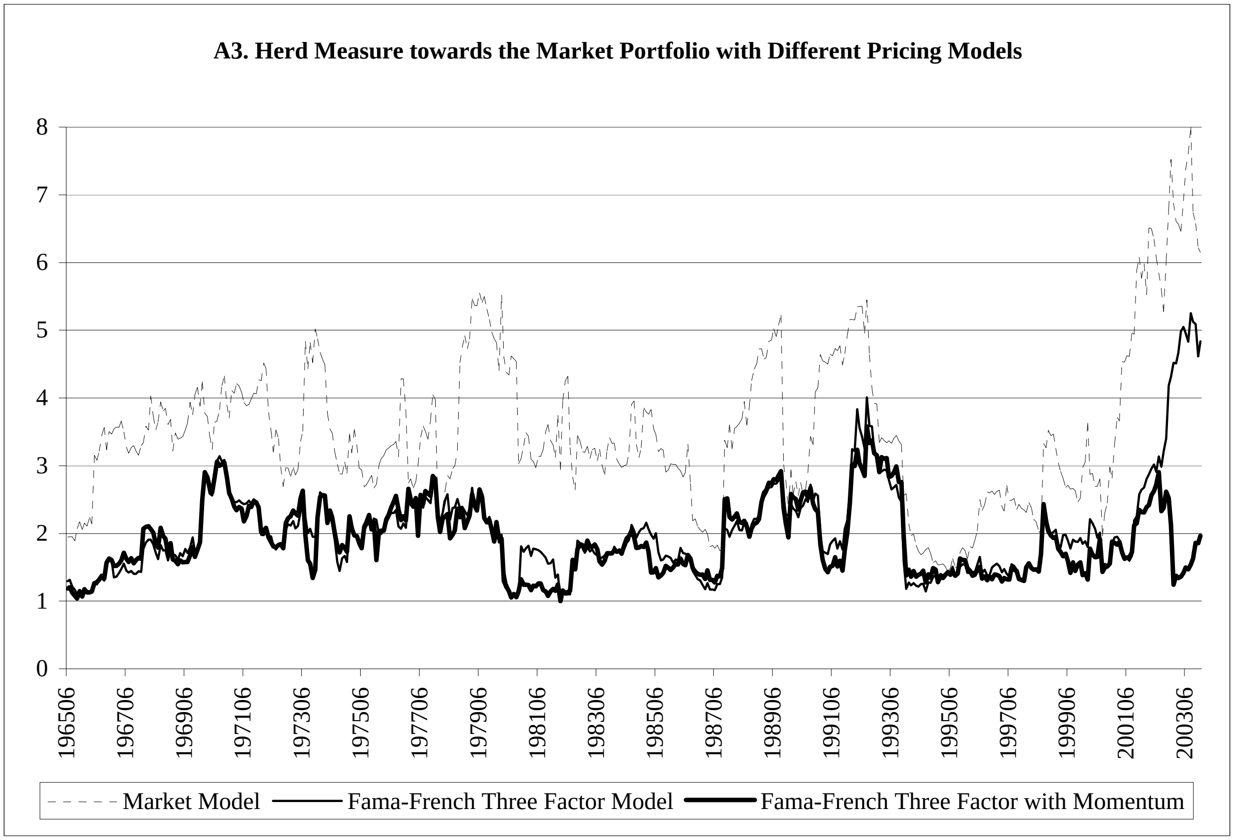
Hearding is measured with t-statistics of the OLS estimates of betas in the Fama-French three factor model with momentum for the Fama-French 25 porttfolios formed on size and book-to-market. For the sentiment index we use bull-bear ratio of Investors Intelligence who collects the information from weekly newsletters that publish their opinion on the future stock market movements. Rolling windows of past 60 months observations is used for the OLS estimation and so we use the same method to construct the moving average sentiment and the market return for the same period.

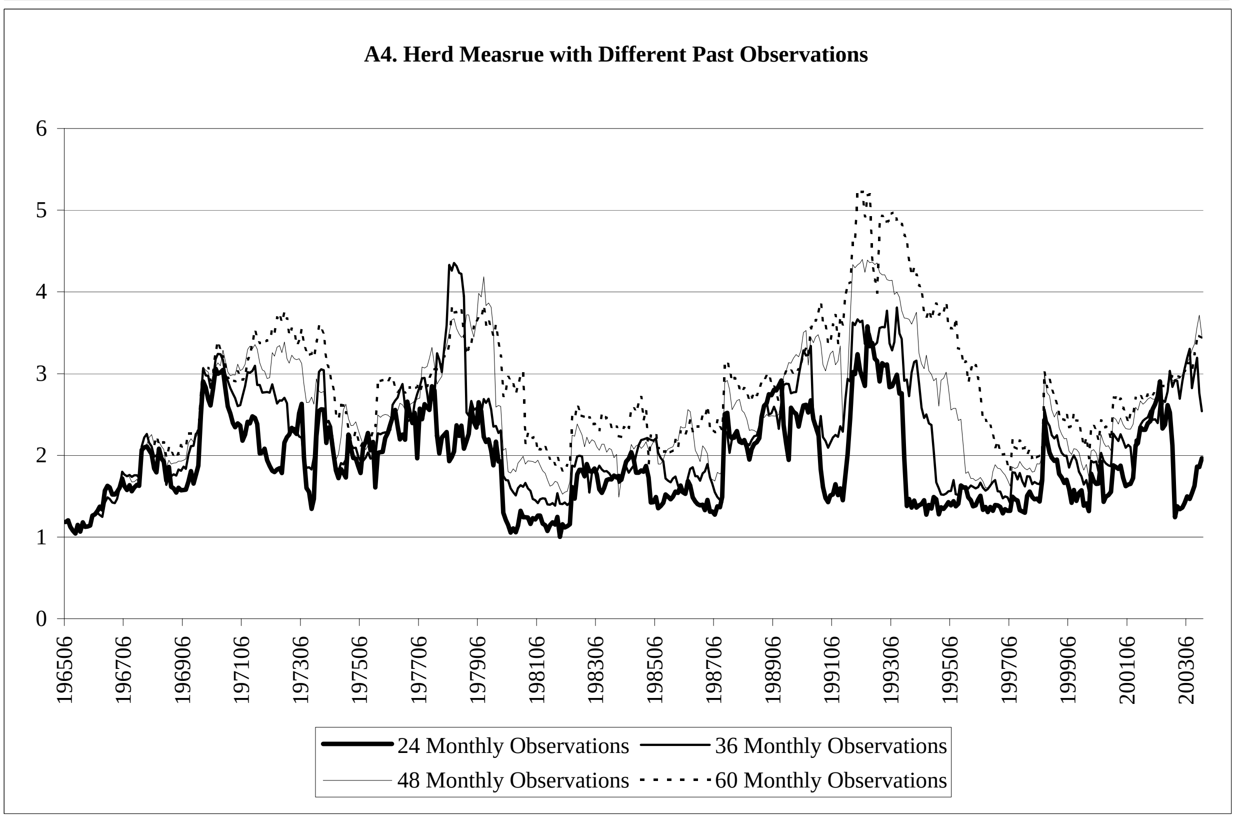


A1. Herd Measure towards the Market Portfolio with Different Size Firms
Herd Measure
Number of Stocks
5
4
3
2
1
0
4000
3000
2000
1000
0
196506
196706
196906
197106
197306
197506
197706
197906
198106
198306
198506
198706
198906
199106
199306
199506
199706
199906
200106
200306
0.1% of Market Capitalisation
0.01% of Market Capitalisation
0.0001% of Market Capitalisation
Stocks Available for the Entire Sample Period
Number of Stocks with 0.1% Rule



A2. A. Herd Measure towards the Market Portfolio in the Presence of Systematic Bias in Betas
3.5
3
2.5
2
1.5
1
196506
196706
196906
197106
197306
197506
197706
197906
198106
198306
198506
198706
198906
199106
199306
199506
199706
199906
200106
200306
Large Beta Stocks (Top 70%)
Small Beta Stocks (Bottom 70%)
Large-Small Beta Stocks (Top 35% and Bottom 35%)
Middle Beta Stocks (Middle 70%)


A3. Herd Measure towards the Market Portfolio with Different Pricing Models
8
7
6
5
4
3
2
1
0
196506
196706
196906
197106
197306
197506
197706
197906
198106
198306
198506
198706
198906
199106
199306
199506
199706
199906
200106
200306
Market Model
Fama-French Three Factor Model
Fama-French Three Factor with Momentum


A4. Herd Measrue with Different Past Observations
6
5
4
3
2
1
0
196506
196706
196906
197106
197306
197506
197706
197906
198106
198306
198506
198706
198906
199106
199306
199506
199706
199906
200106
200306
24 Monthly Observations
36 Monthly Observations
48 Monthly Observations
60 Monthly Observations