Generalized Autoregressive Conditional Intensity Models with Long Range Dependence

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Abstract

In this paper, we generalize Russell's (1999) autoregressive conditional intensity model in several directions. First, we propose a framework which nests both proportional intensity structures as well as accelerated failure time structures. Second, the process dynamics are extended to allow for long range dependence in the intensity process. Third, we account for spill-over effects between consecutive trading days by incorporating inter-day dynamics. Fourth, a semiparametric extension of the Burr hazard rate for the modelling of the baseline intensity component is suggested. Applications of univariate and bivariate versions of the model to trade intensities and price change intensities based on the IBM stock traded at the New York Stock Exchange illustrate the usefulness of the proposed extensions. Significant long memory effects are observed. Furthermore, we find evidence for non-stationary patterns in the intensity series. In contrast, the inter-day dynamics are weak and only identifiable on a sufficiently long time series. Moreover, we observe the presence of acceleration effects and a rejection of proportional intensity structures. Finally, it turns out that a semiparametric specificaton of the baseline intensity component is necessary to capture the distributional properties of the data. The latter is particularly important for trade durations.

Keywords: Multivariate point processes, intensity function, proportional intensity vs. accelerated failure time model, long memory.

JEL Classification: C22, C32, C41

1 Introduction

The modelling of financial transaction data is an ongoing topic in the financial econometrics literature. A key property of transaction data is the irregular spacing in time which

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necessitates to consider the data statistically as point processes. The importance of point processes in financial econometrics has been stressed in the seminal papers by Engle (2000) and Engle and Russell (1998). They proposed the Autoregressive Conditional Duration (ACD) model in which the durations between consecutive events are modelled in terms of an autoregressive accelerated failure time model. Since the ACD model is specified in discrete time, it is not able to appropriately describe multivariate point processes in which the individual processes occur asynchronously. For this reason, Russell (1999) suggested to model point processes in a continuous-time setting by directly parameterizing the (multivariate) intensity function instead of the inter-event durations. He proposed the Autoregressive Conditional Intensity (ACI) model which is a proportional intensity model in which the intensity function is parameterized as a multiplicative function of a baseline intensity and a function following a log-linear vectorial ARMA-type process which is updated at each point of the process.

However, recent literature on financial point processes realized four major deficiencies of basic ACD or ACI type models in applications to financial duration data. First, Jasiak (1998) found clear evidence for long memory in financial duration processes. Moreover, applications of ACD and ACI models to financial duration processes typically reveal autoregressive parameters which are close to non-stationarity and indicate a strong persistence in the process. Second, Bauwens, Giot, Grammig, and Veredas (2004) illustrated that even highly parameterized ACD specifications are not able to fully capture the distributional properties of trade durations. Similar evidence was provided by Hall and Hautsch (2004) who found a Burr parameterization of the baseline intensity to be not sufficient to model trading intensities. Third, Bowsher (2002) illustrated significant evidence for daily spill-overs in trade intensities and price change intensities which is typically ignored in many applications. Fourth, several studies like e.g. Dufour and Engle (2000), Fernandes and Grammig (2000) and Hautsch (2003) among others illustrated the presence of asymmetric news impacts in ACD models.

In this paper, we propose a framework for (multivariate) autoregressive intensity processes which generalizes Russell's ACI model in the aforementioned directions: First, we relax the assumption of a proportional intensity model and allow for accelerated failure time specifications in which components determining the intensity (such as functions of past durations or covariates) might accelerate or decelerate the time to failure. The

distinction between proportional intensity models and accelerated failure time models and thus the motivation for this extensions emanates from the traditional literature on failure times in biostatistics and labor economics[1]. In the literature of financial point processes this distinction is present since the ACD model belongs to the class of accelerated failure time model whereas the ACI model is based on a proportional intensity structure. However, the resulting model proposed in this paper nests both the basic ACI model as well as a special type of ACD model as special cases.

Second, we allow for long range dependence in the intraday intensity process by extending the (short memory) GARCH type dynamics to the case of non-summable autocovariances. By adapting the long memory ARCH(∞) model proposed by Koulikov (2003), we model the process dynamics in terms of a fractionally integrated process which is updated by lagged integrated intensities. Moreover, we capture inter-day spillovers by incorporating a daily intensity component which follows itself an autoregressive structure. This structure allows to separate between intradaily and daily process dynamics.

Third, in order to allow for more distributional flexibility, we propose a semiparametric specification of the baseline intensity component. The major idea is to parameterize the baseline intensity in terms of a Burr hazard function which is augmented by a spline function based on a flexible Fourier form according to Gallant (1981). Fourth, the intraday dynamics are extended to allow for asymmetric news impacts.

The resulting model is called Generalized Long Memory ACI (GLMACI) model and is applied to trade durations and price durations based on the trading process of the IBM stock at the New York Stock Exchange (NYSE). Estimating and evaluating univariate and bivariate specifications of the GLMACI model illustrate the usefulness of the proposed specifications. We find significant evidence for long range dependence in trade durations as well as price durations. In contrast, only weak inter-day dynamics are found. Particularly for trade durations which are analyzed based on a time series capturing one month, the latter effects are hard to identify. For price durations which are investigated on the basis of a two-month sample, weak but significant inter-day effects are found. Furthermore, the analyzed duration processes reveal evidence for acceleration effects in dependence of intraday dynamics and seasonalities. It turns out that the basic proportional intensity

[1] See Kalbfleisch and Prentice (1980) and Kiefer (1988) as well as Lancaster (1997) for classical references in both areas.

structure is rejected. Finally, besides the existence of asymmetric news impact effects we find a semiparametric specification of the baseline intensity to be necessary in order to capture the distributional properties of the financial duration data. In line with empirical evidence (see e.g. Bauwens, Giot, Grammig, and Veredas, 2004) this effect is particularly apparent for trade durations.

The remainder of the paper is structured as follows: In Section 2, we illustrate the univariate GLMACI model. The multivariate extension is given in Section 3. Section 4 describes the statistical inference whereas Section 5 presents the empirical application of the model. Finally, Section 6 concludes.

2 The Univariate GLMACI Model

2.1 Statistical Background on Simple Point Processes

Let t denote the calendar time and let $\{t_i\}_{i\in\{1,2,\ldots,n\}}$ be a sequence of random arrival times with $0 < t_1 < \ldots < t_n$. The series of points $\{t_i\}$ is called a (simple) point process. Define $N(t) := \sum_{i\geq 1} \mathbb{1}_{\{t_i \leq t\}}$ as the right-continuous counting function. Denote $\mathcal{F}_t$ as the history (inclusive possible (exogenous) covariates) of the point process $N(t)$ up to t. Then, the point process $N(t)$ is said to be adapted to the filtration $\mathcal{F}_t$. A key concept in point process theory is the (stochastic) intensity function. Define $\lambda(t)$ as a scalar, positive, left-continuous $\mathcal{F}_t$-predictable process with right hand limits. Then, if

$$\mathrm{E}[N(s) - N(t)|\mathcal{F}_t] = \mathrm{E}\left[\int_t^s \lambda(u)du|\mathcal{F}_t\right] \tag{1}$$

(almost surely) for all t, s with $0 \leq t \leq s$, we call $\lambda(t)$ the intensity of $N(t)$ (see e.g. Brémaud, 1981). A more intuitive representation of the intensity function is obtained by considering the limit case when $s \downarrow t$. Then, we have

$$\lambda(t) := \lim_{\Delta\downarrow 0} \frac{1}{\Delta}\mathrm{E}\left[N(t+\Delta) - N(t)|\,\mathcal{F}_t\right]. \tag{2}$$

A direct consequence of (1) is that $\mathrm{E}[N(t)] = \mathrm{E}\left[\int_0^t \lambda(u)du\right] = \mathrm{E}[\tilde{\Lambda}(t)]$, where $\tilde{\Lambda}(t) := \int_0^t \lambda(u)du$ is referred to as the $\mathcal{F}_t$-compensator of $N(t)$. Furthermore, if the point process $N(t)$ is integrable, i.e. $\mathrm{E}[N(t)] < \infty$, $\forall\, t \geq 0$, Definition (1) implies that the process

$$N(t) - \tilde{\Lambda}(t) \tag{3}$$

is a mean zero martingale.

The stochastic properties of the compensator $\tilde{\Lambda}(t)$ play an important role in the theory of point processes and are exploited to construct diagnostic tests for point process models (see e.g. Bowsher, 2002). A central theorem is the random time change theorem for point processes. Brown and Nair (1988) prove that the point process formed from the (continuous) process $\tilde{\Lambda}(t)$ with event arrival times $\tilde{t}_i := \int_0^t \lambda(s)ds$ is a Poisson process with unit intensity.[2] Consequently, the integrated intensity $\Lambda(t_{i-1}, t_i) := \int_{t_{i-1}}^{t_i} \lambda(s)ds$ corresponds to the increment of a Poisson process and is standard exponentially distributed. Therefore, $\Lambda(t_{i-1}, t_i)$ can be interpreted as a generalized error (e.g. in the spirit of Cox and Snell, 1968) that establishes a link between the intensity function and the implied waiting time until the next event.

2.2 The Basic Model

Define $\breve{N}(t) := \sum_{i\geq 1} \mathbb{1}_{\{t_i<t\}}$ as the left-continuous counting function. Then, the basic structure of the univariate generalized ACI model is given by

$$\lambda(t) = \lambda_0\left(\eta(t)\right)\Phi_{\breve{N}(t)+1}s(t), \tag{4}$$

$$\eta(t) := x(t)\cdot\left[\Phi_{\breve{N}(t)+1}s(t)\right]^{\delta}, \tag{5}$$

where $\Phi_{\breve{N}(t)}$ is a function capturing the model dynamics as well as possible covariates, $x(t) := t - t_{\breve{N}(t)}$ denotes the time elapsed since the last event (the so-called backward recurrence time), and $\lambda_0(\cdot)$ denotes a baseline intensity function. Furthermore, $s(t)$ is a (deterministic) function of time capturing possible seasonality effects. The function $\Phi_{\breve{N}(t)}$ is indexed by the left-continuous counting function and is updated instantaneously *after* the arrival of a new point. Hence, Φ_i is constant for $t_{i-1} < t \leq t_i$. The evolution of the intensity function between two consecutive arrival times is determined by the function $\lambda_0(\cdot)$ depending on a function of time, $\eta(t)$, corresponding to the time elapsed since the last event scaled by $\Phi_{\breve{N}(t)+1}$ and $s(t)$. Hence, $\eta(t)$ can be interpreted as a transformation of the time scale on which the baseline intensity $\lambda_0(\cdot)$ is defined. If $\delta > 0$, the model dynamics and seasonalities accelerate the time until the next event, whereas for $\delta < 0$ the time scale is decelerated. For the special case $\delta = 1$, we obtain the counterpart of the

[2] See Theorem 1 in Brown and Nair (1988).

so-called accelerated failure time (AFT) model. Correspondingly, for $\delta = 0$, we obtain the counterpart of the classical proportional intensity (PI) model.[3]

The non-negativity of $\lambda(t)$ is ensured by parameterizing Φ_i in log-linear form which is augmented by a function of covariates z_i, observable at each event arrival point t_i,

$$\Phi_i = \exp\left(\tilde{\Phi}_i + z'_{i-1}\gamma\right). \tag{6}$$

As proposed by Russell (1999), the key idea of the ACI model is to specify the dynamic function $\tilde{\Phi}_i$ in terms of the past sequence of integrated intensities, $\{\Lambda(t_{j-1}, t_j)\}_{j=2}^{i}$. Exploiting the standard exponential property of $\Lambda(t_{j-1}, t_j)$, a natural i.i.d. innovation term is given by

$$\varepsilon_i := 1 - \Lambda(t_{i-1}, t_i) \tag{7}$$

with the property $\mathrm{E}[\varepsilon_i] = 0$.

In order to extend Russell's parameterization of $\tilde{\Phi}_i$ to the case of long range dependence, we parameterize it in terms of an infinite series representation as proposed by Koulikov (2003) and given by

$$\tilde{\Phi}_i = \omega + \alpha \sum_{j=1}^{\infty} \theta_{j-1}\varepsilon_{i-j}, \tag{8}$$

where $\{\theta_j : j \geq 0\} \subseteq \mathbb{R}_{0+}$ is an infinite sequence of coefficients with $\theta_0 = 1$ and ω and α are model parameters. Koulikov (2003) proves that the process (8) is covariance stationary and ergodic as long as ε_i is a (zero mean) martingale difference, and the coefficients θ_j are square-summable. Following Granger and Joyeux (1980), or Hosking (1981), a possible parameterization of the coefficients θ_j is given by a power series expansion of $(1 - \beta z)^{-1}(1 - z)^{-d}$ as given by

$$\theta_j := \sum_{k=0}^{j} \beta^k \theta^*_{j-k}, \tag{9}$$

where

$$\theta^*_j := \frac{\Gamma(d + j)}{\Gamma(d)\Gamma(1 + j)} \quad \forall\, j \geq 0, \tag{10}$$

[3]See e.g. Kalbfleisch and Prentice (1980), Kiefer (1988) or Lancaster (1997).

are the coefficients of the expansion of $(1-z)^{-d}$, β is a model parameter with $|\beta| < 1$ and $\Gamma(\cdot)$ denotes the gamma function. For $d \in (0,1)$, (10) produces a non-summable autocovariance function, and thus long memory according to the classification of McLeod and Hipel (1978). For $0 < d < \frac{1}{2}$, the power series expansion implies a sequence of square-summable hyperbolic decaying coefficients and thus ensures covariance stationarity. By replacing z by the lag (or backshift) operator L, we can re-write $\tilde{\Phi}_i$ as

$$\tilde{\Phi}_i = \omega + \alpha(1-\beta L)^{-1}(1-L)^{-d}\varepsilon_{i-1} \tag{11}$$

with $(1-L)^{-d}\varepsilon_{i-1} = \sum_{j=1}^{\infty} \theta_{j-1}\varepsilon_{i-j}$.

Special Cases

In the following we discuss several special cases nested in the encompassing model. For convenience, we assume $s(t) = 1$.

(i) For $d = 0$, (11) modifies to the basic ACI(1,1) specification as proposed by Russell (1999), i.e.

$$\begin{aligned} \tilde{\Phi}_i &= \omega + \alpha(1-\beta L)^{-1}\varepsilon_{i-1} \\ &= \omega + \alpha\varepsilon_{i-1} + \beta(\tilde{\Phi}_{i-1} - \omega), \end{aligned} \tag{12}$$

where α denotes the innovation parameter and β the persistence parameter.

(ii) If the baseline intensity $\lambda_0(\cdot)$ is non-specified, and $\delta = 1$, $\omega = \alpha = \beta = 0$, then the model corresponds to the standard (non-dynamic) AFT model (see e.g. Kalbfleisch and Prentice, 1980) as given by

$$\lambda(t) = \lambda_0\left[x(t)\exp(z_{\breve{N}}(t)'\gamma)\right]\exp(z_{\breve{N}}(t)'\gamma). \tag{13}$$

It is well known that it can be alternatively written as a log-linear model in terms of the inter-event durations $x_i := t_i - t_{i-1}$,

$$\ln x_i = -z_{i-1}'\gamma + \xi_i, \quad i = 1, \ldots, n, \tag{14}$$

where ξ_i is an error term that follows a non-specified continuous distribution.

(iii) If the baseline intensity $\lambda_0(\cdot)$ is non-specified, and $\delta = \omega = \alpha = \beta = 0$, then the model corresponds to the well-known class of (non-dynamic) semiparametric PI models (see Cox, 1972) given by

$$\lambda(t) = \lambda_0(x(t)) \exp(z_{\breve{N}}(t)'\gamma). \tag{15}$$

It is well known that the PI and AFT model coincide if the baseline intensity $\lambda_0(\cdot)$ is parameterized according to an exponential distribution ($\lambda_0(\cdot) = \lambda$), or a Weibull distribution, $\lambda_0(\cdot) = p\lambda(x(t)\lambda)^{p-1}$, with $\lambda > 0$, $p > 0$ (see e.g. Kalbfleisch and Prentice, 1980).

(iv) If $\lambda_0(\cdot) = 1$, then Φ_i^{-1} corresponds to the conditional expectation of the duration x_i, i.e.

$$\Phi_i = \mathrm{E}[x_i|\mathcal{F}_{t_{i-1}}]^{-1} := \Upsilon_i^{-1} \tag{16}$$

with

$$\Upsilon_i = \exp\left(\tilde{\Upsilon}_i - z_{i-1}'\gamma\right) \tag{17}$$

$$\tilde{\Upsilon}_i = -\omega + \alpha(1-\beta L)^{-1}(1-L)^{-d}(x_{i-1}/\Upsilon_{i-1} - 1), \tag{18}$$

which corresponds to a (long memory) exponential type Log-ACD model based on centered standardized durations as innovations. In the case of $d = 0$, the model modifies to

$$\tilde{\Upsilon}_i = -\omega + \alpha(1-\beta L)^{-1}\frac{x_{i-1}}{\Upsilon_{i-1}}. \tag{19}$$

(v) If $\lambda_0(\eta(t)) = p \cdot \eta(t)^{p-1}(1+\kappa\eta(t)^p)^{-1}$ and $\delta = 1$, then

$$\Phi_i = \mathrm{E}[x_i|\mathcal{F}_{t_{i-1}}]^{-1}\left[\frac{\kappa^{1+1/p}\Gamma(1+1/\kappa)}{\Gamma(1+1/p)\Gamma(\kappa^{-1}-1/a)}\right] := \Upsilon_i, \tag{20}$$

where Υ_i is given by (17) and

$$\tilde{\Upsilon}_i = -\omega - \alpha(1-\beta L)^{-1}(1-L)^{-d}\varepsilon_{i-1}, \tag{21}$$

corresponding to a special type of Burr type (long memory) ACD model with the integrated intensity as innovation term.

2.3 Extensions of the Basic Model

2.3.1 Long-Term Dynamics

A typical property of transaction data is that it is not continuously observed. On all markets which do not operate on a 24h basis, the intraday trading activity is subjected to overnight effects and weekend or holiday effects. Typically, these effects are taken into account by the inclusion of appropriate overnight dummy variables and by a re-initialization of the model dynamics at the beginning of each trading day. An alternative approach is to take daily spill-overs and possible inter-day dynamics explicitly into account. Bowsher (2002) proposes a generalization of a Hawkes (1971) process by allowing for spill-overs in the intensity from one to the next trading day. Here, we propose an alternative approach which allows to capture possible long-run effects which are updated on a daily level.

Denote $\tau(t)$ as an integer variable indexing the current trading day observed at time t. Furthermore, define $t^{\dagger}_{\tau(t)}$ and $t^{\ddagger}_{\tau(t)}$ as the time of the opening and closure of the trading day $\tau(t)$, respectively. Then, we propose re-specifying (4) as

$$\lambda(t) = \lambda_0\left(\eta(t)\right)\Phi_{\check{N}(t)+1}s(t)\Psi_{\tau(t)}, \tag{22}$$

where $\Psi_{\tau(t)}$ is a function varying only on a daily level and is given by

$$\Psi_{\tau(t)} = a\zeta(t) + b\Psi_{\tau(t)-1}. \tag{23}$$

The innovation term is specified as

$$\zeta(t) = 1 - \frac{\Lambda\left(t^{\dagger}_{\tau(t)-1}, t^{\ddagger}_{\tau(t)-1}\right)}{N\left(t^{\ddagger}_{\tau(t)-1}\right) - N\left(t^{\dagger}_{\tau(t)-1}\right)} \tag{24}$$

and exploits the basic results of the martingale theory of point processes. Following (3), it is shown that $(N(t) - N(s)) - \Lambda(t,s)$ is a mean zero martingale. Consequently, $\zeta(t)$ is a mean zero martingale as well.

Hence, both types of dynamics Φ_i and $\Psi_{\tau(t)}$ are driven by functions of the lagged integrated intensity. In order to distinctly separate both types of dynamics, we assume that Φ_i is exclusively only an intraday component. Consequently, the dynamics of Φ_i do not run over consecutive trading days and are re-initialized at the beginning of each day.

In this sense, we allow for long memory processes within individual trading days and short memory processes across trading days.[4]

2.3.2 Component-Specific Acceleration Effects

A further extension is to allow for component-specific acceleration effects by re-formulating (5) as

$$\eta(t) := x(t) \cdot \Phi^{\delta_\Phi}_{\check{N}(t)+1} s(t)^{\delta_s} \Psi^{\delta_\Psi}_{\tau(t)}, \tag{25}$$

where δ_Φ, δ_s and δ_Ψ are specific acceleration parameters affecting the individual components separately. Hence, this specification allows us to identify whether a possible acceleration/decelaration effect is mainly driven by the inter-day dynamics, seasonality effects or intra-day dynamics. If $\delta_\Phi = \delta_s = \delta_\Psi = \delta$, the model collapses to the basic specification (5).

2.3.3 Flexible Baseline Intensities

Recent literature on financial duration processes (see e.g. Hautsch, 2003 or Bauwens, Giot, Grammig, and Veredas, 2004) illustrate that the distributional properties of financial durations are not easily modelled even by highly flexible distributions. Particularly the distribution of the time between consecutive transactions reveals peculiarities which typically cannot be captured in a fully parametric framework. In the given intensity setting distributional properties of the underlying durations are captured by the specific form of the baseline intensity function. Since in this framework no pseudo-maximum likelihood arguments are available and a valid statistical inference of intensity processes depends on the correct parameterization of $\lambda(t)$, a flexible form of the baseline intensity $\lambda_0(\cdot)$ is particularly important. In order to allow for a higher flexibility we propose to specify $\lambda_0(\cdot)$ in a semiparametric way which encompasses the Burr hazard as a special case.

Define $\nu(t) := 1 - \exp(-x(t))$ as a transformation of the backward recurrence time $x(t)$ with the property $\nu(t) \in [0; 1]$. Then, we propose to augment a Burr parameterization by the exponential transformation of a flexible Fourier form as proposed by Gallant (1981).

[4] A straightforward extension would be also to allow for long memory dynamics across individual trading days by adopting the structure given in (8). However, our empirical results do not provide evidence for inter-day long range dependencies.

Hence, $\lambda_0(\eta(t))$ is given by

$$\lambda_0(\eta(t)) = \frac{p \cdot \eta(t)^{p-1}}{1 + \kappa\eta(t)^p} \cdot \exp\left[\sum_{m=1}^{M} p_{m,s}\sin(2 \cdot m\pi\nu(t)) + p_{m,c}\cos(2 \cdot m\pi\nu(t))\right], \tag{26}$$

where M denotes the order of the process and $p_{m,s}$ and $p_{m,c}$ are coefficients to be estimated. Depending on the choice of M this specification allows to capture any particularly form of the baseline intensity function nesting the case of a pure Burr parameterization for $p_{m,s} = p_{m,v} = 0$ for all $m = 1, \ldots, M$. In the empirical section (Section 5) we will illustrate that this flexible form is needed to capture the distributional properties of financial durations and induces a significant improvement of the goodness-of-fit.

2.3.4 Asymmetric News Response

A simple extension of the basic parameterization of $\tilde{\Phi}_i$ is to allow for an asymmetric news impact function. The recent literature on financial duration models[5] illustrates that the impact of past duration innovations on the conditional expected waiting time until the next event arrival is nonlinear. A simple way to allow for an asymmetric news impact while ensuring the zero mean property of the innovation term is to allow for a kinked news impact function in the spirit of Nelson (1991). Hence, (11) is modified to

$$\tilde{\Phi}_i = \omega + \alpha(1-\beta L)^{-1}(1-L)^{-d}\varepsilon_{i-1} + \varsigma(1-\beta L)^{-1}(1-L)^{-d}\bar{\varepsilon}_{i-1}, \tag{27}$$

where $\bar{\varepsilon}_i := |\varepsilon_i| - \mathrm{E}[|\varepsilon_i|]$ and ς denotes the asymmetry parameter.

3 The Multivariate GLMACI Model

Let $\{Z_i\}_{i\in\{1,\ldots,n\}}$ be a sequence of $\{1, 2, \ldots, K\}$-valued random variables representing K different types of events. If $\{t_i\}_{i\in\{1,\ldots,n\}}$ is a simple point process with $0 < t_1 < \ldots < t_n$, we call the process $\{t_i, Z_i\}$ an K-variate *marked* point process. Let $\{t_i^k\}_{i\in\{1,\ldots,n^k\}}$, $k = 1, \ldots K$, be K sequences of arrival times with corresponding counting processes $N^k(t) := \sum_{i\geq 1} 1\!\!1_{\{t_i^k \leq t\}}$, $k = 1, \ldots K$. Then,

$$\mathrm{E}[N^k(s) - N^k(t)|\mathcal{F}_t] = \mathrm{E}\left[\int_t^s \lambda^k(u)du|\mathcal{F}_t\right] \tag{28}$$

[5] See e.g. Fernandes and Grammig (2001), Dufour and Engle (2000) or Hautsch (2003).

defines the k-type intensity process with compensator $\tilde{\Lambda}^k(t) := \int_0^t \lambda^k(u)du$. In Theorem 1 in Brown and Nair (1988) it is proven that the point processes formed from the multivariate process of compensators $\left(\tilde{\Lambda}^1(t), \tilde{\Lambda}^2(t), \ldots, \tilde{\Lambda}^K(t)\right)$ are independent Poisson processes with unit intensity. As a result, the k-type integrated intensities $\Lambda^k(t_{i-1}^k, t_i^k) := \int_{t_{i-1}^k}^{t_i^k} \lambda^k(s)ds$ $\forall\, k = 1, \ldots, K$ are independently standard exponentially distributed. This property is an important building block of the (generalized) multivariate ACI specification.

Following the notation in Section 2, denote $\Phi^k_{\check{N}(t)}$, $\lambda_0^k(\cdot)$, $s^k(t)$ and $\Psi^k_{\tau(t)}$ as the corresponding k-type intensity components associated with intra-day dynamics, the baseline intensity, seasonality effects as well as inter-day dynamics, respectively. Then, the multivariate GLMACI model is given by

$$\lambda^k(t) = \lambda_0^k \left[\eta^1(t), \ldots, \eta^K(t)\right] \Phi^k_{\check{N}(t)+1} s^k(t) \Psi^k_{\tau(t)}, \tag{29}$$

where

$$\eta^k(t) := \begin{cases} \eta^k(t_{\check{N}(t)}) + x(t) \cdot \Xi(t) & \text{if } t_{\check{N}(t)} \text{ was of type } r \neq k \\ x(t) \cdot \Xi(t) & \text{if } t_{\check{N}(t)} \text{ was of type } k \end{cases} \tag{30}$$

and

$$\Xi(t) := \prod_{r=1}^{K} \left(\Phi^r_{\check{N}(t)+1}\right)^{\delta^k_{r,\Phi}} \left(s^r(t)\right)^{\delta^k_{r,s}} \left(\Psi^r_{\tau(t)}\right)^{\delta^k_{r,\Psi}}. \tag{31}$$

Furthermore, $\delta^k_{r,\Phi}$, $\delta^k_{r,s}$ and $\delta^k_{r,\Psi}$ denote parameters determining the impact of possible acceleration/deceleration effects of the individual r-type intensity components on the k-type process. According to eq. (29) and (30), the acceleration/deceleration effects act piecewise on the backward recurrence times and change at each point of the pooled process. I.e., the time scale might be scaled in different directions between the arrivals of two consecutive points of the same process.

According to the univariate specification we specify Φ_i^k as

$$\Phi_i^k = \exp\left(\tilde{\Phi}_i^k + z_{i-1}'\gamma^k\right), \tag{32}$$

where γ^k denotes the k-type parameter vector associated with explanatory variables.

Define ε_i as a scalar innovation term and y_i^j as an indicator variable taking on the value 1 if the i-th event is of type j. Then, $\tilde{\Phi}_i^k$ is assumed to follow the process

$$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K} \left\{\alpha_j^k (1 - \beta_j^k L)^{-1} (1 - L)^{-d^k} \varepsilon_{i-1}\right\} y_{i-1}^j, \tag{33}$$

where α_j^k, β_j^k and d^k for $k, j = 1, \ldots, K$ are autoregressive parameters depending on the type of the most recent point of the pooled process. In a multivariate framework there are two ways to specify the innovation term. As proposed by Russell (1999), ε_i can be specified in terms of the integrated intensity associated with the type of the most current point. Then, ε_i is given by

$$\varepsilon_i = \sum_{k=1}^{K} \left(1 - \Lambda^k(t_{i-1}^k, t_i^k)\right) y_i^k$$

and corresponds to a (random) mixture of the series of integrated intensity functions $\{\Lambda(t_{i-1}^k, t_i^k)\}_{\{i=1,\ldots,n^k\}}$ for $k = 1, \ldots, K$. Since the latter are mean zero i.i.d. variates, the resulting mixture ε_i is a mean zero i.i.d. innovation term as well. Alternatively, Bowsher (2002) suggested to specify the innovation in terms of the integrated intensity of the pooled process, i.e.

$$\tilde{\varepsilon}_i = 1 - \Lambda(t_{i-1}, t_i),$$

where $\Lambda(t_{i-1}, t_i) := \sum_{k=1}^{K} \Lambda^k(t_{i-1}, t_i)$. Following the arguments above, $\tilde{\varepsilon}_i$ is also a mean zero i.i.d. innovation term.

Therefore, each of the processes $\tilde{\Phi}_i^k$, $k = 1, \ldots, K$, correspond to a univariate long memory process with regime-switching dynamics in dependence of the type of the most recent point. The individual processes are linked together since they are jointly updated by ε_i at each point of the pooled process. Note that we do *not* allow for cross-dependences in the persistence terms, i.e. $\tilde{\Phi}_i^k$ is only updated by ε_i and own lags. Correspondingly, the process $\tilde{\Phi}_i^k$ can be re-formulated as

$$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K} \left\{ \alpha_j^k \sum_{s=1}^{\infty} \theta_{s-1,i}^k \varepsilon_{i-s} \right\} y_{i-1}^j,$$

where

$$\theta_{s,i}^k := \begin{cases} \theta_s^{k*} & \text{if } s = 1 \\ \theta_s^{k*} + \sum_{m=1}^{s} \theta_{s-m}^{k*} \prod_{r=0}^{m-1} \sum_{j=1}^{K} \beta_j^k y_{i-r}^j & \text{if } s > 1, \end{cases} \tag{34}$$

where θ_s^{k*} is given by (10). Hence, due to the regime switching behavior of the persistence parameters β_j^k depending on the type of the previous event, the power series expansion depends on i and thus varies over the time series. A sufficient condition for covariance stationarity of the process $\tilde{\Phi}_i^k$ is that $|\beta_j^k| < 1$, $\forall\, j, k$ and $0 < d^k < \frac{1}{2}$.

The inter-day dynamics $\Psi^k_{\tau(t)}$ are specified as a straightforward multivariate extension of (23). Define $\Psi_{\tau(t)} = \left(\Psi^1_{\tau(t)}, \ldots, \Psi^K_{\tau(t)}\right)'$ as the $(K \times 1)$ vector of inter-day components. Then, we parameterize $\Psi_{\tau(t)}$ as

$$\Psi_{\tau(t)} = A\zeta(t) + B\Psi_{\tau(t)-1},$$

where A and B are $(K \times K)$ matrices of innovation and persistence parameters, respectively, and $\zeta(t) = \left(\zeta^1(t), \ldots, \zeta^K(t)\right)'$ is a $(K \times 1)$ vector of innovation terms with elements

$$\zeta^k(t) = 1 - \frac{\Lambda^k\left(t^\dagger_{\tau(t)-1}, t^\ddagger_{\tau(t)-1}\right)}{N^k\left(t^\ddagger_{\tau(t)-1}\right) - N^k\left(t^\dagger_{\tau(t)-1}\right)}, \quad k = 1, \ldots, K.$$

Finally, the k-type baseline intensity function $\lambda^k_0\left[\eta^1(t), \ldots, \eta^K(t)\right]$ is specified as the product of single hazard functions. Define $\nu^k(t) := 1 - \exp(-x^k(t))$, then a multivariate extension of (26) is given by

$$\lambda^k_0\left[\eta^1(t), \ldots, \eta^K(t)\right] = \prod_{r=1}^{K} \frac{p^k_r \eta^r(t)^{p^k_r - 1}}{1 + \kappa^k_r \eta^r(t)^{p^k_r}} \exp\left[\sum_{m=1}^{M} p^k_{m,r,s} \sin(2 \cdot m\pi\nu^r(t))\right] \tag{35}$$

$$\times \exp\left[p^k_{m,r,c} \cos(2 \cdot m\pi\nu^r(t))\right], \quad p^k_r > 0,\ \kappa^k_r \geq 0, \tag{36}$$

where p^k_r, κ^k_r, $p^k_{m,r,s}$ and $p^k_{m,r,c}$ are distributional parameters reflecting the impact of the r-type backward recurrence time on the k-type baseline intensity.

Hence, the multivariate GLMACI model allows to test for four channels through which the individual point processes are linked together: First, there might be interdependences in short-run dynamics as determined by the autoregressive parameters α^k_j, $j \neq k$. Second, we allow for spill-overs in inter-day dynamics as characterized by the off-diagonal elements in the matrices A and B. Third, there might be cross-relations in accelerations/decelarations of the time scale as captured by the parameters $\delta^k_{r,\Phi}$, $\delta^k_{r,s}$ and $\delta^k_{r,\Psi}$ for $r \neq k$. Finally, we allow for cross-interdepences between the process-specific baseline intensity functions and backward recurrence times. These effects are determined by the parameters p^k_r, κ^k_r, $p^k_{m,r,s}$, and $p^k_{m,r,c}$ for $r \neq k$.

4 Statistical Inference

As shown by Karr (1991), valid statistical inference can be performed based on the intensity function solely, where the log likelihood function of a K-variate marked point process

$\{t_i, Z_i\}_{i\in\{1,\ldots,n\}}$ with covariate series $\{z_i\}_{i\in\{1,\ldots,n\}}$ is given by

$$\ln \mathcal{L}(\{t_i, Z_i, z_i\}_{i\in\{1,\ldots,n\}}) = \sum_{i=1}^{n}\sum_{k=1}^{K}(-\Lambda^k(t_{i-1}, t_i)) + \ln\left[\lambda^k(t_i)\right] y_i^k. \tag{37}$$

Hence, the model is straightforwardly estimated by maximum likelihood. Exploiting the martingale property of compensators, we can conduct various diagnostic tests to assess the goodness-of-fit of the model. In particular, we perform diagnostics based on three different types of model residuals. First, we evaluate the stochastic properties of the estimated integrated intensities

$$e_{i,1}^k := \hat{\Lambda}^k(t_{i-1}^k, t_i^k),$$

which must be i.i.d. standard exponentially distributed under correct model specification. Second, we analyze the properties of the ACI residuals

$$e_{i,2} := \sum_{k=1}^{K}\left(1 - \hat{\Lambda}^k(t_{i-1}^k, t_i^k)\right) y_i^k,$$

which correspond to mixtures of i.i.d. standard exponentially distributed variates and thus must be i.i.d. standard exponentially distributed themselves. Furthermore, we evaluate the properties of the estimated integrated intensity based on the *pooled* process. Hence, a third type of model residuals is obtained by

$$e_{i,3} = 1 - \hat{\Lambda}(t_{i-1}, t_i) = 1 - \sum_{k=1}^{K}\hat{\Lambda}^k(t_{i-1}, t_i).$$

Note that all three types of model residuals coincide in the univariate case ($K = 1$).

Using the residual series, model evaluation can be done by testing the dynamical and distributional properties. The dynamical properties are easily evaluated based on Portmanteau statistics or tests against independence such as proposed by Brock, Scheinkman, Scheinkman, and LeBaron (1996). The distributional properties can be evaluated based on a a test against excess dispersion. Engle and Russell (1998) propose the asymptotically normal test statistic $\sqrt{n_e/8}\,\hat{\sigma}_e^2$, where n_e denotes the number of residuals and $\hat{\sigma}_e^2$ denotes the empirical variance of the corresponding residual series. Alternative checks of the distributional properties of the residuals can be done by the computation of the probability integral transform (PIT) based on the exponential distribution

$$u_i^k := \int_{-\infty}^{e_i^k} \exp(-s)ds = 1 - \exp(-e_i^k). \tag{38}$$

Diebold, Gunther, and Tay (1998) show that under correct model specification, the distribution of the u_i^k series is be i.i.d. uniformly distributed.

5 Empirical Application

The GLMACI model is applied to trade durations and price durations based on the IBM stock traded at the New York Stock Exchange (NYSE). The sample period covers the two months January and February 2001. The trade durations are defined as the time between consecutive transactions. The price durations are defined as the time between absolute cumulative midquote changes of a certain size. The size of the absolute cumulative midquote changes is chosen as five times of the average size of absolute trade-to-trade midquote changes. This results in average price durations of approximately 45 seconds. Overnight spells as well as all trades before 9:45 and after 16:00 are removed.[6]

Table 1 shows the descriptive statistics of the individual trade duration series. It turns out that the IBM stock is heavily traded with on average 7 seconds between two consecutive trades. Furthermore, we observe the well known very persistent serial dependence in both duration series.

Table 2 shows the estimation results of univariate GLMACI models for trade durations. Because of the high liquidity of the IBM stock resulting in 58,332 observations for January 2001 solely, we restrict the analysis for the case of trade durations to the January sample. The function $s(t)$ is assumed to capture intraday seasonalities and is specified in terms of a linear spline function

$$s(t) = 1 + \sum_{j=1}^{S} \nu_j (t - \tau_j) \cdot \mathbb{1}_{\{t > \tau_j\}}, \tag{39}$$

where τ_j, $j = 1 \ldots, S$, denote the S nodes within a trading day and ν_j the corresponding parameters. We use six nodes ($S = 6$) dividing the trading hours from 9:45 to 16:00 into equal-sized time intervals. Panels A shows the results of a basic ACI(1,1) model with asymmetric news impact function. The autoregressive parameters indicate a high persistence and relatively low innovation parameters. Nevertheless, the parameter α is highly significant and positive. Furthermore, no convincing evidence for an asymmetric news

[6]Note that trading starts at 9:30. However, the first 15 minutes are removed in order to avoid particular opening effects.

impact is found. As shown by Figure 1[7], the estimated baseline intensity function reveal a non-monotonic shape of the baseline intensity which is decreasing in the long run. The residual diagnostics reveal that the model is not able to capture the dynamic properties of the data. The Ljung-Box statistics show evidence for clear serial dependence in the residual series. Moreover, the tests against excess dispersion indicate evidence against exponentiality and thus distributional misspecification. The quantile-quantile (QQ) plots shown in Figure 1 illustrate that distributional misspecifications are particularly driven by observations revealing high intensities and thus small durations. This is a well known result in the literature of financial duration models as e.g. illustrated by Bauwens, Giot, Grammig, and Veredas (2004). In Panel B the model is extended by inter-day dynamics. However, as indicated by the corresponding p-values and model diagnostics these effects are insignificant and lead to no improvements of the goodness-of-fit and the dynamic properties of the model. Hence, inter-day dynamics are obviously of minor importance for trade durations. A further reason might be that the number of underlying trading days is not sufficient to clearly identify these effects. For this reason, we skipped these effects in the other specifications. Pancel C shows the estimates of a specification which accounts for long memory. The estimates reveal clear evidence for the existence of long range dependence. Nevertheless, the estimate $\hat{d} \approx 0.6$ indicates that the coefficients of the power series expansion are not square-summable and thus the intensity process is not necessarily covariance stationary. Nevertheless, the model dynamics are clearly improved. In particular, the Ljung-Box statistics show that the null hypothesis of no serial dependence in the residuals cannot be rejected. The BIC value indicates that the inclusion of long range dependence clearly improves the model's goodness-of-fit. Panel D shows the estimates of GLMACI models with a semiparametric specification of the baseline intensity. It turns out that the flexible parameterization of the baseline intensity significantly improves the explanatory power of the model. The plotted estimated baseline intensity (Figure 2) reveals a clear non-monotonous shape for small durations. This pattern is probably caused by the fact that the smallest measurable unit of a duration at the NYSE is one second which leads to a clear discreteness in the left tail of the distribution. Obviously, this discreteness is also reflected by the distribution of the resulting integrated intensities and

[7]The plots shown in Figure 1 are based on specification C in Table 2. The corresponding plots for specifications A and B look very similar and are not shown in the paper.

cannot be sufficiently captured by a Burr distribution. In contrast, the semiparametric specification implies a significant improvement of the goodness-of-fit as revealed by the QQ-plot in Figure 2. Moreover, no evidence for excess dispersion is found anymore.

Panel E displays the results of a specification which accounts for acceleration effects. We allow for component specific acceleration effects associated with the intraday dynamics and seasonalities. The highly significant parameters δ_Φ and δ_s indicate that the intraday dynamics and seasonalities imply clear acceleration and respectively deceleration effects. In particular, in periods of a high intensity given the time of the day the time scale is scaled upwards corresponding to a leftwards shift of the baseline intensity. Since the baseline intensity has a non-monotonous shape this means that the baseline intensity increases for small durations and decreases for long durations. Interestingly, we observe a counter-balancing effect induced by the seasonality component. I.e., in periods of seasonality driven high intensity the time scale is scaled downwards. However, we can conclude that a proportional intensity specification is rejected in favor of an accelerated failure time specification leading to an improved goodness-of-fit as indicated by the BIC. The plot of the estimated baseline intensity function (Figure 3) reveals a similar pattern as shown in Figure 2. Finally, Figure 7 plots the estimated intraday seasonality function based on the estimates in Panel E which are quite representative for all other specifications. It shows the well-known U-shaped intraday pattern.

Table 3 shows the corresponding estimates based on price durations. Because of the lower frequency of price durations, we use the full sample January-February 2001. In contrast to the trade durations, we find significant evidence for a positive serial dependence in the inter-day component (Panel B). Nevertheless, even though the specification leads to a better goodness-of-fit of the model it is not sufficient to fully capture the dynamic properties of the process. Again, the inclusion of a long memory component is necessary to achieve a significant improvement of the model's dynamical properties as indicated by the residual diagnostics and the BIC. However, again we find a value of d ranging between 0.51 and 0.55 indicating the presence of long range dependence, however not necessarily covariance stationarity. The estimates of ς reflect the presence of asymmetric news impact effects. It turns out that positive price intensity shocks have a stronger effect on the future intensity than negative intensity shocks. Hence, in periods where the price intensity, i.e. the instantaneous volatility, is higher than expected, we observe a stronger

impact on the expected intensity. The QQ-plot based on specification C (Figure 4) as well as the test on excess dispersion reflect that the Burr parameterization of the baseline intensity captures the distributional properties of the data quite well. Nevertheless, we find an increased goodness-of-fit based on the semiparametric specifications in Panels D and E. Similarly to the processes of trade durations, evidence for discreteness in the left tail of the distribution is found (Figures 5 and 6) which is captured by the semiparametric specification. Again, clear evidence for acceleration effects (Panel E) are found. Actually, the acceleration effect in periods of a high intensity given the time of the day is clearly stronger as for trade durations. In contrast to the specifications for the trading intensity, we also find significant acceleration effects for the seasonality component. For the inter-day component only insignificant effects are found. Nevertheless, even though specification E implies an increased explanatory power as indicated by the BIC, the QQ-plot reveals a worse goodness-of-fit in terms of the distribution of the estimated integrated intensities. Hence, slight evidence for over-fitting is provided. Overall, the estimated seasonality component (Figure 7) is very similar to that estimated based on trade duration.

Table 4 shows the estimation results of bivariate GLMACI specifications for the trading intensity and the price change intensity based on the sample period January 2001. In order to ensure model parsimonity we do not include asymmetric news impact effects. Furthermore, the backward recurrence function is restricted to a diagonal specification of the Burr parameters κ and the parameters of the semiparametric extension. However, the estimates of the Weibull parameters indicate a strong interdependence in the backward recurrence functions. Hence, the baseline intensity function of one intensity component is driven not only by the own backward recurrence time but also by that of the other component. Panel A displays a basic bivariate ACI(1,1) specification. We find persistence parameters which are very close to unity[8] indicating that the process is highly persistent and nearly integrated. Interestingly, we do not find significant evidence for cross-dependences in the innovations of the intraday component. Hence, intraday shocks in the trading activity or the volatility do not lead to spill-overs in the corresponding other component. This finding is also confirmed by all other specifications. The residual diagnostics indicate that this specification does not sufficiently capture the distributional and dynamical properties of the underlying duration series. In Panel B, the model dynamics

[8] Actually, one of the regime-switching parameters even slighly exceeds one

are extended by long memory effects as well as interday dynamics. Confirming the results based on the univariate specifications, we find clear evidence for long range dependence with parameters d exceeding the value 0.5. Nevertheless, it turns out that the goodness-of-fit is significantly improved. The Ljung-Box statistics of the integrated intensities and GLMACI residuals indicate no evidence for remaining serial dependence. Furthermore, we observe no significant inter-day dependences. As indicated by the significant parameter a_{22} only for the process of price intensities slight evidence for daily dependencies are found. Figure 8 shows the QQ-plots of the GLMACI residuals $e_{i,1}$ as well as of the pooled residuals $e_{i,3}$. We clearly observe distributional misspecifications particularly for high intensities and thus small durations. Panel C shows the results of an GLMACI specification with a semiparametric specification of the baseline intensity function. In order to ensure model parsimonity, we choose an order of the polynomial $M = 2$. As in the univariate specifications we find an increase of the model's explanatory power and goodness-of-fit which is also revealed by the QQ-plots of the residuals $e_{i,3}$ (Figure 9). Nevertheless, the diagnostics and QQ-plots still indicate deviations from the exponentiality for high realizations of the integrated intensity of the pooled process. This indicates that the specification is not able to fully capture the properties of the joint intensity process and requires a higher order M (as it has been used for the univariate specifications). Panel D shows the results of a specification which allows for acceleration/deceleration effects where we assume that $\delta^j_{j,\Phi} = \delta^j_{j,s} = \delta^j_{j,\Psi}\ \forall\ j = 1,2$ and $\delta^j_{k,\Phi} = \delta^j_{k,s} = \delta^j_{k,\Psi}\ \forall\ k,j = 1,2$. For trade durations, we observe that the deceleration effect dominates, whereas for price durations we observe acceleration effects. Furthermore, no significant acceleration effects of the time scale for trade durations induced by price intensities is observed. In contrast, the time scale of price durations is significantly scaled downwards in periods of a high trading intensity. As revealed by the BIC, the inclusion of acceleration/deceleration effects increases the model's goodness-of-fit. As shown in Panel E we find significant evidence for significant component-specific acceleration/deceleration effects. Confirming our results above the time scale underlying trade durations is significantly scaled downwards when the intraday seasonality component is high. However, intraday and interday dynamics do not cause significant acceleration effects. In contrast, the time scale underlying price durations is most strongly accelerated by intraday and interday dynamics whereas seasonality effects do not have significant impacts. Furthermore, we observe decelerating spill-over effects be-

tween both baseline intensity components which are driven by interday dynamics. Hence, the time scales of trade (price) durations are scaled downwards whenever the intraday dynamics of price (trade) intensities are high. In contrast, no spill-over effects implied by the seasonality components are observed. Surprisingly, high inter-day components of price durations significantly accelerate the time scale of trade durations. Hence, this effect acts in opposite direction as the corresponding effect implied by intraday dynamics. However, since the interday dynamics are only weakly identifiable, this result should be treated with caution. Nevertheless, the corresponding coefficient is highly significant. As indicated by the BIC, we find that this specification leads to a further improvement of the goodness-of-fit of the model. Hence, this flexibilization is supported by the data which indicates the importance of including component-specific acceleration/deceleration effects.

For all specifications, the model diagnostics indicate that the model is able to capture the dynamical properties of the data. Nevertheless, as revealed by the QQ-plots of the estimated integrated intensities (Figures 10 and 11) we still find evidence for distributional misspecification. This is not necessarily reflected in excess dispersion, however, particularly by the tails of the distribution of the integrated intensities. These results indicate that also in a multivariate specification, a semiparametric specification of the baseline intensity is indispensable in order to achieve a satisfying goodness-of-fit.

6 Conclusions

In this paper, we proposed a generalization of the Autoregressive Conditional Intensity (ACI) model introduced by Russell (1999). The so-called Generalized Long Memory ACI (GLMACI) model parameterizes the multivariate intensity in terms of a multiplicative function of a multivariate baseline intensity component, two dynamic VAR-type components capturing intraday and interday dynamics as well as a component capturing deterministic seasonality effects. The baseline intensity component is specified as a semiparametric function of the backward recurrence times in the individual components and nests a Burr hazard rate as special case. The backward recurrence times are measured based on a calendar time scale which might be scaled upwards or downwards in dependence of the most recent realizations of the intraday and interday dynamics as well as intraday seasonalities. In this sense, the model nests both a proportional intensity com-

ponent (like the original ACI model) as well as an accelerated failure time model in the spirit of an Autoregressive Conditional Duration (ACD) model proposed by Engle and Russell (1998). Moreover, we allow for long range dependence in the intraday dynamics by adopting the approach by Koulikov (2003) and parameterizing the latter in terms of a fractionally integrated process in terms of past realizations of the integrated intensity. Finally, the interday dynamics are parameterized as a (short memory) VAR type process which is updated by martingale innovations based on the integrated intensity as well as the number of observations computed over the last trading days.

Applications of univariate and bivariate versions of the GLMACI model to trade and price durations based on the IBM stock traded at the New York Stock Exchange show that the proposed extensions are supported by the data and lead to an improved specification. The most important findings can be summarized as following: First, clear evidence for long range dependence in both trade intensities as well as price intensities is found. However, our estimates reveal evidence that the coefficients of the underlying infinite series representation are not necessarily square-summable implying that the intensity processes are not covariance stationarity. Second, we find significant acceleration/deceleration effects. I.e., the underlying time scales on which the baseline intensities are measured are scaled upwards or downwards in dependence of the magnitudes of the individual intensity components. In the bivariate specification, we even find evidence for cross-dependences in the acceleration/deceleration effects. A relatively robust finding is that the time scale of trade durations is primarily negatively affected by intraday seasonalities. In contrast, the time on which price durations are measured is accelerated by intraday and interday dynamics. Third, it turns out that a Burr parameterization of the baseline intensity function is not sufficient to fully capture the distributional properties of the underlying durations. This is particularly true for the distribution of trade durations which reveals peculiarities on the left tail which are not easily captured even by very flexible parameterizations. However, the semiparametric extension proposed in this paper leads to clearly improved goodness-of-fit.

Overall, our results show a clear outperformance of the GLMACI model compared to a basic ACI model. Nevertheless, a few issues still remain and require further research. One particularly important issue is to further study the stationarity properties of intensity processes. Actually, our findings provide evidence that financial intensity processes

are driven by non-trivial dynamics including both stationary as well as non-stationary components accompanied by long memory effects.

Appendix A: Tables

Table 1: Descriptive statistics of trade durations and price durations of the IBM stock traded at the NYSE. Extracted from the 2001 TAQ data base. Sample period 02/01/01 to 28/02/01. The price durations are computed as the time between cumulative absolute midquote changes of the size \$0.059. The following descriptive statistics are shown: Number of observations, mean, standard deviation, minimum, maximum, 1%-, 5%-, 10%-, 50%-, 90%-, 95%-, as well as 99%-quantile, the first five autocorrelations and the Ljung-Box statistic associated with 20 lags.

	trade durations	price durations
Obs	114,910	20468
Mean	7.829	43.823
S.D.	7.442	59.242
Min	1.000	1.000
Max	205.000	1224.000
q01	1.000	1.000
q05	2.000	3.000
q10	2.000	5.000
q50	6.000	25.000
q90	16.000	100.000
q95	22.000	145.000
q99	37.000	289.000
ρ_1	0.089	0.207
ρ_2	0.101	0.172
ρ_3	0.098	0.152
ρ_4	0.093	0.175
ρ_5	0.090	0.143
LB(20)	15041	1132.700

Table 2: Maximum likelihood estimates of univariate GLMACI models. Based on trade durations of the IBM stock traded at the New York Stock Exchange. Sample period: 02/01/01 until 31/01/01, 58,332 observations. The spline function is based on 6 equally spaced nodes between 9:45 a.m. and 4:00 p.m. Standard errors are based on the outer product of gradients.

	A		B		C		D		E	
	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.
Baseline intensity parameters										
ω	1.433	0.000	1.462	0.000	1.213	0.000	4.358	0.000	4.916	0.000
p	2.076	0.000	2.076	0.000	2.074	0.000	3.900	0.000	4.176	0.000
κ	1.432	0.000	1.431	0.000	1.416	0.000	50.130	0.270	61.598	0.289
$p_{1,s}$							-0.180	0.287	-0.160	0.373
$p_{2,s}$							0.136	0.000	0.182	0.000
$p_{3,s}$							0.091	0.000	0.122	0.000
$p_{1,c}$							0.657	0.000	0.718	0.000
$p_{2,c}$							0.228	0.001	0.239	0.002
$p_{3,c}$							0.066	0.002	0.051	0.028
Acceleration parameters										
δ_Φ									0.082	0.031
δ_s									-0.372	0.000
Dynamic parameters										
α	0.022	0.000	0.023	0.000	2.979	0.000	0.030	0.000	0.030	0.000
ς	-0.003	0.074	-0.002	0.132	-0.264	0.269	-0.001	0.524	0.001	0.551
β	0.992	0.000	0.991	0.000	0.580	0.000	0.572	0.000	0.596	0.000
a			0.011	0.807						
b			0.583	0.854						
d					0.602	0.000	0.604	0.000	0.599	0.000
Seasonality parameters										
ν_1	0.241	0.075	-0.111	0.524	-0.197	0.172	-0.279	0.039	-0.524	0.000
ν_2	-0.203	0.731	-0.357	0.256	-0.276	0.256	-0.127	0.575	0.075	0.684
ν_3	-0.254	0.127	-0.052	0.849	-0.008	0.964	-0.152	0.406	0.160	0.300
ν_4	-0.011	0.968	1.034	0.000	0.990	0.000	1.160	0.000	0.661	0.000
ν_5	-0.387	0.144	-0.172	0.531	-0.140	0.481	-0.316	0.086	0.026	0.862
ν_6	1.180	0.000	0.519	0.122	0.482	0.053	0.644	0.006	0.427	0.034
LL	-50085		-50084		-50031		-49179		-49142	
BIC	-50151		-50161		-50114		-49295			
Diagnostics of GLMACI residuals										
Mean of e_i	0.999		0.999		0.999		1.000		0.996	
S.D. of e_i	0.966		0.966		0.967		0.999		0.998	
LB(20) of e_i	60.030	0.000	57.142	0.000	16.067	0.712	15.029	0.774	16.495	0.685
Exc. disp.	5.600	0.000	5.593	0.000	5.514	0.000	0.150	0.880	0.348	0.727

Table 3: Maximum likelihood estimates of univariate GLMACI models. Based on price durations of the IBM stock traded at the New York Stock Exchange. The price durations are computed as the time between cumulative absolute midquote changes of the size \$0.059. Sample period: 02/01/01 until 28/02/01, 20,468 observations. The spline function is based on 6 equally spaced nodes between 9:45 a.m. and 4:00 p.m. Standard errors are based on the outer product of gradients.

	A		B		C		D		E	
	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.
Baseline intensity parameters										
ω	0.769	0.000	0.879	0.000	0.775	0.000	1.064	0.000	1.762	0.000
p	1.218	0.000	1.218	0.000	1.218	0.000	1.420	0.000	1.689	0.000
κ	0.420	0.000	0.410	0.000	0.399	0.000	0.716	0.000	2.611	0.010
$p_{1,s}$							0.114	0.000	-0.078	0.328
$p_{2,s}$							0.030	0.034	-0.015	0.598
$p_{3,s}$							0.015	0.183	-0.009	0.727
$p_{1,c}$							0.151	0.000	0.287	0.000
$p_{2,c}$							0.076	0.000	0.129	0.000
$p_{3,c}$							0.042	0.002	0.106	0.000
Acceleration parameters										
δ_Φ									0.845	0.000
δ_s									0.330	0.002
δ_Ψ									1.125	0.000
Dynamic parameters										
α	0.088	0.000	0.100	0.000	0.142	0.000	0.142	0.000	0.157	0.000
ς	0.032	0.000	0.038	0.000	0.062	0.000	0.061	0.000	0.068	0.000
β	0.969	0.000	0.943	0.000	0.227	0.000	0.208	0.002	0.166	0.009
a			0.155	0.000	0.086	0.000	0.086	0.000	0.069	0.000
b			0.639	0.000	0.826	0.000	0.830	0.000	0.925	0.000
d					0.513	0.000	0.523	0.000	0.550	0.000
Seasonality parameters										
ν_1	-0.279	0.155	-0.370	0.029	-0.513	0.000	-0.481	0.000	-0.392	0.014
ν_2	-0.253	0.479	-0.160	0.603	0.031	0.897	-0.006	0.978	-0.144	0.602
ν_3	0.180	0.538	0.234	0.359	0.168	0.392	0.163	0.409	0.178	0.405
ν_4	0.738	0.005	0.630	0.007	0.641	0.000	0.666	0.000	0.738	0.000
ν_5	-0.033	0.908	-0.011	0.965	0.034	0.859	0.032	0.871	0.054	0.803
ν_6	-0.280	0.441	-0.218	0.482	-0.262	0.287	-0.256	0.310	-0.303	0.278
LL	-18398		-18351		-18280		-18220		-18140	
BIC	-18457		-18421		-18354		-18324		-18259	
Diagnostics of GLMACI residuals										
Mean of e_i	0.999		1.000		0.999		1.000		1.002	
S.D. of e_i	1.010		1.009		1.013		1.012		0.995	
LB(20) of e_i	95.548	0.000	54.256	0.000	23.737	0.254	24.756	0.210	24.850	0.207
Exc. disp.	1.068	0.285	0.992	0.320	1.333	0.182	1.318	0.187	0.436	0.662

Table 4: Maximum likelihood estimates of bivariate GLMACI models. Based on trade durations ($k = 1$) and price durations ($k = 2$) of the IBM stock traded at the New York Stock Exchange. The price durations are computed as the time between cumulative absolute midquote changes of the size \$0.059. Sample period: 02/01/01 until 31/01/01, $79,818$ observations. The spline functions are based on 6 equally spaced nodes between 9:45 a.m. and 4:00 p.m. Standard errors are based on the outer product of gradients.

	A		B		C		D		E	
	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.
Baseline intensity parameters										
ω^1	-1.617	0.000	-1.598	0.000	-1.606	0.000	-1.819	0.000	-1.351	0.000
ω^2	1.112	0.000	1.074	0.000	1.207	0.000	1.688	0.000	1.757	0.000
p_1^1	2.059	0.000	2.057	0.000	2.549	0.000	2.890	0.000	2.826	0.000
p_2^1	0.802	0.000	0.795	0.000	0.795	0.000	0.798	0.000	0.801	0.000
p_1^2	0.958	0.000	0.956	0.000	0.953	0.000	0.952	0.000	0.951	0.000
p_2^2	1.222	0.000	1.218	0.000	1.308	0.000	1.376	0.000	1.358	0.000
κ^1	0.904	0.000	0.893	0.000	0.676	0.004	1.629	0.042	1.652	0.106
κ^2	0.059	0.000	0.046	0.000	0.043	0.000	0.041	0.000	0.057	0.000
$p_{1,s}^1$					0.562	0.000	0.495	0.000	0.510	0.000
$p_{2,s}^1$					0.087	0.000	0.097	0.000	0.095	0.000
$p_{1,s}^2$					0.108	0.001	0.188	0.000	0.146	0.000
$p_{2,s}^2$					0.032	0.112	-0.043	0.476	0.038	0.293
$p_{1,c}^1$					0.048	0.637	0.294	0.015	0.304	0.040
$p_{2,c}^1$					-0.028	0.060	0.007	0.623	-0.007	0.706
$p_{1,c}^2$					0.004	0.850	-0.060	0.138	-0.019	0.433
$p_{2,c}^2$					-0.171	0.000	-0.175	0.000	-0.167	0.000
Acceleration parameters										
$\delta_{1,\Phi}^1$							-0.087	0.011	0.043	0.363
$\delta_{2,\Phi}^2$							1.065	0.000	0.846	0.000
$\delta_{1,s}^1$							-0.087		-0.711	0.005
$\delta_{2,s}^2$							1.065		0.305	0.622
$\delta_{1,\Psi}^1$							-0.087		-0.195	0.536
$\delta_{2,\Psi}^2$							1.065		0.483	0.005
$\delta_{2,\Phi}^1$							-0.022	0.399	-0.106	0.001
$\delta_{1,\Phi}^2$							-0.542	0.000	-0.515	0.000
$\delta_{2,s}^1$							-0.022		0.871	0.056
$\delta_{1,s}^2$							-0.542		-0.100	0.765
$\delta_{2,\Psi}^1$							-0.022		0.477	0.000
$\delta_{1,\Psi}^2$							-0.542		0.171	0.674
Short-term dynamics										
α_1^1	0.023	0.000	0.028	0.000	0.028	0.000	0.031	0.000	0.027	0.000
α_1^2	0.003	0.175	0.007	0.095	0.008	0.061	-0.001	0.755	0.002	0.517
α_2^1	-0.003	0.390	-0.001	0.690	-0.000	0.786	0.005	0.058	0.004	0.142
α_2^2	0.095	0.000	0.175	0.000	0.173	0.000	0.151	0.000	0.177	0.000
β_1^1	0.995	0.000	0.694	0.000	0.685	0.000	0.640	0.000	0.701	0.000
β_1^2	1.007	0.000	0.365	0.000	0.365	0.001	0.411	0.000	0.270	0.013
β_2^1	0.979	0.000	0.493	0.000	0.492	0.000	0.374	0.000	0.523	0.000
β_2^2	0.944	0.000	0.383	0.025	0.387	0.025	0.340	0.032	0.363	0.023

Table 4 continued:

	A		B		C		D		E	
	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.
Long-term dynamics										
d_1			0.574	0.000	0.578	0.000	0.585	0.000	0.558	0.000
d_2			0.543	0.000	0.546	0.000	0.582	0.000	0.589	0.000
a_{11}			0.024	0.679	0.041	0.490	0.047	0.446	0.077	0.088
a_{12}			0.029	0.355	0.027	0.396	0.026	0.377	0.028	0.171
a_{21}			-0.022	0.816	-0.015	0.873	0.018	0.847	-0.240	0.001
a_{22}			0.113	0.010	0.114	0.010	0.126	0.012	0.121	0.000
b_{11}			0.139	0.907	0.142	0.876	0.838	0.433	0.675	0.000
b_{12}			0.160	0.638	0.178	0.532	0.077	0.859	-0.032	0.638
b_{21}			0.545	0.793	0.548	0.733	0.393	0.842	0.295	0.251
b_{22}			0.813	0.126	0.815	0.066	0.896	0.265	0.879	0.000
Seasonality parameters										
ν_1^1	-0.143	0.589	-0.214	0.090	-0.216	0.085	-0.147	0.241	-0.160	0.184
ν_2^1	-0.241	0.546	-0.250	0.251	-0.265	0.216	-0.399	0.063	-0.434	0.036
ν_3^1	-0.148	0.575	0.043	0.812	0.041	0.815	0.118	0.494	0.191	0.252
ν_4^1	1.091	0.000	0.877	0.000	0.900	0.000	0.884	0.000	0.883	0.000
ν_5^1	-0.239	0.375	-0.120	0.513	-0.107	0.537	-0.051	0.766	0.008	0.958
ν_6^1	0.483	0.141	0.455	0.043	0.456	0.039	0.367	0.092	0.279	0.185
ν_1^2	-0.603	0.104	-0.479	0.013	-0.481	0.013	-0.331	0.120	-0.227	0.098
ν_2^2	0.154	0.796	0.122	0.711	0.123	0.710	0.080	0.826	0.074	0.758
ν_3^2	0.128	0.760	0.072	0.790	0.075	0.780	-0.173	0.585	-0.403	0.105
ν_4^2	0.635	0.088	0.751	0.004	0.755	0.004	1.137	0.000	1.131	0.000
ν_5^2	-0.431	0.275	-0.339	0.233	-0.338	0.236	-0.601	0.090	-0.502	0.048
ν_6^2	0.003	0.993	-0.006	0.984	-0.006	0.984	0.022	0.955	0.147	0.494
LL	-93906		-93799		-93012		-92967		-92903	
BIC	-94085		-94011		-93268		-93246		-93227	
Diagnostics of integrated intensities, trade durations										
Mean of $e_{i,1}^1$	1.000		0.999		0.999		1.002		0.998	
S.D. of $e_{i,1}^1$	0.968		0.967		0.994		0.997		0.997	
LB(20) of $e_{i,1}^1$	53.137	0.000	17.248	0.636	16.343	0.695	18.626	0.546	17.939	0.591
Exc. disp.	5.358	0.000	5.405	0.000	16.120	0.709	16.552	0.681	0.372	0.709
Diagnostics of integrated intensities, price durations										
Mean of $e_{i,1}^2$	1.000		1.003		1.002		1.018		1.020	
S.D. of $e_{i,1}^2$	1.000		0.998		0.996		0.996		1.001	
LB(20) of $e_{i,1}^2$	29.848	0.072	16.304	0.697	16.120	0.709	16.552	0.681	16.184	0.705
Exc. disp.	0.017	0.986	0.127	0.898	0.292	0.769	0.292	0.769	0.120	0.904
Diagnostics of GLMACI residuals										
Mean of $e_{i,2}$	1.000		1.000		0.999		1.005		1.002	
S.D. of $e_{i,2}$	0.973		0.973		0.994		0.997		0.998	
LB(20) of $e_{i,2}$	82.737	0.000	21.387	0.374	20.045	0.455	16.853	0.662	21.863	0.348
Exc. disp.	4.857	0.000	4.960	0.000	1.039	0.298	0.516	0.605	0.282	0.777
Diagnostics of GLMACI residuals based on pooled process										
Mean of $e_{i,3}$	1.000		1.000		1.000		1.005		1.003	
S.D. of $e_{i,3}$	1.015		1.015		1.015		1.038		1.038	
LB(20) of $e_{i,3}$	79.154	0.000	69.362	0.000	79.154	0.000	64.011	0.000	57.039	0.000
Exc. disp.	2.917	0.003	2.883	0.003	2.917	0.003	7.355	0.000	7.357	0.000

Appendix B: Figures

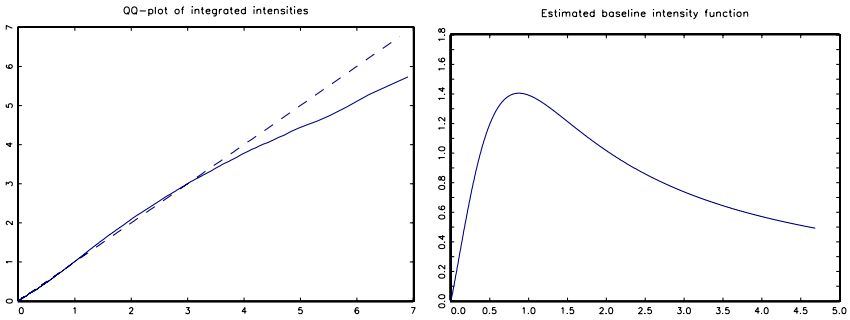

Figure 1: Quantile-quantile plot of the estimated integrated intensities (left) and estimated baseline intensity function (right) based on specification C in Table 2.

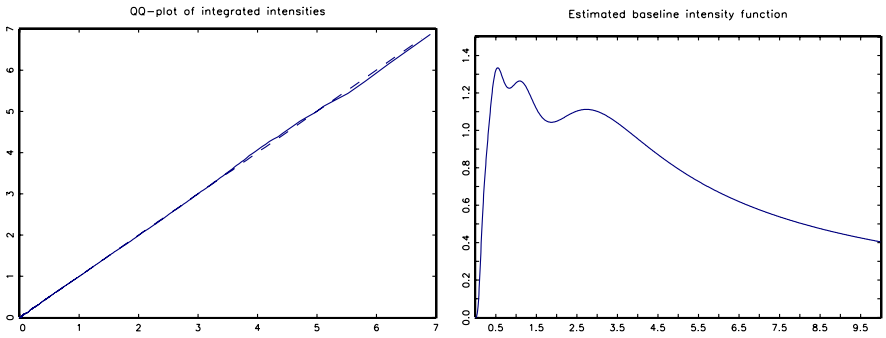

Figure 2: Quantile-quantile plot of the estimated integrated intensities (left) and estimated baseline intensity function (right) based on specification D in Table 2.

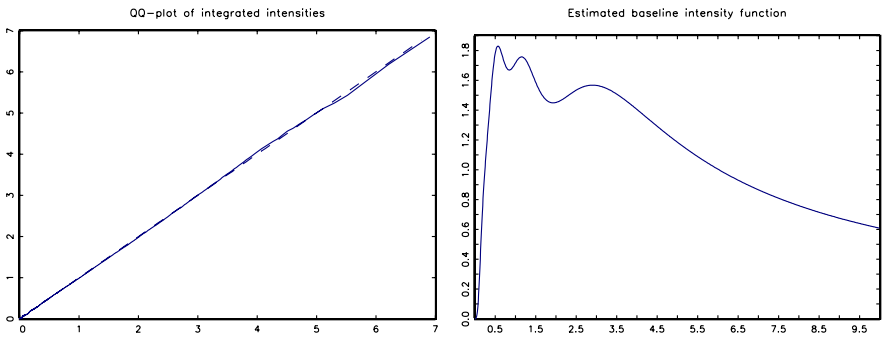

Figure 3: Quantile-quantile plot of the estimated integrated intensities (left) and estimated baseline intensity function (right) based on specification E in Table 2. The baseline intensity function is computed for $\delta_\Phi = \delta_s = \delta_\Psi = 0$.

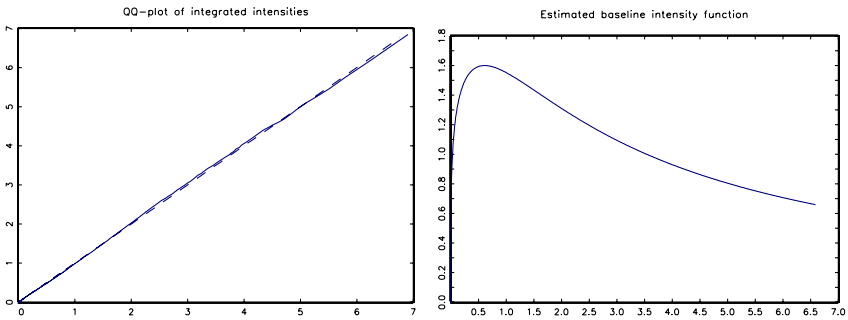

Figure 4: Quantile-quantile plot of the estimated integrated intensities (left) and estimated baseline intensity function (right) based on specification C in Table 3.

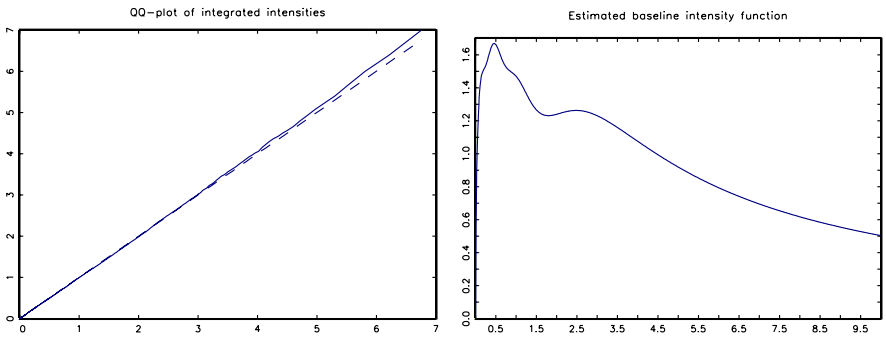

Figure 5: Quantile-quantile plot of the estimated integrated intensities (left) and estimated baseline intensity function (right) based on specification D in Table 3.

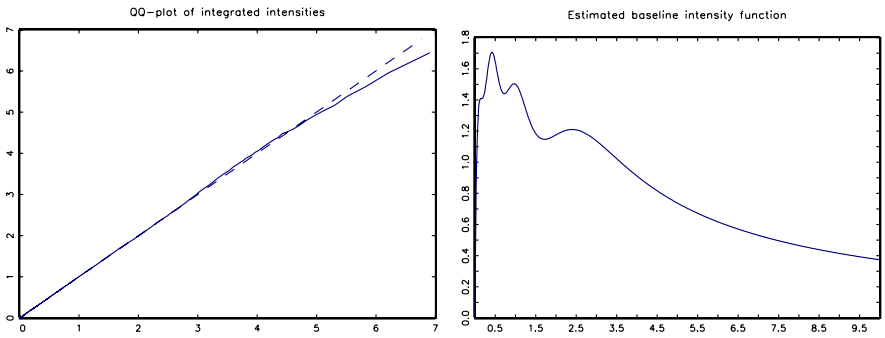

Figure 6: Quantile-quantile plot of the estimated integrated intensities (left) and estimated baseline intensity function (right) based on specification E in Table 3. The baseline intensity function is computed for $\delta_\Phi = \delta_s = \delta_\Psi = 0$.

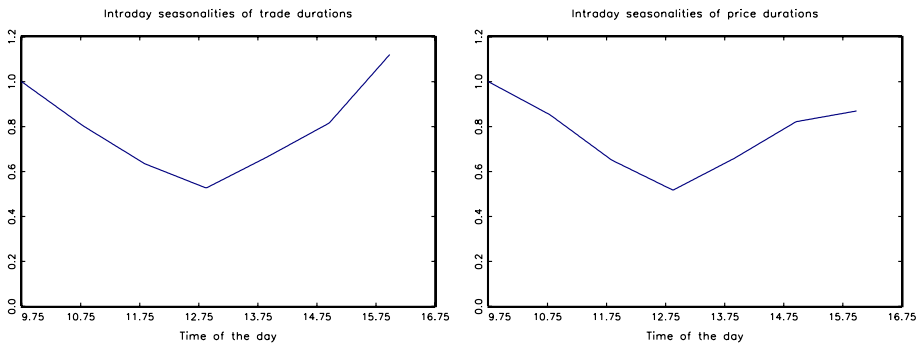

Figure 7: Estimated intraday seasonalities for trade durations (left) and price durations (right).

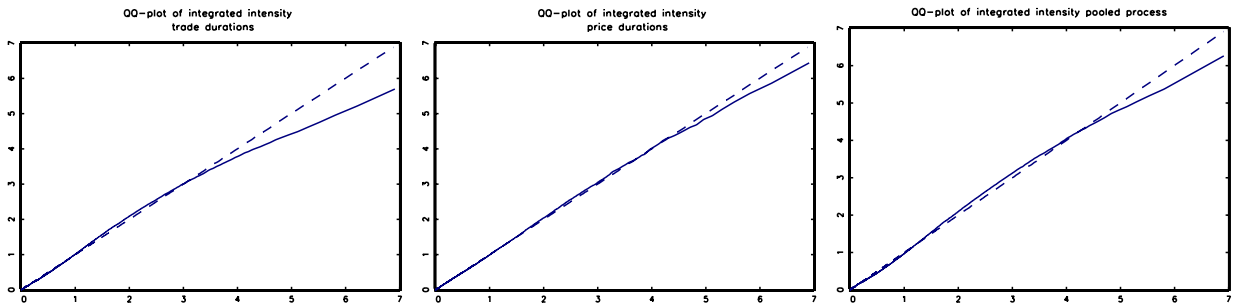

Figure 8: Estimated quantile-quantile plots of the integrated intensities for trade durations, $e^1_{i,2}$, (left) and price durations, $e^2_{i,2}$, (middle) and of the integrated intensities of the pooled process $e_{i,3}$ (right) based on specification B in Table 4.

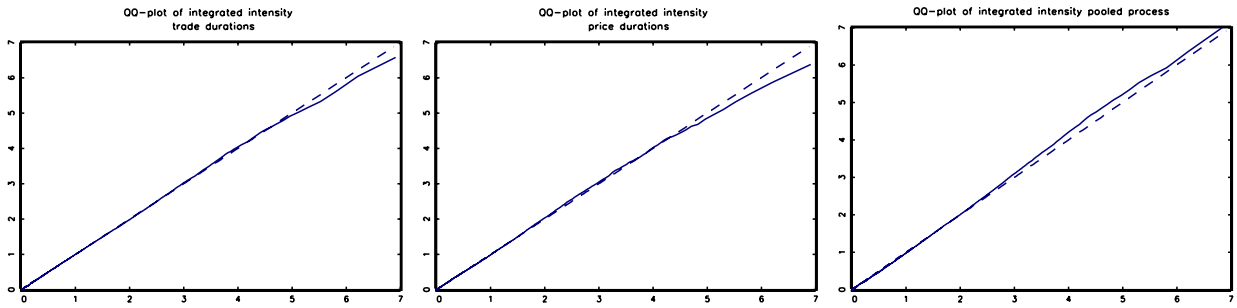

Figure 9: Estimated quantile-quantile plots of the integrated intensities for trade durations, $e^1_{i,2}$, (left) and price durations, $e^2_{i,2}$, (middle) and of the integrated intensities of the pooled process $e_{i,3}$ (right) based on specification C in Table 4.

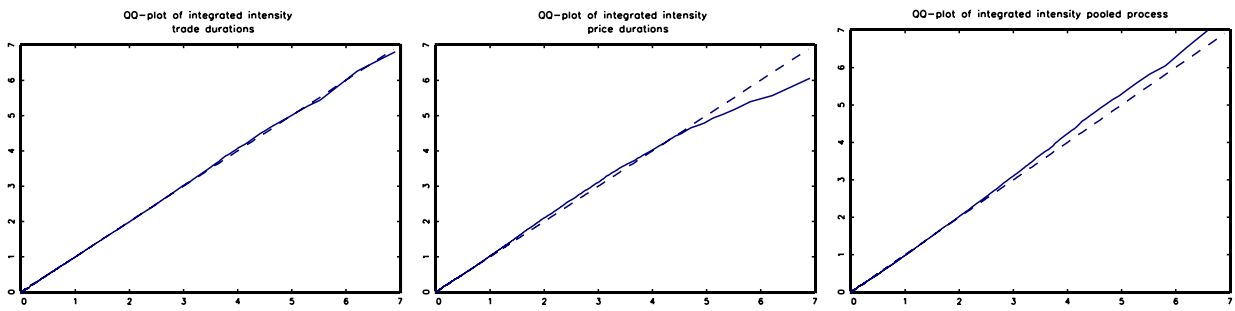

Figure 10: Estimated quantile-quantile plots of the integrated intensities for trade durations, $e^1_{i,2}$, (left) and price durations, $e^2_{i,2}$, (middle) and of the integrated intensities of the pooled process $e_{i,3}$ (right) based on specification D in Table 4.

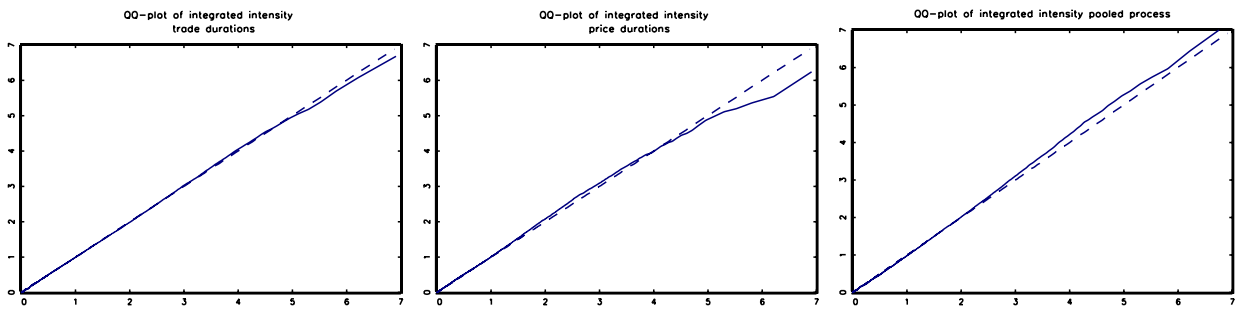

Figure 11: Estimated quantile-quantile plots of the integrated intensities for trade durations, $e^1_{i,2}$, (left) and price durations, $e^2_{i,2}$, (middle) and of the integrated intensities of the pooled process $e_{i,3}$ (right) based on specification E in Table 4.

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