

# Asset Prices and Institutional Investors\*

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## Abstract

Empirical evidence indicates that trades by institutions have sizable effects on asset prices, generating phenomena such as index effects, asset class effects and others. It is difficult to explain such phenomena within standard representative-agent asset pricing models. In this paper, we consider an economy populated by institutional investors alongside standard retail investors. Institutions care about their performance relative a certain index. Our framework is tractable, admitting exact closed-form expressions, and produces the following analytical results. We find that institutions optimally tilt their portfolios towards stocks that comprise their index. The resulting price pressure boosts index stocks, while leaving nonindex stocks unaffected. By demanding a higher fraction of risky stocks than retail investors, institutions amplify the index stock volatilities and the aggregate stock market volatility. Trades by institutions induce excess correlations among stocks that belong to their index, generating an asset class effect. Institutions finance their additional purchases of index stocks by taking on leverage. A policy prescription that calls for a reduction in leverage, while reducing the riskiness of institutional portfolios, would also reduce the ability of institutions to tilt their portfolios towards index stocks, depressing the index level.

**JEL Classifications:** G12, G18, G29

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# 1. Introduction

A significant part of the trading volume in financial markets is attributed to trading by institutional investors: investment banks, mutual funds, hedge funds, sovereign wealth funds, etc. Trades by retail investors constitute only a small fraction of the trading volume.<sup>1</sup> In light of this evidence, it is surprising that the standard theories of asset pricing stipulate that prices in financial markets are determined by households (or by the “representative consumer” aggregated over households) who seek to optimize their consumption and investment over their life cycle. This approach leaves no role for important considerations influencing institutional portfolios such as, for instance, compensation-induced incentives or implicit incentives resulting from the predictability of inflows of capital into the money management business. An analysis of such considerations would add vastly to our understanding of how professional money managers make decisions to buy or sell a particular asset. It would also allow us to explore the implications of these decisions for asset prices. It is now undisputed that the 2007-2008 financial crisis was largely due to the poor incentives given to financial institutions and to the inability of key market participants (institutional investors, hedge funds) to supply liquidity to financial markets in distress. This calls for a closer look at how incentives of institutions may influence financial markets.

In this paper, we focus on the most prominent feature of professional managers’ incentives which may affect the prices of the assets they hold: concern about own performance vis-à-vis some index (e.g., S&P 500). This characteristic is what induces institutional investors to act potentially markedly different from retail investors. Relative performance matters because inflows of new money into funds and payouts to fund managers at year-end depend on the index return that year. Our specific focus is on the incentive of fund managers to do particularly well in bull markets, when payouts to money managers are especially high.<sup>2</sup> Our goal is to demonstrate how in the presence of such incentives institutions optimally tilt their portfolios towards index stocks, influencing the performance of the index, and in the process how they exacerbate leverage in the economy as well as stock market volatility, and boost the correlation among the stocks that are included in the

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<sup>1</sup>See, for example, Griffin, Harris, and Topaloglu (2003).

<sup>2</sup>A good illustrative example for this (well-documented) phenomenon is the spectacular growth of assets under management of the Janus Funds family. During the technology bubble Janus made big bets on technology and telecommunications. At the peak of the bubble, half of the money going into mutual funds went to Janus, bringing its assets under management to \$318B. Janus fund managers, of course, received hefty paychecks. From 2000 to 2005, after the collapse of the bubble, Janus lost 60% of its assets under management. Yet still it had \$132B as of 3/2005 and remained in the top dozen of fund families. For a formal analysis of fund inflows following bull markets, see Karceski (2002).

index.

We consider a dynamic general equilibrium model with two classes of investors: “retail” investors with standard preferences and “institutional” investors who have an additional incentive to do well in bull markets—formally, their marginal utility of wealth is increasing with the level of the index. There are multiple risky stocks, some of which are part of the index, and a riskless bond. The stocks are in positive net supply, while the bond is in zero net supply. The model is designed to capture several important empirical phenomena and to provide the economic mechanisms generating these phenomena. One major advantage of our model is its tractability. All the results described below are based on exact, closed-form expressions.

We first examine the tilt in the portfolios of the institutional investors which is caused by the presence of the index. We find that, relative to the retail investor, the institution increases the fraction of index stocks in the portfolio so as not to fall behind when the index does well. To finance additional purchases of index stocks, the institution takes on leverage. So the institutions in our model always end up borrowing funds from the retail sector, to the extent allowed by the size of their assets under management that serve as collateral. As institutions continue to do well and accumulate assets, they increase the overall leverage in the economy, but only up to a certain turning point determined by the lending capacity of the retail sector.

We next investigate how the presence of institutions influences asset prices. Our first finding is that institutions push up the prices of index stocks. In the economy with institutional investors, the index stock prices are higher both relative to those in the retail-investor-only economy and relative to their (otherwise identical) non-index counterparts. This is because institutions generate excess demand for the index stocks. This finding is well supported by empirical evidence: such an “index effect” occurs in many markets and countries. Starting from Harris and Gurel (1986) and Shleifer (1986), a series of papers documents that prices of stocks that are added to the S&P 500 and other indices increase following the announcement and prices of stocks that are deleted drop. For example, Chen, Noronha, and Singal (2004) find that during 1989-2000, the stock price increases by an average of 5.45% on the day of the S&P 500 inclusion announcement and a further 3.45% between the announcement and the actual addition. The corresponding figures for the S&P 500 deletions are -8.46% and -5.97%, respectively. Moreover, the index effect has become stronger after 1989. While there are possible alternative explanations to this phenomenon, the growth of the institutions who benchmark their performance against the index remains a leading one.

We also find that the price pressure from the institutions boosts the level of the overall stock market. This is because the institutions have a higher demand for risky assets than retail investors, which increases the average risk appetite in the economy relative to that in the retail-investor-only counterpart. This higher risk appetite translates into a higher level of the overall stock market. Moreover, the presence of institutional investors also makes the index stocks and the overall stock market more volatile. This is again because of the increased average risk appetite in the economy. These findings have several policy implications. One popular policy recommendation during the 2007-2008 financial crisis, which was arguably caused by excessive leverage, was to impose leverage caps on institutions. In our model, when the institutions do not control the dominant fraction of wealth in the economy, this policy brings down the riskiness of their portfolios (a desired effect) but also brings down the level of stock prices, creating an undesired side effect. Another set of policy recommendations involved injecting new money into some institutions. In our model, this policy has the opposite effect from the leverage cap. It boosts the level of the stock market, raising the financial wealth of both institutions and the retail investors alike, but at the same time, it may increase leverage in the economy and raise stock market volatility.

Finally, we examine the correlations among stocks included in the index and stocks outside the index. We find that the presence of institutions who care explicitly about their index generates an “asset class” effect: returns on stocks belonging to the index are more correlated amongst themselves than with those of otherwise identical stocks outside the index. This asset class effect is, of course, absent in the retail-investor-only benchmark economy: there, the correlation between any two stocks’ returns is determined simply by the correlation of their fundamentals (dividends). The additional correlation among the index stocks is caused by the additional demand of institutions for the index stocks: the institutions hold a hedging portfolio, consisting of index stocks only, that hedges them against fluctuations in the index. Following a good realization of cash flow news, institutions get wealthier and demand more shares of index stocks relative to the retail-investor-only benchmark. This additional price pressure affects all index stocks at the same time, inducing excess correlations among these stocks. Empirical research lends support to our findings. For example, Barberis, Shleifer, and Wurgler (2005) show that when a stock is added to the S&P 500 index, its beta with respect to the S&P 500 goes up while its non-S&P 500 “rest of the market” beta falls; that the opposite is true for stocks deleted from the index; and that these effects are stronger in more recent data. Boyer (2007) provides similar evidence for BARRA value and growth indices. He finds that “marginal value” stocks—the stocks that just switched from the growth into the

value index—comove significantly more with the value index; the opposite is true for the “marginal growth” stocks. Consistent with the institutional explanation for this phenomenon, Boyer finds that the effect appears only after 1992, which is when BARRA indices were introduced.

It is somewhat surprising that despite extensive empirical work showing that institutions have important effects on asset prices and despite the 2007-2008 financial crisis that has made this point all too obvious, we still have little theoretical work on equilibrium in the presence of professional money management. Brennan (1993) is the first to attempt to introduce institutional investors into an asset pricing model. Within a static mean-variance setting with constant absolute risk aversion (CARA) utility agents, he shows that equilibrium expected returns are given by a two-factor model, with the two factors being the market and the index. More recent related mean-variance models appear in Gomez and Zapatero (2003), Cornell and Roll (2005), Brennan and Li (2008), Leippold and Rohner (2008), and Petajisto (2009). CARA utility, as is well-known, rules out wealth effects, which play a central role in our paper. Cuoco and Kaniel (2010) develop a two-stock dynamic equilibrium model with constant relative risk aversion (CRRA) agents who explicitly care about an index due to performance-based fees.<sup>3</sup> Our model is more general since it features multiple risky stocks, which allows us to derive implications such as an “asset class” effect. It is also more tractable, allowing us to prove analytically the results pertaining to the index effect and equilibrium leverage and to analyze the implications of popular policy measures employed in the recent financial crisis. The closest theoretical model that exhibits the “asset class” effect is by Barberis and Shleifer (2003), whose explanation for this phenomenon is behavioral.

Other related papers that have explored equilibrium effects of delegated portfolio management include He and Krishnamurthy (2008ab), in which poor performance of fund managers triggers portfolio outflows due to contracting frictions, Dasgupta and Prat (2008), Dasgupta, Prat, and Verardo (2008), Guerrerri and Kondor (2010), Vayanos and Woolley (2010), in which outflows following poor performance are due to learning about managerial ability, and Vayanos (2004) in which outflows occur for exogenous reasons, once the fund performance is sufficiently poor. They show that, similarly to our findings, flow-based considerations amplify the effects of exogenous shocks on asset prices. All of these papers model thoroughly various agency frictions. In our model, we simplify this aspect, but offer a richer model of a securities market. We view these papers as complementary to our work.

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<sup>3</sup>See also Kapur and Timmermann (2005) and Arora, Ju, and Ou-Yang (2006) for related dynamic models.

Finally, there is a related literature on the effects of fund flows and benchmarking considerations on portfolio choice of fund managers, at a partial equilibrium level. For example, Carpenter (2000), Basak, Pavlova, and Shapiro (2007, 2008), Hodder and Jackwerth (2007), and Chen and Pennacchi (2009) show that future fund flows induce a manager to tilt her portfolio towards stocks that belong to her benchmark. These works demonstrate that there is a range over which such benchmarking considerations induce her to take more risk. Ross (2004) investigates the relation between the manager’s fee structure and her attitude toward risk within a general class of preferences and compensation structures. The main difference of our paper from this body of work is that we examine the general equilibrium effects of benchmarking.

The remainder of our paper is organized as follows. Section 2 presents a simplified single-stock version of our model for which we establish a number of our results in the clearest possible way. Section 3 discusses the index effect, institutional risk-taking, wealth effects, and the resulting policy implications. Section 4 presents the general multi-stock version of our model and focuses on the asset class effect. Section 5 concludes, and the appendices contain proofs of our results and further generalizations of our model.

## **2. Economy with Institutional Investors**

### **2.1. Economic Setup**

We consider a simple and tractable pure-exchange security market economy with a finite horizon. The economy evolves in continuous time and is populated by two types of market participants: retail investors,  $R$ , and institutional investors,  $I$ . In the general specification of our model, there are  $N$  stocks,  $M$  of which are included in the index against which the performance of institutions is measured, as well as a riskless bond. In this section, however, we specialize the securities market to feature a single risky stock, henceforth referred to as the stock market *index*, and a riskless bond. The index is exposed to a single source of risk represented by a Brownian motion  $\omega$ . The main reason for considering the single-stock case is expositional simplicity. It turns out that a number of key insights of this paper can be illustrated within the single-stock economy. We then build on our baseline intuitions and expand them (Section 4) to demonstrate how our economy behaves in the general case in which there are multiple stocks and multiple sources of risk.

The stock market index,  $S$ , is posited to have dynamics given by

$$dS_t = S_t[\mu_{S_t}dt + \sigma_{S_t}dw_t], \quad (1)$$

with  $\sigma_{S_t} > 0$ . The mean return  $\mu_S$  and volatility  $\sigma_S$  are determined endogenously in equilibrium (Section 3). The bond is in zero net supply. It pays a riskless interest rate  $r$ , which we set to zero without loss of generality.<sup>4</sup> The stock market index is in positive net supply. It is a claim to the terminal payoff (or “dividend”)  $D_T$ , paid at time  $T$ , and hence  $S_T = D_T$ . This payoff  $D_T$  is the terminal value of the process  $D_t$ , with dynamics

$$dD_t = D_t[\mu dt + \sigma dw_t], \quad (2)$$

where  $\mu$  and  $\sigma > 0$  are constant. The process  $D_t$  represents the arrival of news about  $D_T$ . We refer to it as the *cash flow news*. Equation (2) implies that cash flow news arrives continuously and that  $D_T$  is lognormally distributed. The lognormality assumption is made for technical convenience.<sup>5</sup>

Each type of investor  $i = I, R$  in this economy dynamically chooses a portfolio process  $\phi_i$ , where  $\phi_i$  denotes the fraction of the portfolio invested in the stock index, or the *risk exposure*, given the initial assets of  $W_{i0}$ . The wealth process of investor  $i$ ,  $W_i$ , then follows the dynamics

$$dW_{it} = \phi_{it}W_{it}[\mu_{S_t}dt + \sigma_{S_t}dw_t]. \quad (3)$$

The (representative) institutional and retail investors are initially endowed with fractions  $\lambda \in [0, 1]$  and  $(1 - \lambda)$  of the stock market index, providing them with initial assets worth  $W_{I0} = \lambda S_0$  and  $W_{R0} = (1 - \lambda)S_0$ , respectively. The parameter  $\lambda$  thus represents the (initial) fraction of the institutional investors in the economy—or equivalently, how large the institutions are relative to the overall economy. It is an important comparative statics parameter in our analysis, which allows us to illustrate how the growth of the financial sector (or more precisely, funds managed by institutions) can influence asset prices.

The retail investor has standard logarithmic preferences over the terminal value of her portfolio:

$$u_R(W_{RT}) = \log(W_{RT}). \quad (4)$$

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<sup>4</sup>Or equivalently, we may use the riskless bond as the numeraire and denote all prices in terms of this numeraire. Our model does not have intermediate consumption, and so the equilibrium places no restrictions on the interest rate.

<sup>5</sup>In related analysis (not presented here due to space limitations), we relax the lognormality assumption and show that the bulk of our results remains valid for more general stochastic processes, but the characterization of our economy becomes more complex.

In modeling the institutional investor’s objective function, we consider several noteworthy features of the professional money management industry that make institutions behave markedly different from retail investors. The first is that institutions care about their returns relative to the stock market index, against which their performance is evaluated. The second is their incentive to attract new money into the business and hence increase the size of assets under management, and in particular to perform well during bull markets when payouts to money managers are especially high (for empirical evidence, see, e.g., Karceski (2002)). Accordingly, we formulate the institutional investor’s objective function over the terminal value of her portfolio as being given by:

$$u_I(W_{IT}) = (a + bS_T) \log(W_{IT}), \quad (5)$$

where  $a, b > 0$ . This objective function is similar to the retail investor’s objective function, but it additionally incorporates the desire to do well in bull markets. Formally, we require that the institutional investor’s marginal utility of wealth is increasing in the level of the index. There are, of course, multiple alternative specifications that satisfy this property, but the empirical literature to date is unclear as to what the exact form of the dependence on the index should be. In (5) we have chosen a particularly simple affine specification, which renders tractability to our model. It is certainly desirable to extend our specification to a more general class of functions and to provide microfoundations for an objective function to depend on the performance of the index.<sup>6</sup> We leave these exercises for future research.

## 2.2. Investors’ Portfolio Choice

Each type of investor’s dynamic portfolio problem is to maximize her expected objective function in (4) or (5), subject to the dynamic budget constraint (3). Lemma 1 presents the investors’ optimal portfolios explicitly, in closed form.

**Lemma 1.** *The institutional and retail investors’ portfolios are given by*

$$\phi_{It} = \frac{\mu_{St}}{\sigma_{St}^2} + \frac{b e^{\mu(T-t)} D_t}{a + b e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}}, \quad (6)$$

$$\phi_{Rt} = \frac{\mu_{St}}{\sigma_{St}^2}. \quad (7)$$

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<sup>6</sup>In addition to justifying the objective function due to implicit incentives provided by fund flows, one can also motivate it due to explicit incentives provided by compensation contracts. For example, in a moral hazard framework, Dybvig, Farnsworth, and Carpenter (2010) show that under certain conditions, benchmarking the manager against the index emerges as an optimal compensation contract.

Consequently, the institution invests a higher fraction of wealth in the stock market index than the retail investor does,  $\phi_{It} > \phi_{Rt}$ .

The first term in the expression for the institutional investor's portfolio is the standard (instantaneous) mean-variance efficient portfolio. It is the same mean-variance portfolio that the retail investor holds. The wedge between the portfolio holdings of the two groups of investors is created by the second term in (6): the hedging portfolio. This hedging portfolio arises because the institution has an additional incentive to do well in bull markets, and so the hedging portfolio is positively correlated with cash flow news ( $D_t$ ). The instrument that allows the institution to achieve a higher correlation with cash flow news is the stock market index itself, and so the institution holds more of it than does the retail investor. This implies that the institution ends up taking on more risk than the retail investor does. We are going to demonstrate shortly (Section 3.2) that in equilibrium, the institution finances its additional demand for the stock by borrowing from the retail investor. So the higher effective risk appetite of the institutional investor induces her to lever up. One can draw parallels with the 2007-2008 financial crisis, in which leverage of financial institutions was one of the key factors contributing to the instability. Excessive leverage has often been ascribed to the bonus structure of market participants. While we do not dispute the conclusion that an option-like compensation function can generate excessive risk taking, we would like to stress that a simple incentive to do well when the stock market index is high, which we model here, also leads to a higher effective risk appetite.

### 3. Equilibrium in the Presence of Institutional Investors

We are now ready to explore the implications of the presence of institutions in the economy on asset prices and their dynamics. As we have shown in the previous section, institutions have an incentive to take on more risk relative to the retail investors, and hence their presence increases the demand for the risky stock. In this section, we demonstrate how these incentives boost the price and the volatility of the risky stock and how they affect the behavior of all market participants.

Equilibrium in our economy is defined in a standard way: equilibrium portfolios and asset prices are such that (i) both the retail and institutional investors choose their optimal portfolio strategies, and (ii) stock and bond markets clear. We will often make comparisons with equilibrium in a *benchmark economy* with no institutions ( $\lambda = 0$ ) which is populated by a representative retail investor. The benchmark economy is an example of a standard representative agent asset-pricing

model, well-explored in the literature. By increasing the parameter  $\lambda$  above zero, we will then be able to see clearly the distinction between our economy and the standard benchmark.

### 3.1. Stock Price, Volatility, and Index Effect

**Proposition 1.** *In the economy with institutional investors, the equilibrium level of the stock market index is given by*

$$S_t = \bar{S}_t \frac{a + b e^{\mu T} D_0 + \lambda b (e^{\mu(T-t)} D_t - e^{\mu T} D_0)}{a + b e^{\mu T} D_0 + \lambda b (e^{(\mu-\sigma^2)(T-t)} D_t - e^{\mu T} D_0)}, \quad (8)$$

where  $\bar{S}_t$  is the equilibrium index level in the benchmark economy with no institutional investors given by

$$\bar{S}_t = e^{(\mu-\sigma^2)(T-t)} D_t. \quad (9)$$

Consequently, the stock market index level is increased in the presence of institutional investors,  $S_t > \bar{S}_t$ . Moreover, it increases with the fraction  $\lambda$  of the institutional investors in the economy.

The presence of institutions generates price pressure on the stock market index. Recall that institutions in our model have a higher demand for the risky stock than retail investors. Therefore, relative to the benchmark economy, there is an excess demand for the stock market index. The stock is in fixed supply, and so its price must be higher. As the fraction of institutional investors goes up ( $\lambda$  increases), there is more price pressure on the index, pushing it up further. This is the simplest way to capture the “index effect” in our model—the phenomenon widely documented empirically in many markets (see, e.g., Shleifer (1986), Chen, Noronha, and Singal (2004)). (This result is generalizable to the multi-stock case. In that case, only the stocks included in the index trade at a premium due to the excess demand for these stocks by the institutions; prices of the non-index stocks remain unchanged. See Section 4.) Finally, it is worth noting that the expressions for asset prices that we derive here and below are all simple and in closed form. This is a very convenient feature of our framework, which allows us to explore the economic mechanisms in play within our model and comparative statics without resorting to numerical analysis.

Since institutional investors affect the level of the index, it is conceivable that they also influence its volatility. They demand a riskier portfolio relative to that of the retail investors, and so one would expect them to amplify the riskiness of the index. Proposition 2 verifies this conjecture.

**Proposition 2.** *In the equilibrium with institutional investors, the volatility of the stock market*

index returns is given by

$$\sigma_{St} = \bar{\sigma}_{St} + \lambda b \sigma \frac{(1 - e^{-\sigma^2(T-t)}) (a + (1 - \lambda) b e^{\mu T} D_0) e^{\mu(T-t)} D_t}{(a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{(\mu - \sigma^2)(T-t)} D_t) (a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t)}, \quad (10)$$

where  $\bar{\sigma}_{St}$  is the equilibrium index volatility in the benchmark economy with no institutions, given by

$$\bar{\sigma}_{St} = \sigma.$$

Consequently, the index volatility is increased in the presence of institutions,  $\sigma_{St} > \bar{\sigma}_{St}$ .

In the benchmark economy with no institutional investors, the index return volatility is simply a constant. In the presence of institutional investors, it becomes stochastic, and in particular, dependent on the cash flow news. It also depends on the fraction of institutional investors in the economy,  $\lambda$ . The notable implication here is that institutional investors make the stock more volatile. In other words, the effects of cash flow news are amplified by institutional investors. This is again due to the institutions' higher risk appetite. The institutions demand a riskier portfolio, levering up in equilibrium, and so the "representative investor" in this model is effectively more risk tolerant than the one in the retail-investor-only benchmark. The representative investor can thus tolerate more volatility in equilibrium, which in turn leads to a riskier index.

Figure 1 depicts the equilibrium index volatility as a function of the size of the institutions in the economy ( $\lambda$ ) and cash flow news ( $D_t$ ). As institutions become larger, they constitute a larger fraction of the representative investor, and hence the risk appetite of the representative investor increases. Along with that comes an increase in the total leverage taken out by the institutions and an increase in the volatility of index returns. However, the institutions' ability to lever up depends on the lending capacity of retail investors, who in equilibrium provide a counterparty to the institutional investors in the market for borrowing/lending. As the fraction of institutions increases further, there is lesser lending capacity that can be provided by the retail investors. This in turn forces the institutional leverage to go down in equilibrium, pushing down the index volatility along with it. This explains the peak in the volatility in panel (a) of Figure 1. Similarly, as positive cash flow news arrive (higher  $D_t$ ), the institutions get wealthier and lever up more, increasing the index volatility. However, after a while, this increase peaks and the volatility starts to decrease. This is because the counterparty retail investors are getting less wealthy, shrinking in size and therefore in their lending capacity.

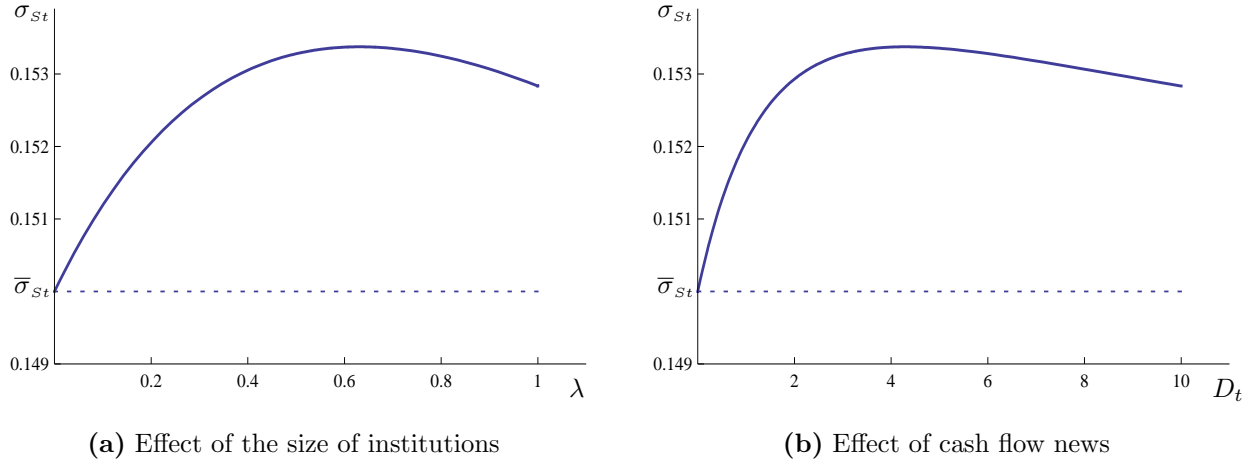


Figure 1: **Equilibrium index volatility.** This figure plots the index volatility in the presence of institutions against the fraction of institutions in the economy  $\lambda$  and against cash flow news  $D_t$ . The dotted lines correspond to the equilibrium index volatility in the benchmark economy with no institutions. The plots are typical. The parameter values are:  $a = 1$ ,  $b = 1$ ,  $D_0 = 1$ ,  $\mu = 0.05$ ,  $\sigma = 0.15$ ,  $t = 1$ ,  $T = 5$ . In panel (a)  $D_t = 2$ , and in panel (b)  $\lambda = 0.2$ .

### 3.2. Risk Taking, Leverage, and Wealth Effects

To further understand the underlying economic mechanisms operating in our model, we look more closely at the investors' portfolios in equilibrium. Towards that, it is more convenient to restate the investors' portfolios in terms of the number of shares of the risky stock, i.e.,

$$\pi_{It} = \phi_{It} \frac{W_{It}}{S_{It}}, \quad \pi_{Rt} = \phi_{Rt} \frac{W_{Rt}}{S_{Rt}},$$

where as before  $\phi_{it}$  denotes the fraction of investor's wealth invested in the index. Proposition 3 reports the investors' equilibrium portfolios, as well as an important property of the institutional portfolio holdings.

**Proposition 3.** *The institutional and retail investors' portfolios in equilibrium are given by*

$$\pi_{It} = \lambda \frac{a + b e^{\mu(T-t)} D_t}{a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t} \times \left( 1 - \frac{\lambda b e^{\mu(T-t)} D_t}{a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}} + \frac{b e^{\mu(T-t)} D_t}{a + b e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}} \right), \quad (11)$$

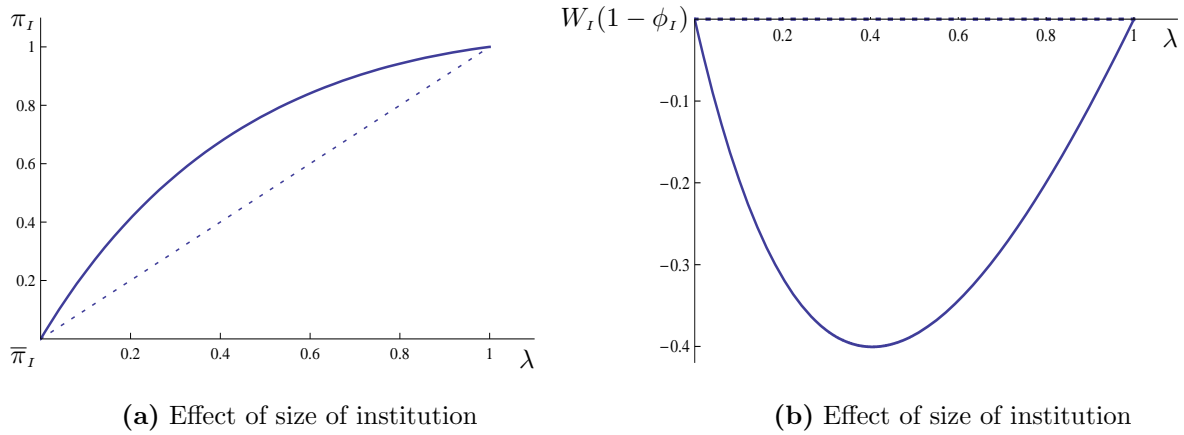
$$\pi_{Rt} = (1 - \lambda) \frac{a + b e^{\mu T} D_0}{a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t} \left( 1 - \frac{\lambda b e^{\mu(T-t)} D_t}{a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}} \right), \quad (12)$$

where  $\sigma_{st}$  is as in Proposition 2.

Consequently, the institutional investor is always levered,  $W_{It}(1 - \phi_{It}) < 0$ .

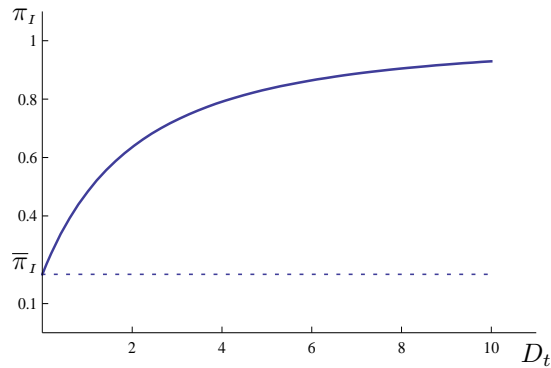
To better highlight the results in Proposition 3, Figure 2 plots the institutional investor's equilibrium portfolio holdings against the size of the institution ( $\lambda$ ) and cash flow news ( $D_t$ ). We see that the institution always “tilts” her portfolio towards the index stock, as compared to an otherwise identical benchmark investor who does not directly care about the index (Figure 2(a)). We have seen this implication at a partial equilibrium level; we now confirm that in equilibrium the institutional investor still ends up holding more shares of the index than his benchmark counterpart. In order to be able to finance those additional index shares, the institution borrows from the retail investor and so it always levers up in equilibrium (Figure 2(b)). The bell-shaped plot in Figure 2(b) is an important illustration of how leverage in the economy depends on the size of the institutional sector. One extreme is when the size of the institutional sector is zero ( $\lambda = 0$ ). In that case, all agents in the economy are retail investors with identical preferences, and so no one is willing to take a counterparty position in the market for borrowing and lending (recall that the bond is in zero net supply). The bondholdings of all investors are then equal to zero. The other extreme is when there are no retail investors in the economy ( $\lambda = 1$ ). Again, there is no heterogeneity to induce borrowing and lending in equilibrium, and the bondholdings are zero. In the intermediate range,  $0 < \lambda < 1$ , the institution borrows from the retail investor, using its initial wealth as collateral. The budget constraint always forces the borrower to repay; the higher the initial wealth, the more leverage the borrower is able to take on. This is why we see an increase in the overall leverage as the size of the institutional sector starts to increase ( $\lambda$  increases). At a certain point, however, it peaks and then starts to fall. This is because, as the institutional sector becomes larger, the size of the retail sector ( $1 - \lambda$ ) shrinks, and therefore the lending capacity of the retail sector progressively reduces. This in turn reduces the equilibrium leverage in the economy.

In Figure 2(c) we illustrate the response of the institutional investors' equilibrium portfolios to cash flow news. Rebalancing following good cash flow news is simply a “wealth effect” (as highlighted by, e.g., Kyle and Xiong (2001)). In equilibrium, both types of investors have positive holdings of the risky stock, and so good cash flow news translates into higher wealth for each investor. As the investors become wealthier, they want to increase the riskiness of their portfolios, which in this model implies buying more shares of the risky stock. Of course, for the stock market to clear, both investors cannot be buying the stock simultaneously; one of them has to sell. To



(a) Effect of size of institution

(b) Effect of size of institution



(c) Effect of cash flow news

Figure 2: **The institutional investor's portfolio holdings.** Panels (a) and (b) of this figure plot the institution's holdings of the shares of the index  $\pi_I$  and the bond  $W_I(1 - \phi_I)$  against the size of the institution  $\lambda$ . Panel (c) plots the institution's holdings of the index against cash flow news  $D_t$ . The lines for  $\bar{\pi}$  correspond to the holdings of an otherwise identical investor in the benchmark economy. The plots are typical. In panels (a) and (b)  $D_t = 2$ , and in panel (c)  $\lambda = 0.2$ . The remaining parameter values are as in Figure 1.

determine who is buying and who is selling, one can look at the change in the wealth distribution in the economy. In this case, as good cash flow news arrives ( $D_t$  increases), the wealth distribution shifts in favor of the institutional investor. Intuitively, this is because the institutional portfolio is over-weighted in the risky stock relative to that of the retail investor, and so good news about the stock produces a higher return on the institutional portfolio relative to that of the retail investor.<sup>7</sup> Hence, following good cash flow news, the institution buys from the retail investor (Figure 2(c)). This wealth effect is an important part of the economic mechanisms that operate in our model. It is useful to stress at this point that the bulk of the related literature, developed in the framework in which investors have CARA preferences, is not able to capture wealth effects. The assumption of CARA utilities is made, of course, for tractability. In our model, tractability is achieved through alternative channels, which we highlight in this section and the next.

### 3.3. Policy Implications

The two main policy implications we would like to explore in the context of our model are the effects of deleveraging (a mandate to reduce leverage) and the effects of a transfer of capital to leveraged institutions. These two policy instruments have widely been employed during the 2007-2008 financial crisis. The objective, of course, was to improve the balance sheets of individual institutions in difficulty. But these policy actions, given their size and scope, inevitably had an effect on the overall economy, including asset prices. Our goal here is to analyze the equilibrium implications of these policies.

At this point, we also draw a distinction between long-only institutions (“real money”) and leveraged institutions (“leveraged money”). So far we have only dealt with the latter category. We model long-only institutions,  $L$ , in a very simple form: these institutions do not solve any optimization problem but simply buy and hold the risky stock they are endowed with. The initial endowments of the retail investors, leveraged institutions, and long-only institutions are now  $W_{R0} = (1 - \lambda)S_0$ ,  $W_{I0} = \lambda\theta S_0$ , and  $W_{L0} = \lambda(1 - \theta)S_0$ , respectively. That is, the endowment of the retail investors is as before, but the endowment of institutions is now divided between the leveraged institutions and long-only institutions in proportions  $\theta$  and  $1 - \theta$ , respectively. The new parameter

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<sup>7</sup>We show formally that the institution becomes wealthier relative to the retail investor following good cash flow news, i.e.,  $W_{It}/W_{Rt}$  increases with  $D_t$ , in the proof of Proposition 3 in the Appendix. In particular, we show that the wealth distribution is given by

$$\frac{W_{It}}{W_{Rt}} = \frac{\lambda}{1 - \lambda} \frac{a + b e^{\mu(T-t)} D_t}{a + b e^{\mu T} D_0}.$$

$\theta \in (0, 1)$  then captures the mass of “leveraged money” as a fraction of funds held (initially) by institutions. By reducing  $\theta$  we can model a transfer of assets from leveraged institutions to long-only, or deleveraging. Proposition 4 summarizes how asset prices and equilibrium portfolios in our model are affected by the introduction of this new class of institutions.

**Proposition 4.** *The equilibrium index level, volatility, and institutional portfolio in the presence of long-only and leveraged institutions are given by*

$$S_t = \bar{S}_t \frac{(1 - \lambda)(a + be^{\mu T} D_0) + \theta \lambda (a + be^{\mu(T-t)} D_t)}{(1 - \lambda)(a + be^{\mu T} D_0) + \theta \lambda (a + be^{(\mu - \sigma^2)(T-t)} D_t)}, \quad (13)$$

$$\sigma_{S_t} = \bar{\sigma}_{S_t} + \theta \lambda b$$

$$\times \frac{(1 - e^{-\sigma^2(T-t)}) ((1 - \lambda)(a + be^{\mu T} D_0) + \theta \lambda a)}{[(1 - \lambda)(a + be^{\mu T} D_0) + \theta \lambda (a + be^{(\mu - \sigma^2)(T-t)} D_t)] [(1 - \lambda)(a + be^{\mu T} D_0) + \theta \lambda (a + be^{\mu(T-t)} D_t)]}, \quad (14)$$

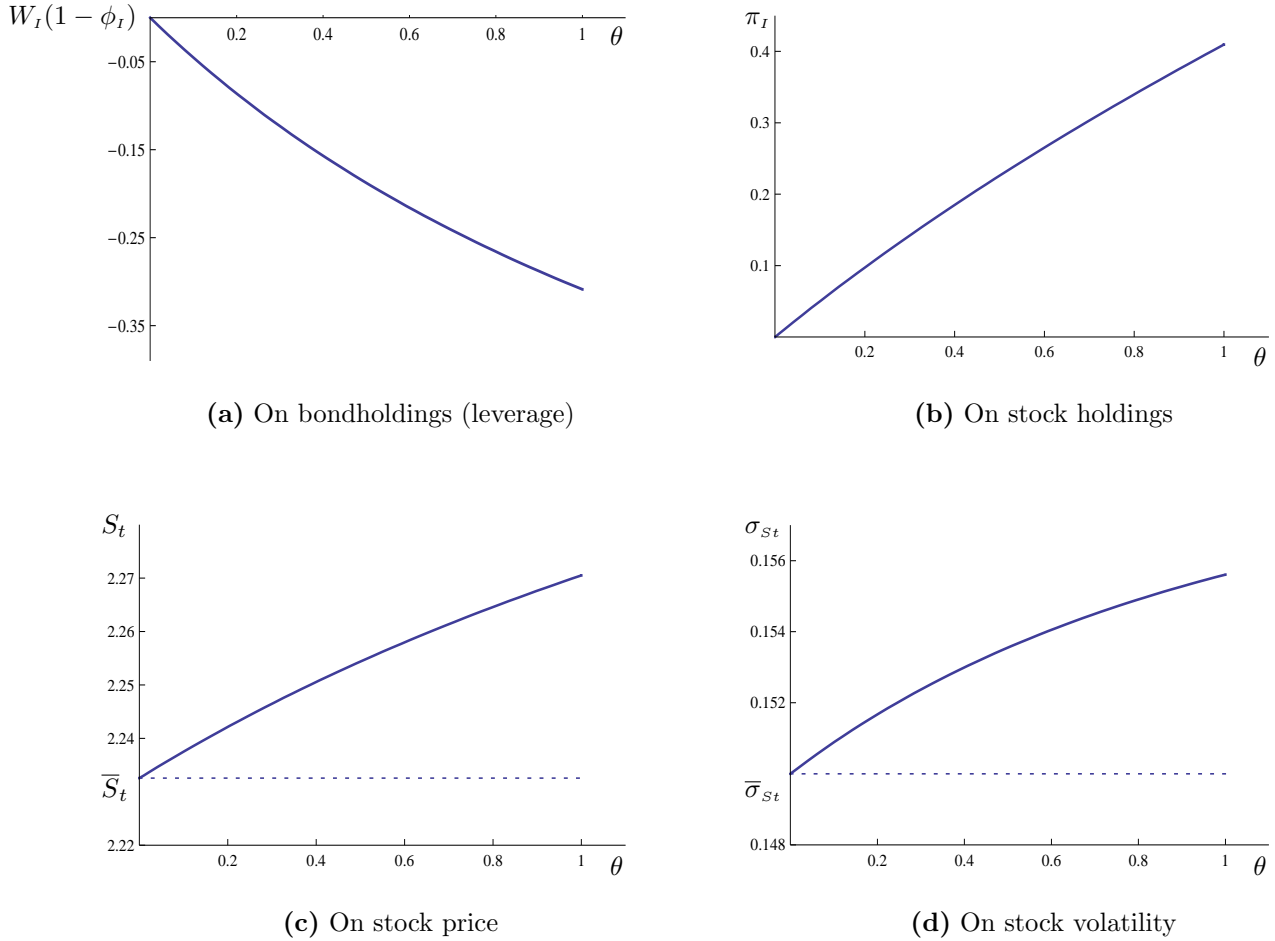
$$\pi_{I_t} = \theta \lambda (1 - \lambda + \theta \lambda) \frac{a + be^{\mu(T-t)} D_t}{(1 - \lambda)(a + be^{\mu T} D_0) + \theta \lambda (a + be^{\mu(T-t)} D_t)} \times \left( 1 - \frac{\theta \lambda b e^{\mu(T-t)} D_t}{(1 - \lambda)(a + be^{\mu T} D_0) + \theta \lambda (a + be^{\mu(T-t)} D_t)} \frac{\sigma}{\sigma_{S_t}} + \frac{b e^{\mu(T-t)} D_t}{a + b e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{S_t}} \right), \quad (15)$$

where the benchmark economy stock and volatility are  $\bar{S}_t = D_t e^{(\mu - \sigma^2)(T-t)}$ ,  $\bar{\sigma}_{S_t} = \sigma$ , respectively.

Consequently, the equilibrium stock price and volatility are higher in the presence of institutions and the stock price increases further as the fraction of leveraged institutions in the economy,  $\theta \lambda$ , increases.

We again find it useful to highlight the results of the proposition in a figure. Figure 3 plots the bond and stock holdings of the leveraged institution, as well as the equilibrium stock market index and its volatility, as functions of the size of the “leveraged money” sector  $\theta$ . The figure confirms that the stock price and the stock holdings of the leveraged institution are unambiguously increasing in  $\theta$ . The effect of  $\theta$  on bondholdings (leverage), however, is not necessarily unambiguous. It depends on the total size of the institutional investors (both real and leveraged money) relative to that of the retail investors. If there is enough lending capacity in the economy—the mass of retail investors is high—then the total amount of borrowing always increases with the size of the leveraged money sector. If, however, the mass of retail investors is relatively high, then leverage in the economy can peak for some  $\theta$  and then start decreasing beyond that point. The economic mechanism generating such a bell-shaped pattern is as in Section 3.2, when we discussed the effects of  $\lambda$  on equilibrium

leverage. For realistic calibrations of the model, we however find that the relevant scenario is the one in which the equilibrium leverage never reaches its maximum (i.e., there are enough retail investors to provide counterparties in the market for borrowing and lending to the leveraged institutions).



**Figure 3: The effects of the size of the leveraged institutions in the economy.** This figure plots the leveraged institution’s holdings of the the bond  $W_R(1 - \phi_R)$  (panel (a)), the leveraged institution’s holdings of the shares of the index  $\pi_R$  (panel (b)), the stock index (panel (c)), the stock index volatility (panel (d)) against the size of the institution  $\theta$ . The plots are typical. The parameter values are:  $a = 1$ ,  $b = 1$ ,  $D_0 = 1$ ,  $\mu = 0.05$ ,  $\sigma = 0.15$ ,  $t = 1$ ,  $T = 5$ . In panel (a)  $D_t = 2$ , and in panel (b)  $\lambda = 0.2$ . The remaining parameter values are as in Figure 1.

### a. Effects of deleveraging

In our framework, we model deleveraging as a transfer of assets from leveraged institutions to long-only institutions at time 0. This policy can be interpreted as a requirement that a fraction of leveraged institutions must convert into “real money” long-only investors. In our model, we capture this as a reduction in the fraction of leveraged institutions  $\theta$ .

Figure 3(a) reveals that a reduction in the mass of leveraged institutions indeed decreases the total leverage in the economy, with the total amount of outstanding bondholdings going down. Not being able to finance a risky asset position of the same size as prior to deleveraging, the institutional sector reduces its demand for the risky stock and the stock holdings of the sector fall (Figure 3(b)). While the deleveraging policy does achieve its desired outcome—the riskiness of the institutional portfolios going down—it does, however, come with side effects. The most notable one is that a reduction in the number of leveraged institutions also brings down the stock market index (Figure 3(c)). This effect is simply a consequence of the drop in demand for the stock index by the institutions.

### **b. Effects of a capital injection**

In our model, a capital injection into leveraged institutions at time 0 is equivalent to an increase in the mass of leveraged institutions  $\theta$ . So the effects of such an injection are the opposite from those of deleveraging. This policy does boost the stock market index (Figure 3(c)) because a capital injection increases the demand of the institutions for the risky stock and they purchase more shares of it (Figure 3(c)). As a result of the stock price increase, everybody in the economy, including retail investors, becomes wealthier. But along with the run-up in the stock market, comes an increase in the leverage of institutional investors (Figure 3(a)). When the institutions do not control a dominant fraction of the financial wealth in the economy ( $\theta \ll 1$ ), the stock price volatility also increases (Figure 3(d)). These side effects could be undesirable.

## **4. Multiple Stocks, Asset Classes, and Correlations**

Our analysis has so far been presented in the context of a single-stock economy. Our goal in this section is to demonstrate how our results generalize in a multi-stock economy and to examine the correlations among stock returns. For the latter, we aim to demonstrate how institutional investors in our model generate an “asset class” effect—i.e., how they make returns of stocks belonging to an index to be more correlated amongst themselves than with those of otherwise identical stocks outside the index. This effect has been documented in the data and has strengthened with the growth of the institutional money management.<sup>8</sup>

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<sup>8</sup>See Barberis, Shleifer, and Wurgler (2005) for evidence on stocks belonging to the S&P 500 index and Boyer (2007) for stocks belonging to BARRA value and growth indices.

## 4.1. Economic Setup

The general version of our economy features  $N$  risky stocks and  $N$  sources of risk, generated by a standard  $N$ -dimensional Brownian motion  $\omega = (\omega_1, \dots, \omega_N)^\top$ . Each stock price,  $S_j$ ,  $j = 1, \dots, N$ , is posited to have dynamics

$$dS_{jt} = S_{jt}[\mu_{S_{jt}}dt + \sigma_{S_{jt}}d\omega_t], \quad (16)$$

where the vector of stock mean returns  $\mu_S \equiv (\mu_{S_1}, \dots, \mu_{S_N})^\top$  and the stock volatility matrix  $\sigma_S \equiv \{\sigma_{S_{j\ell}}, j, \ell = 1, \dots, N\}$  are to be determined in equilibrium. The (instantaneous) correlation between stock  $j$  and  $\ell$  returns,  $\rho_{j\ell t} \equiv \sigma_{S_{jt}}^\top \sigma_{S_{\ell t}} / \sqrt{|\sigma_{S_{jt}}|^2 |\sigma_{S_{\ell t}}|^2}$ , is also to be endogenously determined.<sup>9</sup> The value of the equity *market portfolio*,  $S_{MKT}$ , is the sum of the risky stock prices:

$$S_{MKT t} = \sum_{j=1}^N S_{jt}, \quad (17)$$

with posited dynamics

$$dS_{MKT t} = S_{MKT t}[\mu_{MKT t} dt + \sigma_{MKT t} d\omega_t]. \quad (18)$$

Additionally, there is a value-weighted *index* (in terms of returns) made up of the first  $M$  stocks in the economy:

$$I_t = \frac{1}{M} \sum_{i=1}^M S_{it}.$$

The stock index  $I$  represents a specific asset class in the economy, and we will refer to the first  $M$  stocks as “index stocks” and the remainder  $N - M$  stocks as the nonindex stocks.

Each stock is in positive net supply of one share. Its terminal payoff (or dividend)  $D_{jT}$ , due at time  $T$ , is determined by the process

$$dD_{jt} = D_{jt}[\mu_j dt + \sigma_j d\omega_t], \quad (19)$$

where  $\mu_j$  and  $\sigma_j > 0$  are constant for all stocks except for the last ones in the index and the market (the  $M^{\text{th}}$  and  $N^{\text{th}}$  stocks).<sup>10</sup> The process  $D_{jt}$  represents the cash flow news about the terminal

<sup>9</sup>The notation  $||z||$  denotes the dot product  $z \cdot z$ .

<sup>10</sup>That is, we do not explicitly specify the process of the cash flow news for the last stock in the index and in the market; but, in what follows, we specify processes for the sums of all stocks in the index and in the market. This modeling device is inspired by Menzly, Santos, and Veronesi (2004). It allows us to assume that the stock market and the index cash flow news follow geometric Brownian motion processes (equations (20) and (21)), which improves the tractability of the model considerably. In related analysis, we find that one may alternatively not assume a geometric Brownian motion process for the index cash flow news, but instead assume that stock  $M$ 's dividend follows a geometric Brownian motion process. In that case, the analogs of the expressions that we report below are less elegant, and several results can be obtained only numerically.

stock dividend  $D_{jT}$ , and  $S_{jT} = D_{jT}$ . For the thought experiment that we are going to undertake in this section, it is convenient to assume that the stocks' fundamentals (dividends) are independent. We thus assume that only the  $j$ th element of  $\sigma_j$  in (19) is nonzero, while all other elements are zero, so that the volatility matrix of cash flow news is diagonal. This implies zero correlation among all stocks' cash flow news,  $\sigma_j^\top \sigma_\ell = 0$  for all  $j \neq \ell$ .

The stock market has a terminal payoff  $S_{MKT T} = D_T$ , given by the terminal value of the process

$$dD_t = D_t[\mu dt + \sigma d\omega_t], \quad (20)$$

where  $\mu$  and  $\sigma > 0$  are constant. Similarly, the index has a terminal value  $I_T$ , determined by the process

$$dI_t = I_t[\mu_I dt + \sigma_I d\omega_t], \quad (21)$$

with  $\mu_I, \sigma_I > 0$  constant and with  $\sigma_I$  having its first  $M$  components non-zero and the remainder  $N - M$  components zero. The latter assumption is to make  $\sigma_I$  consistent with our assumptions about the individual stocks' cash flow news processes. Accordingly, while the index stocks' cash flow news have positive correlation with that of the index,  $\sigma_j^\top \sigma_I > 0$ ,  $j = 1, \dots, M$ , the cash flow news of the nonindex stocks have zero correlation,  $\sigma_k^\top \sigma_I > 0$ ,  $k = M + 1, \dots, N$ .

Each type of investor  $i = I, R$  now dynamically chooses a multi-dimensional portfolio process  $\phi_i$ , where  $\phi_i = (\phi_{i1}, \dots, \phi_{iN})^\top$  denotes the portfolio weights in each risky stock. The portfolio value  $W_i$  then has the dynamics

$$dW_{it} = W_{it} \phi_{it}^\top [\mu_{st} dt + \sigma_{st} d\omega_t]. \quad (22)$$

The retail investor is initially endowed with  $1 - \lambda$  fraction of the stock market, providing initial assets  $W_{R0} = (1 - \lambda)S_{MKT0}$ , and has the same objective function as in the single-stock case:  $u_R(W_{RT}) = \log(W_{RT})$ . The institutional investor is initially endowed with  $\lambda$  fraction of the stock market and hence has initial assets worth  $W_{I0} = \lambda S_{MKT0}$ . In this multi-stock version of our economy, the objective function of the institution is given by

$$u_I(W_{IT}) = (a + bI_T) \log(W_{IT}), \quad (23)$$

where  $a, b > 0$  and  $I_T$  is the terminal value of the index (composed of the first  $M$  stocks in the economy). Here, the institutional investor has a benchmark that is distinct from the overall stock market. He now strives to perform particularly well when a specific asset class, represented by the index  $I$ , does well. One can think of this asset class as value stocks, technology stocks, or the stocks included in the S&P 500 index.

## 4.2. Investors' Portfolio Choice

We are now ready to examine how the results derived in the earlier analysis extend to the multi-stock case. We start with Lemma 2, which reports the investors' optimal portfolios in closed form.

**Lemma 2.** *The institutional and retail investors' optimal portfolio processes are given by*

$$\phi_{It} = (\sigma_{st}\sigma_{st}^\top)^{-1}\mu_{st} + \frac{b e^{\mu_I(T-t)}I_t}{a + b e^{\mu_I(T-t)}I_t}(\sigma_{st}^\top)^{-1}\sigma_I, \quad (24)$$

$$\phi_{Rt} = (\sigma_{st}\sigma_{st}^\top)^{-1}\mu_{st}. \quad (25)$$

Moreover,

- (i) *The institutional investor's hedging portfolio, the second term in (24), has positive holdings in the index stocks but zero holdings in the nonindex stocks in equilibrium;*
- (ii) *The institutional investor invests a higher fraction of wealth in the index stocks than the retail investor, while holding the same fractions in the nonindex stocks as the retail investor.*

The investors' portfolios in (24)–(25) are natural multi-stock generalizations of the single-stock case. Again, the institutional investor holds the mean-variance efficient portfolio plus an additional portfolio hedging her against fluctuations in her index. In our single-stock economy, the hedging demand of the institutional investor generates a tilt in her portfolio towards the risky stock, as compared to the retail investor. The multi-stock economy refines this implication. It is not the case that the institutional investor simply desires to take on more risk; rather, she demands a portfolio that is highly correlated with her index. This is why she has the same demand for the nonindex stocks as the retail investor, but demands additional holdings of index stocks, so as to not fall behind when the index does well. As we will see shortly, this excess demand for index stocks by the institution is the key driver of the index effect in our model.

From Lemma 2, we also see that the institution's optimal portfolio satisfies a three-fund separation property, with the three funds being the mean-variance efficient portfolio, the intertemporal hedging portfolio, and the riskless bond. The importance of this decomposition will become apparent later, when we discuss the asset class effect in Section 4.4. For now, we just note that the hedging portfolio has positive holdings of the index stocks, and when the institution gets wealthier—following for example, good cash flow news—she demands more shares of the index stocks (a wealth effect). This additional price pressure (beyond the standard increase in demand for the mean-variance portfolio) is applied to all index stocks simultaneously. There is no additional demand for the nonindex stocks.

Our implications for the higher risk-taking by institutions, who take on leverage in order to finance the hedging portfolio, remain the same as in our earlier analysis. We do not repeat them here and proceed to exploring the additional insights that a multiple stock environment is able to offer.

### 4.3. Stock Prices and Index Effect

Proposition 5 reports the equilibrium stock prices in closed form and highlights the effects of institutions on stock prices.

**Proposition 5.** *In the economy with institutional investors and multiple risky stocks, the equilibrium prices of the market portfolio, index stocks  $j = 1, \dots, M - 1$  and non-index stocks  $k = M + 1, \dots, N - 1$  are given by*

$$S_{MKTt} = \bar{S}_{MKTt} \frac{a + b e^{\mu_I T} I_0 + \lambda b (e^{\mu_I (T-t)} I_t - e^{\mu_I T} I_0)}{a + b e^{\mu_I T} I_0 + \lambda b (e^{(\mu_I - \sigma_I^\top \sigma)(T-t)} I_t - e^{\mu_I T} I_0)}, \quad (26)$$

$$S_{jt} = \bar{S}_{jt} \frac{a + b e^{\mu_I T} I_0 + \lambda b (e^{(\mu_I - \sigma_I^\top \sigma + \sigma_j^\top \sigma_I)(T-t)} I_t - e^{\mu_I T} I_0)}{a + b e^{\mu_I T} I_0 + \lambda b (e^{(\mu_I - \sigma_I^\top \sigma)(T-t)} I_t - e^{\mu_I T} I_0)}, \quad (27)$$

$$S_{kt} = \bar{S}_{kt}, \quad (28)$$

where  $\bar{S}_{MKTt}$ ,  $\bar{S}_{jt}$ , and  $\bar{S}_{kt}$  are the equilibrium prices of the market portfolio, index and nonindex stocks, respectively, in the benchmark economy with no institutions, given by

$$\bar{S}_{MKTt} = e^{(\mu - \|\sigma\|^2)(T-t)} D_t, \quad \bar{S}_{jt} = e^{(\mu_j - \sigma_j^\top \sigma)(T-t)} D_{jt}, \quad \bar{S}_{kt} = e^{(\mu_k - \sigma_k^\top \sigma)(T-t)} D_{kt}. \quad (29)$$

Consequently, the market portfolio and index stock prices are increased in the presence of institutional investors, while nonindex stock prices are unaffected.

Proposition 5 generalizes our earlier discussion in the single-stock case and underscores the index effect occurring in our model. The direction of the effect is as before—the price pressure from the institutions raises the level of the index relative to that in the economy with no institutions. But now we can also make cross-sectional statements. If a stock  $j$  is added to the index  $I$  and a stock  $k$  is dropped, the price of stock  $j$  gets a boost, while that of stock  $k$  falls. This is precisely the empirical regularity that is robustly documented in the data. In our model, however, we cannot make finer predictions which separate announcement-date returns and inclusion-date returns; our results concern only the announcement date. To generate inclusion-date abnormal returns, one could introduce passive indexers who buy at the inclusion date.

Figure 4 presents a plot of the price of an index stock relative to that of an otherwise identical nonindex stock. The plot is drawn as a function of the size of institutions  $\lambda$ . As expected, we see that the stock price is increasing with  $\lambda$ . This is due to the additional price pressure on index stocks as the institutional sector becomes larger. We also observe that the magnitudes are reasonable for our calibration. Chen, Noronha, and Singal (2004) find that during 1989-2000, a stock's price increases by an average of 5.45% on the day of the S&P 500 inclusion announcement and a further 3.45% between the announcement and the actual addition. The effects that we find are smaller, but roughly in line with these figures.

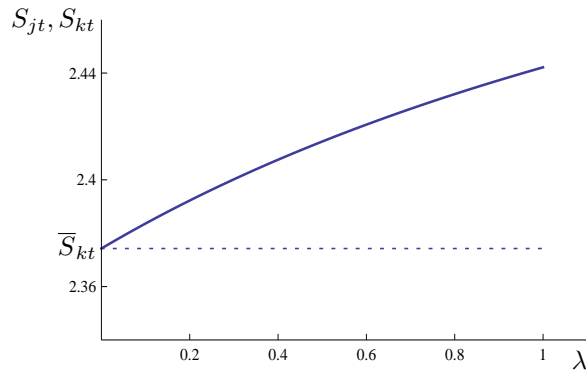


Figure 4: **An index effect.** This figure plots the prices of an index stock  $S_j$  (solid line) and a nonindex stock  $S_k$  (dotted line) in the presence of institutions against the fraction of institutions in the economy  $\lambda$ . The plot is typical. The parameter values are:  $M = 5$ ,  $N = 10$ ,  $j = 1$ ,  $k = 6$ ,  $\mu_I = \mu_j = \mu_k = 0.05$ ,  $\sigma_j = 0.15 i_j$ , where  $i_j$  is an  $N$ -dimensional unit vector with the  $j^{\text{th}}$  element equal to 1 and the remaining elements equal to 0,  $\sigma_k = 0.15 i_k$ ,  $\sigma_I = 0.15 \sum_{j=1}^M i_j / \sqrt{M}$ ,  $\sigma = 0.15 \times \mathbf{1} / \sqrt{N}$ , where  $\mathbf{1}$  is an  $N$ -dimensional vector of ones,  $I_0 = 1$ , and  $I_t = 2$ . The normalizations by  $\sqrt{M}$  and  $\sqrt{N}$  are adopted so as to keep  $\|\sigma_I\|$  and  $\|\sigma\|$  constant as we vary the number of stocks. The remaining parameters are as in Figure 1.

#### 4.4. Stock Volatilities, Correlations, and Asset Class Effects

We now turn to examining the implications of our model for stock return volatilities and correlations. We report them in the following proposition in closed form.

**Proposition 6.** *In the economy with institutional investors and multiple risky stocks, the equilibrium volatilities of the market portfolio, index stocks  $j = 1, \dots, M - 1$ , and nonindex stocks*

$k = M + 1, \dots, N - 1$  are given by

$$\sigma_{MKTt} = \bar{\sigma}_{MKTt} + \lambda b \sigma_I \frac{\left(1 - e^{-\sigma^\top \sigma_I (T-t)}\right) (a + (1 - \lambda) b e^{\mu_I T} I_0) e^{\mu_I (T-t)} I_t}{\left(a + (1 - \lambda) b e^{\mu_I T} I_0 + \lambda b e^{(\mu_I - \sigma_I^\top \sigma)(T-t)} I_t\right) \left(a + (1 - \lambda) b e^{\mu_I T} I_0 + \lambda b e^{\mu_I (T-t)} I_t\right)}, \quad (30)$$

$$\sigma_{S_j t} = \bar{\sigma}_{S_j t} + \lambda b \sigma_I$$

$$\times \frac{\left(1 - e^{-\sigma_j^\top \sigma_I (T-t)}\right) (a + (1 - \lambda) b e^{\mu_I T} I_0) e^{(\mu_I - \sigma_I^\top \sigma + \sigma_j^\top \sigma_I)(T-t)} I_t}{\left(a + (1 - \lambda) b e^{\mu_I T} I_0 + \lambda b e^{(\mu_I - \sigma_I^\top \sigma)(T-t)} I_t\right) \left(a + (1 - \lambda) b e^{\mu_I T} I_0 + \lambda b e^{(\mu_I - \sigma_I^\top \sigma + \sigma_j^\top \sigma_I)(T-t)} I_t\right)}, \quad (31)$$

$$\sigma_{S_k t} = \bar{\sigma}_{S_k t}, \quad (32)$$

where  $\bar{\sigma}_{MKTt}$ ,  $\bar{\sigma}_{S_j t}$ , and  $\bar{\sigma}_{S_k t}$  are the equilibrium market portfolio, index stock, and nonindex stock volatilities, respectively, in the benchmark economy with no institutions, given by

$$\bar{\sigma}_{MKTt} = \sigma, \quad \bar{\sigma}_{S_j t} = \sigma_j, \quad \bar{\sigma}_{S_k t} = \sigma_k. \quad (33)$$

Consequently, in equilibrium:

- (i) The market portfolio and index stock volatilities are increased in the presence of institutional investors, while nonindex stock volatilities are unaffected;
- (ii) The correlations between index stocks are increased in the presence of institutional investors, while the correlations between nonindex stocks and between index and nonindex stocks are unaffected.

As one could expect from our earlier analysis, only the volatilities of the index stocks change in the presence of institutions; the volatilities of the nonindex stocks remain unchanged. The index stocks become riskier for the same reason as in the single-stock economy: the risk appetite of the aggregate investor in the economy is higher in the presence of institutional investors.

The multiple stock formulation offers additional insights, allowing us to explore how the presence of institutions affects the correlations of stock returns. These results, based on fully analytical closed-form expressions, are reported in Proposition 6. Consistent with the empirical evidence on asset class effects, we find that the presence of institutions increases the correlations among the stocks included in their index. The intuition is as follows. In the benchmark retail-investor-only economy, the cash flow news on all stocks are independent, and the stock returns turn out to be independent as well. Now consider the economy with institutions. As we have established in the single-stock case in Section 3.2, following a good realization of cash flow news, institutions demand

more shares of the index. This is simply a wealth effect. In the multi-stock case, the institutions demand more shares of *all* index stocks. This is a consequence of the three-fund separation property, discussed in the context of Lemma 2. It is important to keep in mind that the additional price pressure affects all index stocks, but not the nonindex stocks because the third fund, the hedging portfolio, consists only of index stocks. Hence, as compared to the retail-investor-only benchmark, following good cash flow news, all index stocks get an additional boost and following bad news, they all suffer from an additional selling pressure. This mechanism induces the comovement between index stocks, absent in the retail-investor-only benchmark. The correlation between the nonindex stocks is still zero, as in the benchmark, because these stocks are not part of the hedging portfolio of institutions, and so there is no additional buying or selling pressure on these stocks relative to the benchmark. The same is true for the correlations between the index and nonindex stocks. Figure 5 illustrates these effects. So summing this up, consistent with the empirical evidence, the returns of stocks belonging to an index to be more correlated amongst themselves than with those of otherwise identical stocks outside the index.

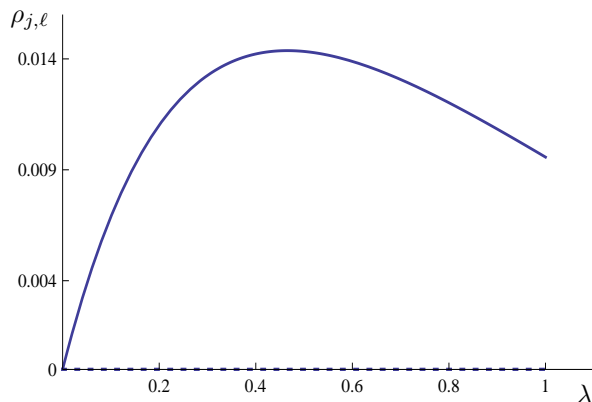


Figure 5: **An asset class effect.** This figure plots the correlation between two index stock returns (solid plot) and the correlation between two nonindex stock returns (dashed line) in the presence of institutions against the fraction of institutions in the economy  $\lambda$ . The plots is typical. The two index stocks are stocks 1 and 2 and the two nonindex stocks are stocks  $M + 1$  and  $M + 2$ . All these four stocks  $j$  have a drift  $\mu_j = 0.05$  and the diffusion  $\sigma_j = 0.15i_j$ . The remaining parameter values are as in Figure 4.

## 5. Concluding Remarks

Institutions and the incentives they face feature prominently in models of corporate finance and banking, but they have largely been ignored in the standard asset pricing theory. We believe that

establishing a role for institutional investors is an important avenue to explore in future asset pricing models, and especially the models that aim to analyze the recent 2007-2008 financial crisis. In this paper, we take a step in this direction by focusing on the incentives of some investors, interpreted as institutional investors, to do well relative to their index. One reason why professional managers may strive to do well relative to a given index is the prospect of receiving fund flows into their business. We demonstrate that this simple ingredient of our model has profound implications for asset prices. For example, it generates index effects and creates excess correlations among stocks belonging to an index (an asset class effect). We also demonstrate that the incentive to do well vis-à-vis an index induces institutional investors to tilt their portfolios towards the index stocks and makes them hold leveraged portfolios, borrowing from the retail sector. We link the amount of leverage in the economy to the size of assets under management by the institutions and evaluate policy recommendations aimed at limiting institutional leverage.

In this paper, we have not explored the ability of our model to generate momentum of stock returns and stock price bubbles. Recently, the link between institutional fund flows and momentum has been established theoretically by Vayanos and Woolley (2009) and empirically by Lou (2009). The explanation in Vayanos and Woolley relies additionally on delayed reaction of traders; it would be interesting to see whether our model can also generate momentum and whether one needs to assume further that traders cannot immediately rebalance. Stock price bubbles is another interesting phenomenon that could possibly be attributed to fund flows in the money management business. The financial press has argued that the recent NASDAQ bubble was fueled by inflows of new money into technology funds following the strong performance of the NASDAQ index. The argument was that the funds were investing this new money into technology stocks, propping up their prices further. Exploring such an institutional explanation of bubbles is another fruitful avenue for future research.

In our analysis, we have adopted a reduced-form approach of modeling the institutional incentives: the incentive to do well vis-à-vis an index is given exogenously. In future work, it would be desirable to provide microfoundations for this assumption and to endogenize the reward for good performance relative to an index (as in, for example, Berk and Green (2004)).

## Appendix

**Proof of Lemma 1.** Since the securities market in our setup is dynamically complete, it is well known that there exists a state price density process,  $\xi$ , such that the time- $t$  value of a payoff  $C_T$  at time  $T$  is given by  $E_t[\xi_T C_T]/\xi_t$ . In our setting, the state price density is a martingale and follows the dynamics

$$d\xi_t = -\xi_t \kappa_t dw_t, \quad (\text{A1})$$

where  $\kappa_t \equiv \mu_{S_t}/\sigma_{S_t}$  is the Sharpe ratio process. Accordingly, investor  $i$ 's dynamic budget constraint (3) can be restated as

$$E_t[\xi_T W_{iT}] = \xi_t W_{it}. \quad (\text{A2})$$

Maximizing the institutional investor's expected objective function (5) subject to (A2) evaluated at time  $t = 0$  leads to the institution's optimal terminal wealth as

$$W_{IT} = \frac{a + bD_T}{y_I \xi_T},$$

where  $1/y_I$  solves (A2) evaluated at  $t = 0$ . Using the fact that  $D_t$  is lognormally distributed for all  $t$ , we obtain

$$\frac{1}{y_I} = \frac{\lambda \xi_0 S_0}{a + b e^{\mu T} D_0}.$$

Consequently, the institution's optimal terminal wealth in terms of primitive is given by

$$W_{IT} = \frac{\lambda \xi_0 S_0}{\xi_T} \frac{a + bD_T}{a + b e^{\mu T} D_0}, \quad (\text{A3})$$

and from (A1) its optimal time- $t$  wealth by

$$\xi_t W_{It} = \lambda \xi_0 S_0 \frac{a + b e^{\mu(T-t)} D_t}{a + b e^{\mu T} D_0}. \quad (\text{A4})$$

Applying Itô's lemma to both sides of (A4), and using (3) and (A1), leads to

$$\xi_t W_{It} (\phi_{It} \sigma_{S_t} - \kappa_t) dw_t = \lambda \xi_0 S_0 \frac{b e^{\mu(T-t)} D_t}{a + b e^{\mu T} D_0} \sigma dw_t,$$

which after matching the diffusion terms and rearranging gives the institutional investor's optimal portfolio (6). Similarly, we obtain the retail investor's optimal terminal and time- $t$  wealth as

$$W_{RT} = \frac{(1 - \lambda) \xi_0 S_0}{\xi_T}, \quad (\text{A5})$$

$$\xi_t W_{Rt} = (1 - \lambda) \xi_0 S_0. \quad (\text{A6})$$

Application of Itô's lemma leads to the standard retail investor's optimal portfolio in (7). *Q.E.D.*

**Proof of Proposition 1.** By no arbitrage, the stock market price in this complete market setup is given by

$$S_t = \frac{E_t[\xi_T D_T]}{\xi_t}. \quad (\text{A7})$$

We proceed by first determining the equilibrium state price density process  $\xi$ . Imposing the market clearing condition  $W_{RT} + W_{IT} = D_T$ , and substituting (A3) and (A5) yields

$$\left( \lambda \frac{a + bD_T}{a + be^{\mu T} D_0} + (1 - \lambda) \right) \frac{\xi_0 S_0}{\xi_T} = D_T,$$

which after rearranging leads to the equilibrium terminal state price density:

$$\xi_T = \frac{\xi_0 S_0}{a + be^{\mu T} D_0} \frac{1}{D_T} (a + be^{\mu T} D_0 + \lambda b (D_T - e^{\mu T} D_0)). \quad (\text{A8})$$

Consequently, the equilibrium state price density at time  $t$  is given by

$$\begin{aligned} \xi_t &= E_t[\xi_T] \\ &= \frac{\xi_0 S_0}{a + be^{\mu T} D_0} E_t[1/D_T] (a + be^{\mu T} D_0 + \lambda b (1/E_t[1/D_T] - e^{\mu T} D_0)) \\ &= \frac{\xi_0 S_0}{a + be^{\mu T} D_0} \frac{e^{(-\mu + \sigma^2)(T-t)}}{D_t} \left( a + be^{\mu T} D_0 + \lambda b \left( e^{(-\mu + \sigma^2)(T-t)} D_t - e^{\mu T} D_0 \right) \right), \end{aligned} \quad (\text{A9})$$

where the last equality employs the fact that  $D_t$  is lognormally distributed.

Finally, to determine the equilibrium stock market level, we substitute (A8)–(A9) into (A7) and manipulate to obtain the stated expression (8). The stock market level  $\bar{S}$  in the benchmark economy with no institutions (9) follows by considering the special case of  $a = 1, b = 0$  in (8). The property that the stock market is higher in the presence of institutions follows from the fact that the factor multiplying  $\bar{S}_t$  in expression (8) is strictly positive, and being increasing in  $\lambda$  from the fact that the numerator in that factor is increasing at a faster rate than the denominator does in  $\lambda$ . *Q.E.D.*

**Proof of Proposition 2.** We write the equilibrium stock price in (8) as

$$S_t = \bar{S}_t \frac{X_t}{Z_t}, \quad (\text{A10})$$

where

$$\begin{aligned} \bar{S}_t &= e^{(\mu - \sigma^2)(T-t)} D_t, \\ X_t &= a + be^{\mu T} D_0 + \lambda b \left( e^{\mu(T-t)} D_t - e^{\mu T} D_0 \right), \\ Z_t &= a + be^{\mu T} D_0 + \lambda b \left( e^{(\mu - \sigma^2)(T-t)} D_t - e^{\mu T} D_0 \right). \end{aligned}$$

Applying Itô's lemma to (A10) we obtain

$$\sigma_{S_t} = \sigma + \sigma_{X_t} - \sigma_{Z_t}, \quad (\text{A11})$$

where

$$\begin{aligned} \sigma_{X_t} &= \frac{\lambda b e^{\mu(T-t)} D_t}{a + b e^{\mu T} D_0 + \lambda b (e^{\mu(T-t)} D_t - e^{\mu T} D_0)} \sigma, \\ \sigma_{Z_t} &= \frac{\lambda b e^{(\mu-\sigma^2)(T-t)} D_t}{a + b e^{\mu T} D_0 + \lambda b (e^{(\mu-\sigma^2)(T-t)} D_t - e^{\mu T} D_0)} \sigma. \end{aligned}$$

We note that  $X_t \sigma_{X_t} = \lambda b e^{\mu(T-t)} D_t \sigma$ ,  $Z_t \sigma_{Z_t} = \lambda b e^{(\mu-\sigma^2)(T-t)} D_t \sigma$ , and so  $X_t \sigma_{X_t} = Z_t \sigma_{Z_t} e^{-\sigma^2(T-t)}$ . Hence, we have

$$X_t \sigma_{X_t} Z_t - Z_t \sigma_{Z_t} X_t = X_t \sigma_{X_t} (1 - e^{-\sigma^2(T-t)}) (a + (1 - \lambda) b e^{\mu T} D_0). \quad (\text{A12})$$

Substituting (A12) into the expression  $\sigma_{X_t} - \sigma_{Z_t} = (X_t \sigma_{X_t} Z_t - Z_t \sigma_{Z_t} X_t) / X_t Z_t$ , and then into (A11) leads to the equilibrium stock index volatility expression in (10). The property that the stock volatility is higher than the volatility in the benchmark with no institutions is immediate since  $\sigma_{X_t} - \sigma_{Z_t} > 0$ . *Q.E.D.*

**Proof of Proposition 3.** We first determine the investors' equilibrium fractions of wealth invested in the stock index,  $\phi_{it}$ ,  $i = I, R$ . From (A7) and (A8) we have

$$\begin{aligned} \xi_t S_t &= E_t [\xi_T D_T] \\ &= \frac{\xi_0 S_0}{a + b e^{\mu T} D_0} (a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t). \end{aligned} \quad (\text{A13})$$

Applying Itô's lemma to both sides of (A13), we obtain

$$\sigma_{S_t} - \kappa_t = \frac{\lambda b e^{\mu(T-t)} D_t}{a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t} \sigma,$$

or

$$\frac{\kappa_t}{\sigma_{S_t}} = 1 - \frac{\lambda b e^{\mu(T-t)} D_t}{a + (1 - \lambda) b e^{\mu T} D_0 + \lambda b e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{S_t}}, \quad (\text{A14})$$

where  $\sigma_{S_t}$  is as given in Proposition 2. Substituting (A14) into the investors' optimal portfolios (6)–(7) in Lemma 1 yields their equilibrium portfolios  $\phi_{it}$ .

Next, we determine the investors' wealth per unit of the stock market level,  $W_{it}/S_t$ , in equilibrium. Substituting the deflated time- $t$  wealth of investors, (A4) and (A6), along with the deflated stock market level (A13), we obtain

$$\frac{W_{It}}{S_t} = \frac{\xi_t W_{It}}{\xi_t S_t} = \lambda \frac{a + b e^{\mu(T-t)} D_t}{a + b e^{\mu T} D_0 + \lambda b (e^{\mu(T-t)} D_t - e^{\mu T} D_0)}, \quad (\text{A15})$$

$$\frac{W_{Rt}}{S_t} = \frac{\xi_t W_{Rt}}{\xi_t S_t} = (1 - \lambda) \frac{a + b e^{\mu T} D_0}{a + b e^{\mu T} D_0 + \lambda b (e^{\mu(T-t)} D_t - e^{\mu T} D_0)}. \quad (\text{A16})$$

As a remark, we here note that the ratio of the two investors' wealth in equilibrium is given by substituting (A15) in (A16):

$$\frac{W_{It}}{W_{Rt}} = \frac{\lambda}{1-\lambda} \frac{a + be^{\mu(T-t)}D_t}{a + be^{\mu T}D_0}, \quad (\text{A17})$$

as highlighted in footnote (7). Finally, the investors' equilibrium portfolio weights  $\phi_{it}$  above, along with their equilibrium per unit of stock index level leads to their equilibrium strategies in units of shares  $\pi_{it}$ , as given by (11)–(12) in Proposition 3.

The leverage property follows by substituting (A14) into (6) and rearranging to get the fraction of wealth invested in the riskless bond as

$$1 - \phi_{It} = \frac{\lambda be^{\mu(T-t)}D_t}{a + (1-\lambda)be^{\mu T}D_0 + \lambda be^{\mu(T-t)}D_t} \frac{\sigma}{\sigma_{St}} - \frac{be^{\mu(T-t)}D_t}{a + be^{\mu(T-t)}D_t} \frac{\sigma}{\sigma_{St}} < 0.$$

*Q.E.D.*

**Proof of Proposition 4.** We first consider the investors' optimal choices in partial equilibrium. The retail investor's optimal terminal wealth and time- $t$  wealth are as in the proof of Lemma 1, given by (A5)–(A6). The “leveraged” institutional investor with initial wealth  $W_{I_0} = \theta\lambda S_0$  now chooses its optimal terminal wealth and time- $t$  wealth as

$$W_{IT} = \frac{\theta\lambda\xi_0 S_0}{\xi_T} \frac{a + bD_T}{a + be^{\mu T}D_0}, \quad (\text{A18})$$

$$\xi_t W_{It} = \theta\lambda\xi_0 S_0 \frac{a + be^{\mu(T-t)}D_t}{a + be^{\mu T}D_0}. \quad (\text{A19})$$

Both the levered institutional and retail investors' optimal portfolios are as before, given by (6)–(7) in Lemma 1.

Moving to general equilibrium, we first note that in the presence of the additional buy-and-hold institutional investor with initial assets  $W_{L_0} = (1-\theta)\lambda S_0$ , the market clearing condition is now:

$$W_{IT} + W_{RT} = (1 - (1-\theta)\lambda)D_T. \quad (\text{A20})$$

Substituting the investors' optimal terminal wealth (A5) and (A18) into (A20) and manipulating, we obtain the equilibrium terminal state price density as

$$\xi_T = \frac{\xi_0 S_0}{a + be^{\mu T}D_0} \frac{1}{(1 - (1-\theta)\lambda)D_T} \left( (1-\lambda)(a + be^{\mu T}D_0) + \theta\lambda(a + bD_T) \right). \quad (\text{A21})$$

Consequently, we get the equilibrium time- $t$  state price density, after some manipulation, as

$$\begin{aligned} \xi_t &= E_t[\xi_T] \\ &= \frac{\xi_0 S_0}{a + be^{\mu T}D_0} \frac{e^{(-\mu+\sigma^2)(T-t)}}{(1 - (1-\theta)\lambda)D_t} \left( (1-\lambda)(a + be^{\mu T}D_0) + \theta\lambda(a + be^{(\mu-\sigma^2)(T-t)}D_t) \right). \end{aligned} \quad (\text{A22})$$

From (A21), we may also derive the deflated stock price process as

$$\begin{aligned}\xi_t S_t &= E_t[\xi_T D_T] \\ &= \frac{\xi_0 S_0}{a + be^{\mu T} D_0} \frac{1}{(1 - (1 - \theta)\lambda)} \left( (1 - \lambda)(a + be^{\mu T} D_0) + \theta\lambda(a + be^{\mu(T-t)} D_t) \right).\end{aligned}\quad (\text{A23})$$

The equilibrium stock market index expression (13), then, follows by substituting (A22) to (A23). To determine the equilibrium stock volatility we proceed as in the proof of Proposition 3. Expressing the stock price as  $S_t \equiv \bar{S}_t X_t / Z_t$  and then applying Itô's lemma we obtain:

$$\sigma_{S_t} = \sigma + \sigma_{X_t} - \sigma_{Z_t},$$

where

$$\begin{aligned}\sigma_{X_t} &= \frac{\theta\lambda be^{\mu(T-t)} D_t}{(1 - \lambda)(a + be^{\mu T} D_0) + \theta\lambda(a + be^{\mu(T-t)} D_t)} \sigma, \\ \sigma_{Z_t} &= \frac{\theta\lambda be^{(\mu - \sigma^2)(T-t)} D_t}{(1 - \lambda)(a + be^{\mu T} D_0) + \theta\lambda(a + be^{(\mu - \sigma^2)(T-t)} D_t)} \sigma.\end{aligned}$$

Hence, we have  $\sigma_{Z_t} Z_t = \sigma_{X_t} X_t e^{-\sigma^2(T-t)}$ , implying after some manipulation

$$(\sigma_{X_t} - \sigma_{Z_t}) X_t Z_t = (1 - e^{-\sigma^2(T-t)}) \left( (1 - \lambda)(a + be^{\mu T} D_0) + \theta\lambda a \right) \sigma_{X_t} X_t,$$

leading to the stock index volatility expression (14).

Finally, to determine the levered institution's equilibrium portfolio, we apply Itô's lemma to both sides of (A23) and match coefficients to obtain

$$\frac{\kappa_t}{\sigma_{S_t}} = 1 - \frac{\theta\lambda be^{\mu(T-t)} D_t}{(1 - \lambda)(a + be^{\mu T} D_0) + \theta\lambda(a + be^{\mu(T-t)} D_t)} \frac{\sigma}{\sigma_{S_t}},\quad (\text{A24})$$

where  $\sigma_{S_t}$  is as in Proposition 4. Substituting (A24) into (6) in Lemma 1 yields the levered institution's equilibrium portfolio weight  $\phi_{I_t}$ . The levered institution's wealth per unit of stock price is found by substituting the deflated wealth (A19) and deflated stock (A23) processes:

$$\frac{W_{I_t}}{S_t} = \theta\lambda(1 - (1 - \theta)\lambda) \frac{a + be^{\mu(T-t)} D_t}{(1 - \lambda)(a + be^{\mu T} D_0) + \theta\lambda(a + be^{\mu(T-t)} D_t)}.\quad (\text{A25})$$

The levered institution's equilibrium weight along with (A25) leads to the equilibrium holdings of the index as reported in equation (15) of Proposition 4. *Q.E.D.*

**Proof of Lemma 2.** The securities market is still dynamically complete in this multi-stock setup with  $N$  risky stocks and  $N$  sources of risk. Hence, there exists a state price density process,  $\xi$ , which is a martingale and follows the dynamics

$$d\xi_t = -\xi_t \kappa_t^T dw_t,\quad (\text{A26})$$

where  $\kappa_t \equiv \sigma_{S_t}^{-1} \mu_{S_t}$  is the  $N$ -dimensional Sharpe ratio process.

Following the same steps as in the proof of Lemma 1, the single-stock case, and using the fact that the index cash flow news  $I$  is lognormally distributed, we obtain the institutional investor's optimal terminal wealth and time- $t$  wealth as

$$W_{IT} = \frac{\lambda \xi_0 S_{MKT0}}{\xi_T} \frac{a + bI_T}{a + be^{\mu_I T} I_0}, \quad (\text{A27})$$

$$\xi_t W_{It} = \lambda \xi_0 S_{MKT0} \frac{a + be^{\mu_I(T-t)} I_t}{a + be^{\mu_I T} I_0}. \quad (\text{A28})$$

Applying Itô's lemma to (A28) leads to

$$\xi_t W_{It} (\phi_{It}^T \sigma_{S_t} - \kappa_t^T) dw_t = \lambda \xi_0 S_{MKT0} \frac{be^{\mu_I(T-t)} I_t}{a + be^{\mu_I T} I_0} \sigma_I dw_t,$$

which after matching coefficients yields the institutional optimal portfolio as reported in (24). The retail investor's optimal terminal wealth and time- $t$  wealth are as in the single-stock case given by (A5) and (A6), which leads to the optimal portfolio in (25).

To prove property (i), consistent with the multi-stock equilibrium as reported in Proposition 6, we first represent the  $M \times N$  stock volatility matrix,  $\sigma_S$ , as:

$$\sigma_{S_t} = \begin{pmatrix} \sigma_{Mt} & 0 \\ 0 & \sigma_{Nt} \end{pmatrix},$$

where  $\sigma_{Mt}$  is the  $M \times M$  volatility matrix of index stocks,  $\sigma_{Nt}$  is the  $(N - M) \times (N - M)$  volatility matrix of nonindex stocks and two 0 matrices are  $(N - M) \times M$  and  $M \times (N - M)$  matrices with all elements zero. Hence we have

$$(\sigma_{S_t}^T)^{-1} = \begin{pmatrix} (\sigma_{Mt}^T)^{-1} & 0 \\ 0 & (\sigma_{Nt}^T)^{-1} \end{pmatrix}.$$

Using the fact that the first  $M$  elements of  $\sigma_I$  are non-zero and the remaining  $N - M$  elements are zero, multiplying out  $(\sigma_{S_t}^T)^{-1} \sigma_I$  we obtain the reported property (i). Property (ii) then follows immediately. *Q.E.D.*

**Proof of Proposition 5.** We first determine the equilibrium state price density process. Imposing the market clearing condition  $W_{IT} + W_{RT} = D_T$ , substituting (A27) and (A5), and manipulating yields the terminal equilibrium state price density as

$$\xi_T = \frac{\xi_0 S_{MKT0}}{a + be^{\mu_I T} I_0} \frac{1}{D_T} (a + be^{\mu_I T} I_0 + \lambda b(I_T - e^{\mu_I T} I_0)). \quad (\text{A29})$$

To obtain the time- $t$  equilibrium state price density, we use the properties of lognormal distribution  $E_t[1/D_T] = e^{(-\mu + \|\sigma\|^2)(T-t)}/D_t$ ,  $E_t[I_T/D_T] = e^{(\mu_I - \mu + \|\sigma\|^2 - \sigma_I^T \sigma)(T-t)} I_t/D_t$ , which along with

(A29) and some manipulations we get

$$\xi_t = \frac{\xi_0 S_{MKT0}}{a + be^{\mu_I T} I_0} \frac{e^{(-\mu + \|\sigma\|^2)(T-t)}}{D_t} \left( a + be^{\mu_I T} I_0 + \lambda b (e^{(\mu_I - \sigma_I^T \sigma)(T-t)} I_t - e^{\mu_I T} I_0) \right). \quad (\text{A30})$$

To determine the equilibrium market portfolio price, we first compute its deflated process from (A29) as, after some manipulation

$$\begin{aligned} \xi_t S_{MKTt} &= E_t[\xi_T D_T] \\ &= \frac{\xi_0 S_{MKT0}}{a + be^{\mu_I T} I_0} \left( a + be^{\mu_I T} I_0 + \lambda b (e^{\mu_I(T-t)} I_t - e^{\mu_I T} I_0) \right). \end{aligned} \quad (\text{A31})$$

Substituting (A30) into (A31) yields the market portfolio level as reported in (26). The price in the benchmark economy with no institution is obtained as a special case by setting  $a = 1, b = 0$ .

To determine the equilibrium price of an index stock  $j = 1, \dots, M - 1$ , we first find its deflated process:

$$\xi_t S_{jt} = E_t[\xi_T D_{jT}]. \quad (\text{A32})$$

From (A29), we have

$$\xi_T D_{jT} = \frac{\xi_0 S_{MKT0}}{a + be^{\mu_I T} I_0} \frac{D_{jT}}{D_T} \left( a + be^{\mu_I T} I_0 + \lambda b (I_T - e^{\mu_I T} I_0) \right). \quad (\text{A33})$$

After some manipulations and substitution of the properties of the properties of lognormally distributed processes

$$\begin{aligned} E_t \left[ \frac{D_{jT}}{D_T} \right] &= e^{(\mu_j - \mu + \|\sigma\|^2 - \sigma_j^T \sigma)(T-t)} \frac{D_{jt}}{D_t}, \\ E_t \left[ \frac{D_{jT} I_T}{D_T} \right] &= e^{(\mu_j + \mu_I - \mu + \sigma_j^T \sigma_I + \|\sigma\|^2 - \sigma_I^T \sigma - \sigma_j^T \sigma)(T-t)} \frac{D_{jt} I_t}{D_t}, \end{aligned}$$

we obtain

$$\begin{aligned} E_t[\xi_T D_{jT}] &= \frac{\xi_0 S_{MKT0}}{a + be^{\mu_I T} I_0} e^{(\mu_j - \mu + \|\sigma\|^2 - \sigma_j^T \sigma)(T-t)} \frac{D_{jt}}{D_t} \\ &\quad \times \left( a + be^{\mu_I T} I_0 + \lambda b (e^{(\mu_I - \sigma_I^T \sigma + \sigma_j^T \sigma_I)(T-t)} I_t - e^{\mu_I T} I_0) \right). \end{aligned} \quad (\text{A34})$$

Finally, substituting (A30) and (A34) into (A32), we obtain the equilibrium price of an index stock as reported in (27) of Proposition 5. The index stock price in the benchmark economy is obtained as a special case by setting  $a = 1, b = 0$ .

To determine the equilibrium price of a non-index stock  $k = M + 1, \dots, N - 1$ , we proceed as in the index stock case and obtain the same stock price equation (27) but now with the correlation with the index  $\sigma_k^T \sigma_I = 0$  substituted in. With this zero correlation, the nonindex stock price collapses to its value in the benchmark economy with no institutions. The stated property of higher market portfolio and index stock prices is immediate from the expressions (26)–(27). *Q.E.D.*

**Proof of Proposition 6.** To determine the equilibrium volatilities in this multi-stock case, we proceed as in Proposition 3. For the market portfolio, we express its equilibrium price as  $S_{MKT,t} \equiv \bar{S}_{MKT,t} X_t / Z_t$  and apply Itô's lemma to obtain

$$\sigma_{MKT,t} = \sigma + \sigma_{X_t} - \sigma_{Z_t},$$

where

$$\begin{aligned}\sigma_{X_t} &= \frac{\lambda b e^{\mu_I(T-t)} I_t}{a + b e^{\mu_I T} I_0 + \lambda b (e^{\mu_I(T-t)} I_t - e^{\mu_I T} I_0)} \sigma_I, \\ \sigma_{Z_t} &= \frac{\lambda b e^{(\mu_I - \sigma_I^T \sigma)(T-t)} I_t}{a + b e^{\mu_I T} I_0 + \lambda b (e^{(\mu_I - \sigma_I^T \sigma)(T-t)} I_t - e^{\mu_I T} I_0)} \sigma_I.\end{aligned}$$

So we have  $\sigma_{Z_t} Z_t = \sigma_{X_t} X_t e^{-\sigma_I^T \sigma (T-t)}$ , implying after some manipulation that

$$(\sigma_{X_t} - \sigma_{Z_t}) X_t Z_t = \lambda b (1 - e^{-\sigma_I^T \sigma (T-t)}) (a + (1 - \lambda) b e^{\mu_I T} I_0) e^{\mu_I(T-t)} I_t \sigma_I,$$

leading to the market portfolio volatility as reported in (30).

For the index stock volatility, analogously we express the equilibrium price of an index stock  $j = 1, \dots, M - 1$  as  $S_{jt} \equiv \bar{S}_{jt} X_{jt} / Z_{jt}$ . Applying Itô's lemma we obtain

$$\sigma_{S_{jt}} = \sigma_j + \sigma_{X_{jt}} - \sigma_{Z_{jt}},$$

where

$$\begin{aligned}\sigma_{X_{jt}} &= \frac{\lambda b e^{(\mu_I - \sigma_I^T \sigma + \sigma_j^T \sigma_I)(T-t)} I_t}{a + (1 - \lambda) b e^{\mu_I T} I_0 + \lambda b e^{(\mu_I - \sigma_I^T \sigma + \sigma_j^T \sigma_I)(T-t)} I_t} \sigma_I, \\ \sigma_{Z_{jt}} &= \frac{\lambda b e^{(\mu_I - \sigma_I^T \sigma)(T-t)} I_t}{a + (1 - \lambda) b e^{\mu_I T} I_0 + \lambda b e^{(\mu_I - \sigma_I^T \sigma)(T-t)} I_t} \sigma_I.\end{aligned}$$

hence, we have  $\sigma_{Z_{jt}} Z_{jt} = \sigma_{X_{jt}} X_{jt} e^{-\sigma_j^T \sigma_I (T-t)}$ , implying

$$(\sigma_{X_{jt}} - \sigma_{Z_{jt}}) X_{jt} Z_{jt} = \lambda b (1 - e^{-\sigma_j^T \sigma_I (T-t)}) (a + (1 - \lambda) b e^{\mu_I T} I_0) e^{(\mu_I - \sigma_I^T \sigma + \sigma_j^T \sigma_I)(T-t)} I_t \sigma_I,$$

leading to the market portfolio volatility as reported in (31).

The implications that the market portfolio and index stock volatilities are higher follow immediately from the expressions (30)–(31). As for the higher correlation property (ii) amongst index stocks, we need to show that for two index stocks  $j$  and  $l$

$$\frac{\sigma_{S_{jt}}^T \sigma_{S_{lt}}}{\sqrt{\|\sigma_{S_{jt}}\|^2 \|\sigma_{S_{lt}}\|^2}} > \frac{\bar{\sigma}_{S_{jt}}^T \bar{\sigma}_{S_{lt}}}{\sqrt{\|\bar{\sigma}_{S_{jt}}\|^2 \|\bar{\sigma}_{S_{lt}}\|^2}}.$$

Since  $\bar{\sigma}_{S_{jt}}^T \bar{\sigma}_{S_{jt}} = \bar{\sigma}_j^T \bar{\sigma}_l = 0$ , above is equivalent to showing  $\sigma_{S_{jt}}^T \sigma_{S_{jt}} > 0$ . From (25), for an index stock we have

$$\sigma_{S_{jt}} = \sigma_j + f_j(I_t)\sigma_I,$$

where  $f_j$  is some strictly positive function of  $I_t$  specific to stock  $j$ . Consequently, we have

$$\sigma_{S_{jt}}^T \sigma_{S_{jt}} = \sigma_j^T \sigma_j + f_j \sigma_j^T \sigma_I + f_l \sigma_l^T \sigma_I + f_j f_l \|\sigma_I\|^2 > 0, \quad (\text{A35})$$

proving the desired result. The correlation property regarding the nonindex stocks is obvious.

*Q.E.D.*

## References

- Arora, N., N. Ju, and H. Ou-Yang, 2006, "A Model of Asset Pricing under Portfolio Delegation and Differential Information," working paper, University of North Carolina.
- Barberis, N., and A. Shleifer, 2003, "Style investing," *Journal of Financial Economics*, 68(2), 161-199.
- Barberis, N., A. Shleifer, and J. Wurgler, 2005, "Comovement," *Journal of Financial Economics*, 75, 283-317.
- Basak, S., A. Pavlova, and A. Shapiro, 2007, "Optimal Asset Allocation and Risk Shifting in Money Management," *Review of Financial Studies*, 20(5), 1583-1621.
- Basak, S., A. Pavlova, and A. Shapiro, 2008, "Offsetting the Implicit Incentives: Benefits of Benchmarking in Money Management," *Journal of Banking and Finance*, 32(9), 1883-1893.
- Berk, J., and R. Green, 2004, "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 112, 1269-1295.
- Boyer, B., 2007, "Comovement among stocks with similar book-to-market ratios," working paper, Brigham Young University.
- Brennan, M. J., 1993, "Agency and Asset Pricing," working paper, UCLA.
- Brennan, M. J., and F. Li, 2008, "Agency and Asset Pricing," working paper, UCLA.
- Chen, H., G. Noronha, and V. Singal, 2004, "The Price Response to S&P 500 Index Additions and Deletions: Evidence of Asymmetry and a New Explanation," *Journal of Finance*, 59, 1901-29.
- Chen, H., and G. Pennacchi, 2009, "Does Prior Performance Affect a Mutual Fund's Choice of Risk?," *Journal of Financial and Quantitative Analysis*, 44, 745-775.
- Cornell, B., and R. Roll, 2005, "A Delegated-Agent Asset-Pricing Model," *Financial Analysts Journal*, 61, 57-69.
- Cuoco, D., and R. Kaniel, 2010, "Equilibrium Prices in the Presence of Delegated Portfolio Management," working paper, Duke University.
- Dasgupta, A., and A. Prat, 2008, "Information Aggregation in Financial Markets with Career Concerns," *Journal of Economic Theory*, 143, 83-113.
- Dasgupta, A., A. Prat, and M. Verardo, 2008, "The Price Impact of Institutional Herding," working paper, London School of Economics.
- Dybvig, P.H., H.K. Farnsworth, and J.N. Carpenter, 2010, "Portfolio Performance and Agency," *Review of Financial Studies*, 23, 1-23.
- Gomez, J., and F. Zapatero, 2003, "Asset Pricing Implications of Benchmarking: A Two-Factor CAPM," *European Journal of Finance*, 9, 343-57.
- Griffin, J.M., Harris, J.H., and S. Topaloglu, 2003, "The Dynamics of Institutional and Individual Trading," *Journal of Finance*, 58, 2285-2320.
- Guerrieri, V., and P. Kondor, 2010, "Fund Managers, Career Concerns, and Asset Price Volatility," working paper, Central European University.
- Harris L., and E. Gurel, 1986, "Price and Volume Effects Associated with Changes in the S&P 500 List: New Evidence for the Existence of Price Pressures," *Journal of Finance*, 41, 815-29.

- He, Z. and A. Krishnamurthy, 2008a, "Intermediary Asset Pricing," working paper, Northwestern University.
- He, Z. and A. Krishnamurthy, 2008b, "A Model of Capital and Crises," working paper, Northwestern University.
- Hodder, J.E., and J.C. Jackwerth, 2007, "Incentive Contracts and Hedge Fund Management," *Journal of Financial and Quantitative Analysis*, 42, 811-826.
- Kapur, S., and A. Timmermann, 2005, "Relative Performance Evaluation Contracts and Asset Market Equilibrium," *Economic Journal*, 115, 1077-1102.
- Karceski, J., 2002, "Returns-Chasing Behavior, Mutual Funds, and Beta's Death," *The Journal of Financial and Quantitative Analysis*, 37, 559-594.
- Kyle, A., and W. Xiong, 2001, "Contagion as a Wealth Effect," *Journal of Finance*, 56, 1401-1440.
- Leippold, M., and P. Rohner, 2008, "Equilibrium Implications of Delegated Asset Management under Benchmarking," working paper, University of Zurich.
- Lou, D., 2009, "A Flow-Based Explanation for Return Predictability," working paper, London School of Economics.
- Menzly, L., T. Santos, and P. Veronesi, 2004, "Understanding Predictability," *Journal of Political Economy*, 112, 1-47.
- Petajisto, A., 2009, "Why Do Demand Curves for Stocks Slope Down?," *Journal of Financial and Quantitative Analysis*, 44, 1013-1044.
- Ross, S., 2004, "Compensation, Incentives, and the Duality of Risk Aversion and Riskiness," *Journal of Finance*, 59, 207-25.
- Shleifer A., 1986, "Do Demand Curves for Stocks Slope Down?," *Journal of Finance*, 41, 579-90.
- Shleifer, A., and R. Vishny, 1997, "The Limits of Arbitrage," *Journal of Finance*, 52, 35-55.
- Vayanos, D., 2004, "Flight to Quality, Flight to Liquidity, and the Pricing of Risk," working paper, London School of Economics.
- Vayanos, D., and P. Woolley, 2010, "An Institutional Theory of Momentum and Reversal," working paper, London School of Economics.