Skewness in Stock Returns: Reconciling the Evidence on Firm versus Aggregate Returns*

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Abstract

Aggregate stock market returns display negative skewness. Firm-level stock returns display positive skewness. The large literature that tries to explain the first stylized fact ignores the second. This paper provides a unified theory that reconciles the two facts by explicitly modeling firm-level heterogeneity. I build a stationary asset pricing model of firm announcement events where firm returns display positive skewness. I then show that cross-sectional heterogeneity in firm announcement events can lead to negative skewness in aggregate returns. I provide evidence consistent with the model predictions.

Key words: Skewness, market returns, firm returns, announcement events, cross-sectional heterogeneity.

JEL Classifications: G12, G14, D82

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1 Introduction

Aggregate stock market returns display negative skewness, the propensity to generate negative returns with greater probability than suggested by a symmetric distribution. A large body of literature has aimed to explain this stylized fact about the distribution of aggregate stock returns (e.g., Fama, 1965, Black, 1976, Christie, 1982, Blanchard and Watson, 1982, Pindyck 1984, French et al., 1987, Hong and Stein, 2003). The evidence on aggregate returns contrasts with another stylized fact, namely, that firm-level returns are positively skewed. For this reason, theories of negative skewness that model single-firm stock markets necessarily depict an incomplete picture. In this paper I provide a unified theory for both stylized facts by explicitly modeling firm-level heterogeneity and present evidence consistent with the theory.

The paper develops a simple stationary asset pricing model of cash payout and earnings announcement events to capture the basic stylized facts on volatility and mean returns around such events. When cash payouts are periodic, cash flow news is discounted according to the time remaining until the next payout. The impact of news on the conditional return volatility is thus greater for news released closer to the payout. This gives rise to a pattern of increasing conditional return volatility, despite homoskedastic news. In addition, discounting also implies that the conditional return volatility increases at an increasing rate. The presence of a risk-return trade-off in the model implies that these properties apply to expected returns and induces positive skewness in conditional mean returns.

Next, consider earnings announcement events that do not coincide with a payout event. At an earnings announcement, the disclosure of new information, which can be used to update old signals, results in conditionally higher return volatility and mean returns. Firm-level returns may thus display sporadic and short-lived periods of high volatility and high mean returns consistent with positive skewness in conditional mean returns.

I show that the equilibrium unconditional distribution of stock returns is a mixture of normals distribution. Under a mixture of normals distribution, skewness in stock returns is given by two components. The first component is skewness in conditional mean returns. The second component captures the association between expected returns and conditional return variance and is positive given the risk-return trade-off imbedded in the model. With both terms positive, the model can generate positive skewness in firm-level stock returns.

To explain the apparent disconnect between firm-level return skewness and aggregate return skewness, consider a portfolio of firms that have positively skewed returns. Skewness of a portfolio return is the sum of firm-level return skewness and various *co-skewness* terms. Co-skewness terms are inherently cross-sectional terms, and loosely speaking, capture co-movement in one firm's conditional mean return with the conditional variance of the portfolio that comprises the remaining firms.[1] Hence, co-skewness terms are negative when on average a low return on one stock coincides with high return volatility in the portfolio of the remaining stocks. When co-skewness is sufficiently negative, the portfolio that consists of these firms has a larger probability of low outcomes than predicted by a symmetric distribution.

I assume that firms' announcement events occur at heterogeneous dates. When firms have different cash payout dates, the high mean return and return volatility of some firms around their event date contrasts with the low return volatility of the portfolio of the remaining firms. This generates negative co-skewness in the market portfolio. A "perfect storm" in the stock market is thus likely to occur during an "announcement season" in which a significant fraction of firms display high return volatility, while the rest display low expected returns and low return volatility.

The paper provides evidence consistent with the above model predictions. Using CRSP daily stock returns to compute skewness over six-month periods from 1973 to 2009, I first document that firm-level skewness is higher than aggregate skewness 96% of the time. Moreover, firm-level return skewness is always positive except once, in the second half of 1987, whereas market skewness is almost always negative. Bakshi et al. (2003) and Conrad et al. (2009) document a similar disconnect in ex-ante skewness in firm and portfolio returns.

The evidence that the cross-sectional dispersion in event dates can produce the correct sign for aggregate return co-skewness uses data on earnings announcement events. As in the model, earnings announcements are associated with brief periods of high volatility and high mean returns (Ball and Kothari, 1991, and Cohen et al., 2007). I use earnings announcement dates over the 1973 to 2009 period from the merged CRSP/Compustat quarterly file. I construct two experiments, both of which use daily return data over six-month periods. In the first experiment, I form portfolios of firms based on the calendar week of their first earnings announcement in each semester. I then group the firms in the first portfolio (first-week announcers) with those firms announcing k weeks later and report the six-month portfolio

[1] Intuitively, the co-skewness in a portfolio return skewness calculation resembles the covariance in a portfolio return variance calculation: A portfolio may have lower skewness than the mean skewness of its component stocks as it may also have lower variance than the mean variance of its component stocks.

return skewness. I show that, as in the model, there is a symmetric U-shaped pattern in skewness: The portfolio of firms that announce in weeks 1 and 2 has similar return skewness to the portfolio of firms that announce in weeks 13 and 1, and their skewness is higher than the skewness in any other portfolio configuration.

In the second experiment, I form portfolios of firms that announce in weeks 1 through k in the quarter for $k = 2, ...13$, and report the respective portfolio return skewness. This experiment constructs stock markets with announcement seasons. I show that, consistent with the model, there is a negative relationship between skewness and the increased heterogeneity that results from adding dispersion in event dates. I also show that portfolio skewness in the model can be negative if sufficient heterogeneity in event dates is allowed. The predictive power of this last result hinges on information flowing to the market in the form of announcement seasons. I show that firms in the U.S. tend to announce between weeks two and eight in each quarter, consistent with the existence of an earnings announcement season.

The beginning of an announcement season is also the period in the model that most contributes to the overall negative skewness in the market. Consistent with this model prediction, I split aggregate skewness into its weekly components and document that aggregate skewness is particularly negative around the beginning of an earnings announcement season. Finally, I compare the level of co-skewness across industries with varying degrees of cross-sectional variation in earnings announcement dates. The evidence suggests that industries with greater dispersion of earnings announcement dates have more negative co-skewness.

An alternative explanation for why market skewness differs in sign from firm skewness is the existence of a negatively skewed return factor (Duffee, 1995). Following Duffee (1995), I remove the market return–a negatively skewed factor–from firm returns to obtain "idiosyncratic" returns. I show that while some results are weaker when CAPM-based idiosyncratic returns are used, the evidence is still broadly consistent with the model. Ideally, the use of structural models that nest various theories of negative aggregate skewness can provide for more statistically powerful identification strategies.

The model in this paper is consistent with the evidence from dividend and earnings announcements. Aharony and Swary (1980), Kalay and Loewenstein (1985), and Amihud and Li (2006) show that dividend announcements are associated with high returns and high volatility of stock returns. Ball and Kothari (1991) and Cohen et al. (2007) show that the high expected returns around earnings announcements are also associated with high volatility. In addition, in this paper, the increase in expected returns and firm volatility is driven by an increase in systematic risk. Using daily firm-level betas, Patton and Verardo (2010) document an economically and statistically significant increase in firm beta on days of earnings announcements. Finally, there is evidence that firm-level stock returns are well described by a mixture of normals distribution (see Kon, 1984, Zangari, 1996, and Haas et al., 2004).

Many studies have focused on asymmetric volatility as an explanation for negative skewness in aggregate stock returns. Black (1976) and Christie (1982) posit the existence of a leverage effect, whereby a low price leads to increased market leverage, which in turn leads to high volatility (see also Veronesi, 1999). Pindyck (1984), French et al. (1987), Campbell and Hentschel (1992), Bekaert and Wu (2000), Wu (2001), and Veronesi (2004) further propose the existence of a volatility feedback effect, whereby high volatility is associated with a high risk premium and a low price. Blanchard and Watson (1982) show that negative skewness can result from the bursting of stock price bubbles. Hong and Stein (2003) hypothesize that short sales constraints limit the market's ability to incorporate bad news. According to their model, when more bad news arrives in the market, the price responds to the cumulative effect of news and falls at a time when volatility may be high (see also Bris et al., 2007). These papers have made important contributions to our understanding of the dynamics of return volatility and skewness. The current paper contributes to this literature by providing a bottom-up theory for negative skewness in aggregate stock returns that explicitly models positive skewness in firm-level returns and firm-level heterogeneity. This paper also contributes to the literature by documenting empirically the sources of negative skewness in aggregate returns.[2]

Finally, the model is related to the literature that analyzes the flow of information in the stock market (e.g., He and Wang, 1995), and the literature that studies properties of stock returns around public news events (e.g., Kim and Verrecchia, 1991, 1994). This paper provides a stationary asset pricing model of events to study skewness in stock returns.

The paper is organized as follows. Section 2 describes the model and Section 3 describes the stock market equilibrium. Section 4 analyzes the skewness properties of aggregate stock returns. Section 5 presents a model with incomplete information and earnings announcement events. Section 6 presents evidence on the paper's main hypotheses and Section 7 concludes.

[2]There is also a literature that documents that skewness is priced; total skewness (e.g., Arditti, 1967); co-skewness (e.g., Kraus and Litzenberger, 1976, and Harvey and Siddique, 2000); or, idiosyncratic skewness (e.g., Boyer et al., 2010). For models of positive skewness at the firm level see Duffee (2002), Grullon et al. (2010), Hong et al. (2008), and Xu (2007). Hong et al. (2007) develop a model that predicts negatively skewed returns for glamour stocks and positively skewed returns for value stocks.

The Appendix contains the proofs of the propositions and some additional results.

2 The Model

I construct a simple model that captures the observed changes in volatility and mean returns around dividend announcement events. I use this model to show that the observed pattern of conditional volatility leads to positive skewness in firm-level returns. In Section 5, I study a model of incomplete information with earnings announcement events and find similar results.

2.1 Investment opportunities

Time is discrete and indexed by $t = 1, 2, ...$ There is a risk-less asset with perfectly elastic supply that can be traded at the gross rate of return of $R > 1$. There are also N firms whose shares are infinitely divisible and trade competitively in the stock market.

Each firm makes a dividend announcement (and simultaneously pays a dividend) at equidistant periods and with equal frequency. Firms are assumed to differ at most by K periods in their announcements which limits the amount of heterogeneity with respect to announcement dates to $K+1$ possible dates. A firm of type $k = 0, 1, ..., K$ is identified in the following manner. I arbitrarily assign firm-type 0 to a group of firms announcing in the same period. All other firm types are identified using the distance of their announcement date to that of firms of type 0. Therefore, a firm's type is set vis-à-vis firm-type 0 event time. It helps to think of a trading period as one week and of $K+1$ periods as one quarter: A firm of type k makes an announcement every quarter at week $k+1$ in the quarter, k weeks after firms of type 0 have made their announcements.

If t corresponds to a dividend event for firm i, then at t firm i announces

$$D_{it} = F_{it} + \sum_{j=0}^{K} \varepsilon_{it-K+j}^{D}. \tag{1}$$

If t corresponds to a non-dividend period, then $D_{it} = 0$. The dividend can be decomposed into a persistent component,

$$F_{it} = \rho_{Fi} F_{it-1} + \varepsilon_{it}^{F}, \qquad 0 \leq \rho_{Fi} \leq 1,$$

with $\varepsilon_{it}^{F} \sim N\left(0, \sigma_{Fi}^{2}\right)$, and a transitory component, $\sum_{j=0}^{K} \varepsilon_{it-K+j}^{D}$, with $\varepsilon_{it}^{D} \sim N\left(0, \sigma_{Di}^{2}\right)$. Cash flows may be correlated across firms. For any two firms i and i', $E\left[\varepsilon_{it}^{D} \varepsilon_{i't-s}^{D}\right] = \sigma_{ii'}^{D}$ and $E\left[\varepsilon_{it}^{F} \varepsilon_{i't-s}^{F}\right] = \sigma_{ii'}^{F}$ when $s = 0$, and zero otherwise. I am interested in the case in which shocks have one or more common components that affect the cash flows of all firms in the economy in the same direction, $\sigma_{ii'}^{D}, \sigma_{ii'}^{F} \geq 0$. For simplicity, $E\left[\varepsilon_{it}^{D}\varepsilon_{i't-s}^{F}\right] = 0$ for any two firms i and i' and any s. Note that dividend shocks are homoskedastic and thus any heteroskedasticity in equilibrium returns is generated endogenously.

The heterogeneity in announcement dates suggests that stock prices may be a function of a firm's event time. Let the index k_i track event time for firm i. Denote by $P_{it}^{k_i}$ and $Q_{it}^{k_i}$ the ex-dividend stock price and excess return that occur in period t, k_i periods after the last dividend event for firm i. If t corresponds to a dividend-paying period for firm i, i.e. $k_i = 0$ at t, then firm i's excess return is

$$Q_{it}^{0} \equiv P_{it}^{0} + D_{it} - RP_{it-1}^{K}.$$

Alternatively, if $k_i > 0$ at t,

$$Q_{it}^{k_i} \equiv P_{it}^{k_i} - RP_{it-1}^{k_i - 1}.$$

Denote by $\mathbf{Q}_t^k = \left(Q_{1t}^{k_1}, ..., Q_{Nt}^{k_N}\right)^{\intercal}$ the column vector of time t stock returns. The superscript k indicates that firms of type k (if there are any) announce at time t. Because heterogeneity in firm announcements is fixed and given by each firm's type, k is a sufficient statistic for the heterogeneity in firm announcements at t.

2.2 Investors' problem

There is a continuum of identical investors with unit mass. Investors choose their time t asset allocation, $\boldsymbol{\theta}_t = \left(\theta_t^1, ..., \theta_t^N\right)^{\intercal}$, to maximize utility over next period wealth, W_{t+1},

$$-E\left[\exp^{-\gamma W_{t+1}} | \mathcal{I}_t\right],$$

where $\gamma > 0$ is the coefficient of absolute risk aversion. The maximization is subject to the budget constraint

$$W_{t+1} = \boldsymbol{\theta}_t^{\intercal}\mathbf{Q}_{t+1}^{k+1} + RW_t,$$

and the information set

$$\mathcal{I}_t = \left\{P_{t-s}, D_{t-s}, F_{t-s}, \varepsilon_{t-s}^{D}\right\}_{s \geq 0}.$$

For simplicity, I adopt the short-hand notation $E_t\left[.\right] = E\left[.|\mathcal{I}_t\right]$.

2.3 Stock market clearing

Each firm has a fixed supply of one share traded in the stock market. Let $\mathbf{1}$ be a column vector of ones. Investors trade the stock competitively, making their asset allocation while taking prices as given. In equilibrium, stock prices are such that the market for each stock clears:

$$\boldsymbol{\theta}_t = \mathbf{1}. \tag{2}$$

3 Stock Market Equilibrium

I start with a characterization of the equilibrium price function.

3.1 Equilibrium stock price

In the Appendix, I show that:

Proposition 1 *The equilibrium price function for firm i is*

$$P_{it}^{k_i} = p_i^{k_i} + \Gamma_{k_i} F_{it} + R^{-(K+1-k_i)} \sum_{j=0}^{k_i-1} \varepsilon_{i,t-j}^D, \tag{3}$$

for $\Gamma_{k_i} \equiv \frac{(\rho_{Fi}/R)^{K+1-k_i}}{1-(\rho_{Fi}/R)^{K+1}}$ and any $k_i = 0, ..., K$. The constants $p_i^{k_i} < 0$, are given by

$$p_i^{k_i} = -\frac{1}{R^{K+1}-1} \sum_{j=0}^{K} R^{K-j} E_t\left[Q_{it+1}^{k_i+1+j}\right], \tag{4}$$

where for any k_i, $E_t\left[Q_{it+1}^{k_i+1+K}\right] = E_t\left[Q_{it+1}^{k_i}\right]$.

The stock price at k_i reflects the present value of dividends conditional on all available information. The present value accounts for the fact that at time t –after k_i periods have elapsed since the last dividend payment– it will take another $K+1-k_i$ periods until dividends are paid again. Consider first the coefficient associated with F_{it}. With $k_i = 0$, the coefficient is, $\left[(R/\rho_{Fi})^{K+1} - 1\right]^{-1}$, and the stock resembles a perpetuity discounted at rate $(R/\rho_{Fi})^{K+1} - 1$. This is because the next payment arises in $K+1$ periods and is discounted by R^{K+1} and by that time F_{it} will have decreased in expectation at the rate ρ_{Fi}^{K+1}. $K+1$ periods later, another payment occurs, which is also discounted at the same rate, and so on.

The transitory shock ε_{it}^D enters the stock price function because investors learn about it before it is paid as a dividend: ε_{it}^D enters the price function at time t with a coefficient of $R^{-(K-k_i)}$, whereas ε_{it+1}^D enters the price function at time $t+1$ with a coefficient of

$R^{-(K-k_i-1)} > R^{-(K-k_i)}$. Despite being transitory, ε_{it}^D has de facto persistence of one until the next dividend payment and persistence of zero thereafter.

3.2 Conditional distribution of stock returns

Define the vector of conditional mean returns as $\boldsymbol{\mu}_k = E_t\left[\mathbf{Q}_{t+1}^{k+1}\right]$ and the conditional covariance matrix of returns as $\mathbf{V}_k = E_t\left[\left(\mathbf{Q}_{t+1}^{k+1} - \boldsymbol{\mu}_k\right)\left(\mathbf{Q}_{t+1}^{k+1} - \boldsymbol{\mu}_k\right)^\intercal\right]$. The investors' first-order condition together with stock market clearing requires that

$$\boldsymbol{\mu}_k = \gamma \mathbf{V}_k \mathbf{1}. \tag{5}$$

To solve for the equilibrium values of $\{\boldsymbol{\mu}_k, \mathbf{V}_k\}_k$, use the price function above to express excess returns as

$$Q_{it}^{k_i} = p_i^{k_i} - R p_i^{k_i-1} + \Gamma_{k_i}\varepsilon_{it}^F + R^{-(K+1-k_i)}\varepsilon_{it}^D, \tag{6}$$

for any k_i. In this expression, Q_{it}^0 is recovered by replacing k_i with $K+1$ and noting that $p_i^{K+1} = p_i^0$ and $Q_{it+1}^{K+1} = Q_{it+1}^0$. Therefore,

Corollary 1 *The conditional distribution of stock returns is*

$$\mathbf{Q}_{t+1}^{k+1}|t \sim N\left(\boldsymbol{\mu}_k, \mathbf{V}_k\right),$$

where the elements of $\mathbf{V}_k$ *are*

$$\sigma_{ik}^2 \equiv Var_t\left[Q_{it+1}^{k_i+1}\right] = \Gamma_{k_i+1}^2\sigma_{Fi}^2 + R^{-2(K-ki)}\sigma_{Di}^2. \tag{7}$$

$$\sigma_{ii',k} \equiv Cov_t\left[Q_{it+1}^{k_i+1}, Q_{i't+1}^{k_{i'}+1}\right] = \Gamma_{k_i+1}\Gamma_{k_{i'}+1}\sigma_{ii'}^F + R^{-(2K-k_i-k_{i'})}\sigma_{ii'}^D. \tag{8}$$

For each firm i, the conditional mean and volatility of the stock return increase monotonically and are convex in k_i, all else equal.

The corollary states that the conditional stock return volatility increases with k_i despite the fact that the shocks ε_{it}^F and ε_{it}^D are conditionally homoskedastic. The intuition is that news that occurs farther away from the dividend payment is more highly discounted and contributes less to risk than news that occurs closer to the dividend payment. Further, geometric discounting penalizes news asymmetrically (i.e., the conditional volatility of stock returns is convex in k). Holding all else constant, the same comparative statics apply to conditional mean returns which are weighted averages of volatility and covariance terms. When $k_{i'}$ is also allowed to change as k_i changes, it is no longer possible to establish the monotonicity of μ_k^i because the covariance $\sigma_{ii',k}$ need not be monotonic in k_i. In the numerical examples below this effect is dominated and μ_k^i is monotonic in k.

Quantitatively, the effect of discounting on conditional heteroskedasticity via the persistent shocks can be very large even for small interest rates. Consider the impact of k_i on the coefficient associated with σ_{Fi}^2 in equation (7). Specifically, evaluate the difference in coefficients at $k_i = 0$ and $k_i = K$ and take the limit as $\rho_{Fi}/R \to 1$. Applying L'Hopital's rule,

$$\lim_{\rho_{Fi}/R \to 1} \frac{(\rho_{Fi}/R)^2 \left(1 - (\rho_{Fi}/R)^{2K}\right)}{\left(1 - (\rho_{Fi}/R)^{K+1}\right)^2} = +\infty.$$

Intuitively, a lower interest rate (and higher persistence ρ_{Fi}) reduces the impact of discounting associated with news that is released before the next payout, but increases the value of the perpetuity associated with the news. The second effect is stronger than the first producing the result. Because transitory shocks lack the second effect, when $R \to 1$ the discounting effect through transitory shocks disappears.

The result in the Corollary shows that the model is consistent with the evidence that dividend announcements are associated with both higher mean returns and higher volatility (e.g., Aharony and Swary, 1980, and Kalay and Loewenstein, 1985). More recently, Amihud and Li (2006) show evidence of a declining, but still significant, dividend announcement effect.

To clarify the sources of risk in the model, I show next that the model admits a CAPM representation of the equilibrium. The stock market dollar return is the return from buying and selling the stock on all N firms. The purchase price is $\sum_{i=1}^{N} P_{it-1}$ and the sale price plus the dividend is $\sum_{i=1}^{N} (P_{it} + D_{it})$. Thus, the per share excess return in the market is $Q_{Mt} = \frac{1}{N}(Q_{1t} + ... + Q_{Nt})$. Let $\boldsymbol{\alpha} \equiv \mathbf{1}/N$ and write $Q_{Mt}^k = \boldsymbol{\alpha}^\intercal \mathbf{Q}_t^k$. Then, $\mu_k^M \equiv E_t\left[Q_{Mt+1}^{k+1}\right] = \boldsymbol{\alpha}^\intercal \boldsymbol{\mu}_k$ and $\sigma_{M,k}^2 \equiv E_t\left[\left(Q_{Mt+1}^{k+1} - \mu_k^M\right)^2\right] = \boldsymbol{\alpha}^\intercal \mathbf{V}_k \boldsymbol{\alpha}$. In the Appendix, I show that:

Proposition 2 *The stock market equilibrium has a conditional CAPM representation:*

$$\boldsymbol{\mu}_k = \boldsymbol{\beta}_k \mu_k^M, \tag{9}$$

where $\boldsymbol{\beta}_k \equiv Cov_k\left(\mathbf{Q}_t^k, Q_{Mt}^k\right)/\sigma_{M,k}^2$ *and* $\boldsymbol{\alpha}^\intercal \boldsymbol{\beta}_k = 1$.

The proposition demonstrates that in equilibrium only systematic risk is priced. In particular, if firm i has a high expected return around its announcement event, it must also be that β_k^i is high around the event. This systematic risk is driven by the volatility associated with the information flow in common factors. For example, if $F_{it} = F_t$ for all t and i, and $\sigma^D_{ii'} = 0$, then the economy has only one common factor, which is persistent. Shocks to this common factor, ε^F_t, affect stock returns of firms differently depending on how far each firm is from its respective payout event. This timing explains the dynamics in the conditional stock return moments because proximity to a payout event determines the impact of (systematic) information on returns.[3] Consistent with this model prediction, Patton and Verardo (2010) show that daily firm betas increase by an economically and statistically significant amount around earnings announcement events.

3.3 Unconditional distribution of stock returns

Proposition 1 shows that firm i stock returns are conditionally normally distributed with mean μ^i_k and variance σ^2_{ik}. The unconditional distribution of firm i returns is not normal because the mean and variance of a randomly drawn return observation depends on k_i. In fact, because a k_i-period stock return is drawn from a normal density $\phi\left(Q^i; \mu^i_k, \sigma^2_{ik}\right)$ and such observations occur with frequency $1/(K+1)$, the unconditional distribution of returns is a mixture of normals distribution. Formally,

Proposition 3 *For $K \geq 1$, the unconditional distribution of stock returns for firm i is a mixture of normals distribution with density*

$$f\left(Q^i\right) = \frac{1}{K+1} \sum_{k=0}^{K} \phi\left(Q^i; \mu^i_k, \sigma^2_{ik}\right), \tag{10}$$

where $\phi(.)$ is the normal density function. For $K = 0$, returns are unconditionally normally distributed.

The periodicity of dividends –by generating time-varying conditional volatility in stock returns– leads to the derived mixture of normals distribution for stock returns for $K \geq 1$. This result provides a theoretical justification for attempting to fit a mixture of normals distribution for stock returns (e.g., Fama, 1965, Granger and Orr, 1972, Kon, 1984, and Tucker 1992).

In the Appendix, I prove the following corollary.

[3] Below I show that the same patterns in conditional stock return moments exist if idiosyncratic risk is also priced (say, because N is small). While some recent literature suggests that idiosyncratic risk is priced, the results in this paper do not rely on it.

Corollary 2 *The unconditional mean and variance of stock returns are*

$$\begin{aligned} E\left(Q_{t+1}^{i}\right) &= \frac{1}{K+1}\sum_{k=0}^{K}\mu_{k}^{i}, \\ Var\left(Q_{t+1}^{i}\right) &= \frac{1}{K+1}\sum_{k=0}^{K}\left[\sigma_{ik}^{2}+\left(\mu_{k}^{i}-E\left(Q_{t+1}^{i}\right)\right)^{2}\right]. \end{aligned}$$

The unconditional (non-standardized) skewness in stock returns is

$$E\left[\left(Q_{t+1}^{i}-E\left(Q_{t+1}^{i}\right)\right)^{3}\right]=\frac{1}{K+1}\sum_{k=0}^{K}\left(\mu_{k}^{i}-E\left(Q_{t+1}^{i}\right)\right)^{3}+\frac{3}{(K+1)^{2}}\sum_{k=0}^{K}\sum_{j<k}\left(\sigma_{ik}^{2}-\sigma_{ij}^{2}\right)\left(\mu_{k}^{i}-\mu_{j}^{i}\right). \tag{11}$$

The unconditional mean return is simply the mean of the k-conditional expected returns. The unconditional mean variance is the mean of the k-conditional variances plus the variance of the k-conditional means.

Skewness in stock returns can be decomposed into two terms. The first term in (11) is the level of skewness in expected returns, μ_k^i. Intuitively, this term is positive if μ_k^i is increasing and convex in k_i which means that a small number of periods display high expected returns relative to the larger number of periods with low expected returns. The second term describes the impact on skewness of the co-movement between return volatility with expected returns. This term is positive if both σ_{ik}^2 and μ_k^i increase with k_i. The risk-return trade off imbedded in the model would normally suffice to generate the necessary association between σ_{ik}^2 and μ_k^i. Unfortunately, it is not possible to sign skewness because when k_i changes other firms' event time, say $k_{i'}$, also changes which may lead to non-monotonicity in the conditional return covariance between i and i' and hence in conditional mean returns for firm i. However, in the numerical examples below this effect is dominated and firm-level skewness is positive.

3.4 Discussion

The model generates skewness in firm-level stock returns by making use of the time-series patterns in volatility that arise from having cash payouts spread out over time. While these patterns in conditional volatility are consistent with the evidence, there could be other explanations for the same facts. For example, it could be the case that the resolution of uncertainty afforded by earnings announcements also results in greater volatility and higher expected returns. I explore this idea below when earnings announcements are introduced separately from cash payouts.

The model produces deterministic patterns in conditional volatility. These patterns are stylized and consistent with the patterns in volatility around certain firm events. A more realistic model would also generate stochastic sources of conditional heteroskedasticity. If the average behavior of conditional volatility is consistent with the deterministic path that results from the equilibrium of the model here, firm level skewness would likewise be positive.

Finally, positive skewness arises despite the fact that prices and returns are conditionally normally distributed. The source of skewness in the model is thus distinct from that which arises mechanically when prices are lognormally distributed due to truncation at zero. This benefit, which arises from the modeling choices of exponential utility and normal shocks, comes at the cost of having negative prices with positive probability. To minimize this probability, it is customary to add a positive long-run mean dividend to the process in equation (1). Because all main results (i.e., patterns in conditional volatility and expected returns in event time) are unchanged, I have assumed away this constant for simplicity of presentation. Nevertheless, one can never rule out the possibility of negative prices in this setting, which is why the model should be understood as an approximation to reality.

4 Skewness in Aggregate Stock Returns

I start by presenting the unconditional distribution of aggregate stock returns and computing skewness in aggregate returns.

4.1 The distribution of aggregate stock returns

Aggregate market returns are conditionally normally distributed with mean μ_k^M and variance $\sigma_{M,k}^2$. The unconditional distribution of aggregate market returns is a mixture of normals distribution with

$$f(Q_M) = \frac{1}{K+1}\sum_{k=0}^{K}\phi\left(Q_M;\mu_k^M,\sigma_{M,k}^2\right). \tag{12}$$

The Appendix shows that skewness in aggregate stock returns is

$$
\begin{aligned}
& E\left[\left(Q_{Mt}-E\left(Q_{Mt}\right)\right)^{3}\right] \qquad (13) \\
= \; & \frac{1}{N^{3}}\sum_{i=1}^{N}E\left[\left(Q_{it}-E\left(Q_{i}\right)\right)^{3}\right] \\
& +\frac{3}{K+1}\frac{1}{N^{3}}\sum_{k=0}^{K}\sum_{i=1}^{N}\left(\mu_{k}^{i}-E\left(Q\right)\right)\left[\sum_{i'\neq i}^{N}\left(\sigma_{k,i'}^{2}+2\sum_{l>i'}\sigma_{i'l,k}+\left(\mu_{k}^{i'}-E\left(Q\right)\right)^{2}\right)+2\sum_{i'>i}^{N}\sigma_{ii',k}\right] \\
& +\frac{6}{K+1}\frac{1}{N^{3}}\sum_{k=0}^{K}\sum_{i=1}^{N}\left(\mu_{k}^{i}-E\left(Q\right)\right)\sum_{i'>i}^{N}\sum_{l>i'}^{N}\left(\mu_{k}^{i'}-E\left(Q\right)\right)\left(\mu_{k}^{l}-E\left(Q\right)\right).
\end{aligned}
$$

Skewness in aggregate stock returns is the sum of average firm-level skewness (first term on the right-hand side of equation (13)) and co-skewness terms (remaining two terms). The first of the co-skewness terms describes the co-movement of one firm's stock with other firms' volatility. I label this term *co-vol*. The second co-skewness term describes the co-movement of one firm's stock with the covariance between any two other firms. I label this term *co-cov* and note that it requires $N \geq 3$ in the stock market to be non-zero. Together, the co-skewness terms describe the co-movement between a firm's conditional mean return with the conditional variance of the portfolio of the remaining firms.

Because firm-level skewness is positive in this model, negative aggregate skewness must come from the co-skewness terms: Negative stock market skewness becomes a cross-sectional phenomenon. The co-skewness terms shift probability mass in the distribution of aggregate stock returns to the left in order to generate a negatively skewed distribution. Intuitively, the portfolio return becomes negatively skewed when a low return for one firm is associated with high volatility in the remaining firms in the portfolio.

4.2 Skewness and cross-sectional heterogeneity in announcement events

To evaluate the effect of cross-sectional heterogeneity in payout dates and co-skewness, I conduct two numerical experiments simulating different stock market configurations. In all simulations and for simplicity, I assume one firm per firm type. In the first experiment, each stock market is composed of two types of firms with cash payouts separated by k periods, where $k \in \{0, 1, ..., K\}$. By varying k, the two firms start off similar, become increasingly dissimilar, and end up similar again. I choose $K = 12$ so that each trading period represents one week and the time from 0 to K corresponds to one calendar quarter. Because $N = 2$, this experiment explores the effect of cross-sectional heterogeneity ignoring the *co-cov* term.

Panel A of Figure 1 plots firm-level and market skewness for the various stock market configurations. Specifically, the figure depicts normalized skewness of aggregate stock returns, i.e., the third centered moment normalized by the standard deviation cubed (solid line). The figure also presents the plot of the corresponding firm-level statistic (dashed line). The plot starts with a stock market composed of two firms that are identical in all respects including the payout date. Ignoring this stock market configuration, the plot is symmetric because having the second firm pay out k periods after the first firm or k periods before the first firm results in identical cross-sectional heterogeneity. Firm-level skewness is positive and varies across stock market configurations because of the correlation in cash flows. Co-skewness can be very large and negative but never sufficiently so in order to offset the individual skewness terms. Co-skewness is particularly negative when the two firms pay out at dates that are farthest apart because then the high volatility of the announcing firm contrasts the most with the contemporaneously low expected return of the non-announcing firm. In summary, the experiment suggests that the *co-vol* terms can significantly reduce market skewness relative to firm-level skewness, but cannot generate negative market skewness. This result is confirmed with many other numerical parameterizations available upon request.

In the second experiment, I allow a full role for the *co-cov* term by having the number of firms in the stock market grow as heterogeneity across firms also changes. Each stock market is indexed by k, meaning it consists of $k+1$ firm types with cash payout dates at periods $0, 1, ...,$ and k. The period from 0 to k thus denotes an announcement season in cash payouts during the window of time $0, ..., K$. Panel B of Figure 1 depicts the market (solid line) and firm-level (dashed line) normalized skewness in each of the stock market configurations. Moving to the right along the x-axis represents an increase in the number of firms in the stock market, but normalized skewness is not directly affected by the number of firms. Market skewness displays a flipped J-curve with respect to k. For $k = 0$ there is only one firm type in the stock market, and firm-level and market skewness are identical. For $k = 1$, the stock market has two firm types, one announcing at 0 and the other at 1. This case is also presented in panel A of the figure. For $k > 1$ skewness drops faster than it did in panel A because of a negative *co-cov* term. As more firm types are added and the range of cash payout dates is widened, market skewness becomes negative. The negative market skewness occurs despite the fact that firm-level skewness is positive. Market skewness remains negative until the stock market consists of one firm of each type. When the stock market consists of one firm of each type, skewness is zero because every period looks the same with equal aggregate stock market conditional mean and volatility of returns.

The driving force for the negative skewness is the asymmetric volatility across announcing and non-announcing firms, which is also apparent when total skewness is decomposed into the contribution from each of the trading periods. Figure 2 presents a decomposition of the negative skewness for the stock market consisting of nine firm types, each firm type announcing at a different period k, with $k = 0, ..., 8$. Most negative skewness occurs around the start of the announcement season when some firms' volatility spikes vis-à-vis that of others.

It is possible to also produce a breakdown of stock market skewness in its various components according to equation (13). Figure 3 plots aggregate skewness in each of the stock market configurations under experiment two as well as the respective *co-cov* term also normalized by market volatility. A common property of the numerical examples studied, and of this one in particular, is that the *co-cov* term is the main driver of negative skewness in the stock market. The symmetry of events in the model also implies that as k approaches K and skewness goes to zero, the *co-cov* term turns positive and the *co-vol* terms turn negative. The *co-vol* terms are negative because, for large k, almost every period t consists of an event period with one firm with the highest conditional volatility (the one with an event at $t+1$) and all the others with low volatility possibly below their respective unconditional means.

In the exercises above, I assume that $K = 12$ so that there are always 13 periods between any two events for the same firm. While the choice is meant to identify each period as one week and each set of 13 periods as one quarter to match the regularity of the events studied, this choice is not innocuous. Taking $K = 0$ means that payouts occur at every period and in the model returns become unconditionally normally distributed with zero skewness. More generally, K controls the amount of firm heterogeneity in payout dates. Small values of K mean that there cannot be much heterogeneity. For example, consider a stock market that consists of two firm types and $K = 2$. When one firm-type has a payout event, the other will either have one next period or the period after. Because of the regularity of the payout events, both configurations would imply the same level of market skewness. Because of the closeness of the announcements, market skewness would generally be positive.

4.3 A simplified model with uncorrelated cash flows

In this subsection, I consider a simplified version of the model where the stock market is composed of firms with uncorrelated cash flows, i.e., $\sigma^D_{ii'} = \sigma^F_{ii'} = 0$. Firms differ only with respect to the timing of their cash payouts, but as is clear from above, equilibrium firm returns are uncorrelated. The main reason to consider this simplified model is to isolate the effect of cross-sectional heterogeneity in cash payout dates on aggregate skewness: With uncorrelated cash flows, negative skewness in market returns can only arise from the cross-sectional heterogeneity in cash payout dates and not from some exogenous, negatively skewed factor in returns. Another reason is that it becomes possible to sign skewness at the firm level. The drawback of this simplification is that it presumes that idiosyncratic risk is priced. However, as shown above, this is not a necessary assumption for the results.

To evaluate the effect of cross-sectional heterogeneity in payout dates and co-skewness, I repeat the same two experiments above which simulate different stock market configurations. In the first experiment, the stock market is composed of two types of firms with cash payouts separated by k periods. Recall that with $N = 2$ in each stock market configuration, the effect of cross-sectional heterogeneity is limited to the *co-vol* terms. Panel A in Figure 4 plots firm-level and market skewness for the various stock market configurations. First, because the timing of the announcements in one firm does not influence the pricing in the other firm, mean firm level skewness (dashed line) is constant across all stock market configurations. Second, skewness in aggregate returns (solid line) behaves very similarly to the case of correlated cash flows. Specifically, there is not enough cross-sectional heterogeneity in payout dates to yield negatively skewed aggregate returns.

In panel B, I reproduce the results from experiment two in which the *co-cov* term plays a role. Recall that in experiment two, each stock market is indexed by k, meaning it consists of $k+1$ firm types with cash payout dates at periods $0, 1, ...,$ and k. As in panel A, firm level skewness (dashed line) is constant. In contrast with panel A, however, market skewness (solid line) displays a flipped J-curve with respect to k. Further, market skewness is below the skewness that results in the correlated cash flow case for most stock market configurations (see panel B of Figure 1). The reason is that with correlated cash flows, skewness is affected by the term

$$\frac{6}{K+1}\frac{1}{N^3}\sum_{k=0}^{K}\left(\mu_k^M - E\left(Q_M\right)\right)\sum_{i=1}^{N}\sum_{i'>i}\sigma_{ii',k}, \tag{14}$$

and this term is likely to be positive because the return covariance is likely to be highest at event dates k, where mean returns μ_k^M are also likely to be higher.

5 A Model with Earnings Announcements

Earnings announcements are important firm events with similar return and volatility properties to dividend announcements. In this section, I allow for an intermediate earnings announcement event at event date $1 < K_a < K$. It helps to introduce earnings announcements in the model with uncorrelated cash flows of subsection 4.3 for two reasons. First, it is easier to keep track of what has been learned about each firm avoiding excessive notation: When cash flows are uncorrelated, announcements on one firm do not provide any information about other firms. Second, it avoids having to introduce correlated noise in the signals that investors obtain in order to keep aggregate shocks from being fully revealed. Allowing for correlated cash flows would not however change the main results below.

For the earnings announcement to be informative, I introduce incomplete information in the model. To do this with minimal deviation from the model above, I assume that for any $1 \leq k \leq K_a - 1$, investors learn

$$\begin{aligned} S_t^F &= \varepsilon_t^F + \varepsilon_t^{SF}, \\ S_t^D &= \varepsilon_t^D + \varepsilon_t^{SD}, \end{aligned}$$

with the information noise $\varepsilon_t^{SF} \sim N\left(0, \sigma_{SF}^2\right)$ and $\varepsilon_t^{SD} \sim N\left(0, \sigma_{SD}^2\right)$ independent of each other and of all other shocks. It is assumed that the earnings announcement at event date K_a reveals all current and past shocks. Also, for simplicity, shocks are known with certainty after K_a. This gives rise to the following information structure. Let t be any trading period and k be the corresponding date in event time. For any $k = 0$ or $k > K_a - 1$,

$$\mathcal{I}_t^k = \left\{P_{t+k-s}, D_{t+k-s}, F_{t+k-s}, \varepsilon_{t+k-s}^D\right\}_{s \geq 0},$$

and for any $1 \leq k \leq K_a - 1$,

$$\mathcal{I}_t^k = \left\{P_{t+k-s}, S_{t+k-s}^F, S_{t+k-s}^D, \mathcal{I}_t\right\}_{s=0,\ldots,k-1}.$$

The Appendix shows the following proposition:

Proposition 4 *The equilibrium price function is*

$$P_t^k = p^k + \Gamma_k E_t\left(F_t\right) + R^{-(K+1-k)} \sum_{j=0}^{k-1} E_t\left(\varepsilon_{t-j}^D\right),$$

for any $k = 0, ..., K$.

The stock price function takes the same form as before with the actual values of the random variables replaced by their conditional expectations. After K_a, the expectations operators drop out because the shocks are in the investors' information set. With the equilibrium prices, it is possible to derive the equilibrium stock return. For any period $1 \leq k \leq K_a - 1$,

$$Q_t^k = p^k - Rp^{k-1} + \Gamma_k E_t\left(\varepsilon_t^F\right) + R^{-(K+1-k)} E_t\left(\varepsilon_t^D\right).$$

When the signals that investors get are infinitely precise and $\sigma_{SD}^2 = \sigma_{SF}^2 = 0$, equation (6) is recovered. For $k = K_a$,

$$\begin{aligned} Q_t^k &= p^k - Rp^{k-1} + \Gamma_k \varepsilon_t^F + R^{-(K+1-k)} \varepsilon_t^D \\ &\quad + \rho_F \Gamma_k \left[F_{t-1} - E_{t-1}\left(F_{t-1}\right)\right] + R^{-(K+1-k)} \sum_{j=0}^{k-2} \left[\varepsilon_{t-1-j}^D - E_{t-1}\left(\varepsilon_{t-1-j}^D\right)\right]. \end{aligned}$$

The resolution of uncertainty with the earnings announcement implies that the stock return at K_a responds to the unanticipated realizations of the past shocks. Finally, for $k > K_a$, returns take the same form with the same conditional moments as before.

To conclude the derivation of the equilibrium, use the return process above to get the conditional stock return variance, and equation (5) to obtain the conditional mean stock return. It is straightforward to show that for any period $1 \leq k \leq K_a - 1$,

$$Var_{t-1}\left(Q_t^k\right) = \Gamma_k^2 \frac{\sigma_F^4}{\sigma_F^2 + \sigma_{SF}^2} + R^{-2(K+1-k)} \frac{\sigma_D^4}{\sigma_D^2 + \sigma_{SD}^2},$$

and for period $k = K_a$,

$$\begin{aligned} Var_{t-1}\left(Q_t^k\right) &= \Gamma_k^2 \sigma_F^2 + R^{-2(K+1-k)} \sigma_D^2 \\ &\quad + \Gamma_k^2 \rho_F^2 Var_{t-1}\left(F_{t-1}\right) + R^{-2(K+1-k)} \sum_{j=0}^{k-2} Var_{t-1}\left(\varepsilon_{t-1-j}^D\right). \end{aligned}$$

The process for the conditional variance of firm returns is increasing and convex up to K_a. At K_a, the conditional variance may drop so that

$$Var_{t-1}\left(Q_t^{K_a}\right) > Var_t\left(Q_{t+1}^{K_a+1}\right).$$

This case arises for sufficiently low precision of the signals prior to the earnings announcement, which generates significant resolution of uncertainty at K_a. This pattern resembles that of the non-stationary event model of He and Wang (1995). It is then possible to have the conditional variance, and thus also the conditional mean return, displaying two distinct periods of convexity in the event time from 0 to K (one for the earnings announcement and another for the cash payout). By making the periods of high conditional mean returns more likely, the conditional mean return distribution shifts to the right and returns become less positively skewed. By itself this feature cannot generate negative skewness in aggregate returns, but may contribute to more negative skewness in market returns relative to the benchmark model.

The patterns in conditional volatility and mean returns described above are consistent with the evidence as described in Ball and Kothari (1991). Considering a more recent sample, Cohen et al. (2007) report persistent, significant earnings announcement premia, albeit a smaller one in the later part of the sample. They associate the more recent lower premia with increased voluntary disclosures, which is also consistent with the model above.

To further understand the impact of earnings announcements on aggregate return skewness, I repeat the numerical experiments above. Recall that in experiment one, the stock market consists of two types of firms that have their payout dates separated by k periods, where $k \in \{0, ..., K\}$. In experiment two, the stock market consists of firms with payout at event dates 0 through k, with $k \in \{0, ..., K\}$. Likewise, and for both experiments, the earnings announcement dates are separated by the same number of periods for any two firms. The earnings announcement date occurs at $K_a = 6 < K = 12$ in event time for each firm.

In Figure 5, I plot normalized skewness in stock returns with incomplete information. Panel A depicts the results for experiment one and Panel B depicts the results for experiment two. The plot in Panel A shows a symmetric pattern for skewness, which results from the symmetry of event dates. For example, a stock market consisting of two firms with payout dates separated by five periods results in the same return properties regardless of whether the two firms announce at periods 0 and 5, or at periods 0 and 8. Panels A and B show that firm-level skewness is lower in the presence of the additional event than when only payout events are allowed. Comparing with the plots in the model with complete information above, it is possible that lower levels of skewness are achieved with less payout heterogeneity. In particular, with incomplete information, this numerical example shows that it is enough to have seven different types of firms in order to generate negative market skewness, whereas in the complete information model the same parameters require eight different firm types (see subsection 4.3).

6 Empirical Evidence

This section presents evidence on the three main predictions of the model. First, earnings announcement events are neither uniformly distributed on average in a quarter nor concentrated in one week in the quarter. If the former were true, the model would predict zero unconditional skewness. If the latter were true, the model would predict positive skewness in aggregate returns because of the clustering in volatility in the same week for all firms. Second, cross-sectional dispersion in earnings announcement events can generate large enough negative co-skewness and negatively skewed aggregate returns. I demonstrate this by replicating experiments one and two developed above. I also show that skewness is most negative around the start of an earnings announcement season. Third, negative skewness arises due to co-skewness and in particular the *co-cov* term. In addition, in a robustness exercise, I repeat the analysis allowing for a negatively skewed factor in returns.

6.1 Data

I use daily return data on AMEX/NASDAQ/NYSE stocks from CRSP for the period between 1/1/1973 and 12/31/2009. Returns are inclusive of dividends. I also obtain from CRSP the SIC industry classification and the dividend distribution information. I use variable DCLRDT to retrieve the date the board declares a distribution and variable DISTCD to select ordinary dividends and notation of issuance. Information about earnings announcement events is from the merged CRSP/Compustat quarterly file for the period 1/1/1973 through 6/30/2009 (variable RDQ). Below, skewness is estimated using six months of daily return data. Firms are required to have complete return data within each semester to be included in the sample.

6.2 The stylized facts

I start by documenting several well-known facts about firm-level and aggregate return skewness. Figure 6 plots the time series of the mean firm-level stock return skewness and of skewness in the equally weighted market return. Four salient facts emerge from the figure. First, firm-level skewness is always positive, except in the second half of 1987. Second, skewness in market returns is almost always negative, representing 77% of the observations. Third, and as a combination of the two facts above, most semesters of large negative skewness in market returns are not accompanied by negative skewness in firm-level returns. Fourth, firm-level skewness is higher than aggregate skewness in 96% of the semesters. The results using median firm-level skewness are similar and available upon request. Because skewness is generally lower and more often negative for larger firms, I also reproduce the same statistics using value-weighted mean (or median) firm-level skewness and value-weighted aggregate return skewness. Not surprisingly, the value-weighted mean (or median) of firm-level skewness is lower, but the general gist of the results above is unaffected (available upon request).

To better understand these results it is useful to write the sample (non-standardized) skewness for a portfolio with N firms. Assuming equal weights for simplicity, let $r_{pt} = N^{-1}\sum_{i=1}^{N} r_{it}$ be the time-t portfolio return, $\bar{r}_i = T^{-1}\sum_{t=1}^{T} r_{it}$ be the mean sample return for firm i, and $\bar{r}_p = T^{-1}\sum_{t=1}^{T} r_{pt}$ be the mean sample portfolio return. Then, sample non-standardized skewness is (or, the sample estimate of the third-centered moment of returns):

$$\begin{aligned} T^{-1}\sum_t \left(r_{pt}-\bar{r}_p\right)^3 &= \frac{1}{N^3}\sum_{i=1}^{N}\frac{1}{T}\sum_t \left(r_{it}-\bar{r}_i\right)^3 \\ &+\frac{3}{TN^3}\sum_t\sum_{i=1}^{N}\left(r_{it}-\bar{r}_i\right)\sum_{i'\neq i}^{N}\left(r_{i't}-\bar{r}_{i'}\right)^2 \\ &+\frac{6}{TN^3}\sum_t\sum_{i=1}^{N}\left(r_{it}-\bar{r}_i\right)\sum_{i'>i}^{N}\sum_{l>i'}^{N}\left(r_{i't}-\bar{r}_{i'}\right)\left(r_{lt}-\bar{r}_l\right). \end{aligned} \tag{15}$$

The first term in (15) is the mean of firm-level skewness and, as Figure 6 shows, it is positive. The second and third terms in (15) are the sample equivalent to the co-skewness terms *co-vol* and *co-cov*, respectively. Together, they must be negative for skewness in market returns to be negative.

The skewness measure I report is the standardized skewness equal to $T^{-1}\sum_t \left(r_{pt}-\bar{r}_p\right)^3 / [T^{-1}\sum_t \left(r_{pt}-\bar{r}_p\right)^2]^{3/2}$. Formally, standardizing the third-centered moment introduces a discrepancy between mean firm-level skewness and the component of aggregate skewness related to firm-level skewness. When normalized skewness is used, the first term in (15) becomes the volatility-weighted average of firm-level skewness (with weights $\omega_i = [\sum_t \left(r_{it}-\bar{r}_i\right)^2 / \sum_t \left(r_{pt}-\bar{r}_p\right)^2]^{3/2}$). Because small firms tend to be more volatile and also have returns with more positive skew, this term is also positive, and negative normalized skewness can only arise from negative normalized *co-vol* and *co-cov* terms (see also Figure 11 below).

6.3 Cross-sectional heterogeneity in event dates

The main prediction of the model is that negative co-skewness can arise from heterogeneity in event dates. As demonstrated above, the special nature of the events discussed in the model (payout and earnings announcement events) is that they are associated with increased return volatility. Further, cross-sectional dispersion in event dates and the corresponding cross-sectional dispersion in return volatility is shown to lead to negative co-skewness.

Before producing evidence on the model, I describe the cross-sectional dispersion in cash payout announcements and in earnings announcements. I am interested in the calendar week of the announcement within the quarter. Figure 7 plots the histograms of the announcement week for cash payouts (Panel A) and of the announcement week for earnings announcements (Panel B).[4] Cash payouts are close to uniformly distributed across the quarter. In contrast, earnings announcements are on average concentrated between weeks two and eight in the calendar quarter, leaving the other half of the quarter with less than 20% of the announcements. These patterns are consistent across various subsamples and also across the various quarters. This evidence suggests that cross-sectional dispersion in payout dates may not be able to explain the negative skewness in aggregate returns, but that cross-sectional dispersion in earnings announcement events may explain the negative skewness in aggregate returns.[5] I use data on earnings announcements below.

Next, I use data to reproduce the experiments that give rise to Figures 1 and 5. For every semester, I group firms by week of first earnings announcement in the semester. This gives rise to 13 portfolios, $P1$ through $P13$, one for each of the weeks in the first quarter of the semester. The portfolios vary greatly in the number of firms that comprise them because of the concentration of earnings announcement events during the quarter (see Figure 7). To keep a constant number of firms across portfolios, I randomly drop firms from portfolios to match the number of firms in the smallest portfolio. It is important to note that it is not possible to replicate in the data the absolute symmetry that exists in the model because firms do not consistently announce in the same week in every quarter. Forcing firms in portfolio Pk to contain only firms that announce in week k in both quarters in the semester would lead to a significant loss of observations.

Figure 8 replicates experiment one above and Figure 9 replicates experiment two. I consider two samples, namely, the full sample since 1973 and the subsample with data from 1988. Figure 8 plots the sample skewness in the equally weighted portfolio return for the portfolios consisting of the firms in $P1$ and Pk against the index $k = 1, 2, ..., 13$. I also plot

[4]For earnings announcements, observations with an announcement date before the end of the quarter are dropped.

[5]In addition, many firms do not pay dividends, which results in a much smaller sample relative to the earnings announcement sample with a consequent decrease in the precision of estimates.

the corresponding 10% confidence bands constructed assuming the data from each semester are drawn randomly. The figure shows that portfolio return skewness displays a U-shaped pattern in both samples, consistent with the symmetric U-shaped pattern in Figures 1 and 5. Mean firm-level return skewness in each of the portfolios is approximately constant and always positive (available upon request).

Figure 9 plots the sample skewness in the equally weighted portfolio return for the portfolios that result from the unions $P1\text{U}P2\text{U}...\text{U}Pk$ against the index $k = 1, 2, ..., 13$, and the corresponding 10% confidence bands. In both sample periods, there is a negative relationship between skewness and the increased heterogeneity that results from adding dispersion in earnings announcement dates into the portfolio. This evidence is consistent with the model prediction as depicted in Figures 1 and 5. Mean firm-level return skewness for each of the portfolios is approximately constant and positive (available upon request).

There are two main differences between the evidence presented and the model predictions. First, in Figure 9, skewness strictly declines with k whereas in the model, when all firm types are allowed, skewness becomes zero. The result in the model relies on the assumption of symmetry where a firm always announces in the same calendar week in every quarter. This assumption is not validated in the data. Second, in both Figures 8 and 9, portfolio return skewness is always negative. One possible explanation for the negative portfolio skewness is that even the firms in the same portfolio Pk differ in the week of earnings announcement in the second quarter of the semester. Another explanation is that the cross-sectional heterogeneity in events is not subsumed in the cross-sectional heterogeneity of earnings announcement events. Finally, it could be the case that firm returns are exposed to a common factor that is negatively skewed. I return to this last possibility below.

To test the influence of the timing of announcements on skewness, I decompose market skewness computed using six months of data into its weekly components. The decomposition guarantees that adding up the weekly components yields the market skewness for the six-month period. Recalling Panel B of Figure 7, an earnings announcement season starts shortly after the beginning of every quarter. Figure 10 shows that an earnings announcement season is also when skewness has its largest (i.e., most negative) components during the quarter, consistent with the model prediction.

Lastly, I look at co-skewness across industries. Sinha and Fried (2008) show that industries vary significantly with respect to the cross-sectional dispersion in fiscal year-ends and therefore also in their earnings announcement calendar. I use the Herfindahl index of fiscal year-ends as a measure of dispersion in earnings announcements and choose two 2-digit SIC sectors from the extremes of its distribution (using the standard deviation produces a similar ordering). A low value of the index means greater dispersion. The choice of sector also must yield a large enough (and comparable) number of firms in each sector in order to better estimate skewness. Using the sample from 1973 through 2009, I find that SIC 49 "Electric, Gas, and Sanitary Services" has an average index of 0.74 and SIC 38 "Measuring, Analyzing and Controlling Instruments; Photographic, Medical and Optical Goods; Watches and Clocks" has an average index of 0.30. These sectors have a similar average number of firms, around 420.[6] The average six-month co-skewness in SIC 38 is -0.38, which is statistically significantly lower at the 10% significance level than the average six-month co-skewness in SIC 49 of -0.24 (untabulated). The more negative co-skewness in the sector with greater dispersion in fiscal year-ends is consistent with the model. This analysis is necessarily preliminary because little is known about other drivers of skewness across industries. Further development of theory may allow for a full-fledged conditional analysis.

6.4 The number of firms in a portfolio

The number of firms in a portfolio does not directly affect the calculation of sample skewness. Inspection of equation (15) reveals that N^{-3} multiplies every term. At the same time N^{-3} also multiplies every term in $[T^{-1} \sum_t (r_{pt} - \bar{r}_p)^2]^{3/2}$, cancelling off in the calculation of normalized skewness. Where the number of firms matters is in the breakdown of skewness. Observe that there are N firm-level skewness terms, $N(N-1)$ terms in *co-vol*, and $N!/[3!(N-3)!]$ terms in *co-cov*. Hence, as the number of firms increases, the number of terms associated with *co-cov* increases faster than the number of terms associated with any other component of skewness. This does not imply that the *co-cov* terms dominate the sum, because it may be the case that their component terms cancel each other out. In Figure 11, I plot the ratio of the standardized *co-cov* term to the sample skewness of market returns. With a ratio close to 100%, on average, the figure suggests that indeed it is the *co-cov* term that drives negative skewness at the market level, providing additional support to the model (see Figure 3 above). The *co-vol* term makes up for the remaining difference between *co-cov* and skewness of market returns.

There are two additional facts about how the number of firms, N, in a portfolio relates

[6] Two sectors have a higher Herfindahl index than SIC 49: SIC 60 "Depositary Institutions" and SIC 63 "Insurance Carriers," with approximately 600 and 200 firms, respectively. Several sectors have somewhat lower Herfindahl index than SIC 38, but also a much lower number of firms.

to skewness in the portfolio return. The first is that the co-skewness terms are important and negative even when portfolios are composed of a small number of firms. The second is that the co-skewness terms appear to be monotonically decreasing in N. To show these two facts, I construct equally weighted portfolios of $N = 1, 25, 625$, or $Market$ firms.[7] The portfolio labelled $Market$ comprises all of the firms in CRSP in a given semester. For $N = 1$, mean skewness equals mean firm-level skewness. For N equal to 25 and 625, the portfolios are constructed in the following way. First, I assign a random number to each firm and rank firms accordingly. Second, non-overlapping portfolios are formed by taking each consecutive group of N firms according to their ranking. This procedure guarantees that if two firms are in the same portfolio for $N = 25$ they are also in the same portfolio for $N = 625$ –a property that is needed to capture the effect of increasing N. Finally, mean portfolio skewness is computed across the N-firm portfolios. The procedure is then repeated for every semester.

The upshot of the exercise is Figure 12. The figure shows that the co-skewness terms are important even for small N and that they appear to be monotonic in N. Using median skewness produces a similar observation. The observed monotonicity pattern can be fully attributed to monotonicity in co-skewness to N because the mean return skewness across portfolios is the same no matter how many firms are in a portfolio (provided the variance of the portfolio does not change much). While this evidence is consistent with the model, because adding firms to a portfolio could also produce additional heterogeneity in firm events, it is also consistent with the existence of a negatively skewed common factor in returns. If returns follow $r_{it} = \beta_i f_{it} + \varepsilon_{it}$ where f_t is the common factor, it can be shown that, as $N \to \infty$, non-normalized sample skewness converges to $\bar{\beta}^3 T^{-1} \sum_t \left(f_t - \bar{f}\right)^3$, where $\bar{\beta}$ is the average exposure to the common factor.

6.5 Negatively skewed factor in returns

Duffee (1995) proposes that the discrepancy in measured skewness in firm-level and aggregate-level returns can be accounted for by the existence of a negatively skewed factor in returns. Duffee suggests looking at the market factor, but does not try to explain the negative skewness in the market return. While the model here can explain such skewness in market returns from the cross-sectional pattern of volatility in firm-level returns, it is also possible that market returns are negatively skewed due to, for example, peso problems or jumps in the cash

[7] Constructing value-weigthed returns for the portfolios affects the exercise only for small N due to the known negative correlation between skewness and firm size.

flow process. Separating these different explanations is important but difficult because the inclusion of factors, especially those driven by statistical validation, introduces the possibility of "throwing the baby out with the bath water", that is of a false rejection of the paper's null hypothesis.

In this paper, I choose to remove one common factor from returns for the following reasons. First, the model is a one-factor model (see Proposition 2). Second, the market factor may capture the effect of peso problems or jumps in (common factors in) cash flows that would arise in a more general model, whereas a second factor may capture the skewness induced by cross-sectional heterogeneity in firm events. Third, Engle and Mistry (2007) suggest that the ICAPM is inconsistent with priced risk factors that do not display asymmetric volatility or for which time aggregation changes the sign of skewness. In their paper, the market factor is negatively skewed across all frequencies. The size and momentum factors are negatively skewed at high frequencies but positively skewed at lower frequencies and the book-to-market factor is positively skewed across all frequencies. Because Engle and Mistry focus on the post-1988 period, I present the empirical results for the full sample and the post-1988 sample.

To remove the market factor, I run a regression of firm-level daily returns on market returns,

$$q_{it} = a_i + b_{i1} q_{Mt} + b_{i2} q_{Mt-1} + b_{i3} q_{Mt-2} + \varepsilon_{it},$$

over the largest possible sample period from 1963 to 2009 for each firm, from which I obtain the estimated "idiosyncratic" returns, $\hat{\varepsilon}_{it}$. I use logarithmic returns, q_{it} and q_{Mt}, as opposed to simple returns and allow for two lags of the market return because of microstructure effects such as non-synchronous trading (Duffee, 1995). The use of logarithmic returns eliminates the positive skewness that arises mechanically because prices are bounded below at zero (see Duffee, 1995, and Chen et al., 2001). However, the overall impact on the level of skewness is unclear because removing the market factor acts in the opposite direction to increase the level of skewness.

Having obtained the residuals $\hat{\varepsilon}_{it}$, I proceed as in subsection 6.3, creating portfolios of firms according to the calendar week of their first earnings announcement in each semester. I label these portfolios $P1$ through $P13$, one for each of the weeks in the first quarter of the semester. I then repeat experiments one and two using the estimated residuals $\hat{\varepsilon}_{it}$.

Figure 13 depicts skewness across the various portfolios for experiment one. Removal of the market factor contributes to less negative portfolio skewness as compared to Figure

8. In the full sample, portfolio skewness is always insignificant though the point estimate for $P1$ is positive. However, the symmetry present in Figure 8 is lost. In the post-1988 sample period, not only is the skewness in $P1$ significantly positive, but there also is a more symmetric relation in the point estimates. Figure 14 depicts the results for experiment two. Again, compared to Figure 9, portfolio skewness is higher after the removal of the market factor. Consistent with the model there are now several positive point estimates for portfolio skewness and there also is a more pronounced flattening of the skewness curve as k increases.

7 Conclusion

The main contribution of this paper is to model and provide evidence on a new source of negative skewness in aggregate stock returns. This source consists of the cross-sectional heterogeneity in the timing of certain firm announcement events. The paper develops a simple model to capture the observed changes in volatility and mean returns around such firm announcement events. The model shows that periodicity in these events gives rise to conditional heteroskedasticity and positive skewness in firm-level returns. The model also shows that heterogeneity in the timing of these events can lead to negative skewness in aggregate returns despite the positive skewness in firm-level returns.

The results in this paper are informative to the literature on rare disasters that tries to explain the equity premium puzzle and also predicts negative skewness in aggregate stock returns (e.g., Rietz, 1988, and Barro, 2006). Chang et al. (2009) present evidence suggestive that aggregate skewness does not appear to be related to jump risk. This evidence points to the need to develop structural models that nest various explanations for asymmetric volatility to identify the sources of negative skewness in aggregate returns.

The results in this paper are also pertinent to the large literature that tries to model the dynamics of aggregate return volatility. The model predicts that aggregate return volatility is partly explained by the cross-sectional heterogeneity of firm-level volatility. Testing this prediction is left for future research.

Appendix: Proofs

This appendix collects the proofs of the propositions in the text.

Proof of Proposition 1: Guess that firm i's equilibrium stock price is as given in equation (3). From the price function it is easy to obtain

$$E_t\left[Q_{it+1}^{k_i+1}\right] = p_i^{k_i+1} - Rp_i^{k_i}, \tag{A1}$$

$$Var_t\left[Q_{it+1}^{k_i+1}\right] = \Gamma_{k_i+1}^2\sigma_{Fi}^2 + R^{-2(K-k_i)}\sigma_{Di}^2. \tag{A2}$$

Also,

$$Cov_t\left[Q_{it+1}^{k_i+1}, Q_{i't+1}^{k_{i'}+1}\right] = \Gamma_{k_i+1}\Gamma_{k_{i'}+1}\sigma_{ii'}^F + R^{-(2K-k_i-k_{i'})}\sigma_{ii'}^D. \tag{A3}$$

This information can be summarized in the conditional return distribution:

$$\mathbf{Q}_{t+1}^{k+1}|t \sim N\left(\boldsymbol{\mu}_k, \mathbf{V}_k\right).$$

The representative investor solves

$$\max_{\boldsymbol{\theta}_t} -E_t\left(\exp^{-\gamma W_{t+1}}\right)$$

subject to $\boldsymbol{\theta}_t^\intercal = \left(\theta_t^1, ..., \theta_t^N\right)$ and the resource constraint

$$W_{t+1} = \boldsymbol{\theta}_t^\intercal \mathbf{Q}_{t+1}^{k+1} + RW_t.$$

The problem yields the first-order necessary and sufficient conditions

$$\boldsymbol{\theta}_t = \gamma^{-1}\mathbf{V}_k^{-1}\boldsymbol{\mu}_k.$$

Imposing the equilibrium condition that the representative investor holds all shares in the market, $\boldsymbol{\theta}_t = \mathbf{1}$, gives equation (5), $\boldsymbol{\mu}_k = \gamma\mathbf{V}_k\mathbf{1}$. Using equation (5), and assuming without loss of generality that time $t+1$ corresponds to a payout period, it is possible to write the following set of equilibrium conditions for each single firm:

$$\begin{aligned}
P_t^K &= R^{-1}\left[-\gamma\sigma_K^2 + E_t\left[P_{t+1}^0 + D_{t+1}\right]\right] \\
P_{t-1}^{K-1} &= R^{-1}\left[-\gamma\sigma_{K-1}^2 + E_{t-1}\left[P_t^K\right]\right] \\
&\dots \\
P_{t-K}^0 &= R^{-1}\left[-\gamma\sigma_0^2 + E_{t-K}\left[P_{t-K+1}^1\right]\right] \\
P_{t-K-1}^K &= R^{-1}\left[-\gamma\sigma_K^2 + E_{t-K-1}\left[P_{t-K} + D_{t-K}\right]\right],
\end{aligned}$$

where σ_k^2 is the component of $\mathbf{V}_k\mathbf{1}$, $k = 0, ..., K$, associated with the firm. Assuming a stationary solution, recursive substitution yields equation (3) in the proposition. The values for p^k obey the recursion in equation (A1) where stationarity implies that for any k_i, $E_t\left[Q_{it+1}^{k_i+1+K}\right] = E_t\left[Q_{it+1}^{k_i}\right]$.■

Proof of Corollary 1: Note that the variances and covariances in (A2)-(A3) are increasing and convex in k_i for firm i because Γ_{k_i} is increasing and convex in k_i ($\rho_{Fi}/R < 1$), holding all else constant. The mean return is a weighted sum of the variances and covariances and thus is also increasing and convex in k_i.■

Proof of Proposition 2: Combining $\mu_k^M = \boldsymbol{\alpha}^\intercal \boldsymbol{\mu}_k$ and equation (5) gives

$$\begin{aligned}\mu_k^M &= \gamma \boldsymbol{\alpha}^\intercal \mathbf{V}_k \mathbf{1} \\ &= \gamma N \sigma_{M,k}^2,\end{aligned}$$

where the second line follows from $\sigma_{M,k}^2 = \boldsymbol{\alpha}^\intercal \mathbf{V}_k \boldsymbol{\alpha}$ and the definition of $\boldsymbol{\alpha}$. Using equation (5) again:

$$\begin{aligned}\boldsymbol{\mu}_k &= \gamma \mathbf{V}_k \mathbf{1} \\ &= \frac{\mathbf{V}_k \boldsymbol{\alpha}}{\sigma_{M,k}^2}\mu_k^M,\end{aligned}$$

from which $\boldsymbol{\beta}_k \equiv Cov_k\left(\mathbf{Q}_t^k, Q_{Mt}^k\right)/\sigma_{M,k}^2 = \mathbf{V}_k\boldsymbol{\alpha}/\sigma_{M,k}^2$. Also, $\boldsymbol{\alpha}^\intercal \boldsymbol{\beta}_k = \boldsymbol{\alpha}^\intercal \mathbf{V}_k \boldsymbol{\alpha}/\sigma_{M,k}^2 = 1$. ■

Proof of Corollary 2: Without loss of generality drop the firm index i. Using the definition of $f(Q)$, the unconditional mean stock return is

$$E(Q_{t+1}) = \frac{1}{K+1}\sum_{k=0}^{K} E_k(Q_{t+1}) = \frac{1}{K+1}\sum_{k=0}^{K}\mu_k.$$

The unconditional variance in stock returns is

$$\begin{aligned}Var(Q_{t+1}) &= \frac{1}{K+1}\sum_{k=0}^{K}\int (Q - E(Q_{t+1}))^2 \phi\left(Q; \mu_k, \sigma_k^2\right) dQ \\ &= \frac{1}{K+1}\sum_{k=0}^{K}\int (Q - \mu_k + \mu_k - E(Q_{t+1}))^2 \phi\left(Q; \mu_k, \sigma_k^2\right) dQ \\ &= \frac{1}{K+1}\sum_{k=0}^{K}\left(\sigma_k^2 + (\mu_k - E(Q_{t+1}))^2\right).\end{aligned}$$

Finally, unconditional skewness is

$$\begin{aligned} E\left[(Q - E(Q_{t+1}))^3\right] &= \frac{1}{K+1}\sum_{k=0}^{K}\int (Q - E(Q_{t+1}))^3\,\phi\left(Q;\mu_k,\sigma_k^2\right)dQ \\ &= \frac{1}{K+1}\sum_{k=0}^{K}\int (Q - \mu_k + \mu_k - E(Q_{t+1}))^3\,\phi\left(Q;\mu_k,\sigma_k^2\right)dQ \\ &= \frac{1}{K+1}\sum_{k=0}^{K}\left[(\mu_k - E(Q_{t+1}))^3 + 3\sigma_k^2(\mu_k - E(Q_{t+1}))\right]. \qquad \text{(A4)} \end{aligned}$$

The third equality uses $\int (Q-\mu_k)\,\phi\left(Q;\mu_k,\sigma_k^2\right)dQ = 0$ and the fact that skewness is zero for a normal variable, $\int (Q-\mu_k)^3\,\phi\left(Q;\mu_k,\sigma_k^2\right)dQ = 0$. The second term under the summation sign in (A4) can be manipulated to yield the expression in the corollary by noting that

$$\mu_k - E(Q_{t+1}) = \frac{1}{K+1}\sum_{j=0, j\neq k}^{K}\left(\mu_k - \mu_j\right),$$

and grouping terms together under the last summation sign.■

Other calculations in the correlated cash flow case: Here I derive several unconditional moments of aggregate returns including skewness, which is given in the main text in equation (13). Using the definition of $f(Q)$, for a stock market composed of N firms, the unconditional mean stock return is

$$E(Q_{Mt+1}) = \frac{1}{K+1}\sum_{k=0}^{K}E_k(Q_{Mt+1}) = \frac{1}{K+1}\sum_{k=0}^{K}\frac{1}{N}\sum_{i=1}^{N}\mu_k^i.$$

The unconditional variance in stock returns is

$$\begin{aligned} Var(Q_{Mt+1}) &= \frac{1}{K+1}\sum_{k=0}^{K}\int (Q - E(Q_{t+1}))^2\,\phi\left(Q;\mu_k^M,\sigma_{M,k}^2\right)dQ \\ &= \frac{1}{K+1}\sum_{k=0}^{K}\int \left(Q - \mu_k^M + \mu_k^M - E(Q_{t+1})\right)^2\phi\left(Q;\mu_k^M,\sigma_{M,k}^2\right)dQ \\ &= \frac{1}{K+1}\sum_{k=0}^{K}\left(\sigma_{M,k}^2 + \left(\mu_k^M - E(Q_{t+1})\right)^2\right). \end{aligned}$$

Unconditional skewness is

$$\begin{aligned}
E\left[\left(Q_{Mt}-E\left(Q_{Mt}\right)\right)^{3}\right] &= \frac{1}{K+1}\sum_{k=0}^{K}\int\left(Q-E\left(Q\right)\right)^{3}\phi\left(Q;k\right)dQ \\
&= \frac{1}{K+1}\sum_{k=0}^{K}\int\left(\left(Q-\mu_{k}^{M}\right)^{3}+\left(\mu_{k}^{M}-E\left(Q\right)\right)^{3}\right)\phi\left(Q;k\right)dQ \\
&\quad +\frac{3}{K+1}\sum_{k=0}^{K}\int\left(Q-\mu_{k}^{M}\right)^{2}\left(\mu_{k}^{M}-E\left(Q\right)\right)\phi\left(Q;k\right)dQ \\
&\quad +\frac{3}{K+1}\sum_{k=0}^{K}\int\left(Q-\mu_{k}^{M}\right)\left(\mu_{k}^{M}-E\left(Q\right)\right)^{2}\phi\left(Q;k\right)dQ,
\end{aligned}$$

or

$$=\frac{1}{K+1}\sum_{k=0}^{K}\left(\mu_{k}^{M}-E\left(Q\right)\right)^{3}+\frac{3}{K+1}\sum_{k=0}^{K}\left(\mu_{k}^{M}-E\left(Q\right)\right)\sigma_{M,k}^{2}.$$

Expressing market returns as a sum of firm-level returns leads to

$$\begin{aligned}
= &\ \frac{1}{N^{3}}\sum_{i=1}^{N}E\left[\left(Q_{it}-E\left(Q_{it}\right)\right)^{3}\right] \\
&+\frac{3}{K+1}\frac{1}{N^{3}}\sum_{k=0}^{K}\sum_{i=1}^{N}\sum_{i'\neq i}^{N}\left(\mu_{k}^{i}-E\left(Q_{i}\right)\right)^{2}\left(\mu_{k}^{i'}-E\left(Q_{i'}\right)\right) \\
&+\frac{3}{K+1}\frac{1}{N^{3}}\sum_{k=0}^{K}\sum_{i=1}^{N}\left(\mu_{k}^{i}-E\left(Q_{i}\right)\right)\sum_{i'\neq i}^{N}\sigma_{i'k}^{2}+\frac{6}{K+1}\frac{1}{N^{3}}\sum_{k=0}^{K}\left(\mu_{M,k}-E\left(Q_{M}\right)\right)\sum_{i=1}^{N}\sum_{i'>i}\sigma_{ii',k} \\
&+\frac{6}{K+1}\frac{1}{N^{3}}\sum_{k=0}^{K}\sum_{i=1}^{N}\sum_{i'>i}^{N}\sum_{l>i'}^{N}\left(\mu_{k}^{i}-E\left(Q_{i}\right)\right)\left(\mu_{k}^{i'}-E\left(Q_{i'}\right)\right)\left(\mu_{k}^{l}-E\left(Q_{l}\right)\right).
\end{aligned}$$

Proof of Proposition 4: Guess prices to be

$$P_{t}^{k}=p^{k}+\Gamma_{k}E_{t}\left(F_{t}\right)+R^{-(K+1-k)}\sum_{j=0}^{k-1}E_{t}\left(\varepsilon_{t-j}^{D}\right),$$

for all k. Obviously for $k\geq K_{a}$, the expectations operators drop out because the shocks are in investors' information set. Excess stock returns are

$$\begin{aligned}
Q_{t}^{k} &= P_{t}^{k}-RP_{t-1}^{k-1} \\
&= p^{k}+\Gamma_{k}E_{t}\left(F_{t}\right)+R^{-(K+1-k)}\sum_{j=0}^{k-1}E_{t}\left(\varepsilon_{t-j}^{D}\right) \\
&\quad -R\left(p^{k-1}+\frac{\left(\rho_{F}/R\right)^{K+2-k}}{1-\left(\rho_{F}/R\right)^{K+1}}E_{t-1}\left(F_{t-1}\right)+R^{-(K+2-k)}\sum_{j=0}^{k-2}E_{t-1}\left(\varepsilon_{t-1-j}^{D}\right)\right),
\end{aligned}$$

for any period $1 \leq k \leq K_a - 1$. Because

$$\begin{aligned} E_t(F_t) &= \rho_F E_{t-1}(F_{t-1}) + E_t\left(\varepsilon_t^F\right) \\ &= \rho_F E_{t-1}(F_{t-1}) + \frac{\sigma_F^2}{\sigma_F^2 + \sigma_{SF}^2} S_t^F, \end{aligned}$$

the expression for returns reduces to

$$Q_t^k = p^k - Rp^{k-1} + \Gamma_k \frac{\sigma_F^2}{\sigma_F^2 + \sigma_{SF}^2} S_t^F + R^{-(K+1-k)} \frac{\sigma_D^2}{\sigma_D^2 + \sigma_{SD}^2} S_t^D.$$

Above, I used

$$E_t(F_{t-1}) = E_{t-1}(F_{t-1}), E_t\left(\varepsilon_{t-1}^D\right) = E_{t-1}\left(\varepsilon_{t-1}^D\right), ..., E_t\left(\varepsilon_{t-k+1}^D\right) = E_{t-1}\left(\varepsilon_{t-k+1}^D\right),$$

knowing that time t signals are not informative about $t-n$ shocks for any $n > 0$. For period $k = K_a$,

$$\begin{aligned} Q_t^k &= P_t^k - RP_{t-1}^{k-1} \\ &= p^k + \Gamma_k F_t + R^{-(K+1-k)} \sum_{j=0}^{k-1} \varepsilon_{t-j}^D \\ &\quad -R\left(p^{k-1} + \frac{(\rho_F/R)^{K+2-k}}{1-(\rho_F/R)^{K+1}} E_{t-1}(F_{t-1}) + R^{-(K+2-k)} \sum_{j=0}^{k-2} E_{t-1}\left(\varepsilon_{t-1-j}^D\right)\right), \end{aligned}$$

or rearranging,

$$\begin{aligned} Q_t^k &= p^k - Rp^{k-1} + \Gamma_k \rho_F \left[F_{t-1} - E_{t-1}(F_{t-1})\right] + \Gamma_k \varepsilon_t^F \\ &\quad + R^{-(K+1-k)} \left\{\varepsilon_t^D + \sum_{j=0}^{k-2} \left[\varepsilon_{t-1-j}^D - E_{t-1}\left(\varepsilon_{t-1-j}^D\right)\right]\right\}. \end{aligned}$$

Finally, for $k > K_a$, returns take the same form with the same conditional moments as in Proposition 1.

It is now easy to construct conditional return moments. For variance, and for any period $1 \leq k \leq K_a - 1$,

$$Var_{t-1}\left(Q_t^k\right) = \Gamma_k^2 \frac{\sigma_F^4}{\sigma_F^2 + \sigma_{SF}^2} + R^{-2(K+1-k)} \frac{\sigma_D^4}{\sigma_D^2 + \sigma_{SD}^2},$$

which is increasing and convex in k. For period $k = K_a$,

$$\begin{aligned} Var_{t-1}\left(Q_t^k\right) &= \Gamma_k^2 \rho_F^2 Var_{t-1}\left[F_{t-1} - E_{t-1}(F_{t-1})\right] + \Gamma_k^2 \sigma_F^2 \\ &\quad + R^{-2(K+1-k)} \left\{\sigma_D^2 + \sum_{j=0}^{k-2} Var_{t-1}\left[\varepsilon_{t-1-j}^D - E_{t-1}\left(\varepsilon_{t-1-j}^D\right)\right]\right\}. \end{aligned}$$

In addition,

$$
\begin{aligned}
E_{t-1}\left(\varepsilon_{t-1}^{D}\right) &= \frac{\sigma_D^2}{\sigma_D^2+\sigma_{SD}^2} S_{t-1}^{D}, \\
Var_{t-1}\left(\varepsilon_{t-1}^{D}\right) &= \frac{\sigma_D^2 \sigma_{SD}^2}{\sigma_D^2+\sigma_{SD}^2},
\end{aligned}
$$

and

$$
\begin{aligned}
& Var_{t-1,K_a-1}\left[F_{t-1}-E_{t-1}\left(F_{t-1}\right)\right] \\
= \; & Var_{t-1,K_a-1}\left[\varepsilon_{t-1}^{F}-E_{t-1}\left(\varepsilon_{t-1}^{F}\right)+...+\rho_F^{K_a-2}\left(\varepsilon_{t-K_a+1}^{F}-E_{t-K_a+1}\left(\varepsilon_{t-K_a+1}^{F}\right)\right)\right] \\
= \; & \frac{\sigma_F^2 \sigma_{SF}^2}{\sigma_F^2+\sigma_{SF}^2}\left\{1+\rho_F^2+...+\rho_F^{K_a-2}\right\}.
\end{aligned}
$$

For $k \leq K_a$,

$$
Var_{t-1}\left(Q_t^k\right) > Var_{t-2}\left(Q_{t-1}^{k-1}\right).
$$

Furthermore,

$$
Var_{t-1}\left(Q_t^{K_a}\right) > Var_t\left(Q_{t+1}^{K_a+1}\right)
$$

is possible if the arrival of information from past shocks is relevant enough. In that case the path of conditional variance displays two distinct periods of convexity.

Finally, knowing that $\mu_k = \gamma Var_k\left(Q_{t+1}^{k+1}\right)$, it is then possible to recover the constants p^k verifying that the price function above is an equilibrium price.■

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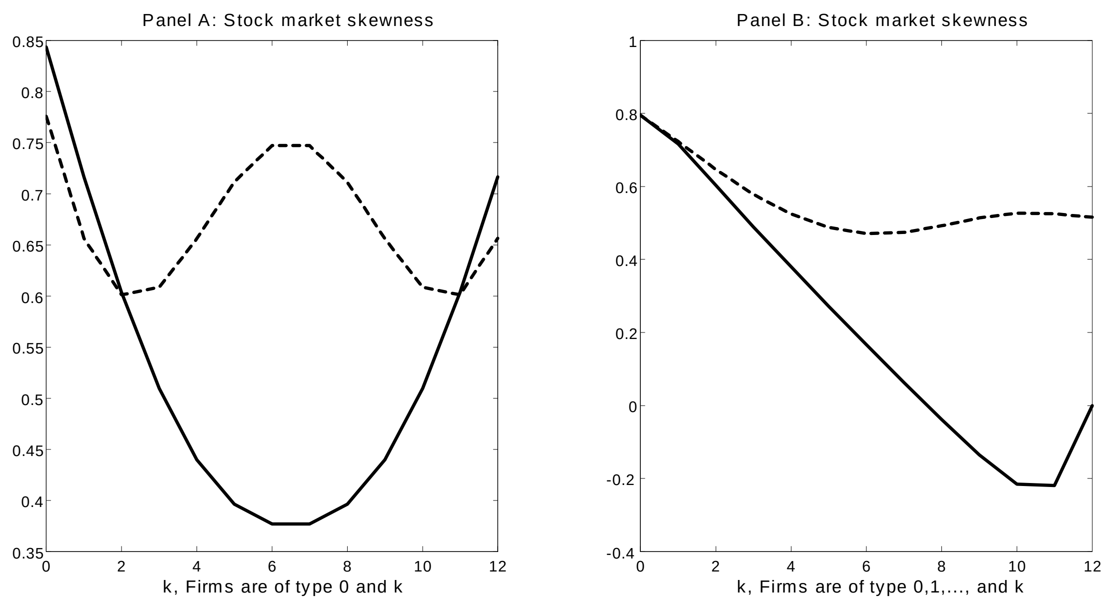

Figure 1: **Stock market skewness in various stock market configurations.** In Panel A, each stock market consists of two types of firms with cash payout dates separated by k periods, where $k \in \{0, 1, ..., K\}$. In Panel B, each stock market consists of $k+1$ different types of firms with cash payout dates of $0, 1, ...,$ and k. Each panel depicts market skewness (solid line) and firm-level skewness (dashed line). Parameters are: $K = 12$, $\sigma_D^2 = \sigma_F^2 = 1$, $\rho_F = 0.9$, $\gamma = 5$, $R = 1.0025$, and $\sigma_{D,ij} = \sigma_{F,ij} = 0.5$.

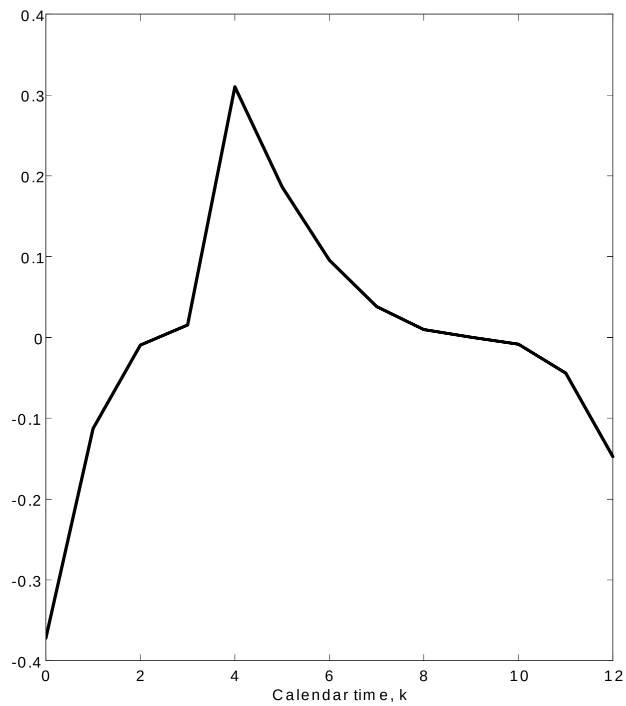

Figure 2: **Contribution of each trading period to stock market skewness.** The stock market consists of firms with cash payout dates of $0, 1, ...,$ and 8. The figure plots the component of normalized skewness, $E\left(Q_t - E\left(Q_t\right)\right)^3 / [E\left(Q_t - E\left(Q_t\right)\right)^2]^{3/2}$, due to each trading period. Parameters are: $K = 12$, $\sigma_D^2 = \sigma_F^2 = 1$, $\rho_F = 0.9$, $\gamma = 5$, $R = 1.0025$, and $\sigma_{D,ij} = \sigma_{F,ij} = 0.5$.

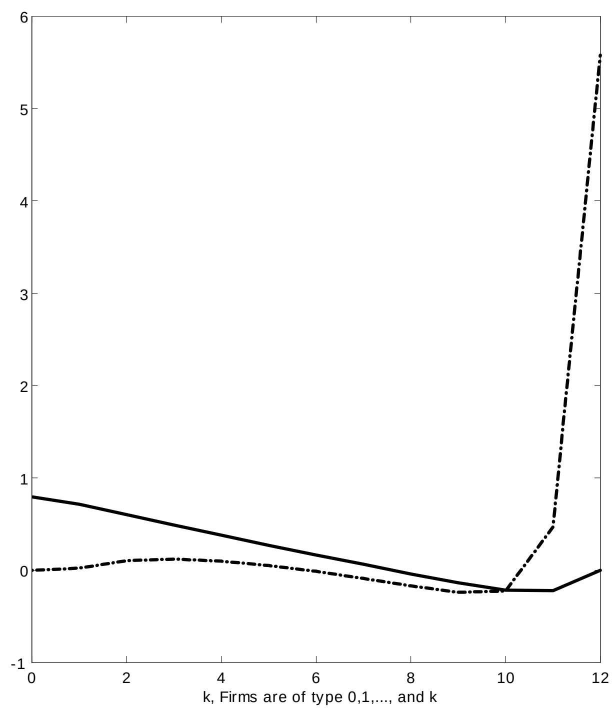

Figure 3: **Decomposing Stock Market Skewness.** Each stock market consists of $k+1$ different types of firms with cash payout dates of $0, 1, ...,$ and k as in Panel B of Figure 1. The figure depicts aggregate skewness (solid) and its co-skewness component term *co-cov* (dashed). Parameters are: $K = 12$, $\sigma_D^2 = \sigma_F^2 = 1$, $\rho_F = 0.9$, $\gamma = 5$, and $R = 1.0025$.

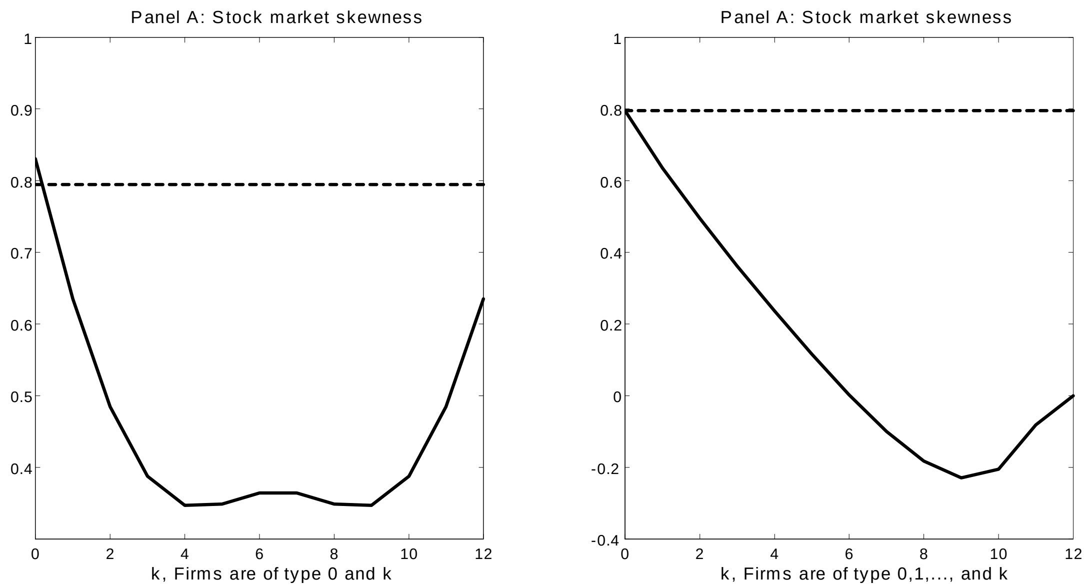

Figure 4: **Stock market skewness in various stock market configurations with uncorrelated cash flows.** In Panel A, each stock market consists of two types of firms with cash payout dates separated by k periods, where $k \in \{0, 1, ..., K\}$. In Panel B, each stock market consists of $k + 1$ different types of firms with cash payout dates of $0, 1, ...,$ and k. Each panel depicts market skewness (solid line) and mean firm-level skewness (dashed line). Parameters are: $K = 12$, $\sigma_D^2 = \sigma_F^2 = 1$, $\rho_F = 0.9$, $\gamma = 5$ and $R = 1.0025$.

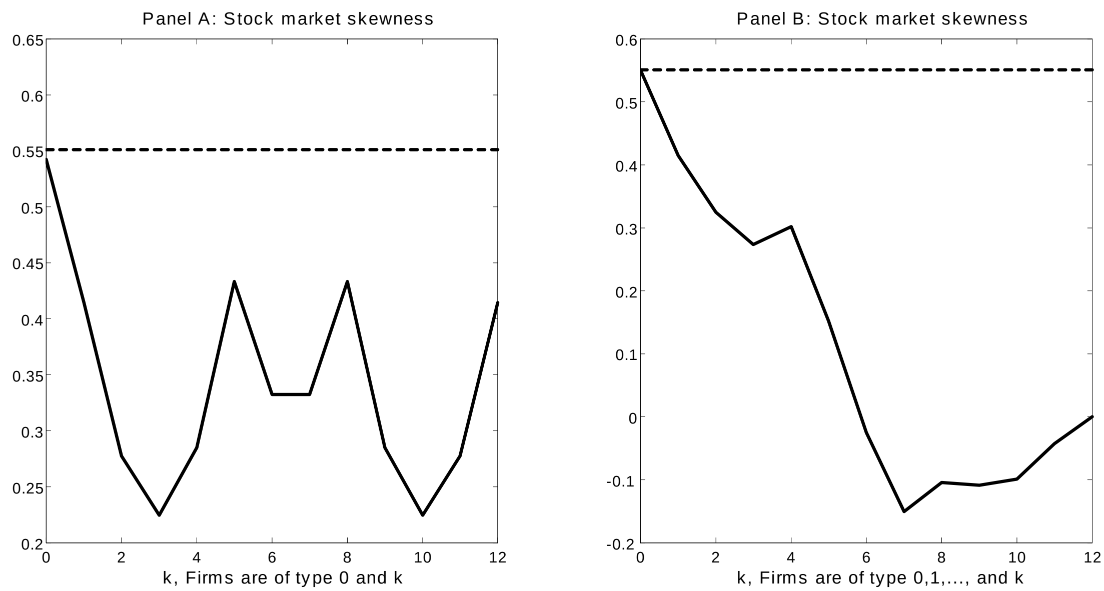

Figure 5: **Stock market skewness in various stock market configurations with earnings announcement events.** In Panel A, each stock market consists of two types of firms with cash payout dates separated by k periods, where $k \in \{0, 1, ..., K\}$. In Panel B, each stock market consists of $k+1$ different types of firms with cash payout dates of $0, 1, ...,$ and k. Each panel depicts market skewness (solid line) and firm-level skewness (dashed line). Parameters are: $K = 12$, $K_a = 6$, $\sigma_D^2 = \sigma_F^2 = 1$, $\rho_F = 0.9$, $\gamma = 5$, $R = 1.0025$, and $\sigma_{SD} = \sigma_{SF} = 0.3$.

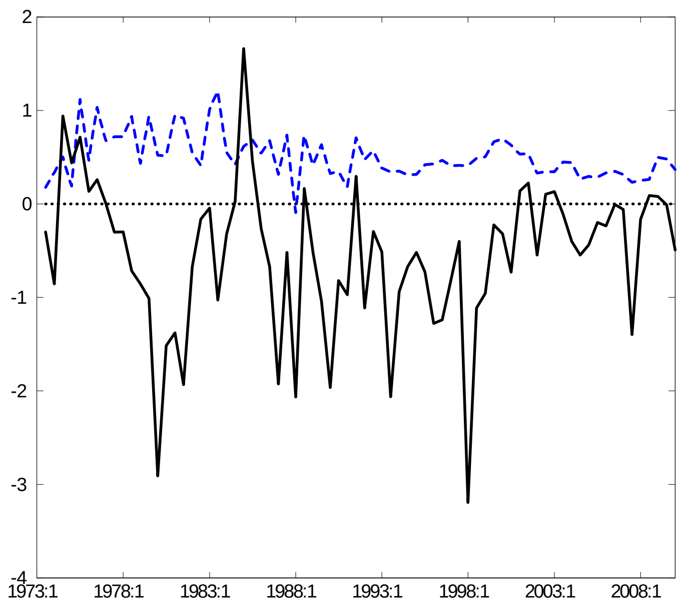

Figure 6: **Skewness in firm-level and aggregate stock returns.** The figure plots mean skewness in daily firm-level returns (dashed line) and skewness in the equally weighted market return (solid line), both computed using six months of trading data. Data comprise all firms in CRSP with complete daily return data by semester. Period of analysis is 1/1/1973 through 31/12/2009.

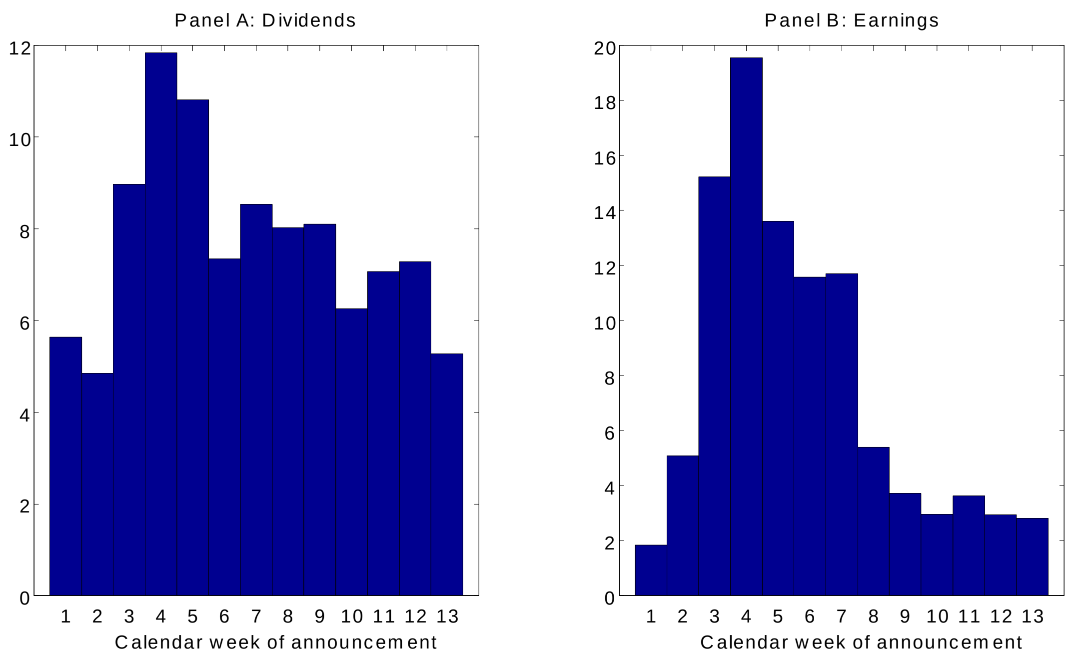

Figure 7: **Histogram of announcement week.** The figure plots the empirical frequency by calendar week of cash payouts (Panel A) and earnings (Panel B) announcements. Data come from the merged Compustat/CRSP quarterly files. The sample period is 1973:Q1 to 2009:Q2. Observations with announcement date before the end of the quarter are dropped.

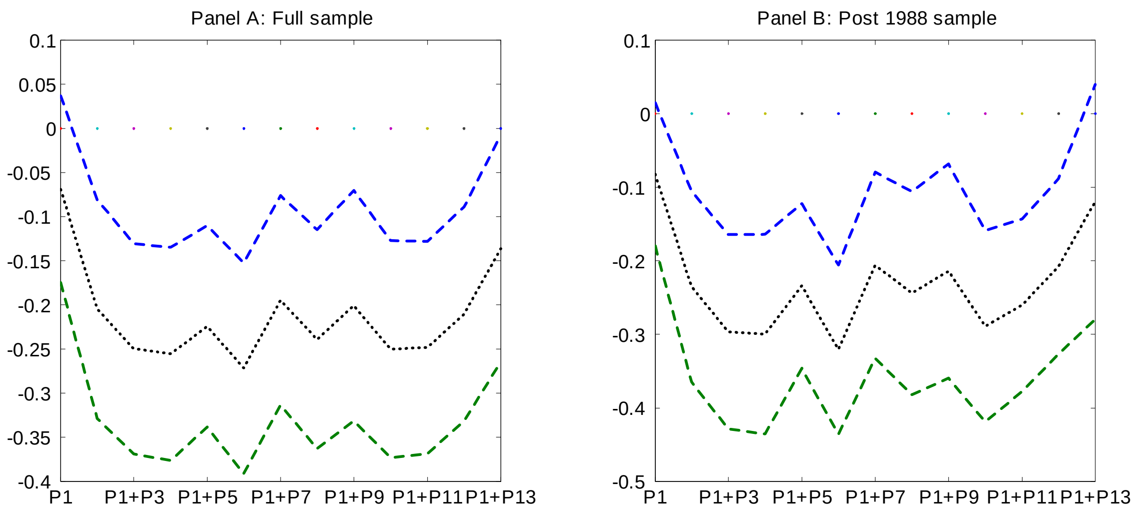

Figure 8: **Skewness and announcement portfolios.** The figure plots portfolio return skewness with 10% confidence bands. Portfolios are constructed by grouping firms that announce in the first week of the first quarter in the semester ($P1$) with firms that announce in week k of the first quarter in the semester (Pk), $k = 2, ..., 13$. Skewness is calculated using daily returns over six months. Portfolio returns are equally weigthed. Portfolios are constrained to have the same number of firms, which is done by randomly dropping firms from the larger portfolios. Data are obtained from the merged Compustat/CRSP quarterly file and the CRSP daily return file. The sample period is January 1, 1973 to December 31, 2009.

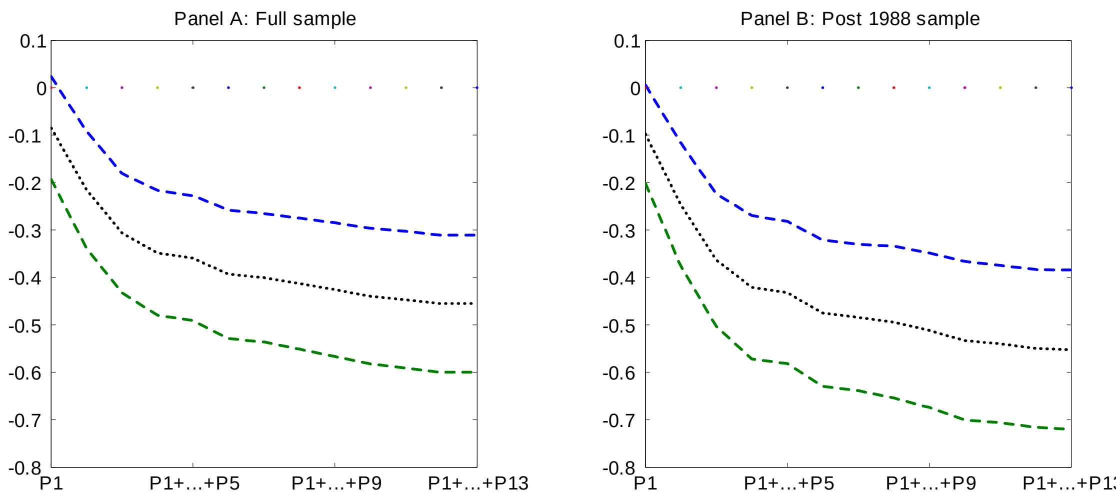

Figure 9: **Skewness and announcement portfolios.** The figure plots portfolio return skewness with 10% confidence bands. Portfolios are constructed by grouping firms that announce between the first week of the first quarter in the semester ($P1$) and week k of the first quarter in the semester (Pk), $k = 2, ..., 13$. Skewness is calculated using daily returns over six months. Portfolio returns are equally weigthed. Portfolios are constrained to have the same number of firms, which is done by randomly dropping firms from the larger portfolios. Data are obtained from the merged Compustat/CRSP quarterly file and the CRSP daily return file. The sample period is January 1, 1973 to December 31, 2009.

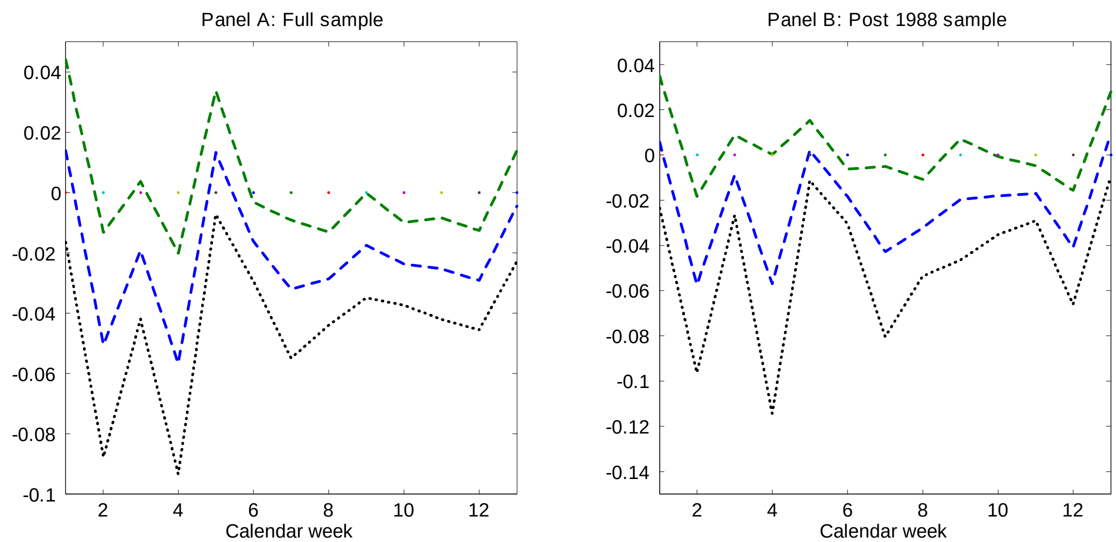

Figure 10: **Skewness and calendar week.** The figure plots the weekly component of market skewness with 10% confidence bands. Skewness is calculated using daily returns over six months. Portfolio returns are equally weigthed. Data are obtained from the CRSP daily return file. The sample period is January 1, 1973 to December 31, 2009.

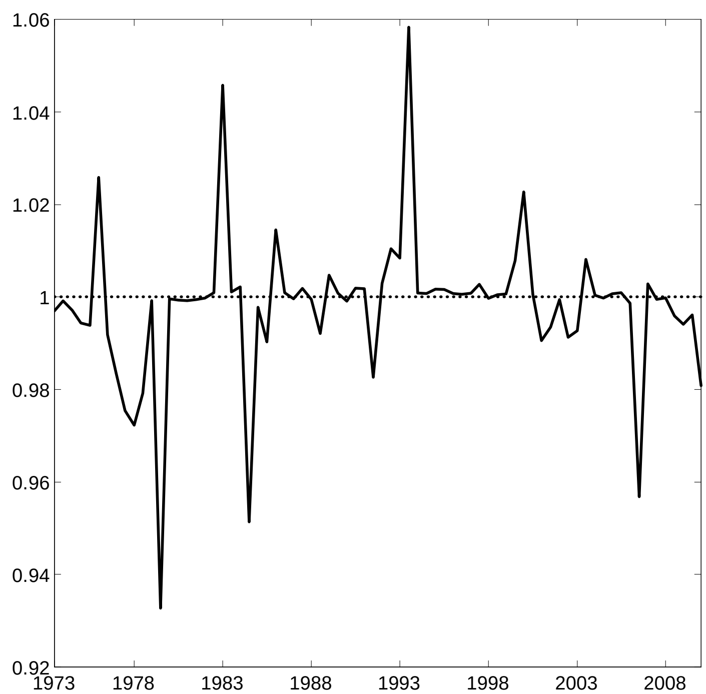

Figure 11: **Skewness decomposition.** The figure plots *co-cov* as a fraction of overall market skewness. Skewness is computed using equally weighted portfolios and six months of trading data. Data comprise all firms in CRSP with complete daily return in the specific year and semester. The sample period is January 1, 1973 to December 31, 2009.

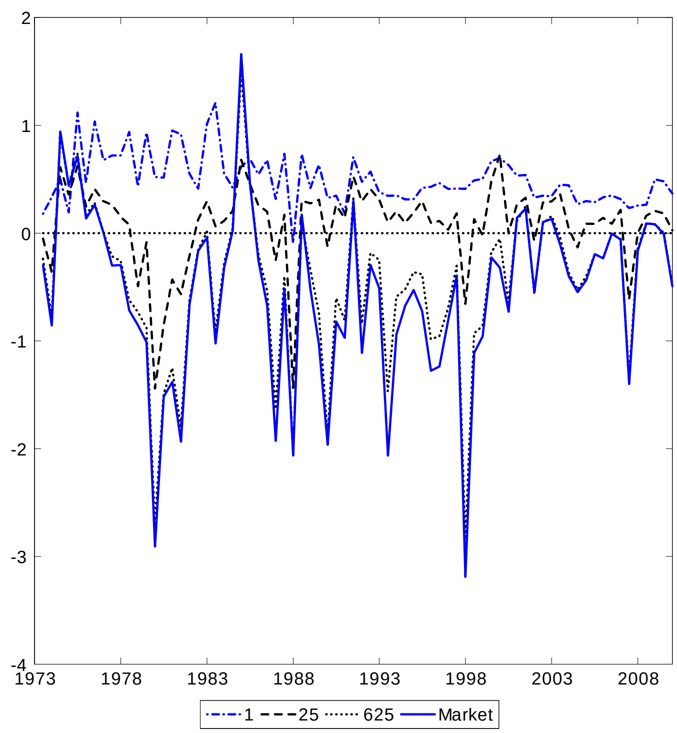

Figure 12: **Skewness in portfolios of varying size.** The figure plots mean skewness in daily returns from portfolios of size N. Skewness is computed using equally weighted portfolios and six months of daily data. When $N = 1$, the figure plots firm-level skewness. The portfolios are constructed by randomly ranking the firms and then grouping them. If two firms are in the same portfolio when $N = 25$, then they will also be in the same portfolio for $N = 625$. The larger portfolio, labeled "$Market$", comprises all firms in CRSP. The sample period is January 1, 1973 to December 31, 2009.

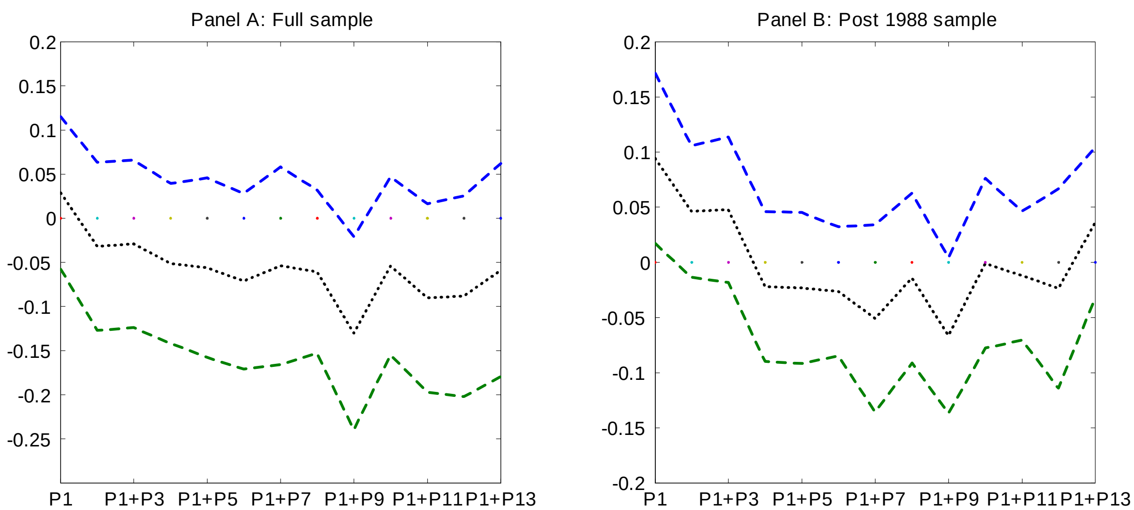

Figure 13: **Skewness and announcement portfolios using CAPM residuals.** The figure plots portfolio return skewness with 10% confidence bands. Portfolios are constructed by grouping firms that announce in the first week of the first quarter in the semester ($P1$) with firms that announce in week k of the first quarter in the semester (Pk), $k = 2, ..., 13$. Skewness is calculated using daily idiosyncratic returns over six months. Portfolio returns are equally weigthed. Portfolios are constrained to have the same number of firms, which is done by randomly dropping firms from the larger portfolios. Data are obtained from the merged Compustat/CRSP quarterly file and the CRSP daily return file. The sample period is January 1, 1973 to December 31, 2009.

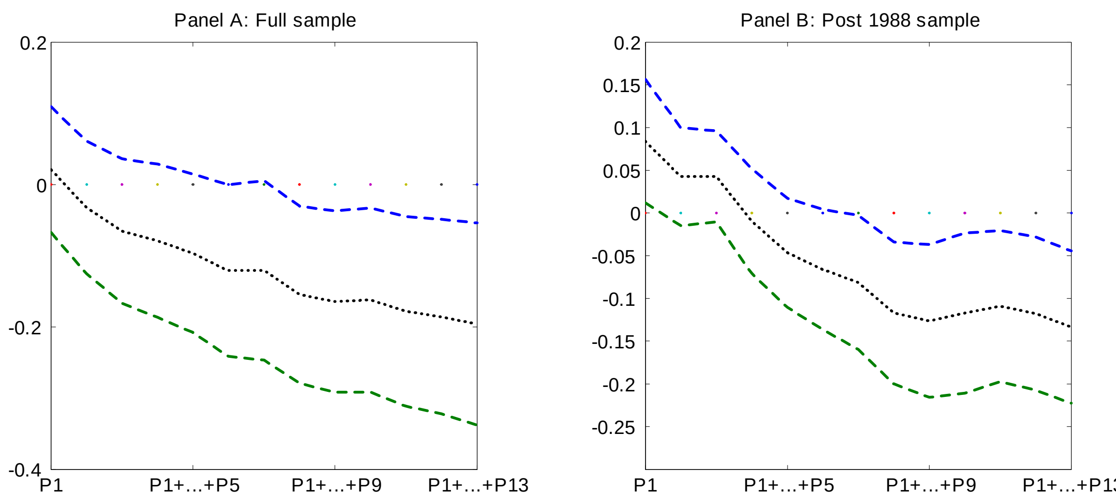

Figure 14: **Skewness and announcement portfolios using CAPM residuals.** The figure plots portfolio return skewness with 10% confidence bands. Portfolios are constructed by grouping firms that announce between the first week of the first quarter in the semester ($P1$) and week k of the first quarter in the semester (Pk), $k = 2, ..., 13$. Skewness is calculated using daily idiosyncratic returns over six months. Portfolio returns are equally weigthed. Portfolios are constrained to have the same number of firms, which is done by randomly dropping firms from the larger portfolios. Data are obtained from the merged Compustat/CRSP quarterly file and the CRSP daily return file. The sample period is January 1, 1973 to December 31, 2009.