Covered Warrant Valuation: A Costly Hedging Model *

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Abstract

We provide a new, supply-side explanation for the consistent, statistically significant, empirical observation that covered warrant prices are higher than those of corresponding traded options. Covered warrant market-makers, who set prices, are also the issuers and always have net short positions. Their reservation prices for redeeming or issuing more warrants reflect the change in their total hedging-related costs. For overall net short positions we show both bid and ask reservation prices lie above perfect-market values, since transaction costs increase both issuers' marginal costs of warrant issuance and the marginal benefits of warrant redemption. The model generates prices and bid-ask spreads consistent with existing empirical evidence and also new testable implications.

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1 Introduction

Empirical studies of covered warrants have consistently found they trade at prices higher than those of comparable exchange-traded options. In this paper we show that the costs incurred by a covered warrant issuer/market-maker in order to dynamically hedge their warrant position generate reservation bid and ask warrant prices which are consistent with such empirical evidence on the values and bid-ask spreads of covered warrants. In addition, our model has wider implications for characteristics of bid and ask prices for structured products and other traded option-like securities.

Covered warrants are bank-issued vanilla options, generally traded on exchanges, where the issuer commits to making a market in the product it has issued. Markets for covered warrants[1] developed rapidly in Europe and Asia in the 1990s. EuWax, the largest European covered warrant market, reported trading volume in covered warrants in 2006 of 17.3 billion Euros (EuWax (2007)).

Key features which distinguish covered warrants from exchange-traded options include the fact that they cannot be held short by the retail investors to whom they are generally issued. Thus issuers have a net short position at all times. Furthermore, in combination with the issuer's commitment to making a market, this means the issuer effectively sets both bid (redemption) and ask (issue) prices for the warrants, since they take one side of virtually all transactions in the warrants issued by them (Bartram & Fehle (2004)). In contrast, trades on options exchanges may be

[1]There are now active markets in Germany, Amsterdam, Italy, Switzerland, Sweden, Spain, Luxembourg, Australia and London (see Bartram & Fehle (2006)). For descriptions of specific covered warrant markets see Bartram & Fehle (2004), Horst & Veld (2003),Chan & Pinder (2000), Abad & Nieto (2007).

made with any of a number of competing market-makers or directly with another investor. Additionally, terms in covered warrants' prospectus[2] suggest there is either an obligation on the issuer to hedge the covered warrants issued or an expectation that such hedging will occur. Finally, the minimum trade size for covered warrants is generally much smaller and the maturity of covered warrants is generally longer than exchange-traded options on the same underlying asset.

Recently, a number of empirical studies have examined covered warrant markets around the world. Comparisons of covered warrant prices with the prices of corresponding exchange-traded options have consistently shown that covered warrant prices are both economically and statistically significantly *higher* than those of the corresponding exchange-traded options (equivalently implied volatilities for covered warrants are higher). Empirical findings are summarised in section 2. Most studies also find that bid-ask spreads are lower in covered warrant markets and that both the bid-ask spread and the difference between prices of covered warrants and equivalent traded options increase with time to maturity.

Potential explanations for the relative overpricing of covered warrants have generally concentrated on the demand side, *i.e.* why investors should be willing to pay more for covered warrants than comparable traded options. These explanations include liquidity premia (Chan & Pinder (2000)), investor clienteles (Bartram &

[2]See Bartram & Fehle (2004, 2006). Some more recent prospectus *e.g.* Goldman Sachs (2007) refer explicitly to the issuer's hedging transactions but do not commit to the form these will take. The term 'covered' would traditionally imply a constant hedge ratio of 1, which is non-optimal, but in which case the model in this paper would not be relevant. There may be such obligations in some markets (*e.g.* Australia), but in other markets there is no obligation in exchange rules or in prospectuses to cover in the traditional sense.

Fehle (2004)), relative transaction costs for investors (Horst & Veld (2003)) and behavioural explanations (Horst & Veld (2003), Abad & Nieto (2007)). However none of these explanations has found universal acceptance.

The supply-side model in this paper complements these demand-side explanations by showing the issuer's costs of hedging their natural large short position in covered warrants results in both the price above which the issuer is willing to issue more covered warrants, and the price below which they are willing to redeem warrants they have already issued (the issuer's reservation ask and bid prices respectively) being greater than the perfect market price and increasing in the magnitude of the issuer's existing position. Thus, relative to traded options prices set by market makers with inventory positions of a much smaller magnitude, issuer-determined covered warrant prices are likely to be higher for both buy and sell transactions.

Whilst some empirical papers on the relative pricing of covered warrants (*e.g.* Bartram & Fehle (2004)) acknowledge that issuers will want to hedge, and that there will be costs associated with this hedging, they do not model these costs or incorporate them explicitly in their empirical tests. The detailed modeling of optimal hedging costs in this paper results in empirical predictions which correspond more closely to existing evidence than initial consideration would suggest and also allows us to draw new testable empirical implications about relative prices and bid-ask spreads in the covered warrant market.

There is a large and growing theoretical literature on the optimal dynamic hedging of derivatives incorporating transaction costs, a tractable example of which we use as the basis for our analysis.[3] However, these models have focused on the valua-

[3]Hodges & Neuberger (1989) first used utility-based valuation to determine optimal hedging strategies

tion of and determination of the optimal hedging strategy for individual options or option portfolios and have not to date been applied to consider the relative pricing of different derivative products or the determination of bid-ask spreads.

We build a utility-based model of reservation bid and ask prices for covered warrant issuers who have an existing short position and who hedge dynamically using the underlying security, thereby incurring transaction costs.[4] Reservation prices should have greater relevance for covered warrant issuers in determining the bid and ask prices they set, as there is potential for direct competition from other covered warrant issuers on ask prices only,[5] whereas options market-makers need to take more account of direct competitive influences.

We model dynamic hedging by covered warrant issuers using the underlying asset. Issuers could also hedge statically using a traded option with identical characteristics.[6] However, since covered warrants often have a much longer maturity

and optimised values for options under transaction costs. Other papers using utility-based methods to value options using transaction costs include Davis, Panas & Zariphopoulou (1993), Clewlow & Hodges (1997), Whalley & Wilmott (1997, 1999), Damgaard (2003) and Zakamouline (2006).

[4] If covered warrant issuers sub-optimally choose not to hedge their covered warrant position, a modified version of our main finding would still hold: reservation bid and ask prices would still both be greater than the perfect market price and the difference would increase with the size of the issuer's existing holding due to a cost of unhedged risk effect, although the functional form would differ. Further details are available from the author on request.

[5] Since warrants from different issuers are not exchangeable, retail investors can generally sell their warrants only back to the original issuer. Thus competition on bid prices is only indirect, through the impact on original choice of issuer. Additionally issuers have no obligation to issue covered warrants with specific characteristics, so there may be no competition on ask prices.

[6] Statically hedging using a non-identical option would still involve optimal dynamic hedging of the residual exposure.

(and lower trade sizes) than exchange-traded options on the same underlying asset, for many covered warrants this is infeasible. Moreover, even if feasible, static hedging using traded options is only worthwhile if it provides greater utility for the issuer net of transaction costs. However, Bartram & Fehle (2004) found higher bid-ask spreads for comparable traded options than for covered warrants, which, combined with fixed costs of trading in options, suggests the incidence of static hedging may be limited, and that dynamic hedging is likely to remain a significant component of hedging activity for covered warrant issuers. Hedging part of their position statically would reduce the net size of the issuer's portfolio requiring dynamic hedging and thus reduce, but not eliminate, the effects of costly dynamic hedging on warrant prices described in this paper. Note that the empirical studies described in section 2 which compare covered warrant and traded option prices generally restrict their samples to covered warrants for which a comparable (in terms of strike price and maturity) traded option exists: exactly those for which the effects may potentially be reduced due to static hedging. This would bias against finding support for the optimal costly hedging model from these studies, which nevertheless we do.

The underlying intuition for our results relies on two points: firstly that the reservation value of a portfolio of hedged warrants incorporates the certainty equivalent value of the transaction costs and residual unhedged risk incurred in hedging the portfolio optimally during its lifetime, and secondly that the reservation value of a *marginal* warrant position to an issuer with an existing portfolio of warrants is given by the difference between the reservation values of the portfolios including and excluding the marginal warrant position.

Transaction costs always reduce the reservation value of a portfolio of covered warrants.[7] However the reservation value of further issues or repurchases depends on the *change* in the certainty equivalent value of the transaction costs involved in optimal hedging. If a covered warrant issuer issues more warrants, the costs of hedging the new, enlarged, portfolio will be greater and this increases the minimum writing price per incremental warrant the issuer would accept. Alternatively, if previously issued warrants are redeemed by the issuer, the costs of hedging the new, smaller, portfolio are *lower* and saving of transaction costs *increases* the maximum redemption price per incremental warrant the issuer is willing to pay. So both reservation purchase (bid) and sale (ask) prices for the covered warrant issuer are greater than the perfect market value, because of their existing short position.[8]

The model produces more detailed predictions for the difference between covered warrant and equivalent traded option prices: the difference increases with the size of the issuer's existing portfolio (though at a decreasing rate) and with the asset's bid-ask spread and, controlling for moneyness, with the time to maturity and asset volatility. Moreover bid-ask spreads for covered warrants are smaller than those for equivalent traded options. Detailed predictions for proportional and absolute bid-ask spreads are summarised in section 4.4. Our key results comparing covered warrant and traded option prices and bid-ask spreads are consistent with the existing empirical literature, which also provides broad support for those of our comparative

[7]The cost of unhedged risk also always reduces reservation values. Thus whilst the functional form may differ, the intuition remains the same if issuers choose not to hedge.

[8]For example, issuing warrants, starting from a zero existing position, increases the issuer's total future transaction costs so he requires a higher minimum price for each warrant to compensate for these. Similarly, if a covered warrant issuer redeems all the warrants he has issued, he is willing to pay a higher price to buy back the warrants, because of the cost saving resulting from the trade.

static results which have been addressed empirically to date.

Section 2 summarises the empirical evidence on covered warrant values and bid-ask spreads. The model for reservation bid and ask prices for a covered warrants issuer is set out in section 3. Section 4 develops testable implications and relates them to the existing empirical literature. Section 5 concludes.

2 Empirical evidence

2.1 Relative prices of options and covered warrants

Empirical studies of covered warrant markets have consistently found that both ask and bid prices for warrants issued by banks are higher than those of comparable traded options. For the largest covered warrant market, EuWax, Bartram & Fehle (2004) found ask prices were on average 4.7% and bid prices 9.9% higher than prices for comparable options traded on the EuReX options exchange during 2000, a statistically significant difference. For the Australian market, Chan & Pinder (2000) found statistically significant median pricing differences of $3-4\%$ and mean pricing differences of $6-7\%$. Horst & Veld (2003)'s results for the Amsterdam market (relative overpricing of more than 25% on average) are not completely comparable, as they are measured over only the first five days of trading, but are of similar magnitude to the Spanish case (Abad & Nieto (2007)) which had a median overpricing of 17% for warrants with similar volumes to those traded on the options market (and $19-25\%$ median overpricing more generally).

Several potential explanations for the overpricing have been put forward. Chan & Pinder (2000) interpret their results as evidence of a liquidity premium: investors are willing to pay a higher price for a warrant in the relatively more liquid Australian warrant market than a corresponding exchange traded option. This liquidity premium hypothesis is supported for the Australian markets, where the covered warrant market is more liquid than the exchange-traded options market. However, as noted by Bartram & Fehle (2004), the EuWax covered warrant market generally has lower volume and liquidity than the corresponding EuRex traded options market, so a liquidity premium would suggest relative underpricing of warrants, the opposite of what actually occurs. For the Spanish case Abad & Nieto (2007) find that the relative price difference is larger when the bid-ask spread for warrants is relatively smaller, which they interpret as a liquidity effect, but which is also consistent with the model in this paper (see section 4).

Bartram & Fehle (2004) suggest a clientele effect, where covered warrants are held by more speculative investors, who are more likely to reverse their position before expiry and are thus more concerned with the bid-ask spread than the initial level of the ask price. This is consistent with the relative characteristics of German covered warrant and traded options markets (EuWax and EuRex), where not only were covered warrant ask and bid prices found to be greater than comparable traded option prices, but also covered warrant bid-ask spreads were significantly smaller than those of equivalent traded options. However Abad & Nieto (2007) found that bid-ask spreads in the Spanish covered warrant and options markets are similar in size, providing only weak grounds to explain why investors would be willing to pay higher prices to buy warrants rather than corresponding options.

Horst & Veld (2003) consider the relative trading costs and flexibility faced by potential investors in each market. They suggest these can explain some of the overpricing (for low warrant prices), but are not large enough to explain the full relative pricing difference. Bartram & Fehle (2004) also consider transaction costs for investors trading on each market and find that whilst these are lower for warrant trades the difference is very small (less than 1% of the trade value).

Horst & Veld (2003) also suggest a behavioural explanation for the willingness of investors to buy more costly warrants: 'financial institutions have managed to create an image for call warrants that is different from call options.', and Abad & Nieto (2007) suggest differences between the level of overpricing between different issuers, which cannot be explained by liquidity, clientele or credit risk arguments, may have a behavioural explanation. This is however difficult to test in practice.

2.2 Bid-ask spreads in covered warrant markets

To date there have been few studies of bid-ask spreads in covered warrant markets. As mentioned above, Bartram & Fehle (2004) found that proportional bid-ask spreads for covered warrants were smaller than those for corresponding traded options (2.8% *vs* 7.1% on average, with the bid-ask spread differences almost all significant at the 1% level or better). Using the same data, Bartram & Fehle (2006) investigated relationships between bid-ask spreads on each market. They found that bid-ask spreads on either market were lowered by $1-2\%$ by competition from the other market, which they interpret as evidence of competition between markets even though contracts are not fungible between them. They also find proportional bid-ask spreads are statistically significantly lower for covered warrants than traded options and suggest the difference could be due to the greater depth offered by EuRex market-makers as compared to EuWax issuers. The theoretical costly hedging model in this paper demonstrates this directly.

More recently, Bartram, Fehle & Shrider (2007) also found lower bid-ask spreads for covered warrants than traded options. They argue informally that this results from the lower levels of adverse selection faced by marketmakers in covered warrant markets relative to their traded option counterparts due to the lack of anonymity in the covered warrant market. In general bid-ask spreads will contain both inventory and adverse selection components.

Petrella (2006) considers the market-making cost determinants of proportional bid-ask spreads on the Italian covered warrants market during December 2000 - January 2001. He finds that initial costs (proxied by $kS\Delta$), rebalancing costs (given by a measure of stock price variability multiplied by the warrant's Gamma) and a 'reservation proportional bid-ask spread', or minimum spread required to avoid scalping[9] (proportional to the tick-size $\times m\Delta/V$ where m is the number of assets underlying one warrant contract) are all significantly positively related to the warrant proportional bid-ask spread. He interprets his results as showing that warrants market makers hedge their positions by rebalancing to keep their portfolio delta-neutral and suggests that thus 'representative market maker does not fear to trade with informed traders, because his position is hedged'. However he does not test for adverse selection explicitly, and his analysis is not specific to covered warrants as opposed to traded options markets.

[9]If the warrant's bid price after a one tick upward movement in the underlying asset would be greater than the warrant's current ask price short term unhedged speculators can make a short term profit from small movements in the underlying

3 Model

3.1 Setting: transaction cost models

We start by summarising briefly the results of Hodges & Neuberger (1989) (HN), who first investigated optimal hedging of options in the presence of transction costs using a utility-based framework. They allowed the holder of an option to hedge optimally, using the underlying asset and the riskless bond, taking into account the transaction costs associated with the optimal hedging strategy, in order to maximise his expected utility of wealth at some date at or after maturity of the option. They considered only transaction costs proportional to the amount traded, $k(S, dy) = kS|dy|$, where k is the percentage transaction cost fee, including the proportional component of the underlying asset's bid-ask spread, S is the value of one unit of the underlying asset and dy is the change in the number of the underlying asset held by the option holder, and assumed the option holder had exponential utility function with absolute risk aversion γ.

HN showed that, with transaction costs, the optimal hedging strategy is to transact only when the actual number of the underlying asset held moves outside a 'hedging band', that the width of the hedging band increases with the level of transaction costs and that the centre of this band can differ systematically from the Black-Scholes delta. They also showed that using this hedging strategy affected option values, decreasing the reservation value of the long vanilla options they considered. Many of their results were, however, purely numerical. We base our model on a later, more tractable model in the same framework (utility-based model of option valuation under transaction costs), Whalley & Wilmott (1997, 1999).[10]

A covered warrant issuer holds a portfolio of warrants with payoff $\Lambda_P < 0$ and maturity T written on an underlying asset which follows Geometric Brownian Motion

$$dS = \mu S dt + \sigma S dz$$

The warrant issuer hedges optimally as in HN in order to maximise his expected utility of final wealth. Like them, we consider only proportional transaction costs[11] and assume the covered warrant issuer has an exponential utility function with absolute risk aversion γ. Assuming additionally that the level of transaction costs is small, $k \ll 1$, Whalley & Wilmott (1997) and Whalley (1998)[12] were able to approximate the option value with relatively simple expressions for the hedging bandwidth and linear partial differential equations satisfied by components of the option value. For a general option portfolio with payoff $\Lambda_P(S,T)$, they showed that

Proposition 1 *(Whalley & Wilmott (1997), Whalley (1998))*

The reservation value of an optimally hedged option portfolio with maturity T and final payoff $\Lambda_P(S,T)$ held by such an investor can be approximated by

$$G(\Lambda_P) \approx G^{BS}(\Lambda_P) + G_b(|\Gamma^P|) + G_f(|\Delta^P|) \tag{1}$$

[10] The particular formulation allows us to derive simple formulae which illustrate explicitly the effects we describe rather than relying exclusively on numerical simulations. Apart from the assumption of small transaction costs, the basic formulation of the problem is the same as that in HN and most subsequent utility-based papers so the results we obtain do not depend on the particular model.

[11] As the size of an option portfolio increases, proportional costs become increasingly important in comparison to fixed costs in determining both the hedging strategy and the reservation price.

[12] Whalley & Wilmott (1997) showed the leading order correction was $G_b(|\Gamma^P|)$ and also included initial costs (see later). Whalley (1998) extended the expansion to higher orders; the next term is $G_f(|\Delta^P|)$.

where

1. $G^{BS}(\Lambda_P)$ *is the Black-Scholes value associated with final payoff* $\Lambda_P(S,T)$*,*

2. $G_b(|\Gamma^P|)$ *satisfies*

$$G_{b_t} + rSG_{b_S} + \frac{\sigma^2 S^2}{2} G_{b_{SS}} - rG_b = \frac{\hat{\gamma}(t)\sigma^2 S^2}{2}\left(H_P^{*^2}(|\Gamma^P|) - H_0^{*^2}\right) \tag{2}$$

s.t. $G_b(S,T) = 0$ *where* $\hat{\gamma} \equiv \gamma e^{-r(T-t)}$ *and* $H_P^*(|\Gamma^P|)$, $H_0^*(S,t)$ *represent the optimal 'hedging semi-bandwidths' associated with an option portfolio with final payoff* Λ_P *and Black-Scholes Gamma* Γ^P *and with no option holdings respectively defined below*

3. $G_f(|\Delta^P|)$ *satisfies*

$$G_{f_t} + rSG_{f_S} + \frac{\sigma^2 S^2}{2} G_{f_{SS}} - rG_f = 0$$

s.t. $G_f(S,T) = -k(|\xi(T) - SG_S^{BS}(S,T)| - |\xi(T)|) = -k(|\xi(T) - S\Delta^P(S,T)| - |\xi(T)|)$ *where* $\xi(t) = \frac{\mu - r}{\hat{\gamma}(t)\sigma^2}$ *and* Δ^P *is the Black-Scholes Delta.*

The optimal hedging strategy is to transact only when the actual number of the underlying asset held differs by more than the 'hedging semi-bandwidth', H_P^**, from the ideal number,* $y_P^*(S,t)$*, and to trade the minimum required in order to bring the actual number back within this no-transaction band* $[y_P^* - H_P^*, y_P^* + H_P^*]$ *where*

$$\begin{aligned} y_P^*(S,t) &= \frac{\xi(t)}{S} - G_S^{BS} = \frac{\xi(t)}{S} - \Delta^P & (3)\\ H_P^*(S,t) &= \left(\frac{3kS}{2\hat{\gamma}}\right)^{\frac{1}{3}} \left|\frac{\partial y_P^*}{\partial S}\right|^{\frac{2}{3}} = \left(\frac{3kS}{2\hat{\gamma}}\right)^{\frac{1}{3}} \left|G_{SS}^{BS} + \frac{\xi}{S^2}\right|^{\frac{2}{3}} = \left(\frac{3kS}{2\hat{\gamma}}\right)^{\frac{1}{3}} \left|\Gamma^P + \frac{\xi}{S^2}\right|^{\frac{2}{3}} & (4) \end{aligned}$$

The optimal hedging semi-bandwidth, H_P^*, is the outcome of the tradeoff between the reduction in hedging error resulting from trading and the transaction costs incurred in doing so and depends on the absolute value of the leading order Gamma of the option portfolio being hedged, $\Gamma^P \equiv G^{BS}_{SS}$, to a fractional power.[13] $G_b(|\Gamma^P|)$ represents the leading order effect of costs and hedging error associated with the hedging strategy during the lifetime of the option. It captures the effects of 'bandwidth hedging' (transacting in order to remain within the no-transaction band), and so depends on the bandwidth and is a function of the option's absolute Gamma integrated over the remaining life of the option. It thus has greatest value for close-to-the-money asset prices. $G_f(|\Delta^P|)$ represents the leading order effect of the final costs of unwinding the hedge, depends on the size of the option's Delta and so is greater when the option is in-the-money.

Applying Proposition 1 to value a covered warrant issuer's portfolio, note the fractional powers in the equations for H^*_P and $G_b(|\Gamma^P|)$ mean the hedging strategy and warrant value are nonlinear. Hence warrant values are not additive and portfolios must be valued and hedged as a whole. This also means the value of a given warrant position differs depending on the composition of the issuer's existing portfolio, since they value it at its *marginal reservation value*, *i.e.* the difference in the value of their portfolio overall due to the change in the portfolio's composition.

We define $g(\Lambda_Q|\Lambda_P)$ as the marginal reservation value of a covered warrant portfolio with final payoff $\Lambda_Q(S,T)$ to a covered warrant issuer with an existing portfolio of covered warrants on the same underlying asset with final payoff $\Lambda_P(S,T)$.

$$g(\Lambda_Q|\Lambda_P) = G(\Lambda_P + \Lambda_Q) - G(\Lambda_P) + g_i(|\Lambda_Q|)$$

[13]The Gamma dependence arises because both the transaction costs incurred in maintaining a holding in the underlying asset close to the option's Delta and the hedging error resulting from discrete hedging are functions of the size of the change in the Delta, $\left|\frac{\partial \Delta}{\partial S}\right| = |\Gamma|$, under the optimal hedging strategy. See *e.g.* Rogers (2000) for a heuristic explanation.

where $g_i(|\Lambda_Q|) = -kS|Q_S^{BS}|$ is the leading order initial cost of changing the number of the underlying asset held.

We assume for simplicity that both the existing and marginal portfolios are positions of various magnitudes in a single European covered warrant, so $\Lambda_P = N\Lambda_V$, $\Lambda_Q = n\Lambda_V$, where Λ_V represents the payoff to a single long European covered warrant. Since warrant issuers hae a net short position, $N < 0$, so $\Lambda_P \leq 0 \forall (S, T)$,[14] $g(n\Lambda_V|N\Lambda_V)$ represents the marginal reservation value of an incremental covered warrant position and can be positive (if n is positive, so the incremental transaction for the issuer is to buy or redeem warrants) or negative (if n is negative, so the marginal transaction for the issuer involves issuing warrants). We define the *marginal bid reservation price* per option for a marginal purchase of $m > 0$ options to an issuer with an existing position of N identical warrants is defined as $V^{bid}(m|N) = g(m\Lambda_V|N\Lambda_V)/m > 0$ and the *marginal ask reservation price* per warrant for taking a short position in $m > 0$ warrants (so $\Lambda_Q = -m\Lambda_V$) for an issuer with an existing position of N identical warrants is defined as $V^{ask}(m|N) = -g(-m\Lambda_V|N\Lambda_V)/m > 0$.

Rather than marginal bid and ask reservation prices, it is easier to work with the reservation mid price, $V^{mid}(m|N)$, and reservation bid-ask spread, $B(m|N)$ (for a quote depth $m > 0$ and an existing position of N warrants), defined as

$$V^{mid}(m|N) = \frac{V^{bid}(m|N) + V^{ask}(m|N)}{2} \tag{5}$$

$$B(m|N) = V^{ask}(m|N) - V^{bid}(m|N) \tag{6}$$

We assume the existing portfolio is large relative both to the marginal portfolio ($|N| \gg |n| = m$) and to the optimal holding of the underlying asset in the absence

[14]The analysis below holds for general N.

of any covered warrant position, ξ/S, which ensures we concentrate on effects due to warrant hedging rather than optimal investment. Then expanding in $|m/N| \ll 1$ and taking leading order terms, we find[15]

Proposition 2 [16]

The reservation mid price per warrant for a quote depth of m for an issuer with an existing position of N identical warrants is, to leading order,

$$\begin{aligned} V^{mid}(m|N) &\approx V^{BS} + V_b^{mid}(N) + V_f^{mid}(N) \\ &\approx V^{BS} - \frac{4}{3} sgn(N)|N|^{\frac{1}{3}} k^{\frac{2}{3}} L_b(|\Gamma|) - sgn(N) kS|\Delta| \\ &= V^{BS} + \frac{4}{3}|N|^{\frac{1}{3}} k^{\frac{2}{3}} L_b(|\Gamma|) + kS|\Delta| \end{aligned} \tag{7}$$

where $L_b(|\Gamma|) > 0$ satisfies

$$L_{b_t} + rSL_{b_S} + \frac{\sigma^2 S^2}{2} L_{b_{SS}} - rL_b = -\frac{\hat{\gamma}(t)\sigma^2 S^2}{2} \left(\frac{3S}{2\hat{\gamma}(t)}\right)^{\frac{2}{3}} |\Gamma|^{\frac{4}{3}} \tag{8}$$

s.t. $L_b(S,T) = 0$ and $\Gamma = V_{SS}^{BS}$ and $\Delta = V_S^{BS}$ are the Black-Scholes Gamma and Delta of a single long warrant respectively.

The absolute reservation bid-ask spread per warrant for a quote depth of m for an issuer with an existing position of N identical warrants is, to leading order,

$$\begin{aligned} B(m|N) &\approx B_b(m|N) + B_i \\ &\approx \frac{4}{9} m|N|^{-\frac{2}{3}} k^{\frac{2}{3}} L_b(|\Gamma|) + 2kS|\Delta| \end{aligned} \tag{9}$$

[15] Details of the derivation are given in the Appendix.

[16] This can be extended to more general portfolios of covered warrants on the same underlying asset as long as the covered warrant issuer has a net short position both before and after any transaction. Reservation ask prices (at which a warrant issuer would issue new warrants) are consistent with redemption before maturity at the relevant future reservation bid price.

The proportional reservation bid-ask spread per warrant for a quote depth of m for an issuer with an existing position of N identical warrants is, to leading order,

$$\frac{B(m|N)}{V^{mid}(m|N)} \approx \frac{\frac{4}{9}m|N|^{-\frac{2}{3}}k^{\frac{2}{3}}L_b(|\Gamma|) + 2kS|\Delta|}{V^{BS} + \frac{4}{3}|N|^{\frac{1}{3}}k^{\frac{2}{3}}L_b(|\Gamma|) + kS|\Delta|} \tag{10}$$

Both the the difference between the reservation mid price and the perfect market (Black-Scholes world) price, $V^{mid}(m|N) - V^{BS}$, and the bid-ask spread, B, have a component relating to the lifetime or bandwidth costs of implementing the dynamic hedging strategy during the warrant's lifetime (V_b^{mid} and B_b respectively).[17] In addition the reservation mid price has a final cost component of $V_f^{mid} = kS|\Delta|$ and no initial cost component, whereas the bid-ask spread has no final but an initial cost component of $B_i = 2kS|\Delta|$.[18] Intuitively, issuing an additional warrant at the ask price gives rise to both initial costs and also future costs of unwinding the hedge, since the sale increases the overall size of the issuer's holding in the underlying asset. In contrast, redeeming a warrant at the bid price still incurs initial costs but saves on future costs, since the repurchase reduces the size of the transaction required to unwind the issuer's position in the future.

4 Empirical implications

The primary implication of the model is that warrant issuers' reservation prices for redeeming and for selling additional warrants are both strictly greater than the

[17]These are nonlinear in the size of the issuer's warrant portfolio, $|N|$, and increment through time.

[18]Higher order terms may reduce the magnitude of B_i and V_f^{mid} if the number of the underlying asset which need to be traded initially or at the final date is lower, *e.g.* because the positions after these trades are at the edge of the no-transaction band rather than the centre. However, the overall magnitude of initial and final costs will always remain non-negative.

reservation price.

Corollary 1 *Both the reservation bid price per warrant and the reservation ask price per warrant for a quote depth of m for an issuer with an existing position of N identical warrants, $V^{bid}(m|N)$ and $V^{ask}(m|N)$ respectively, are strictly greater than the perfect market (Black-Scholes) price for all m and all $N < 0$:*

$$V^{bid}(m|N) - V^{BS} > 0, \qquad V^{ask}(m|N) - V^{BS} > 0 \qquad \forall (S, t)$$

This is a direct consequence of the fact that warrant issuers always have a net short position in warrants.[19] Effectively, the reservation price reflects the marginal cost of each warrant including transaction costs; since the net position in warrants is short, transaction costs increase this marginal cost above the perfect market price.

This relationship should also hold between actual covered warrant and traded option prices. Whilst prices set by options market-makers may also differ from perfect-market values, a covered warrant issuer's portfolio is always net short, whereas options market-makers' holdings can be both positive and negative. Moreover, since they will generally prefer to reduce the magnitude of their inventory position, options market-makers' holdings are smaller in magnitude than a covered warrant issuer/market-maker's inventory holdings of warrants, which represents the total number of warrants outstanding. So hedging costs imply that covered warrant bid and ask prices should be higher than prices for equivalent traded options. This is consistent with all the empirical evidence across different markets and issuers (Bartram & Fehle (2004), Chan & Pinder (2000), Horst & Veld (2003) and Abad & Nieto (2007)).

[19] The signs of both lifetime and final cost components of $V^{mid} - V^{BS}$ depend on $\mathrm{sgn}(N)$.

In sections 4.1 - 4.3 we implement the model numerically and examine the effects of parameters associated with the issuer (N, m), the warrant (T) and the market (σ, k) respectively. For our base case we take, as far as possible, parameters matching the summary data for covered warrants on EuWaX reported in Bartram & Fehle (2004). Consistent with this, base case volatility is set at 32% and warrant maturity at 2 years.[20] Transaction cots in the underlying market are assumed to be 0.5%. Obtaining data on cevered warrant issuers' holdings and risk-aversion is difficult. Assuming $\gamma|N| = 4$ and $m/|N| = 10^{-4}$ gives rise to proportional price differences $(V^{mid} - V^{BS})/V^{BS}$ of 5.6 - 6.5% and proportional bid-ask spreads B/V^{BS} of 2.7 - 3.1% for close to the money warrants $(0.9 < S/K < 1.1)$. Bartram & Fehle (2004) found the average proportional price difference between EuWaX and EuReX to be 7.3% and proportional bid-ask spreads on EuWaX of $2 - 3\%$.

The model can thus generate price differences and bid-ask spreads of similar orders of magnitude as actual bid and ask prices. In practice, other factors suggested in the literature such as adverse selection, liquidity or behavioural considerations may also play a role in determining actual covered warrant reservation prices. Moreover, the model in this paper provides reservation prices, at which issuers are indifferent between transacting or not, *i.e.* minimum ask prices and maximum bid prices. Actual prices may differ from these, if economic rents are available in the market. Thus whilst hedging costs may not be the sole determinant of the difference between covered warrant and traded option prices, the model demonstrates that they generate some, if not all, of such a difference, and hence factors which influence covered warrant *reservation* prices, as discussed in sections 4.1 - 4.3, should also have an

[20]Bartram & Fehle (2004) find average volatility of 32% and maturities ranges of 14-730 days.

effect on their *actual* prices.

4.1 Issuer portfolio effects

Overall we would expect relationships between covered warrant and traded option prices to be consistent with covered warrant market-makers/issuers having larger (negative) inventory positions (more negative N) and smaller quote depths (m). However, issuers of covered warrants vary in the size of their (short) holdings of particular warrants ($|N|$), and potentially also in the quote depth they offer (m).[21] Corollary 2 summarises comparative statics with respect to $|N|$ and m.

Corollary 2 *1. The magnitude of the difference between the reservation mid price and the perfect market price increases with the size of the issuer's existing portfolio, whereas both absolute and proportional bid-ask spreads decrease with the size of the issuer's existing portfolio, $|N|$:*

$$\frac{\partial |V^{mid}(m|N) - V^{BS}|}{\partial |N|} > 0, \qquad \frac{\partial B(m|N)}{\partial |N|} < 0, \qquad \frac{\partial \left(\frac{B}{V^{mid}}\right)}{\partial |N|} < 0 \tag{11}$$

2. The magnitude of the difference between the reservation mid price and the perfect market price is independent of the quote depth, whereas both absolute and proportional bid-ask spreads increase with the quote depth, m:

$$\frac{\partial |V^{mid}(m|N) - V^{BS}|}{\partial m} = 0, \qquad \frac{\partial B(m|N)}{\partial m} > 0, \qquad \frac{\partial \left(\frac{B}{V^{mid}}\right)}{\partial m} > 0 \tag{12}$$

Intuitively, $V^{mid} - V^{BS}$ represents the cost of hedging the existing portfolio optimally and B represents the marginal change in that cost. Thus $V^{mid} - V^{BS}$ is

[21] Both V_b^{mid} and B_b, and hence V^{mid} and B, are also positively related to the issuer's risk aversion, γ. However, this is not empirically testable, so we do not include it in our discussion.

independent of the quote depth and increases with the size of the existing portfolio due to the nonlinear lifetime cost component, whereas B increases with the quote depth and decreases with existing portfolio size since a larger existing portfolio means a change of a given size has less relative impact on total transaction costs.

The implications for covered warrants are that mid-prices should increase (relative to corresponding traded option prices) and that both absolute and proportional bid-ask spreads should decrease with the number of warrants the issuer has already issued, and that both absolute and proportional bid-ask spreads should increase with quote depth.

Figure 1 shows how $V^{mid} - V^{BS}$ varies with moneyness and the size of the issuer's portfolio. Note that since the non-linearity with respect to $|N|$ arises solely from the lifetime cost component, which depends on the size of the warrant's Gamma, the effects are concentrated close-to-the-money. For in-the-money asset prices the cost terms proportional to the size of the warrant's Delta, which do not depend on portfolio size, are more important, so the sensitivity of mid prices and bid-ask spreads to the size of the issuer's existing holding is small. Figure 2 shows the effect of the size of the issuer's existing portfolio, $|N|$, on the proportional price difference (top graph) and proportional bid-ask spread (bottom graph).

Insert Figure 1 here

Insert Figure 2 here

A number of difficulties arise in testing implications of the model in practice. Empirical studies compare actual covered warrant and traded option prices rather than reservation covered warrant and perfect market values. Obtaining data on covered warrant issuers' holdings is likely to be difficult, so it may only be possible to obtain indirect support for some implications. Finally, since the theoretical results are obtained holding all else constant it is important to include appropriate controls in empirical tests.

However, the implications, that, because of the larger (short) positions and lower quote depths, warrant prices should be greater and warrant bid-ask spreads smaller than those of equivalent traded options, are exactly what was found by Bartram & Fehle (2004) for both relative prices and bid-ask spreads in German markets. Whilst not a direct test of sensitivity to portfolio size, this is consistent with the model's implications. Higher prices for covered warrants than equivalent traded options have also been found by Chan & Pinder (2000), Horst & Veld (2003) and Abad & Nieto (2007) in Australian, Dutch and Spanish markets.

Abad & Nieto (2007) regress the relative price difference between warrant and option markets on the ratio of the bid-ask spreads, amongst other characteristics. They find warrants are more expensive when the warrant bid-ask spread just before the transaction is smaller. They interpret the bid-ask spread ratio as a proxy for the relative liquidity of the two markets. However, this is also consistent with the effects of hedging costs: covered warrant issuers with larger portfolios (larger $|N|$) have relatively higher mid-prices and also lower bid-ask spreads.

One potentially testable implication of the extent of the influence of hedging cost issues on covered warrant prices[22] is the predicted positive sensitivity of both proportional and absolute bid-ask spreads to quote depth. This not been addressed directly, however there is some indirect support. Bartram & Fehle (2004) find

[22] Other factors such as competition and adverse selection may also influence bid and ask prices.

minimum trade sizes are generally much smaller and proportional bid-ask spreads are also significantly smaller for covered warrants than traded options. Bartram & Fehle (2006) and Bartram, Fehle & Shrider (2007) also find statistically significantly lower proportional bid-ask spreads for covered warrants.[23] Finally Petrella (2006) finds a significant positive relationship between the proportional bid ask spread and the 'reservation proportional bid-ask spread', proportional to the tick-size $\times m\Delta/V$ where m is the number of assets underlying one warrant contract. This is consistent with the costly hedging model, which predicts a larger quote depth m should be associated with a larger proportional bid-ask spread.

4.2 Warrant characteristics

Corollary 3 *The magnitude of the lifetime or bandwidth cost components of both the reservation mid price and the reservation bid-ask spread increase with the warrant's remaining life.*

The bandwidth cost components V_b^{mid} and B_b represent the effect of the costs of following the hedging strategy during the lifetime of the warrant.[24] Both increase with time to maturity, as the longer the hedging strategy is followed, potentially the more transactions and the greater the total cost.[25]

[23]Bartram & Fehle (2006) suggest the difference could be due to the greater depth offered by options market-makers but do not control for depth in their regressions; Bartram, Fehle & Shrider (2007) argue it is due to lower adverse selection in covered warrant markets. The implications of adverse selection and issuer hedging costs for bid-ask spreads are similar and hence difficult to disentangle.

[24]The hedging strategy involves keeping the number of the underlying asset within a band about the warrant's Delta. The width of the band is proportional to the absolute value of the Gamma.

[25]Formally, this is because $L_b(|\Gamma|)$ increases with time to maturity.

Insert Figure 3 here

The top graphs in Figure 3 show the effect is greatest for both V_b^{mid} and B_b for close to the money asset prices. This is because the bandwidth cost component is a function of the warrant's absolute Gamma integrated over the remaining life of the warrant. A difficulty in testing for this is that the initial and final cost components are both one-off costs and so do not increment with time to maturity. As shown in the middle graphs in Figure 3, depending on the warrant's moneyness, overall bid-ask spreads or $V^{mid} - V^{BS}$ can either increase or decrease with maturity, T. It is thus important to control for moneyness, potentially via initial/final costs, or $|\Delta|$, when testing for the positive dependence of either relative prices or bid-ask spreads on time-to-maturity.

A number of studies support the predicted relationship between relative warrant and option prices and time to maturity. Bartram & Fehle (2004) regress the ratio of warrant to option ask prices (AR) on, amongst other characteristics, the corresponding ratio of bid prices, time to maturity, moneyness, asset volatility and the number of competing warrants. Once cross-sectional variation has been removed they find a positive relationship between the ask ratio and time to maturity. Similarly Abad & Nieto (2007) find the relative price difference between warrant and option prices is significantly positively related to time to maturity.

Most empirical studies of bid-ask spreads use the proportional rather than the absolute bid-ask spread. The bottom graph in Figure 3 shows that, unlike the absolute bid-ask spread, the proportional bid-ask spread decreases with time-to-maturity, and by more, the lower the moneyness: moreover this result does not require controls for $|\Delta|$.

A number of empirical studies in both option and covered warrant markets have consistently found that proportional bid-ask spreads decrease with moneyness and time to maturity.[26] Kaul *et al* (2004), the only study on either covered warrant or traded option spreads to use absolute bid-ask spread, showed empirically that whilst average absolute bid-ask spreads increase across moneyness groupings, proportional bid-ask spreads generally decrease across the same moneyness groups. His results with respect to time-to-maturity were also broadly consistent with the theoretical and numerical results in Corollary 3 and Figure 3.

4.3 Characteristics of the underlying asset market

Insert Figure 4 here

Figure 4 shows the effect of asset volatility on the difference between the reservation mid price and perfect market value and on the per-warrant bid-ask spread. The bandwidth terms, in the top graphs, increase monotonically with asset volatility; however no unambiguous statement is possible for the combined terms, in the middle graphs, as it depends on the relative sizes of the lifetime and initial/final cost terms and also the moneyness. Thus, as for time to maturity, it is necessary to control for *e.g.* the initial/final cost level $S|\Delta|$ in testing for the positive relationships between asset volatility and both absolute bid-ask spreads and the difference

[26] Cho & Engle (1999), Bartram & Fehle (2006) and Petrella (2006) all find significantly negative coefficients in their regressions of proportional bid-ask spreads on moneyness. Jameson & Wilhelm (1992) do not include moneyness but do include elasticity $S\Delta/V$, which is strictly decreasing in moneyness for call warrants and find a significant negative coefficient. Similarly George & Longstaff (1993), Cho & Engle (1999) and Petrella (2006) all find significantly negative coefficients for time-to-maturity.

between covered warrant and equivalent option prices. Moreover, the bottom graph shows that proportional bid-ask spreads decrease with volatility: as with time to maturity, warrant values increase more with volatility than absolute bid-ask spreads.

It is clear from the functional forms for V^{mid} and B that both increase with the underlying asset bid-ask spread, k.[27] The effect is greater for the bandwidth cost terms, which scale as $k^{2/3}$, than the initial or final cost terms, which scale as k. So whilst the relationship between asset bid-ask spread and both warrant bid-ask spreads and the difference between covered warrant and equivalent exchange traded option prices is positive, it should be strongest for close-to-the-money asset prices and when transaction costs are small.

Corollary 4 *The magnitude of the difference between the reservation mid price and the perfect market price and the absolute and proportional reservation bid-ask spreads all increase with the proportional bid-ask spread in the underlying asset*

To date there is no empirical evidence on the relationship between either asset volatility or bid-ask spread on the relative pricing of covered warrants and traded options.[28] In contrast, the general principle that dynamic hedging costs affect bid-ask spreads in both covered warrant and traded option markets has relatively wide-spread support. Cho & Engle (1999), Kaul *et al* (2004) and Petrella (2006) all found that option or warrant proportional bid-ask spreads are significantly positively related to a measure of the initial hedging costs $kS\Delta$. Proxies for rebalancing

[27] After some algebra we find this is also the case for the proportional bid-ask spread, B/V^{mid}.

[28] Whilst Bartram & Fehle (2004) include asset volatility in their regression of relative ask prices on relative bid prices, since they also include asset dummies, they interpret the significantly negative coefficient as evidence of lower sensitivity of warrants than options to changes in liquidity. .

costs have been considered explicitly for options markets by Jameson & Wilhelm (1992) and Kaul *et al* (2004), and, for covered warrant markets, Petrella (2006). All found the proxies to be positively and statistically significantly related to spreads. Similarly, regressions which have included proxies for option or underlying asset price risk as (a component) of explanatory variables[29] have found results consistent with a positive impact of asset price volatility on option spreads.

4.4 Summary

The warrant issuer costly hedging model has the following empirical implications. Covered warrant prices are greater than those of equivalent traded options, whereas covered warrant bid-ask spreads are smaller than those for equivalent traded options. The difference between covered warrant and equivalent traded option prices increases with the size of the issuer's existing portfolio (though at a decreasing rate) and with the asset's bid-ask spread and, controlling for moneyness, with the time to maturity and asset volatility. Similarly both absolute and proportional bid-ask spreads increase with quote depth and the asset's bid-ask spread and decrease with the size of the issuer's existing portfolio. Holding moneyness constant, the absolute bid-ask spread increases and the proportional bid-ask spread decreases with time to maturity. The proportional bid-ask spread also decreases with moneyness.

5 Conclusion and further work

In this paper we developed a model for the reservation bid and ask prices of covered warrants, taking into account hedging costs and a covered warrant issuer's portfolio

[29] Jameson & Wilhelm (1992), Cho & Engle (1999), Petrella (2006)

composition and risk aversion. This provided a new supply-side explanation for the empirically documented regularity that covered warrant prices are consistently higher than those of equivalent exchange-traded options.

The existing empirical evidence on the relative pricing of covered warrants and exchange traded options and on bid-ask spreads in options and warrant markets is broadly supportive of the wider implications of the model. Further empirical work needs to be done to investigate some novel implications of the model which have not yet been addressed explicitly, *e.g.* the effects of the magnitude of a covered warrant issuer's overall portfolio on the relative overpricing of covered warrants.

The characteristics of covered warrant markets which drive the model are also present in markets for many structured products offered to retail investors in recent years:[30] issuers are also market makers and hold net short positions. Interestingly, similar findings about the relative pricing of structured products (that they are greater than the price of equivalent exchange traded products, for both convex and concave payoffs[31]) have been documented for a range of such products. The supply-side cost argument in this paper may also give a partial explanation for this.

More generally, the model in section 3, extended to positive as well as negative N, can be viewed in the spirit of an inventory cost model for options market-makers[32]:

[30]See Stoimenov & Wilkens (2006) for a recent review.

[31]See *e.g.* Burth, Kraus & Wohlwend (2001) for empirical evidence on a structured product with a concave payoff. The dependence of the reservation mid price on the *absolute value* of the warrant's Gamma or Delta, combined with the sign of the (net) number of warrants held in the portfolio, means the issuers of structured products (SPs), who generally have a net short position, will have a reservation mid-price for any SP which is *higher* than the SP's perfect market value (since $\text{sgn}(N) = -1$) irrespective of the sign of the Gamma of a single long SP.

[32]The model gives reservation prices, at which a market-maker would be *indifferent* between selling

inventory costs (*i.e.* the effect of the number of options held, N) are reflected in reservation bid and ask prices indirectly through the optimal hedging bandwidth and hence the residual risk the market-maker optimally incurs. Thus, as in inventory cost models of underlying asset markets (*e.g.* Ho & Stoll (1981)), the mid-price is negatively related to the inventory level (though for options this relationship is non-linear), the bid-ask spread is positively related to the quote depth and both bid-ask spread and mid price are positively related to the risk of the marginal (existing portfolio) position. It would be interesting to explore this relationship further in future work.

6 Appendix

Proof of Proposition 2

We have $g(\Lambda_Q|\Lambda_P) = G(\Lambda_P + \Lambda_Q) - G(\Lambda_P) + g_i(|\Lambda_Q|)$, where $g_i(|\Lambda_Q|) = -kS|Q_S^{BS}|$ is the leading order initial cost of changing the number of the underlying asset held, and from Proposition 1

$$\begin{aligned} G(\Lambda_P) &\approx G^{BS}(\Lambda_P) + G_b(|\Gamma^P|) + G_f(|\Delta^P|) \\ G(\Lambda_P + \Lambda_Q) &\approx G^{BS}(\Lambda_P + \Lambda_Q) + G_b(|\Gamma^P + \Gamma^Q|) + G_f(|\Delta^P + \Delta^Q|) \end{aligned}$$

We can thus write $g(\Lambda_Q|\Lambda_P) \approx g^{BS}(\Lambda_Q) + g_b(\Lambda_Q|\Lambda_P) + g_f(\Lambda_Q|\Lambda_P) + g_i(\Lambda_Q)$ where $g^{BS}(\Lambda_Q) \equiv G^{BS}(\Lambda_P + \Lambda_Q) - G^{BS}(\Lambda_P)$, $g_b(\Lambda_Q|\Lambda_P) \equiv G_b(|\Gamma^P + \Gamma^Q|) - G_b(|\Gamma^P|)$ and

(ask price) or buying (bid price) the quoted number of options and hedging optimally, or not. Hence it does not incorporate expected order flow or maximise the market-maker's profit. It also does not consider adverse selection issues. A fuller model is beyond the scope of this paper; however, we would expect utility-maximising ask and bid prices to incorporate the same factors which underlie reservation prices based on inventory considerations and to lie outside the reservation bid-ask spread.

$g_f(\Lambda_Q|\Lambda_P) \equiv G_f(|\Delta^P + \Delta^Q|) - G_f(|\Delta^P|)$. $g^{BS}(\Lambda_Q)$ is the Black-Scholes value of the portfolio with payoff Λ_Q since the Black-Scholes differential equation is linear. $g_b(\Lambda_Q|\Lambda_P)$ satisfies

$$\begin{aligned}
\mathcal{L}_{BS}(g_b(\Lambda_Q|\Lambda_P)) &= \left(\frac{\hat{\gamma}\sigma^2 S^2}{2}\right)\left(H_{P+Q}^{*^2} - H_0^{*^2}\right) - \left(\frac{\hat{\gamma}\sigma^2 S^2}{2}\right)\left(H_P^{*^2} - H_0^{*^2}\right) \\
&= \left(\frac{\hat{\gamma}\sigma^2 S^2}{2}\right)\left(H_{P+Q}^{*^2} - H_P^{*^2}\right)
\end{aligned}$$

subject to $g_b(\Lambda_Q|\Lambda_P)(S,T) = G_b(|\Gamma^P + \Gamma^Q|)(S,T) - G_b(|\Gamma^P|)(S,T) = 0 - 0 = 0$, where $\hat{\gamma} \equiv \gamma e^{-r(T-t)}$ and H_{P+Q}^*, H_P^* represent the semi-bandwidths associated with payoffs $\Lambda_P + \Lambda_Q$ and Λ_P respectively and are given by $H_{P+Q}^* = \left(\frac{3kS}{2\hat{\gamma}}\right)^{\frac{1}{3}} |\Gamma^P + \Gamma^Q + \frac{\xi}{S^2}|^{\frac{2}{3}}$ and $H_P^* = \left(\frac{3kS}{2\hat{\gamma}}\right)^{\frac{1}{3}} |\Gamma^P + \frac{\xi}{S^2}|^{\frac{2}{3}}$ with $\xi(t) = \frac{\mu - r}{\hat{\gamma}(t)\sigma^2}$. $g_f(\Lambda_Q|\Lambda_P)$ satisfies $\mathcal{L}_{BS}(g_f(\Lambda_Q|\Lambda_P)) = 0$ subject to

$$\begin{aligned}
g_f(\Lambda_Q|\Lambda_P)(S,T) &= -k(|\xi(T) - S(\Delta^P(S,T) + \Delta^Q(S,T))| - |\xi(T)|) \\
&\qquad + k(|\xi(T) - S\Delta^P(S,T)| - |\xi(T)|) \\
&= -k(|\xi(T) - S(\Delta^P(S,T) + \Delta^Q(S,T))| - |\xi(T) - S\Delta^P(S,T)|)
\end{aligned}$$

If P and Q represent portfolios of a single European covered warrant, so $\Lambda_P = N\Lambda_V$, $\Lambda_Q = n\Lambda_V$, let $V^{BS}(S,t)$ be the Black-Scholes value of a single long warrant with payoff Λ_V at maturity T and $\Delta(S,t)$, $\Gamma(S,t)$ be the Black-Scholes Delta and Gamma of the single long warrant respectively. Then the Black-Scholes values of the portfolios' Deltas and Gammas are given by $\Delta^P = N\Delta$, $\Delta^Q = n\Delta$, $\Gamma^P = N\Gamma$ and $\Gamma^Q = n\Gamma$. Writing also $g(n\Lambda_V|N\Lambda_V) \equiv g(n|N) \approx nV^{BS} + g_b(n|N) + g_f(n|N) + g_i(n)$ we find $g^{BS}(\Lambda_Q) = nV^{BS}$, $g_i(n) = -k|n|S|\Delta(S,T)|$, $g_b(n|N)$ satisfies

$$\mathcal{L}_{BS}(g_b(n|N)) = \left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}}\left(|(N+n)S^2\Gamma + \xi|^{\frac{4}{3}} - |NS^2\Gamma + \xi|^{\frac{4}{3}}\right)$$

subject to $g_b(n|N)(S,T) = 0$. Assuming $|NS^2\Gamma + \xi| \gg |nS^2\Gamma|$ this becomes

$$\begin{aligned}
\mathcal{L}_{BS}(g_b(n|N)) &= \left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}} |NS^2\Gamma + \xi|^{\frac{4}{3}} \left(\left(1 + \frac{nS^2\Gamma}{NS^2\Gamma + \xi}\right)^{\frac{4}{3}} - 1\right) \\
&\approx \left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}} \Big(\mathrm{sgn}[NS^2\Gamma + \xi]\ \frac{4}{3}n(S^2\Gamma)|NS^2\Gamma + \xi|^{\frac{1}{3}} \\
&\qquad + \frac{2}{9}n^2(S^2\Gamma)^2|NS^2\Gamma + \xi|^{-\frac{2}{3}}\Big)
\end{aligned}$$

and $g_f(n|N)$ satisfies $\mathcal{L}_{BS}(g_f(n|N)) = 0$ with final condition, assuming $\mathrm{sgn}[\xi(T) - (N+n)S\Delta(S,T)] = \mathrm{sgn}[\xi(T) - NS\Delta(S,T)] = -\mathrm{sgn}[N]\mathrm{sgn}[\Delta]$

$$\begin{aligned}
g_f(n|N)(S,T) &= -k(|\xi(T) - (N+n)S\Delta(S,T)| - |\xi(T) - NS\Delta(S,T)|) \\
&= -k\ \mathrm{sgn}[\xi(T) - (N+n)S\Delta(S,T)]\ (-nS\Delta(S,T)) \\
&= -\mathrm{sgn}[N]\ knS|\Delta(S,T)|
\end{aligned}$$

The bid price is defined as $V^{bid}(m|N) = g(m|N)/m$ with $m > 0$, so setting $n = +m$ and writing $V^{bid}(m|N) \approx V^{BS} + V_b^{bid}(m|N) + V_f^{bid}(m|N) + V_i^{bid}(m)$ we find

$$\begin{aligned}
\mathcal{L}_{BS}(V_b^{bid}(m|N)) &\approx \left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}} \Big(\mathrm{sgn}[NS^2\Gamma + \xi]\ \frac{4}{3}(S^2\Gamma)|NS^2\Gamma + \xi|^{\frac{1}{3}} \\
&\qquad + \frac{2}{9}m(S^2\Gamma)^2|NS^2\Gamma + \xi|^{-\frac{2}{3}}\Big)
\end{aligned}$$

s.t. $V_b^{bid}(m|N)(S,T) = 0$ and $\mathcal{L}_{BS}(V_f^{bid}(m|N)) = 0$ s.t. $V_f^{bid}(m|N)(S,T) = -k\mathrm{sgn}[N]S|\Delta(S,T)|$. $V_i^{bid}(m) = -kS|\Delta(S,T)|$.

Similarly the ask price is defined as $V^{ask}(m|N) = -g(-m|N)/m$ with $m > 0$, so setting $n = -m$ and writing $V^{ask}(m|N) \approx V^{BS} + V_b^{ask}(m|N) + V_f^{ask}(m|N) + V_i^{ask}(m)$ we find

$$\begin{aligned}
\mathcal{L}_{BS}(V_b^{ask}(m|N)) &\approx \left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}} \Big(\mathrm{sgn}[NS^2\Gamma + \xi]\ \frac{4}{3}(S^2\Gamma)|NS^2\Gamma + \xi|^{\frac{1}{3}} \\
&\qquad - \frac{2}{9}m(S^2\Gamma)^2|NS^2\Gamma + \xi|^{-\frac{2}{3}}\Big)
\end{aligned}$$

s.t. $V_b^{ask}(m|N)(S,T) = 0$ and $\mathcal{L}_{BS}(V_f^{ask}(m|N)) = 0$ s.t. $V_f^{ask}(m|N)(S,T) = -k\text{sgn}[N]S|\Delta(S,T)|$ so $V_f^{ask} = V_f^{bid}$. However $V_i^{ask}(m) = -k\frac{|-m|}{(-m)}S|\Delta(S,t)| = +kS|\Delta(S,t)|$ so $V_i^{ask} = -V_i^{bid}$.

Finally, since all the differential equations are linear, $V^{mid}(m|N) = \frac{V^{bid}(m|N)+V^{ask}(m|N)}{2}$ can be represented by $V^{mid}(m|N) \approx V^{BS} + V_b^{mid}(m|N) + V_f^{mid}(m|N) + V_i^{mid}(m)$, where $V_b^{mid}(m|N)$ satisfies

$$\mathcal{L}_{BS}(V_b^{mid}(m|N)) \approx \left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}} \text{sgn}[NS^2\Gamma+\xi]\,\frac{4}{3}(S^2\Gamma)|NS^2\Gamma+\xi|^{\frac{1}{3}} \quad (13)$$

s.t. $V^{mid}(m|N)(S,T) = 0$. If $NS^2\Gamma \gg \xi$ then, recalling $\Gamma > 0$ for vanilla options, this can be further simplified to

$$\mathcal{L}_{BS}(V_b^{mid}(m|N)) \approx \left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}} \text{sgn}[N]\frac{4}{3}|N|^{\frac{1}{3}}|S^2\Gamma|^{\frac{4}{3}}$$

s.t. $V^{mid}(m|N)(S,T) = 0$, which is equivalent to $V^{mid}(m|N) = -\text{sgn}[N]\frac{4}{3}k^{\frac{2}{3}}|N|^{\frac{1}{3}}L_b(|\Gamma|)$ with L_b satisfying

$$\mathcal{L}_{BS}(L_b) = -\left(\frac{\hat{\gamma}\sigma^2 S^2}{2}\right)\left(\frac{3S}{2\hat{\gamma}}\right)^{\frac{2}{3}}|\Gamma|^{\frac{4}{3}} \quad (14)$$

s.t. $L_b(S,T) = 0$. Numerical simulations using (13) confirm our results are not sensitive to the inclusion of ξ. $V_f^{mid}(m|N)$ satisfies $\mathcal{L}_{BS}(V_f^{mid}(m|N)) = 0$ s.t. $V_f^{mid}(m|N)(S,T) = -k\text{sgn}[N]S|\Delta(S,T)|$ and if $\Delta(S,T)$ is one-signed, as is the case for long call warrant then $V_f^{mid}(S,t) = -k\text{sgn}[N]S|\Delta(S,t)|$. However $V_i^{ask} = -V_i^{bid}$ so $V_i^{mid} = 0$.

Similarly $B(m|N) = V^{ask}(m|N) - V^{bid}(m|N)$ can be represented by $B(m|N) \approx B_b(m|N) + B_f(m|N) + B_i(m)$, where $B_b(m|N)$ satisfies

$$\mathcal{L}_{BS}(B_b(m|N)) \approx -\left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}}\frac{4}{9}m(S^2\Gamma)^2|NS^2\Gamma+\xi|^{-\frac{2}{3}}$$

s.t. $B_b(m|N)(S,T)=0$. Again, if $NS^2\Gamma \gg \xi$ then this can be further simplified to

$$\mathcal{L}_{BS}(B_b(m|N)) \approx -\frac{4}{9}m|N|^{-\frac{2}{3}}\left(\frac{\hat{\gamma}\sigma^2}{2}\right)\left(\frac{3k}{2\hat{\gamma}}\right)^{\frac{2}{3}}|S^2\Gamma|^{\frac{4}{3}}$$

s.t. $V^{mid}(m|N)(S,T)=0$, which is equivalent to $B_b(m|N)=\frac{4}{9}m|N|^{-\frac{2}{3}}k^{\frac{2}{3}}L_b(|\Gamma|)$ with L_b satisfying (14), $V_f^{ask}=V_f^{bid}$ so $B_f=0$ and $B_i(m)=2kS|\Delta(S,t)|$.

Sketch of Proof of Corollary 1

It is immediate from (7), noting $L_b(|\Gamma|)>0$ and $N<0$, that $V^{mid}(m|N)-V^{BS}>0$. Similarly from (9), $B(m|N)>0$. Hence clearly $V^{ask}(m|N)=V^{mid}(m|N)+\frac{1}{2}B(m|N)>V^{BS}$, and

$$V^{bid}(m|N)=V^{mid}(m|N)-\frac{1}{2}B(m|N)=V^{BS}+\frac{4}{4}|N|^{\frac{1}{3}}k^{\frac{2}{3}}L_b(|\Gamma|)\left(1-\frac{m}{6|N|}\right)>V^{BS}$$

since $m/|N|\ll 1$ by assumption.

Sketch of Proof of Corollary 2

Differentiating (7) and (9) w.r.t. $|N|$ gives $\frac{\partial(V^{mid}(m|N)-V^{BS})}{\partial|N|}=\frac{\partial V^{mid}(m|N)}{\partial|N|}\approx\frac{4}{9}|N|^{\frac{-2}{3}}k^{\frac{2}{3}}L_b(|\Gamma|)>0$; $\frac{\partial B(m|N)}{\partial|N|}\approx-\frac{8}{27}m|N|^{-\frac{5}{3}}k^{\frac{2}{3}}L_b(|\Gamma|)<0\Rightarrow\frac{\partial\left(\frac{B}{V^{mid}}\right)}{\partial|N|}\approx\frac{1}{V^{mid}}\frac{\partial B}{\partial|N|}-\frac{B}{(V^{mid})^2}\frac{\partial V^{mid}}{\partial|N|}<0$.

Differentiating (7) and (9) w.r.t. m gives $\frac{\partial(V^{mid}(m|N)-V^{BS})}{\partial m}=0$; $\frac{\partial B(m|N)}{\partial m}\approx\frac{4}{9}|N|^{-\frac{2}{3}}k^{\frac{2}{3}}L_b(|\Gamma|)>0\Rightarrow\frac{\partial\left(\frac{B}{V^{mid}}\right)}{\partial m}\approx\frac{1}{V^{mid}}\frac{\partial B}{\partial m}-\frac{B}{(V^{mid})^2}\frac{\partial V^{mid}}{\partial m}>0$.

Sketch of Proof of Corollary 3

V_b^{mid} and B_b depend on time to maturity only through $L_b(|\Gamma|)$, which increases with time to maturity since $\frac{\hat{\gamma}(t)\sigma^2S^2}{2}\left(\frac{3S}{2\hat{\gamma}(t)}\right)^{\frac{2}{3}}|\Gamma(S,t)|^{\frac{4}{3}}>0\forall(S,t)$.

Sketch of Proof of Corollary 4

Differentiating (7) and (9) w.r.t. k gives $\frac{\partial(V^{mid}(m|N)-V^{BS})}{\partial k}\approx\frac{8}{9}|N|^{\frac{1}{3}}k^{-\frac{1}{3}}L_b(|\Gamma|)+S|\Delta|>0$; $\frac{\partial B(m|N)}{\partial k}\approx\frac{8}{27}m|N|^{-\frac{2}{3}}k^{-\frac{1}{3}}L_b(|\Gamma|)+2S|\Delta|>0$ and
$\frac{\partial\left(\frac{B}{V^{mid}}\right)}{\partial k}\approx\frac{V^{BS}\left(\frac{8}{27}m|N|^{-\frac{2}{3}}k^{-\frac{1}{3}}L_b(|\Gamma|)+2S|\Delta|\right)+\frac{20}{27}|N|^{\frac{1}{3}}k^{\frac{2}{3}}L_b(|\Gamma|)S|\Delta|}{(V^{mid})^2}>0$

References

[1] Abad, D and B Nieto (2007) The Unavoidable Task of Understanding Warrants Pricing. Working paper

[2] Bartram, SM and F Fehle (2004) Alternative Market Structures for Derivatives. Working paper

[3] Bartram, SM and F Fehle (2006) Competition without Fungibility: Evidence from Alternative Market Structures for Derivatives. Journal of Banking and Finance 31, 659-677

[4] Bartram, SM, Fehle, F and D Shrider (2007) Does Adverse Selection Affect Bid-Ask Spreads for Options? Working paper

[5] Burth, S, Kraus, T and H Wohlwend (2001) The Pricing of Structured Products in the Swiss Market. Working paper

[6] Chan, HW and SM Pinder (2000) The value of liquidity: Evidence from the derivatives market. Pacific-Basin Finance Journal 8, 483-503

[7] Cho, Y and R Engle (1999) Modeling the impacts of market activity on bid-ask spreads in the option market NBER Working Paper 7331

[8] Clewlow, L and S.D. Hodges (1997) Optimal delta-hedging under transaction costs Journal of Economic Dynamics and Control 21, 1353-1376

[9] Damgaard, A (2003) Utility based option evaluation with proportional transaction costs. Journal of Economic Dynamics and Control 27, 667-700

[10] Davis, MHA, V.G. Panas and T. Zariphopoulou (1993) European option pricing with transaction costs. SIAM Journal of Control and Optimisation 31, 470-493

[11] George, T and F Longstaff (1993) 'Bid-Ask Spreads and Trading Activity in the S&P100 Index warrants Market. Journal of Financial and Quantitative Analysis 28, 381-397

[12] Goldman Sachs (2007) Base prospectus: Programme for the Issuance of Warrants and Certificates 26 March, 2007

[13] Ho, T and HR Stoll (1981) Optimal dealer pricing under transactions and return uncertainty. Journal of Financial Economics 9, 47-73

[14] Hodges, SD and A. Neuberger (1989) Optimal replication of contingent claims under transaction costs. Review of Futures Markets 8, 222-239

[15] Horst, J and C Veld (2003) Behavioral Preferences for Individual Securities: The Case for Call Warrants and Call options. Working paper

[16] Jameson, M and W Wilhelm (1992) Market making in the options markets and the costs of discrete hedge rebalancing. Journal of Finance 47, 765-779

[17] Kaul, G, Nimalendran, M and D Zhang (2004) Informed trading and option spreads. Working paper

[18] Petrella, G (2006) Option bid-ask spread and scalping risk: Evidence from a covered warrants market. Journal of Futures Markets 26, 843-867

[19] Rogers, LCH (2000) Why is the effect of proportional transaction costs $O(\delta^{2/3})$? Working paper

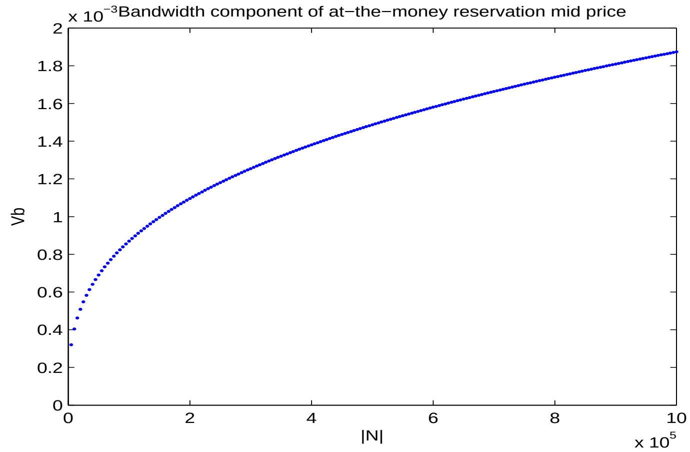
[20] Stoimenov, and Wilkens (2006) Are structured products fairly priced? An analysis of the German market for equity-linked instruments. Journal of Banking and Finance 29, 2971-2993

[21] Whalley, AE (1998) Option pricing with transaction costs. D. Phil. thesis, Oxford University

[22] Whalley, AE and P. Wilmott (1997) An asymptotic analysis of the Davis, Panas & Zariphopoulou model for option pricing with transaction costs. Mathematical Finance 7, 307-324

[23] Whalley, AE and P. Wilmott (1999) Optimal hedging of options with small but arbitrary transaction cost structure. European Journal of Applied Mathematics 10, 117-139

[24] Zakamouline, VI (2006) European option pricing with both fixed and proportional transaction costs. Journal of Economic Dynamics and Control 30, 1-25



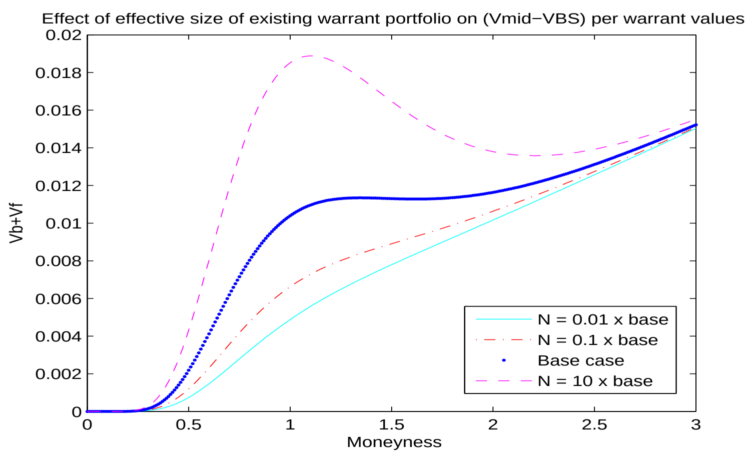
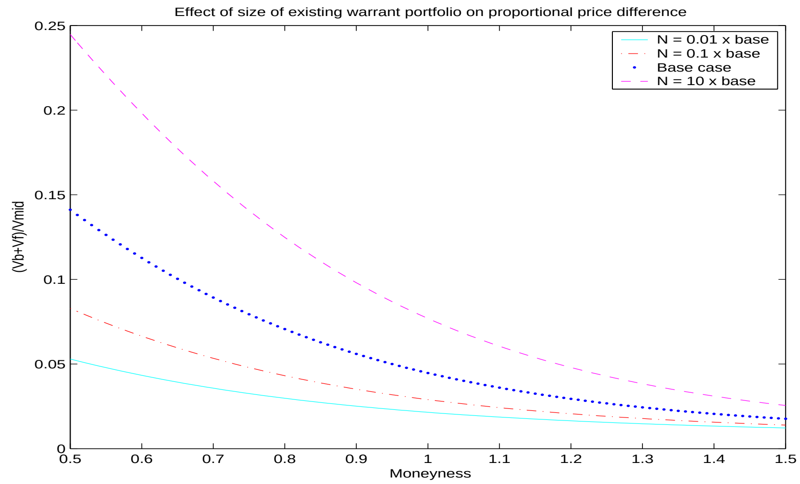


Figure 1: Top graph: Bandwidth cost component of reservation mid price *vs* size of existing portfolio for at-the-money warrants with 2 years to expiry.

Bottom graph: Difference between reservation mid price and Black-Scholes price *vs* moneyness for different sizes of existing portfolio using (7) for warrants with 2 years to expiry. Base case has an existing short position of $N = -1 \times 10^6$ warrants. Other parameter values: $\sigma = 0.32$, $r = 0.05$, $T = 2$, $\gamma|N| = 4$ and $k = 0.005$.



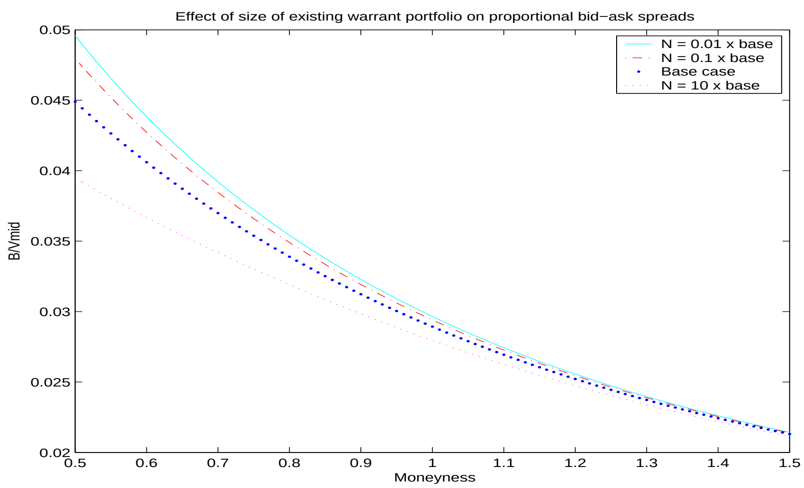


Figure 2: Difference between per-warrant reservation mid-price and Black-Scholes price as proportion of per-warrant reservation mid-price (top graph) and Bid-ask spread as a proportion of per-warrant reservation mid-price (bottom graph) both *vs* moneyness, S/K, for a portfolio of European covered warrants for different portfolio sizes. Base case parameter values: $\gamma|N| = 4$, $m/|N| = 10^{-4}$, $\sigma = 0.32$, $r = 0.05$, $k = 0.005$ and $T = 2$.

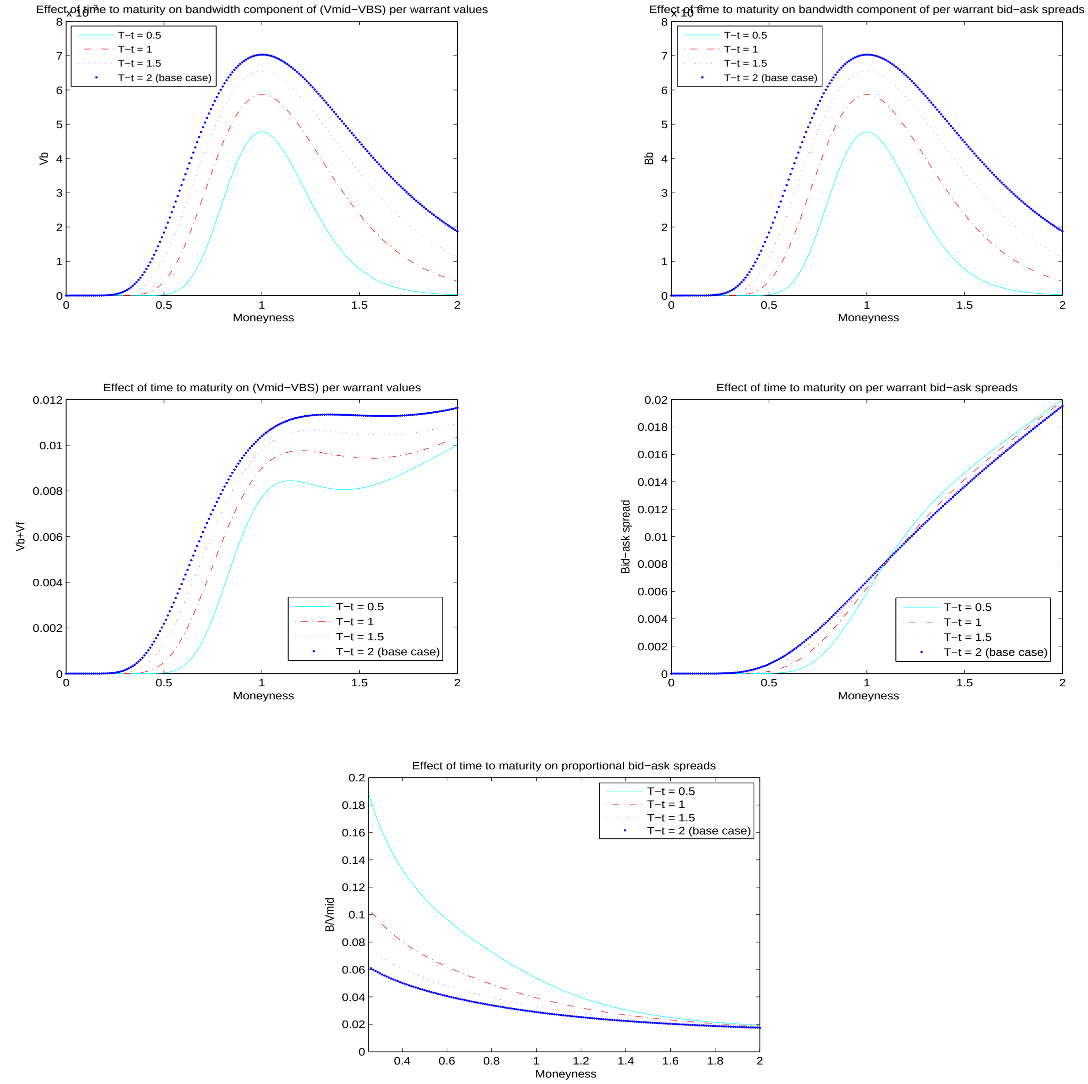


Figure 3: Bandwidth component (top) and total (bottom) of the difference between the per-warrant reservation mid price and the Black-Scholes price (left) and the bid-ask spread (right). Proportional bid-ask spread (bottom). All *vs* moneyness, S/K for a portfolio of European call warrants for different times to maturity. Parameter values where not stated: $\sigma = 0.32$, $r = 0.05$, $\gamma|N| = 4$, $m/|N| = 1 \times 10^{-4}$ and $k = 0.005$.

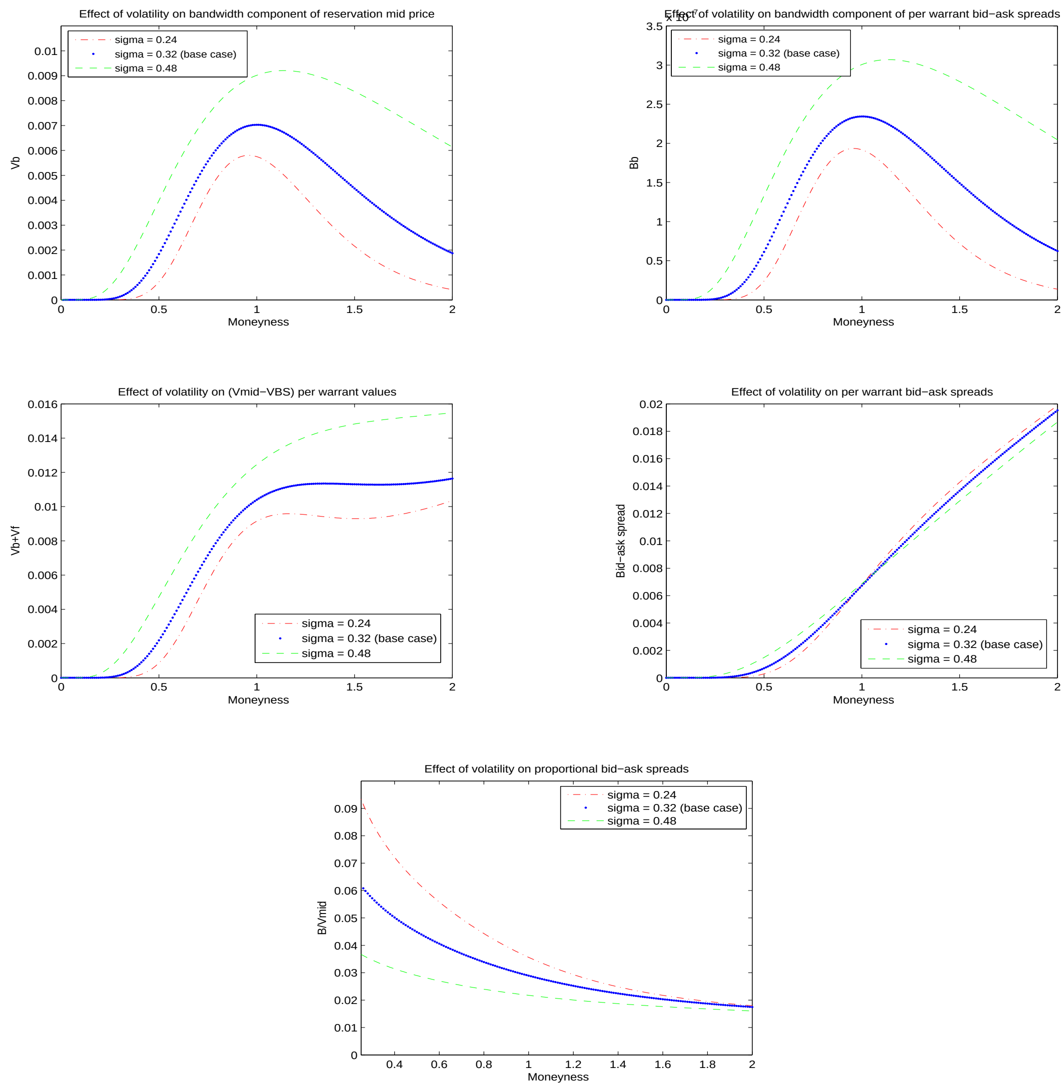


Figure 4: Bandwidth component (top) and total (middle) of the difference between the per-warrant reservation mid price and the Black-Scholes price (left) and the absolute bid-ask spread (right). Proportional bid-ask spread (bottom). All *vs* moneyness, S/K for a portfolio of European call warrants for different asset volatilities. Parameter values where not stated: $T = 2$, $r = 0.05$, $\gamma|N| = 4$, $m/|N| = 1 \times 10^{-4}$ and $k = 0.005$.