Modelling The Probability of UK Housing Market Events (Crashes) using Extreme Value Theory

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1. Introduction to Extreme Value Theory

- The convenient assumption of normality only applies very roughly at very low frequencies in practice -as asset returns exhibit leptokurtosis. ie. *heavier tails* than those predicted under a normal distribution- this becomes a critical issue when modelling risk- as we are then specifically interested in the tails.
- However, most risk measures used in the finance industry have traditionally relied on the assumption of normality, or more generally elliptic distributions.
- Value-at-Risk (**VaR**) has been criticised by Artzner et al. (1997, 1998) on two grounds.
    1. First, VaR may not be sub-additive, in other words somewhat unrealistically using VaR as a measure of risk we could find that risk increases with diversification.
    2. Secondly, VaR says nothing about the potential size of loss *given* that a loss exceeds the given VaR threshold. By contrast, **Expected Shortfall** or the tail conditional expectation measures the expected loss given that the loss exceeds the VaR.
- In what follows, we explore methods that take account of extreme risks and that also model high quantiles which focus not on the whole distribution of the asset returns but only on the tails of the distribution. We can then assess the probabilities of loss in value of the housing stock by examining the probabilities of large price falls using standard risk measures such as VaR and Expected Shortfall based on EVT.
- There are two approaches available to us modelling block maxima or minima using the **Generalised Extreme Value Distribution** (GEV) and the **Peaks Over Threshold** (POT) method; both approaches rest on asymptotic theory regarding limiting distributions for extrema just as the central limit theorem applies to the asymptotic behaviour of sample averages.
- We first describe the theoretical basis for these EVT methods and then apply the theory to UK Housing Market data.
- The approach taken is therefore entirely based on the statistical behaviour of the historical price index itself and does not take into account of economic issues such as the ability to service mortgage repayments.
- One implication that appears from this analysis is that dramatic downward price shifts are not a particularly high probability event. The price index used appears to be one that historically has not suffered from many large price reversals.

1.1. Modelling Maxima and Worst Cases

We take $X_{1,} X_{2,} \ldots$ to be an iid sequence of random variables that represent losses with an unknown cumulative distribution function $F(x) = P[X \leq x]$ and throughout we treat losses as a positive number and hence we are interested in the right hand tail of the loss distribution.

1.1.1. The Fisher- Tippett Theorem

The normal distribution is the important limiting distribution for sample averages as summarized in a central limit theorem. Similarly, the family of extreme value distributions is the one to study the limiting distributions of the sample extrema. This family can be presented under a single parameterization known as the generalized extreme value distribution (GEV). The theory deals with the convergence of maxima. Suppose that $X_t, t = 1, 2, ..., n$ is a sequence of independently and identically distributed random variables with a common distribution function

$$F(x) = \Pr[X \leq x]$$

which has mean (location parameter) μ and variance (scale parameter) σ^2. We treat losses as a positive number and hence we are interested in the right hand tail of the loss distribution. Denote the sample maxima of X_t by $M_1 = X_1,\ M_n = max(X_1, ..., X_n), n \geq 2$ and let R denote the real line. Extreme Value Theory (EVT) then focuses on minima and maxima of distributions. The seminal result of EVT is the Fisher-Tippett theorem that tells us that extrema will be described, under general conditions, by one of three different parametric families:
For iid rvs (X_n), if there exist constants $a_n > 0$ and $b_n \in R$ and a non-degenerate distribution function H such that

$$a_n^{-1}(M_n - b_n) \xrightarrow{d} H$$

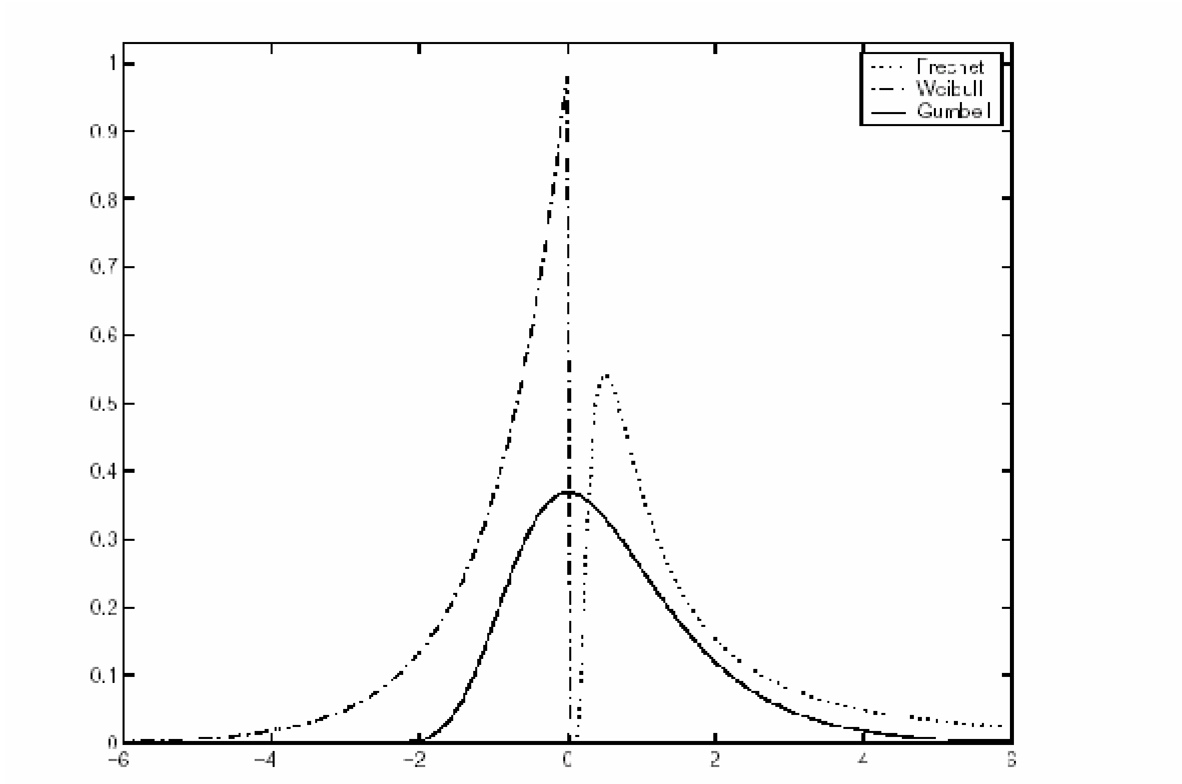
where $M_n = \max(X_{1,..,} X_n)$, then H belongs to one of the three following types:

$$\text{Type I (Gumbel)} \quad H(x) = \exp(-e^{-x}) \quad x \in R$$

$$\text{Type II (Frechet)} \quad H(x) = \begin{cases} 0 & x \leq 0 \\ \exp(-x^{-\alpha}) & x > 0 \end{cases} \quad \alpha > 0$$

$$\text{Type III (Weibull)} \quad H(x) = \begin{cases} \exp(-(-x)^{\alpha}) & x \leq 0 \\ 1 & x > 0 \end{cases} \quad \alpha > 0$$

Notice that no detailed knowledge of the underlying distribution F is required for this result to hold. Essentially we have three different tail shapes, loosely speaking the tail either declines exponentially or by a power law. In the first case all moments exist but in the second the higher moments do not decay rapidly enough when weighted by the tail probabilities to be integrable, i.e. the distribution function $F(x)$ has fat tails. Given that stock returns are fat tailed and in principle unbounded the focus of attention is the Type II limit law, the Fréchet Law.



Notice that the Weibull distribution starts from zero and has a finite left-tail and therefore is a thin-tailed distribution. The Fréchet distribution, on the other hand, starts from zero and has a persistent right tail while the Gumbel distribution has a tail behaviour which lies in between a thin-tail (Weibull) and a heavy-tail (Fréchet).

Jenkinson and Von Mises developed what is known as the Generalized Extreme Value distribution (GEV)) which provides a single parametric form for the distribution of extrema:

$$H(\theta; M_n) = \exp\left\{ -\left(1 + \xi \frac{M_n - b}{a}\right)^{-1/\xi} \right\}$$

with $\left(1+\xi \frac{M-b}{a}\right) > 0$ and $\theta = (\xi, a, b)$. We have the three cases: $\xi = 0$ (Gumbel), $\xi = \alpha^{-1} > 0$ (Fréchet) and $\xi = \alpha^{-1} < 0$ (Weibull).

So every max-stable non-degenerate distribution function $G(x)$ is one of the three extreme value types (see e.g. Koedijk, Schafgans and De Vries (1990)), : the Gumbel law (type I), the Fréchet law (type II) and the Weibull law (type III). Conversely, every distribution of extreme value type is max stable.

The advantage of the theory of extremes is that several relatively fat-tailed distributions (such as Student-t and ARCH process) are in the domain of attraction of the type II distribution. Under the regular variation condition,

$$\forall x > 0, \quad \begin{cases} x_{SUP} = \sup\{x; F(x) < 1\} = +\infty \\ t \to \infty \lim \dfrac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \alpha > 0 \end{cases}$$

the type II is such that:

$$G(x)=\begin{cases}0 & x\le 0\\ \exp(-x^{-\alpha}) & x>0 \quad \alpha>0\end{cases}$$

where α measures the tail index. The lower α the fatter the tail of the distribution.

The class of distributions $F(x)$ where the Fisher-Tippett theorem holds is quite large. One of the conditions is that $F(x)$ has to be in the domain of attraction for the Fréchet distribution ($\frac{1}{\alpha}>0$) which in general holds for the financial time series. Gnedenko (1943) shows that if the tail of $F(x)$ decays like a power function, it is in the domain of attraction for the Fréchet distribution. The class of distributions whose tails decay like a power function is large and includes the Pareto, Cauchy, Student-t and mixture distributions. These distributions are the well-known heavy-tailed distributions. The distributions in the domain of attraction of the Weibull distribution ($\frac{1}{\alpha}<0$) are the thin-tailed distributions such as uniform and beta distributions which do not have much power in explaining financial time series. The distributions in the domain of attraction of the Gumbel distribution ($\frac{1}{\alpha}=0$) include the normal, exponential, gamma and lognormal distributions where only the lognormal distribution has a moderately heavy-tail. Estimation of the GEV is carried out by Maximum likelihood methods.

1.1.2. Hill Estimator and Quantile Estimation

Empirically, the tail index $\alpha=\frac{1}{\xi}$ can be estimated using a range of different estimators: Hill(1975), Pickands (1975), de Haan and Resnick (1980) and Dekkers et al. (1990). Hill (1975) proposed an estimator of ξ when $\xi>0$ (Fréchet Case). By ordering the data with respect to their values, we get the order statistics $X_{(1)},X_{(2)},X_{(3),},...,X_{(n)}$ where $X_{(1)}\ge X_{(2)}\ge ...\ge X_{(n)}$ then Hill's Estimator of the tail index ξ is

$$\hat{\xi}(k)=\frac{1}{k}\sum_{j=1}^{k}\left(\ln X_{(j)}-\ln X_{(k)}\right)$$

where $k\to\infty$ is upper order statistics (the number of exceedences), n is the sample size, and $\alpha=1/\gamma$ is the tail index. A Hill-plot is constructed such that estimated ξ is plotted as a function of k upper order statistics or the threshold. A threshold is selected from the plot where the shape parameter ξ is fairly stable. The Hill estimator is often used in practice since its first and second moments can be approximated using asymptotic series expansions and it has a well defined asymptotic distribution..

The Hill estimator is shown to be a consistent estimator of $\xi=1/\alpha$ for fat-tailed distributions in Mason (1982). The conditions on k and n for weak consistency of the Hill's estimator are given in Mason (1982) and Rootzen et al. (1992). Deheuvels et al. (1988) investigates the conditions for the strong consistency of the Hill's estimator. From Hall (1982) and Goldie and Smith (1987), it follows that $(\hat{\xi}-\xi)k^{1/2}$ is asymptotically normally distributed with zero mean and variance ξ^2.

A difficulty of the Hill's estimator is the ambiguity of the value of threshold parameter, k. In threshold determination, we face a trade off between bias and variance. If we choose a low threshold, the number of

observations beyond the threshold increases and the estimation becomes more precise. However, choosing a low threshold also introduces some observations from the centre of the distribution and the estimation becomes biased. While the estimates of ξ based on a few largest observations are highly sensitive to the number of observations used and the estimates based on too few elements from the top of the ordering may be biased. Therefore, a careful combination of several techniques, such as the QQ-plot, the Hill-plot should be considered in threshold determination. The choice of k determines how much probability mass of the empirical distribution is estimated. The choice of k and hence ξ determines the curvature of the tail.
Given the estimate of the tail index parameter the Value at Risk or quantile at a particular probability level can be calculated as

$$VaR_p = X_{k+1,n}\left(\frac{k}{np}\right)^{1/\gamma}$$

1.1.3. Return Level

The 100 α% level quantile of a distribution function F(.) can be written $q_\alpha = F^{-1}(x)$ and a useful risk measure for block maxima is related to the high quantile and is the so called return level. The k n block return level $R_{n,k}$ is defined to be that level which is exceeded in one out of every k blocks of size n. So $R_{n,k}$ is the loss value given by $\Pr[M_n > R_{n,k}] = 1/k$ and the block length is known as the stress period. Once the GEV has been estimated by MLE we can compute these quantiles relatively simply. ie. What is the loss (return level) which we should expect to be exceeded only once in say 40 years.

1.1.4. GPD estimation

Modelling Extremes Over Threshold

Parametric models of ***peaks-over-threshold*** (POT) can be estimated from which we can compute standard risk measures, such as VaR or ESfall. In building POT models we model all large observations, typically losses, that exceed a given threshold, e.g. the 99th quantile or a given VaR level.

<u>The Generalised Pareto Distribution</u>

The GPD is a two parameter distribution with the following function

$$G_{\xi,\beta}(x) = \begin{cases} 1-(1+\xi x/\beta)^{-1/\xi} & \xi \neq 0 \\ 1-\exp(-x/\beta) & \xi = 0 \end{cases}$$

where $\beta > 0$, and where $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi<0$.

If $\xi>0$, the shape parameter indicates *heavy-tails*.

The set of moments, $E[X^k]$, will be infinite for $k \geq 1/\xi$.

So when $\xi=1/2$, the GPD has an infinite variance (second moment) distribution;

When $\xi=1/4$, the GPD has an infinite fourth moment.

1.1.5. Estimating Tails of Loss Distributions

Fitting the tails of distribution

Let $X_{1,} X_{2,} \ldots$ be a set of iid. random variables representing losses with unknown CDF $F(.)$

- The distribution of (excess) losses, y, over a threshold μ, e.g. 95%, is defined to be

$$F_{\mu}(y) = P\{X - \mu \leq y \,|\, X > \mu\} = \frac{F(y+\mu) - F(\mu)}{1 - F(\mu)}$$

for $0 \leq y \leq x_0 - \mu$, where $x_0 \leq \infty$ is the right endpoint of F. The excess distribution represents the probability that a loss exceeds the threshold μ by at most an amount y, given that it exceeds the threshold.

- The following limit result shows that under certain regularity conditions such excess distributions converge to a Generalised Pareto Distribution.

$$\lim_{u \to x_0} \sup_{0 \leq y \leq x_0 - \mu} \left| \mathrm{F_u}(y) - G_{\xi, \beta(u)}(y) \right| = 0$$

That is, for a large class of underlying distributions F, as the threshold μ is progressively raised, the excess distribution F_{μ} converges to a generalised Pareto distribution.

- In particular, for a risk X_i, having distribution F we assume that, for a certain μ, the excess distribution above this threshold may be taken to be exactly GPD for some threshold μ and β.

$$\mathrm{F_u}(y) = G_{\xi, \beta(\mu)}(y)$$

- Assuming that N_{μ} out of total n data points exceed the threshold, the GPD is fitted to the N_{μ} excess values using maximum likelihood methods.

- Choice of the threshold is a trade-off between choosing a sufficiently high threshold so that the limiting distribution can be considered to be essentially exact and choosing a sufficiently low threshold so that we have enough observations for the estimation of the parameters. There is a trade off between taking too many observations in the body of the distribution and not enough in the tail.

The GPD distribution function can also be written as:

$$F(x) = (1 - F(\mu))G_{\xi,\beta}(x-\mu) + F(\mu)$$

for x> μ. Where the tail estimator $F(\mu)$, can be constructed using an empirical estimator, in other words $(n\text{-}N_{\mu})/n$.

- Putting together the empirical estimate of $F(\mu)$ and the maximum likelihood estimates of the parameters of the GPD we obtain the tail estimator.

$$\hat{F}(x) = 1 - \frac{N_{\mu}}{n}\left(1 + \hat{\xi}\frac{x-\mu}{\hat{\beta}}\right)^{-1/\hat{\xi}}$$

1.1.6. Risk Measures: Value at Risk and Expected Shortfall

<u>Estimating VaR and ESfall</u>

For given $q > F(\mu)$, the VaR estimate is calculated by inverting the tail estimate shown above, so as to get:

$$VaR_q = \mu + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_{\mu}}(1-q)\right)^{-\hat{\xi}} - 1\right)$$

and the Expected Shortfall (ESfall) can be expressed simply as follows:

$$ES_q = VaR_q + E[X - VaR_q \mid X > VaR_q]$$

The second term represents the mean excess over the threshold VaR_q.

- Exploiting the fact that the excess distribution above the higher threshold is also a GPD with the same shape but different scale parameter, the VaR can also be rewritten as:

$$F_{VaRq}(y) = G_{\xi,\beta+\xi(VaRq-\mu)}(y)$$

- This expression can be rewritten in such a way that an explicit estimate of ESfall is found. That is, provided $\xi<1$, the mean of $F_{VaR_q}(y)$ is $(\beta + \xi(VaR_q - \mu))/(1-\xi)$, and the ratio of expected shortfall to VaR is then :

$$\frac{ES_q}{VaR_q} = \frac{1}{1-\xi} + \frac{\beta - \xi\mu}{(1-\xi)VaR_q}$$

Finally, the Expected Shortfall is estimated by substituting data-based estimates for everything which is unknown in this expression to obtain

$$ES_q = \frac{\widehat{VaR_q}}{1-\hat{\xi}} + \frac{\hat{\beta} - \hat{\xi}\hat{\mu}}{(1-\hat{\xi})}$$

2. Application of EVT to the UK Housing Market

In what follows we apply the Extreme Value methods described above to the Halifax Monthly House Price index from Jan 1983 to May 2004 - as plotted below- in order to assess the probabilities of any rapid realignments in the housing market.

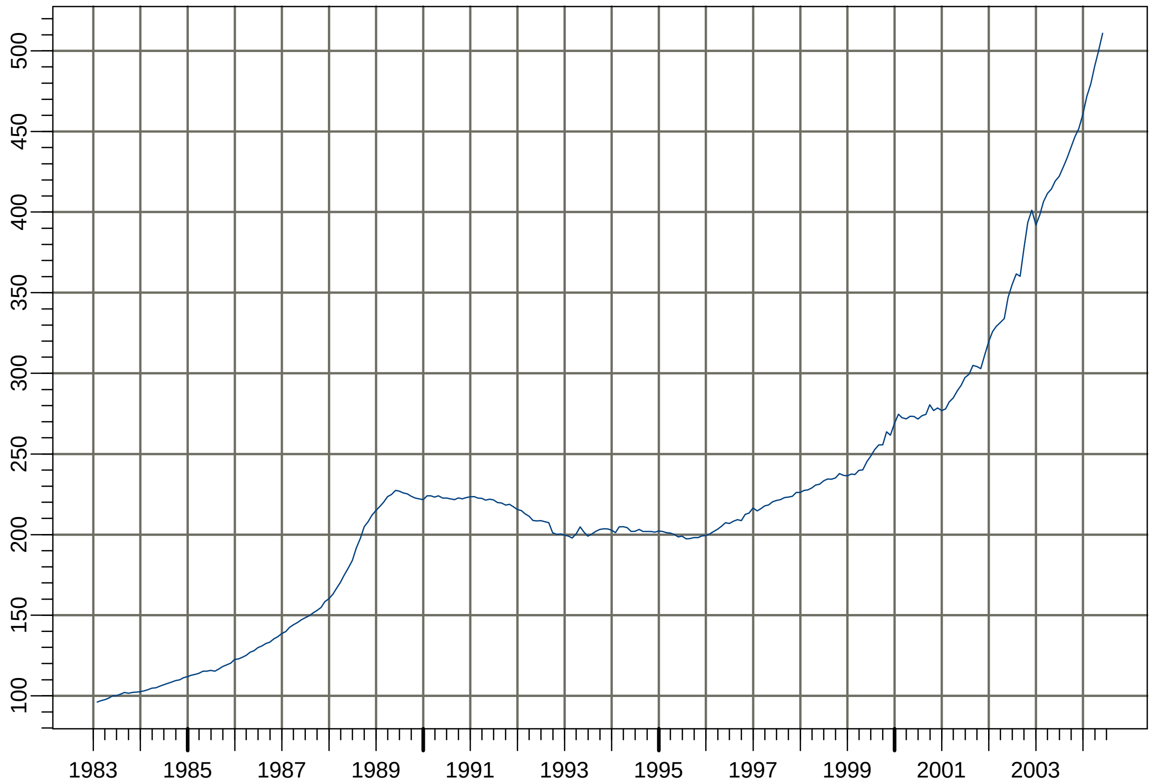


Figure 1 The Halifax Monthly House Price Index

The continuously compounded quarterly returns of this series is shown in the next graph.

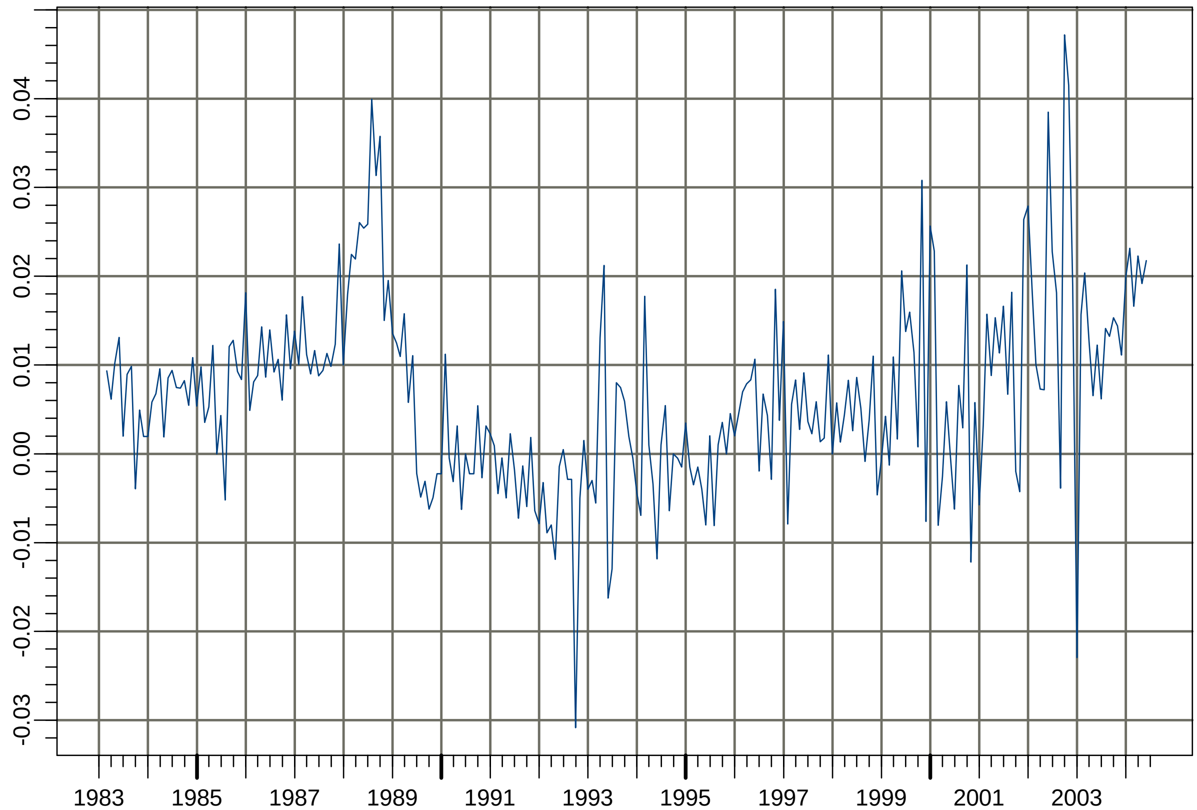


Figure 2 Monthly Housing Returns

I have used returns in the analysis below to remove stationarity issues and to provide scale free measures of possible future (percentage) price changes. If we explore the distributional properties of these house price returns we can see that they deviate substantially from Gaussianity . A QQ plot below indicates that it is particularly in the tails as opposed to the concentration of returns around the mean that the greatest deviations occur. This emphasises that substantial errors would be made if the probability of any tail event were to be calculated assuming Gaussianity.

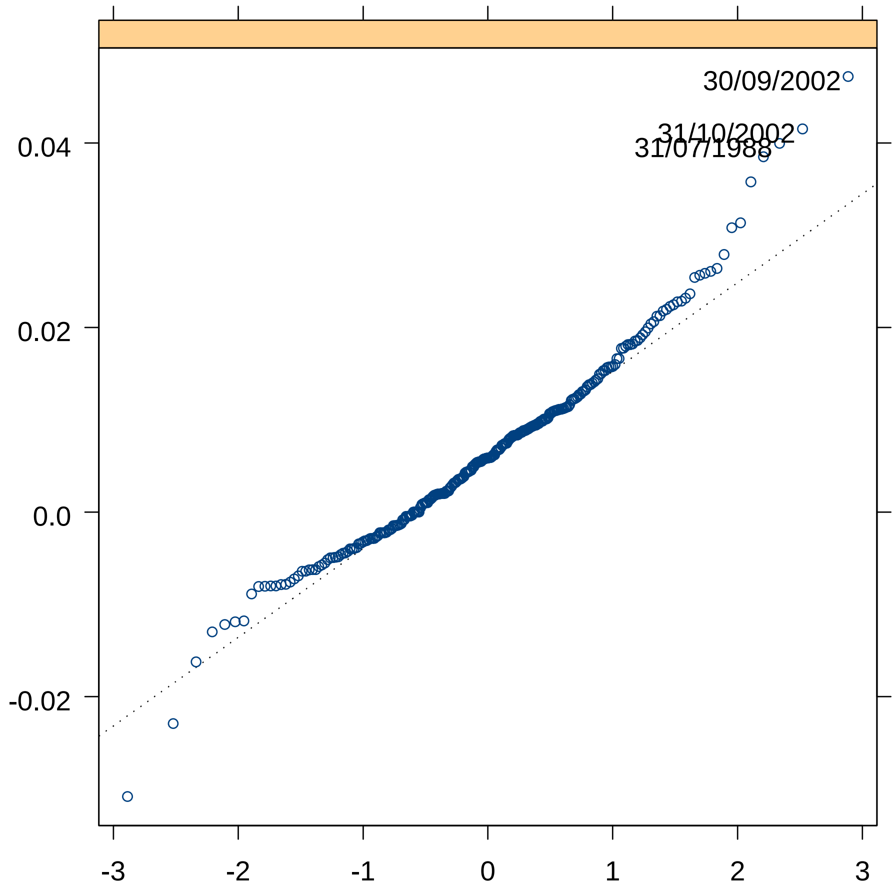


Figure 3 QQ plot against Gaussian Quantiles

We can compute block maxima by taking the largest monthly negative return over an annual block and from the first panel below we can see that that occurred in 1992 with a monthly negative return of 3% followed by 2002 with 2.3%. The Histogram in the next panel indicates the right skew in the (negative return distribution somewhat similar to a Fréchet density. The QQ plot in the next panel uses the Gumbel as the reference distribution and the upward sloping curve indicates a GEV with $\xi < 0$. The final panel shows the development of new maxima (records) for the monthly negative returns along with the expected number of records for *i.i.d.* data and a 95% confidence interval. From this final panel we can see a dramatic deviation from *i.i.d.* behaviour towards the end of the sample and we should be aware that the computations below have relied on an *i.i.d.* assumption for the whole sample.

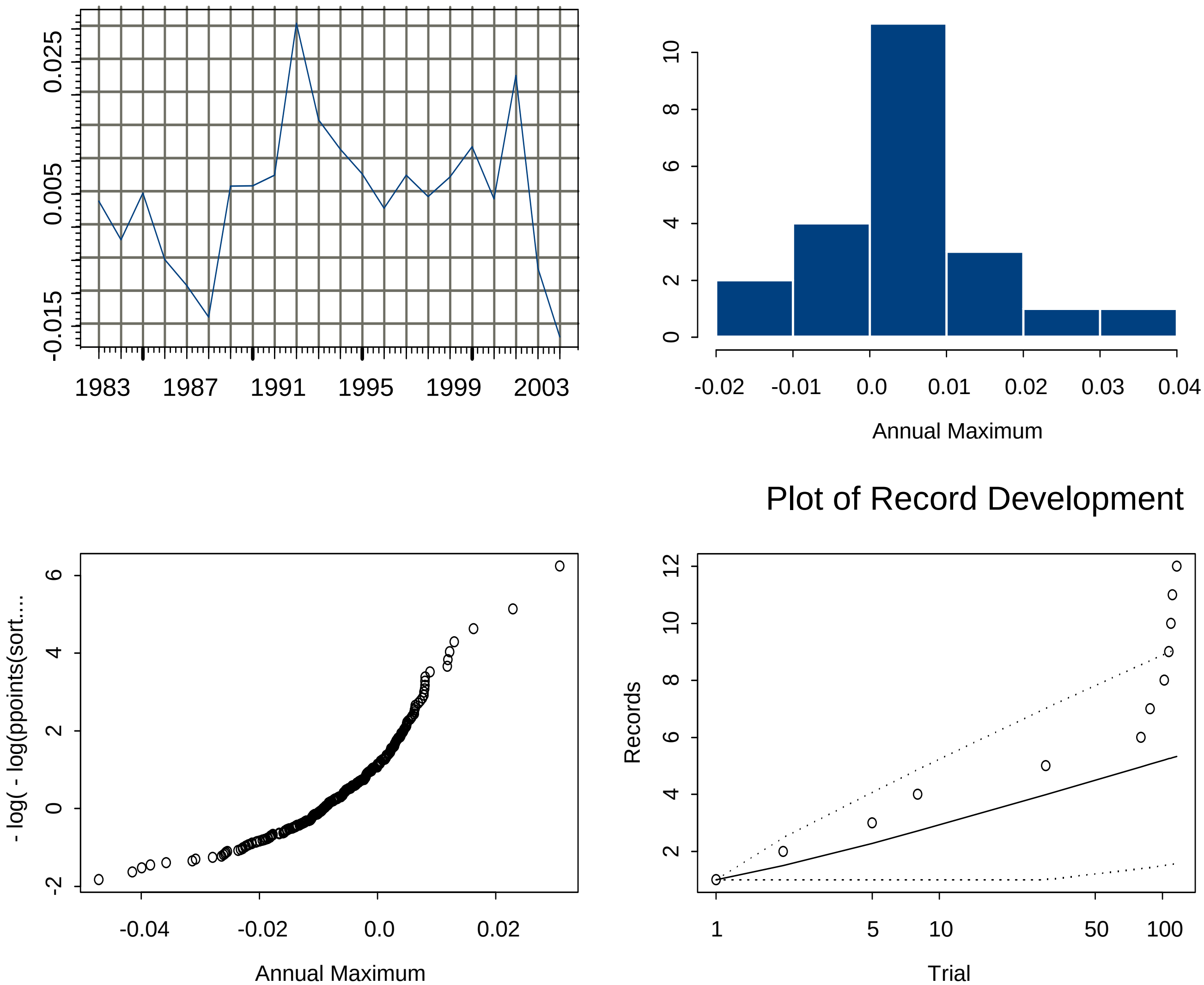


Figure 4 Annual Block Maxima Empirics

We can now turn to consider the MLE's of the parameters of a GEV distribution using these annual block maxima. Using SPLUS the ML parameter estimates for μ, σ, ξ together with their asymptotic standard errors are:-

$\hat{\xi}$ =-0.229 (0.1207)
$\hat{\sigma}$ =0.011 (0.00167)
$\hat{\mu}$ =0.00083 (0.00246)

The 95% confidence interval for ξ is then [-0.4704,0.0124] which suggests a finite tail and a Weibull type as opposed to most financial time series which lie in the fat tailed Fréchet class of distributions. Distributions in the Weibull class included distributions with bounded support such as the uniform and beta distributions. All moments exist for these distributions. The location parameter is not significantly different from zero.

We can evaluate this model by plotting the crude residuals $W_i = \left(1+\hat{\xi}\frac{M_n^i-\hat{\mu}}{\hat{\sigma}}\right)^{-1/\hat{\xi}}$ which should be iid unit exponentially distributed if fitted model is correct and show no trend.

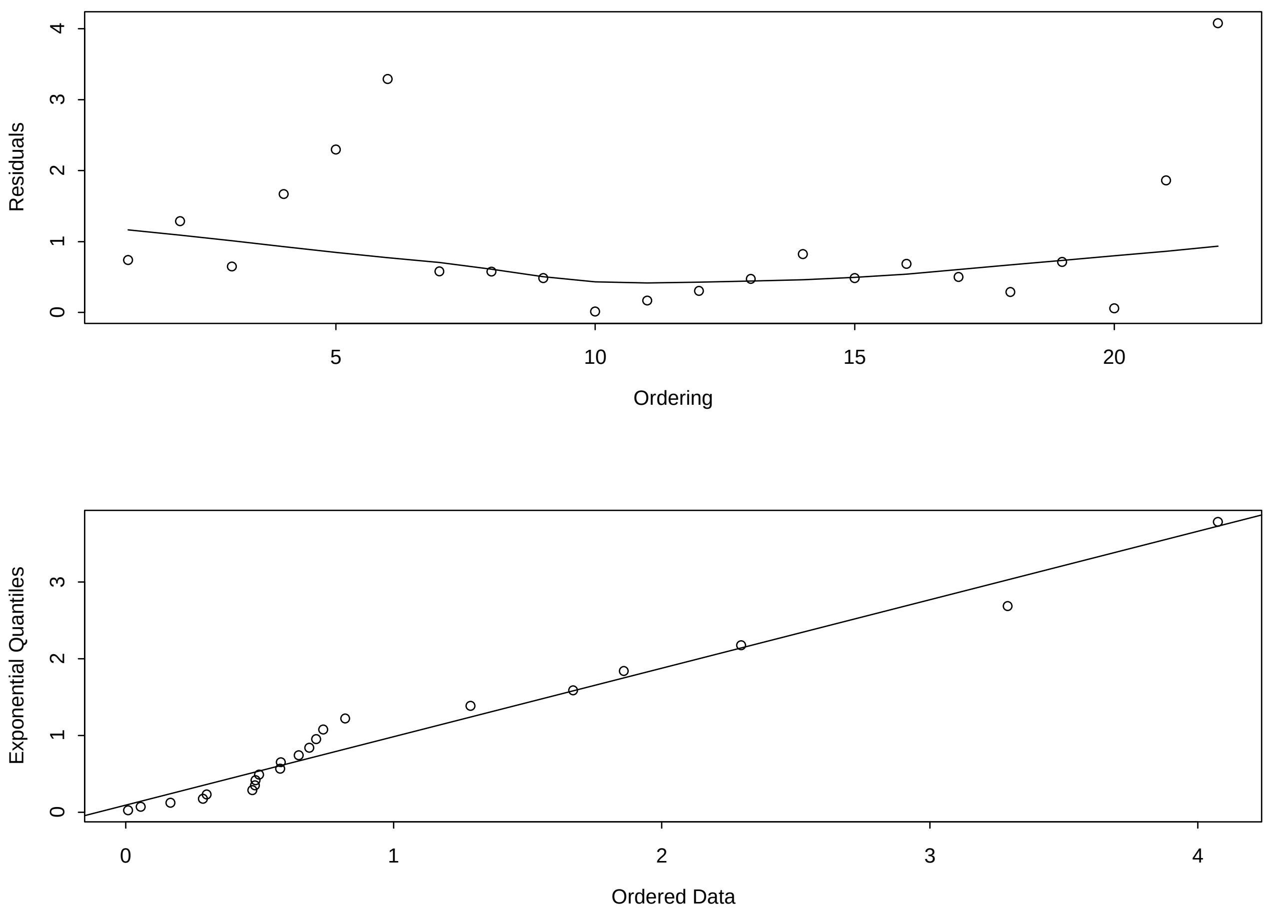


Figure 5 Evaluation of GEV Residuals

The scatterplot of the residuals shows two periods of significant deviations from the trend—in fact the estimated trend line does not indicate any significant unmodelled trend in the data- just two periods of somewhat different behaviour. The QQ plot uses the exponential distribution as the reference distribution and this appears to validate the estimated GEV distribution indicating the GEV distribution has done a relatively good job of modeling the maxima. All in all with the lack of trend- this does suggest the estimated GEV distribution fits well.

Using the estimated GEV distribution we are now in a position to ask questions such as ; what is the probability that next year's annual maximum negative return will exceed all previous negative returns?

ie. $\Pr(M^{(23)} > \max(M^{(1)} \cdots M^{(22)}) = 1 - H_{\hat{\mu},\hat{\xi},\hat{\sigma}}(\max M)$

Remember we have 22 years of monthly data. This probability can be computed from the estimated GEV and is 0.0098 so that there is approximately a 1% chance that a new record monthly maximum will be established next year.

This calculation has been based on annual maxima and we could instead compute the estimated GEV distribution using quarterly block maxima. The parameter estimates do not differ dramatically in this case and both the scale and tail index parameters are virtually the same as the annual case- however the location parameter is now significantly negative (negative returns). The corresponding estimate of the probability of the next quarter's maximum exceeds all previous maxima is now 0.05%. Clearly we are in a highly unusual period … as of May 2004.

We can also compute return levels- which provide a risk measure for Block Maxima. So we can ask the questions such as what is the negative return level that will be exceeded once in every say 10 years? The answer turns out to be 0.019; in other words the negative monthly return level that should be exceeded once in every 10 years is 1.9% which corresponds to the 0.995 quantile. The plot of the profile log likelihood provides an asymmetric 95% confidence interval for this estimate between roughly 1.5 and 3.0%

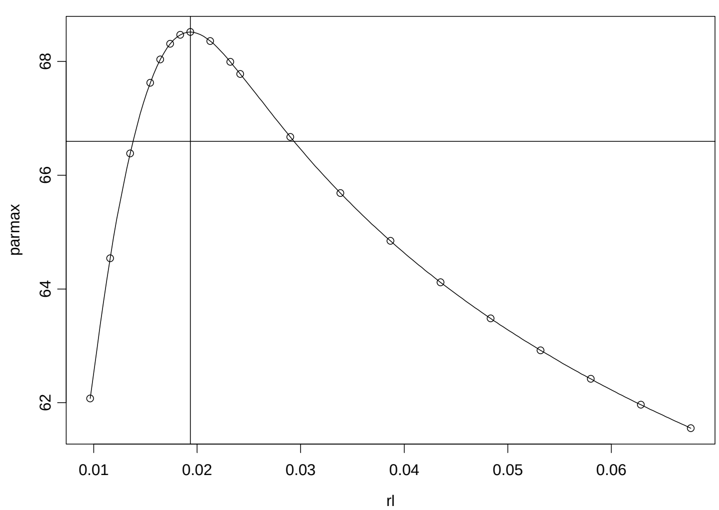


Figure 6 Profile Log Likelihood for Return Level

Alternatively we can get a plot of the return level and block maxima with 95% confidence intervals

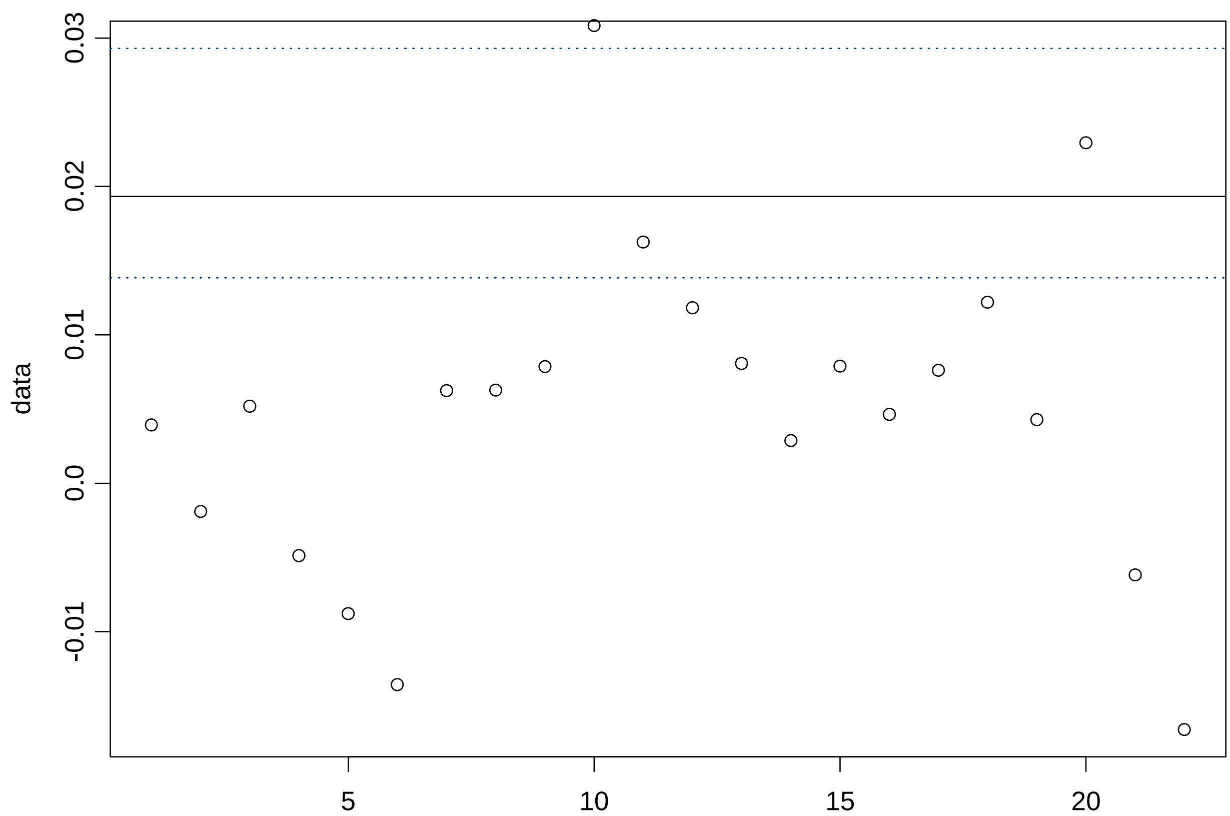


Figure 7 Return Level and Block Maxima

These computations are illustrative of the type of calculations we can carry out to assess the probabilities of particular extreme events using the limiting distribution of block maxima. We now return to consider the alternative approach to extremes that we described above given by looking at exceedences over a threshold and the Generalised Pareto Distribution.

We can use QQplots to infer tail behaviour using the exponential as the reference distribution. If excesses over thresholds are from a thin tailed distribution then the GPD is exponential with ξ=0 and the QQplot should be linear and we can vary the threshold we consider in these computations.

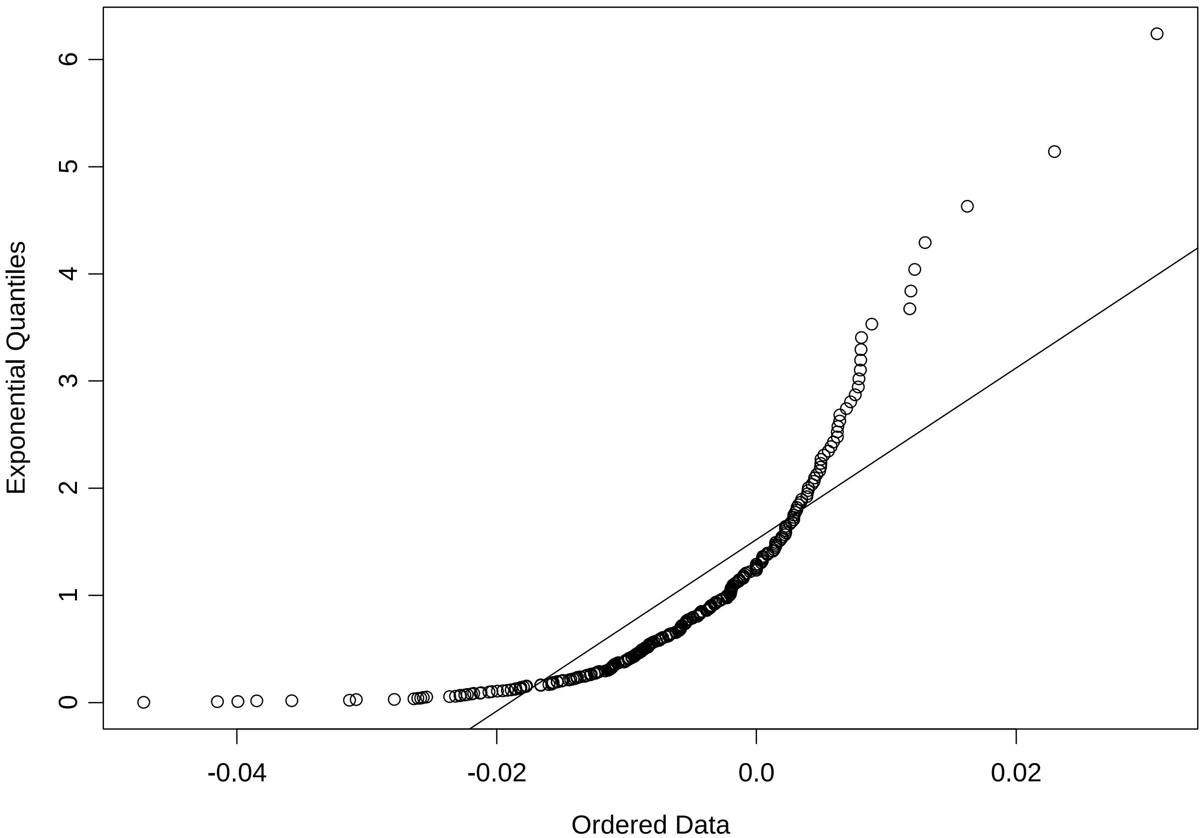


Figure 8 Untruncated exponential QQ plot

Which shows fairly substantial departures from linearity when the threshold is not set and the data is not left truncated. However when we use a truncation level of 0.001 we find a much better approximation. We shall come back to this issue a number of times when we try to assess where within the body of the distribution the tail behaviour takes over.

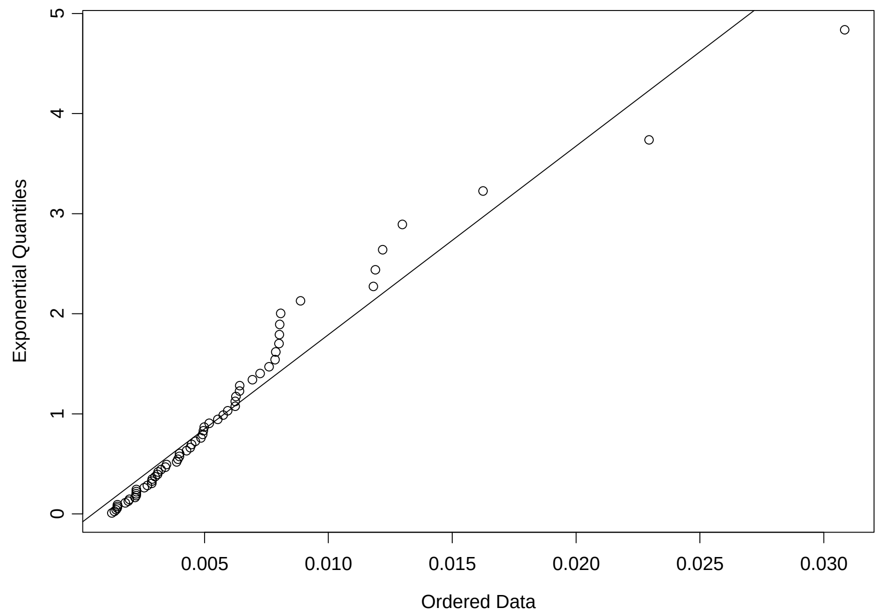


Figure 9 Truncated Exponential QQplot (0.001)

We can next compute the mean excess function over a chosen threshold which should be linear over threshold. The next plot shows how mean excess (Expected Shortfall) varies with the threshold.

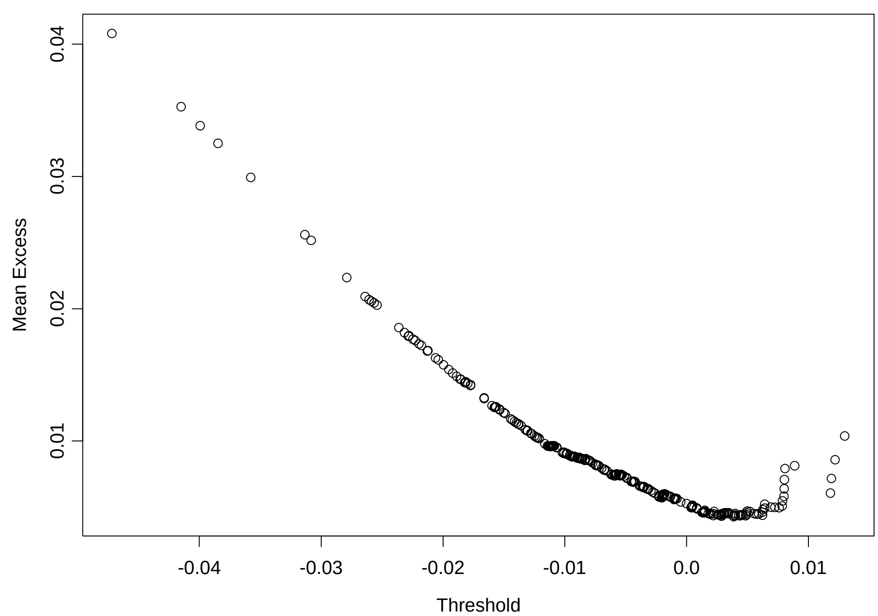


Figure 10 Expected Shortfall and Threshold

Instead of just guessing a value for the threshold such as 0.001 we can use a particular quantile value such as the 95% quantile which in our case is 0.00788. Given this value we can estimate the GPD using maximum likelihood. The 12 exceedences over the estimated 95% quantile are given by the following values at the associated dates.

29/02/1992	0.008880610
31/03/1992	0.008013227
30/04/1992	0.011902068
30/09/1992	0.030846939
31/05/1993	0.016244527
30/06/1993	0.012987226
31/05/1994	0.011817022
31/05/1995	0.008028172
31/07/1995	0.008076727
29/02/2000	0.008041023
31/10/2000	0.012199627
31/12/2002	0.022948948

The MLE estimates of the parameters of the GPD are given by:-

$\hat{\xi}$=0.752 (0.883)

$\hat{\beta}$=0.0024 (0.002)

The difference between the point estimates of ξ and the value we found when estimating the GEV appears large and this in part reflects the difficulty in selecting a truncation point within the body of the distribution in order to define where the tail starts- however both estimates in this case in fact indicate a lack of significance at the 95% level from zero. I interpret this more as a difficulty of statistical inference in this situation where several regimes may be in force. Given these estimates we can plot the excess distribution, the tail of the underlying distribution, the scatter plot of the residuals and a QQplot of the residuals as shown in the following panel.

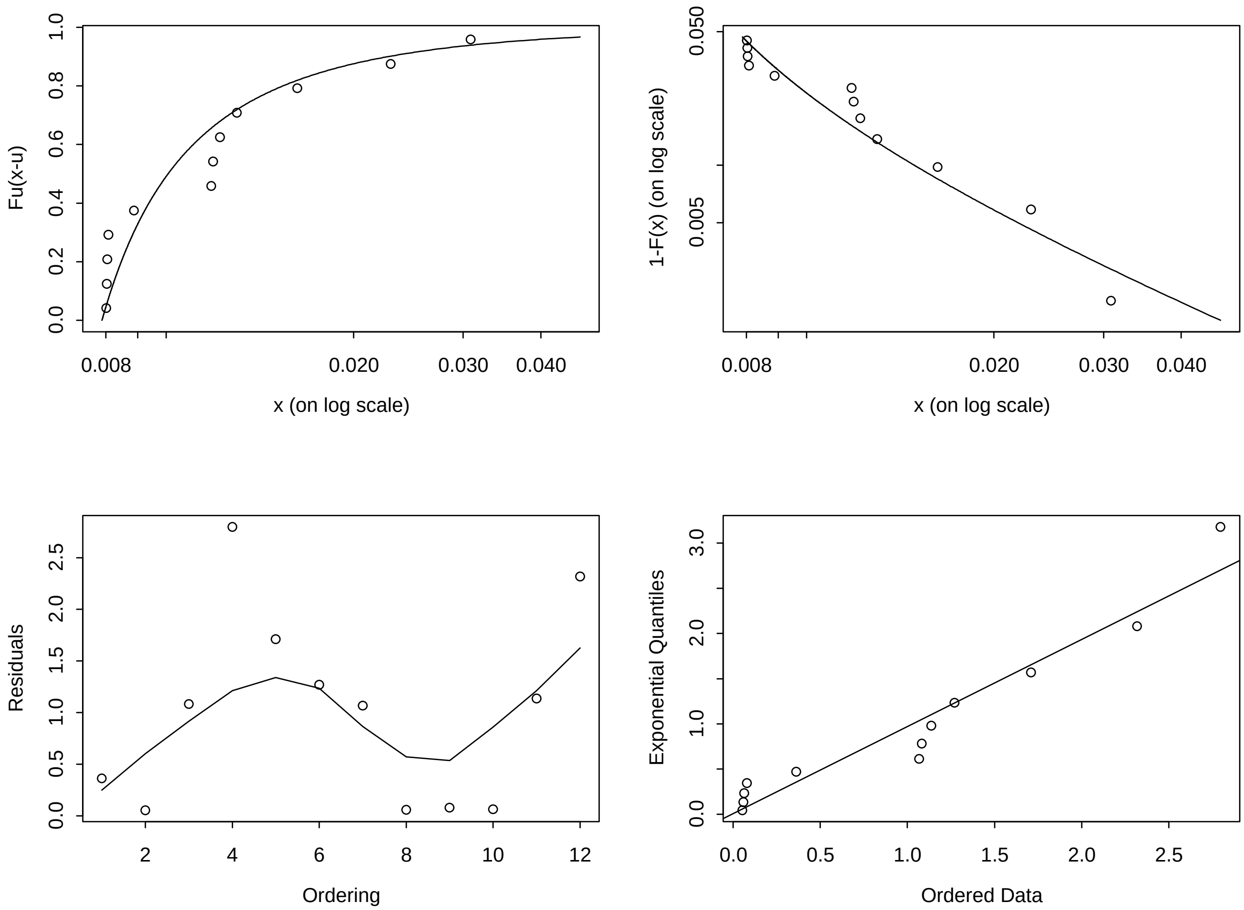


Figure 11 GPD Diagnotics

All of these seem to suggest a fairly good fit to the tail with some outliers. A “shape” function can be used to consider the sensitivity to the choice of threshold as shown in the following figure with the estimated tail index parameter on the left hand axis and the corresponding number of exceedences on the horizontal axis. Also show on figure 12 is the 95 % confidence banc which shows we have been too conservative in selecting the 95% quantile as the shape parameter is relatively stable and significantly different from zero after around 180 exceedences and suggests the earlier negative estimate of ξ from the GEV estimation at around -0.229 is sustainable and sensible.

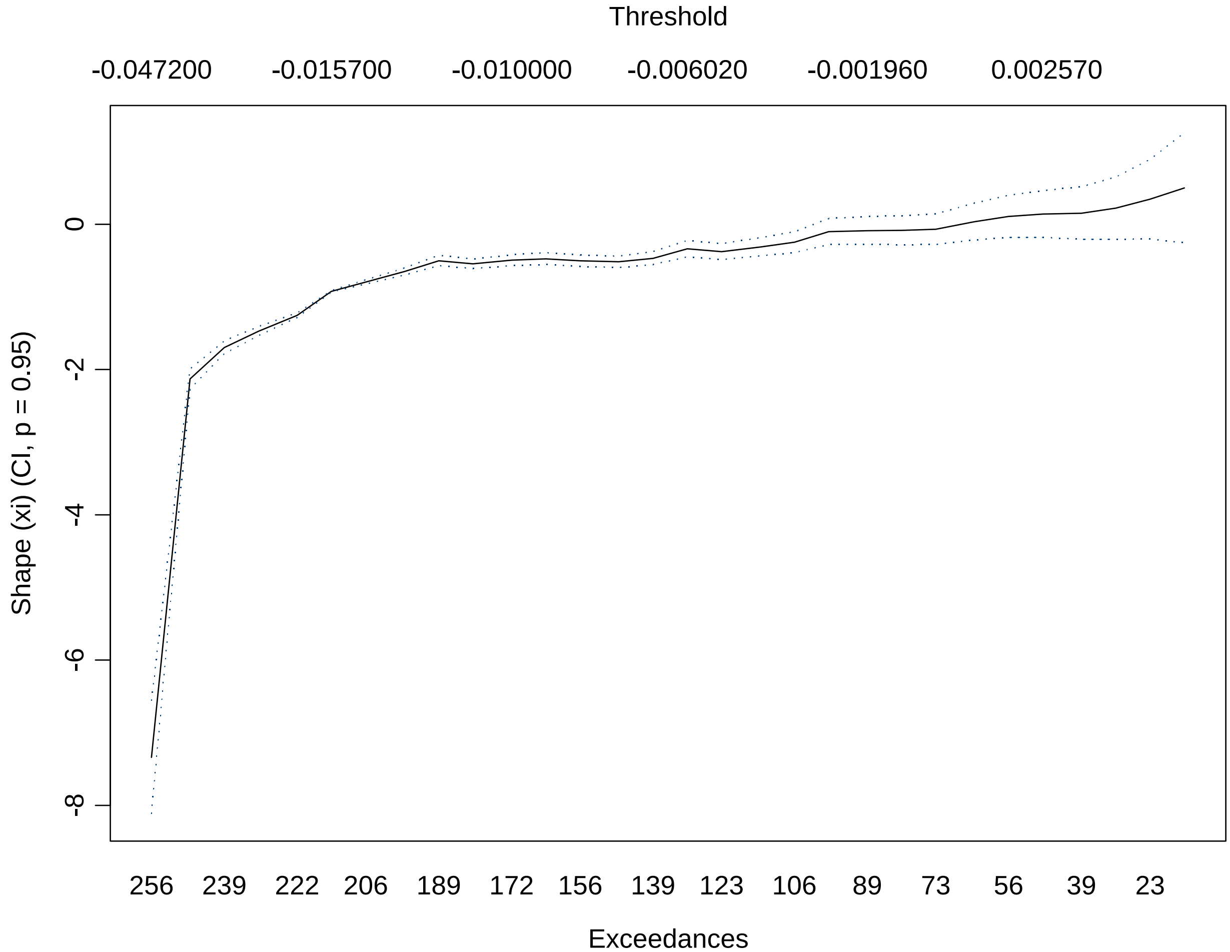


Figure 12 Shape function to determine the threshold

As a compromise we have taken a threshold value that provides 63 exceedences in the tail and provides more reliable estimates of the parameters closer to the GEV estimates with diagnostic plots given by

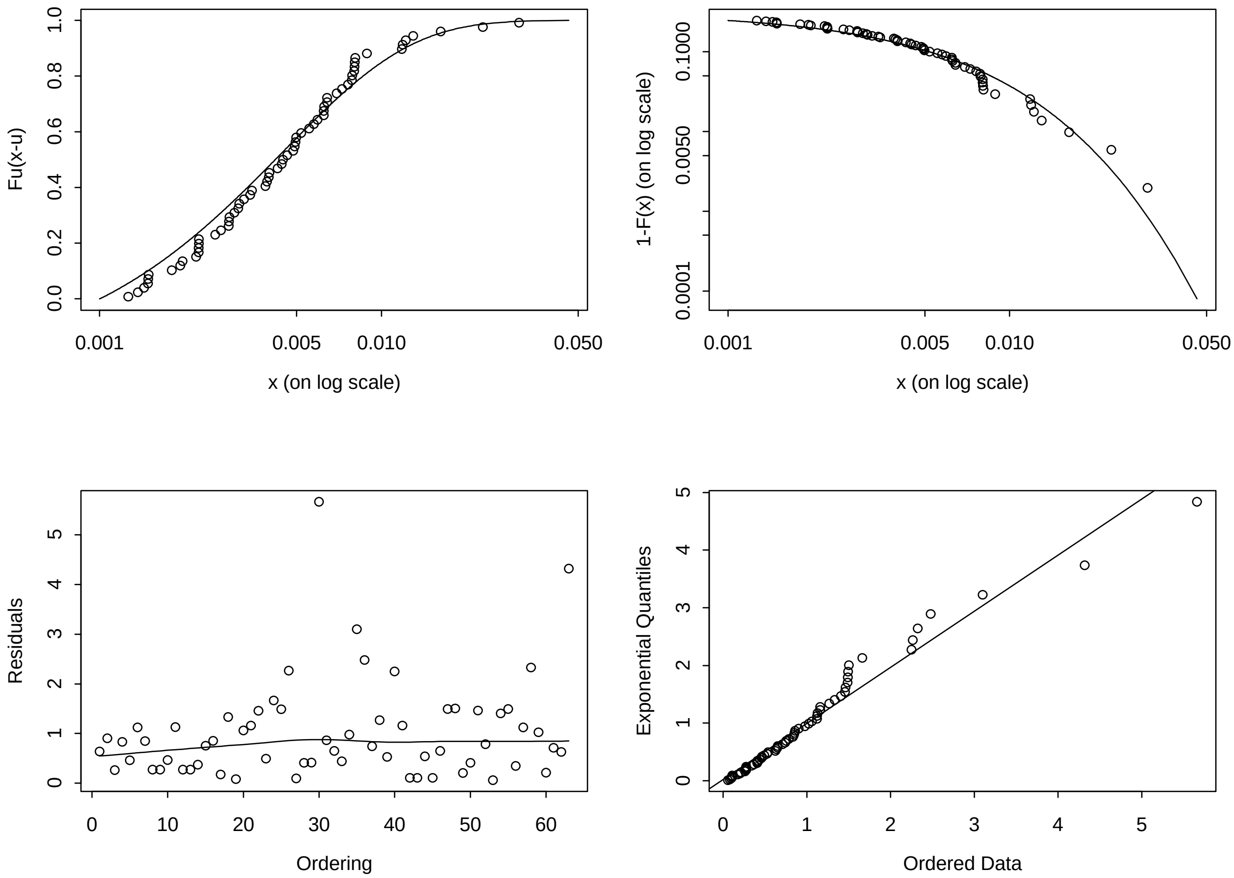


Figure 13 Diagnostic plots with lower threshold (more observations taken into tail estimation)

This figure gives some confidence in now carrying forward the analysis to compute the risk measures in which we are interested; as shown above VaR and Expected Shortfall can be easily computed from this estimated GPD.

The 95% and 99% VaR and Expected Shortfall values are given by

Table 1 Risk Measures based on EVT

PROBABILITY	**VAR**	**EXPECTED SHORTFALL**
0.95	0.0085	0.0137
0.99	0.0168	0.0224

So based on the historical data series of house prices we find that a monthly return in house prices, ie next month's return could be as low as -0.85% with a 5% probability and given that the return is less than 0.85% , the average return – ie in the tail - would be -1.37%. Similarly with a 1% probability the monthly return could be as low as-1.68% and given this the average value is -2.24%. These computations could be inverted to provide estimated probabilities of different sized falls in the housing market. Estimates for asymptotically valid 99% confidence intervals for the VaR and Expected Shortfall can then be constructed using the Delta method and are shown in this final graph.

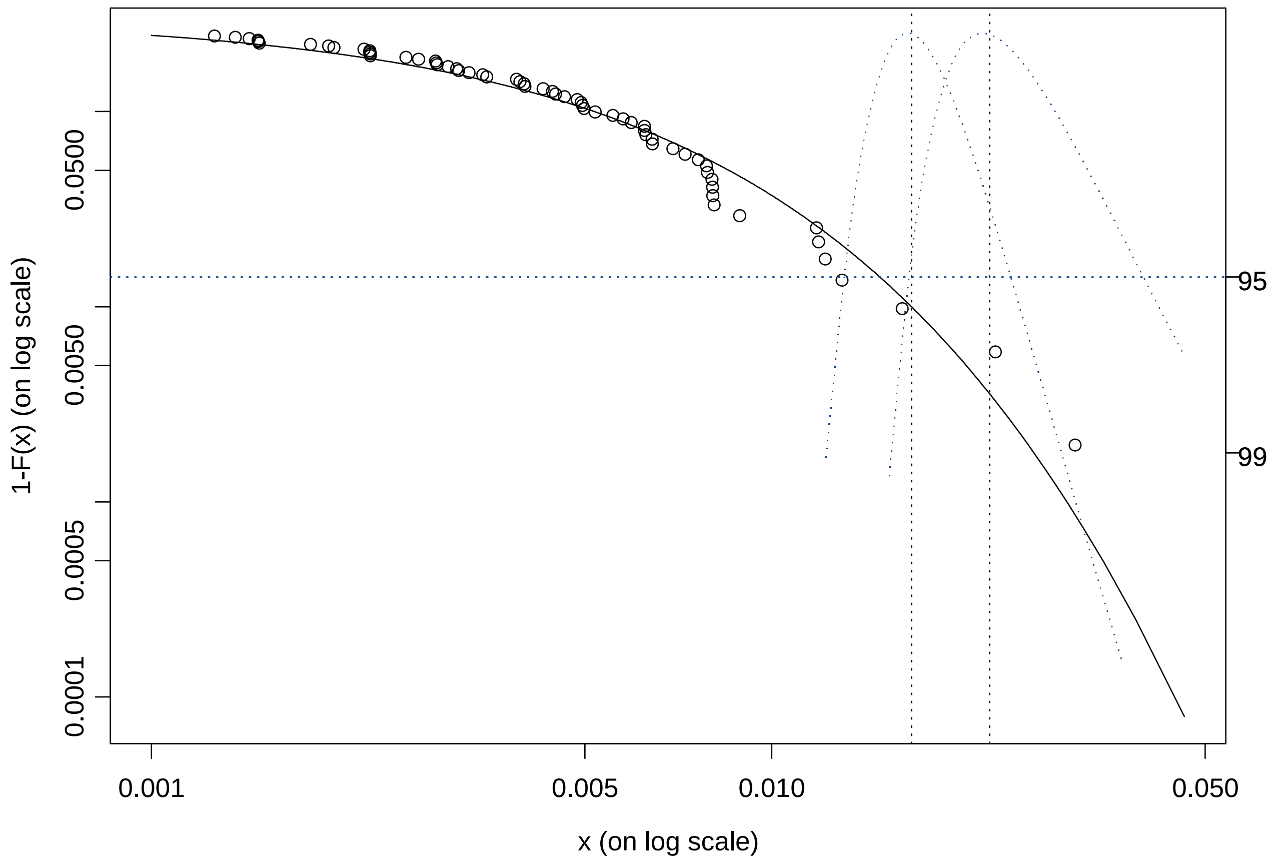


Figure 14Confidence Intervals For VaR and ESFall Estimates

Giving for the VaR:

```
     Lower CI   Estimate   Upper CI
     0.01310464 0.01680462 0.02424438
and for the Expected ShortFall
     Lower CI   Estimate   Upper CI
     0.0167297 0.02245737 0.03956688
```

3. Concluding Remarks

The main observation that seems to appear from this analysis is that large price reversals are not high probability events based on the historical evidence of the Halifax price index. All of this analysis could be repeated for the upper tail to assess the probability of future price rises. It appears that there is temporal dependence in the Halifax series which would qualify these results somewhat. The results above are based on the entire historical series of the price index and a real question exists as to whether we are in a different regime than observed historically and hence how relevant history is in the present situation.