Information Flow along the Yield Curve; an analysis using transaction level data

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Information Flow and the Shape of the Yield Curve

Different information hits the yield curve at different maturities; so measuring market activity at different yields should be central to understanding the shape of the Yield curve and how it evolves over time.

1. Use tick by tick level Gov PX data to measure market activity and information flow at different yields and use this to build a market model of the yield curve.

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4. Examine how information flow as measured by instantaneous volatility affects the shape of the 5 min Yield Curve

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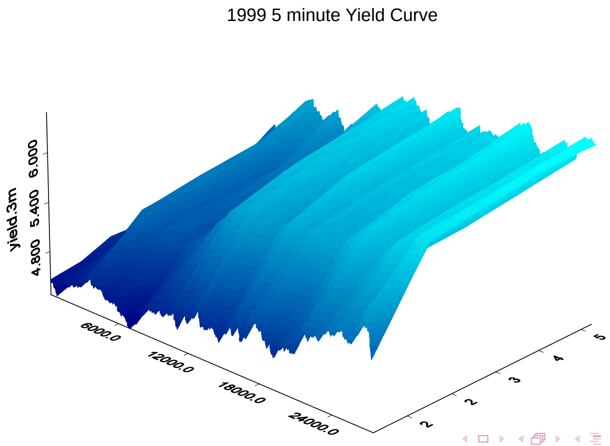
3. Use Hawkes Processes - univariate and multivariate- to measure information flow - information clustering
4. Examine how information flow as measured by instanteneous volatility affects the shape of the 5 min Yield Curve
5. Use the instanteneous volatility derived from the Hawkes models to calibrate an HJM model and price Caplets.

5 Minute Yield Curves



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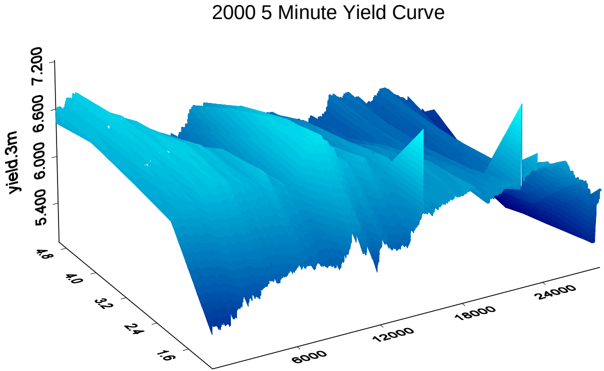
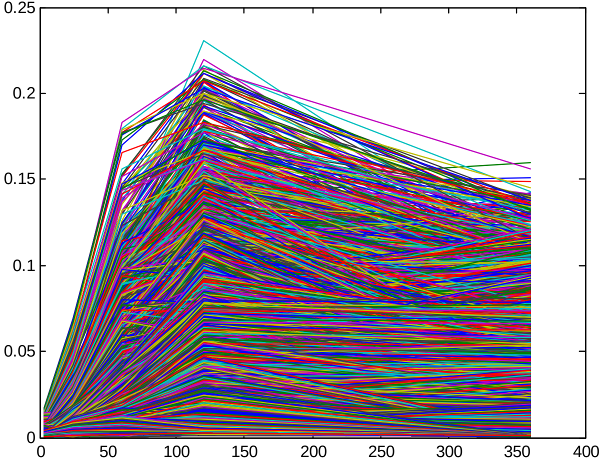
5 minute Yield Curves



Instantaneous Volatility Term Structure



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Research Objectives

So can we model the yield curve by measuring information flow at different points along the yield curve?

If so:

1. How do we measure volatility?

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3. How accurate are option prices that are priced off the volatility Yield Curve model based on Hawkes processes?
4. Develop an approach to pricing fixed income derivatives based on an estimated instantaneous volatility.

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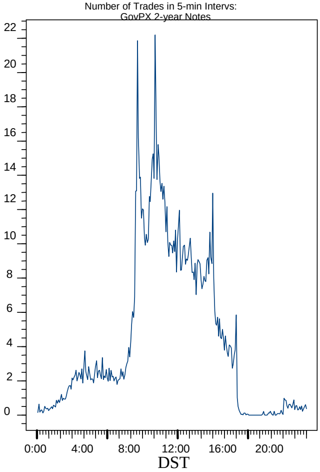
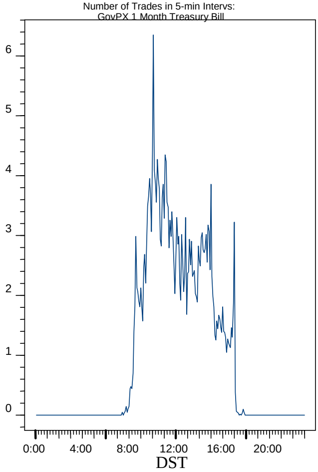
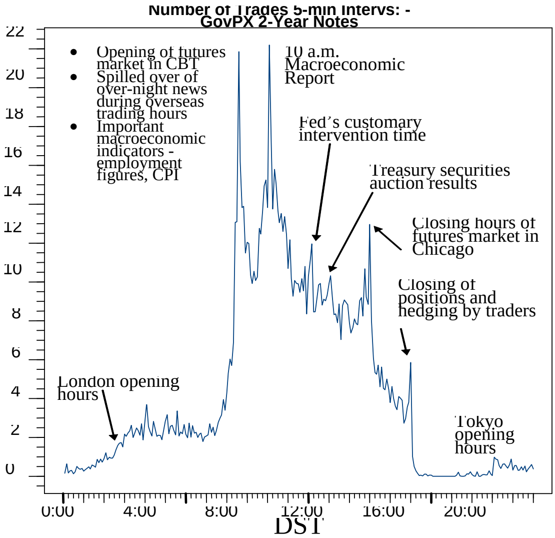
U.S. Treasury Securities - GovPX

1. One of the most important financial markets in the world
2. Daily trading volume in the secondary market of about averages $200 billion.
3. Almost round-the-clock trading - New York, Tokyo and London
4. Trade sizes starting at $1 million for bonds and $5 million for bills
5. Almost no high frequency analysis of this important market
6. About 1,700 brokers and dealers trade in the secondary market, the 39 primary government securities dealers account for the majority of trading volume.


Number of Trades in 5-min Intervs:
GovPX 2-year Notes
22
20
18
16
14
12
10
8
6
4
2
0
0:00
4:00
8:00
12:00
16:00
20:00
DST


Number of Trades in 5-min Intervs:
GovPX 1 Month Treasury Bill
6
5
4
3
2
1
0
0:00
4:00
8:00
12:00
16:00
20:00
DST


Number of Trades 5-min Intervs: - GovPX 2-Year Notes
Opening of futures market in CBT
Spilled over of over-night news during overseas trading hours
Important macroeconomic indicators - employment figures, CPI
10 a.m. Macroeconomic Report
Fed's customary intervention time
Treasury securities auction results
Closing hours of futures market in Chicago
Closing of positions and hedging by traders
London opening hours
Tokyo opening hours
0
2
4
6
8
10
12
14
16
18
20
22
0:00
4:00
8:00
12:00
16:00
20:00
DST

Hawkes Processes

Bowsher (2002), Hautsch (2004), Large(2005), McCulloch and Salmon (2004)(2005)

1. Model the rate that financial events take place as a conditional random intensity with self-excited and cross excited dependence.

$$\lambda(t) = \nu + \int_{-\infty}^{t} g\left(t-u\right) dN\left(u\right) \qquad (1)$$

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Hawkes Processes

Bowsher (2002), Hautsch (2004), Large(2005), McCulloch and Salmon (2004)(2005)

1. Model the rate that financial events take place as a conditional random intensity with self-excited and cross excited dependence.
2. The conditional intensity function can be modelled as a function of its backward recurrence time,Hawkes(1971).

$$\lambda(t) = \nu + \int_{-\infty}^{t} g(t-u)\, dN(u) \qquad (1)$$

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An exponential decay for univariate Hawkes can be modelled as

$$\lambda(t) = \mu + \sum_{i=\max(1,N(t-\epsilon)-R+1)}^{N(t-\epsilon)} a e^{-b(t-t_i)} \tag{2}$$

where R is the number of lags in backwards recurrence time

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Hawkes Processes II

The unknown parameters can be estimated using MLE (Daley and Vere-Jones, 2003)

$$\mathcal{L} = \int_{0}^{N} \log(\lambda(t \mid \mathcal{F}_t)) - \int_0^T (1 - \lambda(t \mid \mathcal{F}_t))\, dt \tag{3}$$

$$\mathcal{L} = \sum_{i=1}^{N} \log(\lambda(t_i)) - \int_0^{T_2} \lambda(t) dt \tag{4}$$

Multivariate Hawkes Process

The multivariate-multidimensional intensity function reads

$$\lambda_s(t) = \mu_s + \sum_{r=1}^{P} \sum_{j=1}^{D} \sum_{k=1}^{\mathcal{N}_r(t)} \alpha^j_{s,r} \exp\left[-\beta^j_{s,r}(t - \tau_{r,k})\right]$$

where P- number of processes, D number of dimensions, N_s number of data points of process s,

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which can be equivalently written in terms of a pooled process

$$\lambda_s(t) = \mu_s + \sum_{j=1}^{D} \sum_{k=1}^{\mathcal{N}^*(t)} \alpha^j_{s,\sigma^*_k} \exp\left[-\beta^j_{s,\sigma^*_k}(t - \tau^*_k)\right] \tag{5}$$

$$= \mu_s + \sum_{r=1}^{P} \sum_{j=1}^{D} \sum_{k=1}^{\mathcal{N}^*(t)} \delta_{r,\sigma^*_k} \alpha^j_{s,r} \exp\left[-\beta^j_{s,r}(t - \tau^*_k)\right] \tag{6}$$

with the Kronecker δ-function defined as $\delta_{a,b} = 1$ if $a = b$ and 0 otherwise.

The LLF for process s

$$s = \sum_{i=1}^{N_s} \log\left[\lambda_s(\tau_{s,i})\right] - \int_{\tau_{s,1}}^{\tau_{s,N_s}} \lambda_s(t)dt$$

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Stationarity Conditions in the Univariate Case

The process needs to be stationary. We have a constant average rate which is the expectation of the process

$$\bar{\lambda} dt = E\left\{dN(t)\right\}$$

moreover it is the expectation of the time-varying intensity function $\lambda(t)$

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$$\bar{\lambda} = E\{\lambda(t)\} \tag{7}$$
$$= E\left\{\mu + \int_{-\infty}^{t} g(t-u)dN(u)\right\} \tag{8}$$
$$= \mu + \int_{-\infty}^{t} g(t-u)E\{dN(u)\} \tag{9}$$
$$= \mu + \bar{\lambda}\int_{-\infty}^{t} g(t-u)du \tag{10}$$

which can only be true if the integral is between 0 and 1 (since $\mu, \bar{\lambda} > 0$)

$$0 < \int_0^\infty g(u)du < 1 \tag{11}$$

$$\tag{12}$$

and in our case

$$g(u) = \alpha e^{-\beta u}$$

this gives the stationarity condition

$$0 < \frac{\alpha}{\beta} < 1$$

Stationarity in the Multivariate Case

In the MV case we have the average rates $\bar{\lambda}_s$ as

$$\bar{\lambda}_s dt = E\left\{dN_s(t)\right\}$$

which is

$$\bar{\lambda}_s = E\left\{\lambda_s(t)\right\} \tag{13}$$

$$= E\left\{\mu_s + \sum_{r=1}^{P} \int_{-\infty}^{t} g_{sr}(t-u)dN_s(u)\right\} \tag{14}$$

$$= \mu_s + \sum_{r=1}^{P} \int_{-\infty}^{t} g_{sr}(t-u)E\left\{dN_s(u)\right\} \tag{15}$$

$$= \mu_s + \sum_{r=1}^{P} \bar{\lambda}_r \int_{-\infty}^{t} g_{sr}(t-u)du \tag{16}$$

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using our assumed model

$$g_{sr}(u) = \alpha_{sr} e^{-\beta_{sr} u}$$

gives P equations

$$\bar{\lambda}_s = \mu_s + \sum_{r=1}^{P} \frac{\alpha_{sr}}{\beta_{sr}} \bar{\lambda}_r$$

where conditions on the parameters can be extracted either by directly reading them off or putting the equation for $\bar{\lambda}_t$ into the equation for $\bar{\lambda}_s$, hence the following

$$\frac{\alpha_{st}}{\beta_{st}} \frac{\alpha_{ts}}{\beta_{ts}} < 1 - \frac{\alpha_{ss}}{\beta_{ss}} \quad \text{for} \quad s \neq t \tag{17}$$

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Results

Estimates of scaled parameters	3m (S.E)	2y (S.E)	5y (S.E)	10y (S.E)	30y (S.E)
μ	0.134374 (0.004)	0.387 (0.008)	0.413 (0.007)	0.375 (0.007)	0.1351 (0.004)
α	2.816 (0.046)	4.459 (0.04)	5.578 (0.039)	5.566 (0.05)	4.323 (0.087)
β	3.197 (0.045)	4.757 (0.041)	5.848 (0.04)	5.856 (0.048)	4.79 (0.092)
$L(\theta)$	5188.2	125223	246605	204826	11533
mean of residuals	1	1	1	0.99	0.99
σ^2 of residuals	1.05	1.04	1.01	1.03	1.01
LB (20 lags)	260.94 (0.000)	406.46(0.000)	345.98(0.000)	324.45(0.000)	245.37(0.000)
Disp	1.33 (0.000)	1.31 (0.000)	1.24 (0.000)	1.11 (0.00)	1.47(0.00)
Obs	14163	77442	112375	95189	17321

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Results Multivariate

	3m (S.E)	2y (S.E)	5y (S.E)	10y (S.E)	30y (S.E)
$\alpha_{3m}(t-stats)$	2.352826(44.256)	0.362883(2.88)	0.583034(3.83)	0.165256(2.12)	-0.021324(-1.53)
$\alpha_{2y}(t-stats)$	2.684901(160.20)	3.137544(84.12)	24.224321(273.44)	10.344241(176.09)	1.122166(188.86)
$\alpha_{5y}(t-stats)$	2.089888(62.71)	3.604475(9.98)	3.726708(92.93)	21.557467(61.63)	1.878735(126.66)
$\alpha_{10y}(t-stats)$	0.852139(54.35)	1.62365(8.08)	19.194776(54.81)	4.300337(94.88)	4.674444(242.09)
$\alpha_{30y}(t-stats)$	-0.084583(-1.40)	-0.004175(-0.04)	0.230784(2.30)	0.391175(2.45)	4.893357(53.68)
$\beta_{3m}(S.E.)$	4.668581(0.11)	5.78676(0.38)	47.042115(0.35)	21.533799(0.23)	38.260298(0.05)
$\beta_{2y}(S.E.)$	9.628071(0.23)	4.826595(0.05)	245.569308(0.40)	127.577995(0.60)	39.771582(0.26)
$\beta_{5y}(S.E.)$	21.791448(0.72)	14.834681(4.00)	6.034754(0.06)	128.587993(2.67)	7.808274(0.24)
$\beta_{10y}(S.E.)$	15.409595(0.23)	15.260858(1.81)	171.609503(1.93)	6.113891(0.06)	19.334739(0.55)
$\beta_{30y}(S.E.)$	33.568557(0.12)	14.778038(0.17)	31.920671(0.41)	11.979099(0.18)	6.557604(0.13)
$\mu(S.E.)$	0.004383(0.00)	0.307226(0.01)	0.213167(0.01)	0.195392(0.01)	0.042997(0.00)

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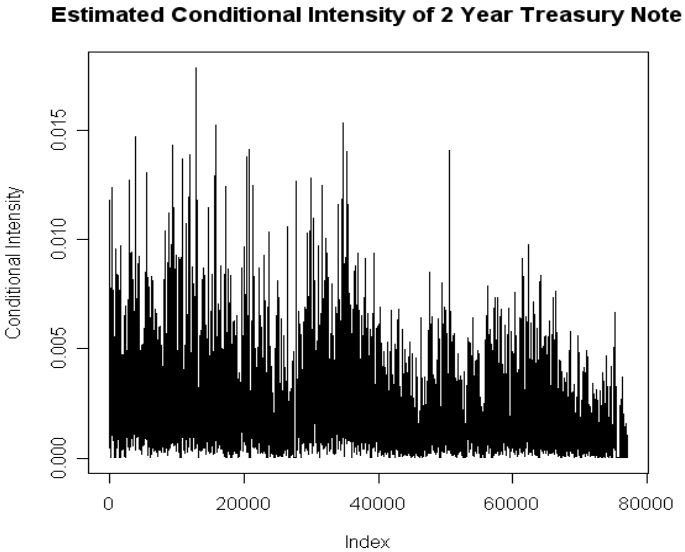
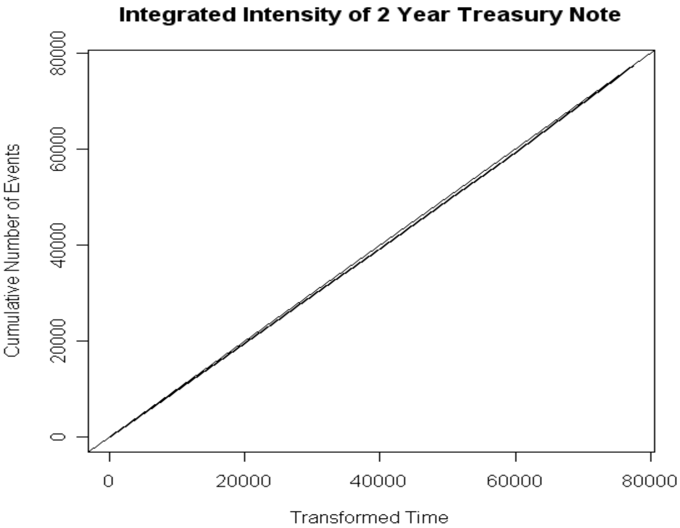

Estimated Conditional Intensity of 2 Year Treasury Note
Conditional Intensity
0.000
0.005
0.010
0.015
0
20000
40000
60000
80000
Index


Integrated Intensity of 2 Year Treasury Note
Cumulative Number of Events
0
20000
40000
60000
80000
Transformed Time

Instantaneous volatility

1. Relationship between trade arrivals and volatilty
2. Instantaneous volatility can be defined by

$$\tilde{\sigma}^2(t) = \lim_{\Delta\to 0} E\left[\frac{1}{\Delta}\left(\frac{P(t+\Delta)-P(t)}{P(t)}\right)^2 \mid \mathcal{F}_t\right]$$

$$\tilde{\sigma}^2_{(x^{dp})}(t) = \lim_{\Delta\to 0}\frac{1}{\Delta}[prob|P(t+\Delta)-P(t)| \geq dp \mid \mathcal{F}^2_t]E\left(\frac{P(t+\Delta)-P(t)}{P(t)} \mid \mathcal{F}^2_t\right)^2$$

$$\tilde{\sigma}^2_{(x^{dp})}(t) = \lambda^{dp}(t;\mathcal{F}^2_t)E\left(\frac{P(t+\Delta)-P(t)}{P(t)} \mid \mathcal{F}^2_t\right)^2$$

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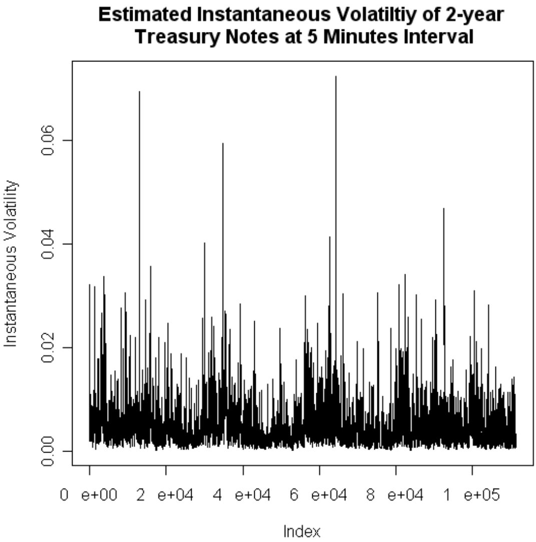
Estimated Instantaneous Volatiltiy of 2-year Treasury Notes at 5 Minutes Interval
Instantaneous Volatility
0.00
0.02
0.04
0.06
0 e+00
2 e+04
4 e+04
6 e+04
8 e+04
1 e+05
Index

Heath Jarrow Morton

Suppose that

$$\mathrm{d}f_t(T) = \alpha(t,T)\mathrm{d}t + \sum_{i=1}^{n} \sigma_i(t,T)\mathrm{d}z_{i,t},$$

Set $a_i(t,T) = -\int_t^T \sigma_i(t,s)\mathrm{d}s$, $i = 1, \ldots, n$.
Pure discount bond prices then follow the process

$$\frac{\mathrm{d}B_t(T)}{B_t(T)} = (r_t + b\,(t,T))\,\mathrm{d}t + a\,(t,T)\,\mathrm{d}z_t, \tag{18}$$

under the objective measure Q, where

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$$a = (a_1, \ldots, a_n)' \tag{19}$$

$$a_i(t, T, \omega) = -\int_t^T \sigma_i(t, s, \omega)\mathrm{d}s, i = 1, \ldots, n, \tag{20}$$

$$b(t, T, \omega) = -\int_t^T \alpha(t, s, \omega)\mathrm{d}s + \sum_{i=1}^{n} a_i^2(t, T, \omega). \tag{21}$$

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Factor Models

1. Three basic approaches to Yield Curve Modelling

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   1. **No-arbitrage models**-model the yield curve at one point in time to ensure no arbitrage possibilities exist-pricing derivatives; Hull White (1990), HJM (1992)

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   1. **No-arbitrage models**-model the yield curve at one point in time to ensure no arbitrage possibilities exist-pricing derivatives; Hull White (1990), HJM (1992)
   2. **Equilibrium models**- model the dynamics of the short rate using affine models after which other maturities can be derived- Vasicek(1977), CIR(1985), Duffie Kan (1996).

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1. Three basic approaches to Yield Curve Modelling
   1. **No-arbitrage models**-model the yield curve at one point in time to ensure no arbitrage possibilities exist-pricing derivatives; Hull White (1990), HJM (1992)
   2. **Equilibrium models**- model the dynamics of the short rate using affine models after which other maturities can be derived- Vasicek(1977), CIR(1985), Duffie Kan (1996).
   3. **Factor models**- distill entire yield cuve period by period into a finite dimensional space - typically three-that evolves dynamically- used for forecasting-Nelson Siegel (1987), Litterman and Scheinkman(1991)-level slope curvature - first three principal components of the yield space–forecast the yield curve by forecasting the factors but where do the factors come from- what do they mean?

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Forward Rate Curve:

$$f_t(\tau) = \beta_{1t} + \beta_{2t} e^{-\lambda_t \tau} + \beta_{3t} \lambda_t e^{-\lambda_t \tau}$$

and corresponding
Yield Curve

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right)$$

Diebold and Li (2006) interpret β_{1t}, β_{2t},and β_{3t} as three latent factors; long term, short term and medium term. Also numerical factors representing level-$\beta_{1t} = y_t(\infty)$; slope $\beta_{2t} = y_t(\infty) - y_t(0)$ for us $(y(120) - y(3))$, curvature $2y_t(24) - y_t(3) - y_t(120)$

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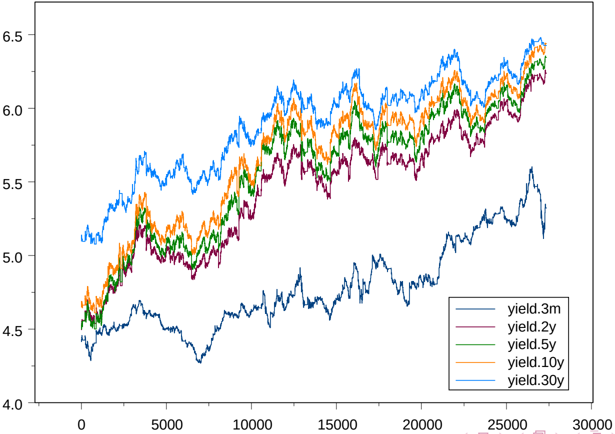
Yields 1999



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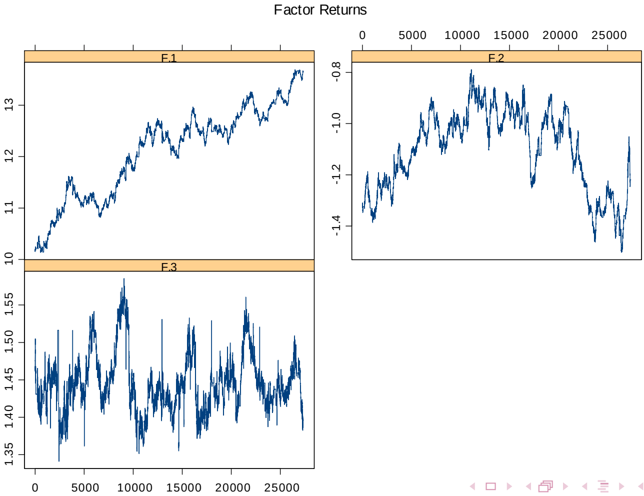
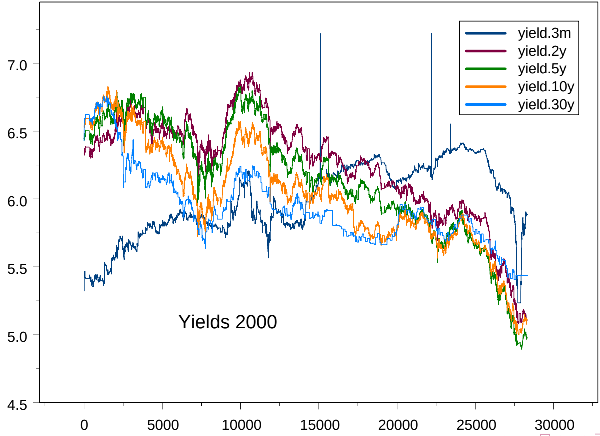
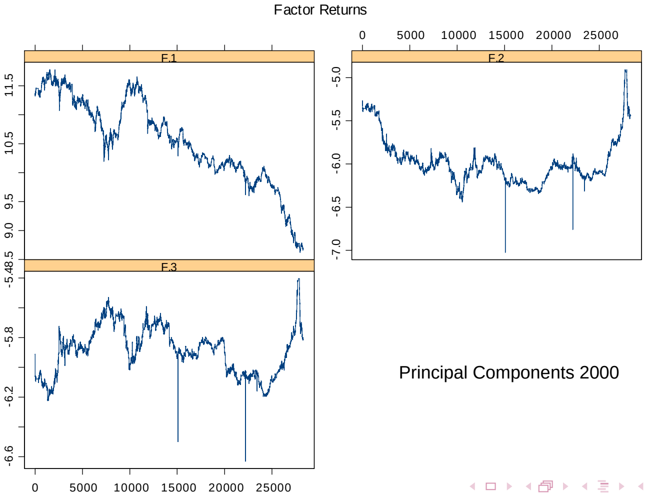
Principal Components 1999



Yields 2000


yield.3m
yield.2y
yield.5y
yield.10y
yield.30y
Yields 2000
7.0
6.5
6.0
5.5
5.0
4.5
0
5000
10000
15000
20000
25000
30000

Principal Components 2000



Principal Components 2000

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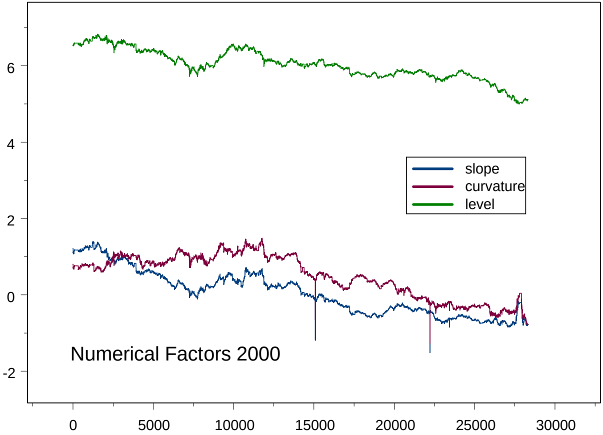
6
4
2
0
-2
0
5000
10000
15000
20000
25000
30000
slope
curvature
level
Numerical Factors 2000

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Can volatility explain standard factors?

dep variable	const	vol3m	vol2yr	vol5yr	vol10yr	vol30yr	R^2
pc1	0.0016 (23.06)	0.0231 (2.61).	0.2328 (11.15)	0.0376 (3.56)	0.0183 (3.62)	0.0224 (4.33)	0.07
pc2	0.001 (9.65)	0.1933 (14.48)	0.0457 (1.45)	0.0431 (2.70)	0.0012 (0.16)	0.0073 (0.92)	0.01
pc3	0.0009 (11.83)	0.1373 (13.84)	0.0171 (0.73)	0.0287 (2.43)	0.0046 (0.80)	0.0161 (2.77)	0.01

of Δprincipal components on the volatilities

No!

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Can volatility explain the Shape of the Yield Curve?

We estimate a modified Nelson-Siegel function following Diebold and Li (2006) using volatility in place of the maturity.

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda vol(\tau)}}{\lambda vol(\tau)}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda vol(\tau)}}{\lambda vol(\tau)} - e^{-\lambda vol(\tau)}\right)$$

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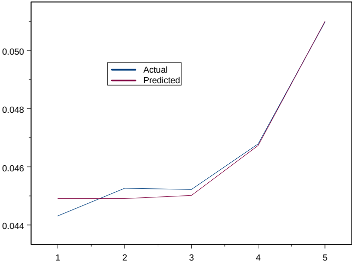
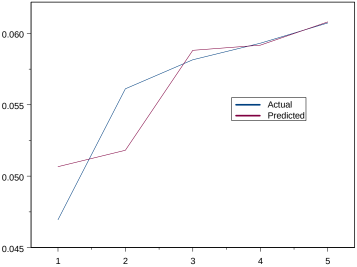
Fitting the Yield Curve with volatility

Yes!





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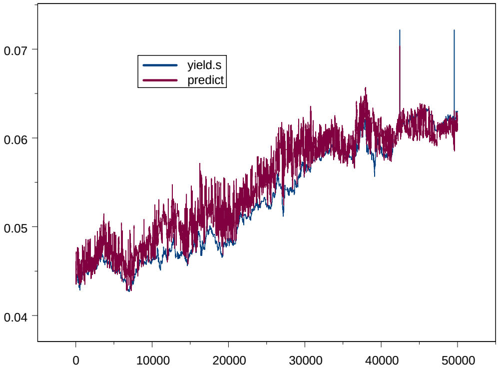
Fit to 3m over 50,000 obs



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Pricing Caps using intensity based volatility

Suppose that

$$\mathrm{d}f_t(T) = \alpha(t, T)\mathrm{d}t + \sum_{i=1}^{n} \sigma_i(t, T)\mathrm{d}z_{i,t},$$

as usual, where $\sigma_i(t, T)$, $i = 1, \ldots, n$, are Gaussian.
Set $a_i(t, T) = -\int_t^T \sigma_i(t, s)\mathrm{d}s$, $i = 1, \ldots, n$.
Pure discount bond prices then follow the process

$$\frac{\mathrm{d}B_t(T)}{B_t(T)} = (r_t + b\,(t, T))\,\mathrm{d}t + a\,(t, T)\,\mathrm{d}z_t, \tag{22}$$

under the objective measure Q, where

$$a = (a_1, \ldots, a_n)' \tag{23}$$

$$a_i(t, T, \omega) = -\int_t^T \sigma_i(t, s, \omega)\mathrm{d}s, i = 1, \ldots, n, \tag{24}$$

When forward rate volatilities are Gaussian it possible to obtain formulae for some simpler instruments. Brenner and Jarrow (1993) and Au and Thurston(1994) showed that there is a standard Black formula for a caplet $c_t(T_1, T_2)$, in order to hedge interest rate risk.

$$c_t(T_1, T_2) = B_t(T_2)N(d) - XB_t(T_1)N(d - w) \tag{26}$$

where

$$d = \frac{1}{\sqrt{w}} \ln\left(\frac{B_t(T_2)}{XB_t(T_1)}\right) + \frac{1}{2}\sqrt{w}, \tag{27}$$

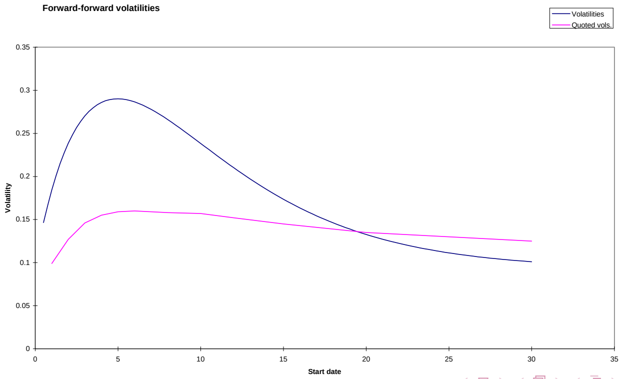
$$w = \sum_{i=1}^{n} \int_t^{T_1} (a_i(u, T_2) - a_i(u, T_1))^2 \, \mathrm{d}u, \tag{28}$$

and the initial forward curve has been fitted to match the market values $B_t(T)$ of PDBs.

Suppose that the bond volatility curve has been fitted by a curve of Nelson and Siegel type so that

$$a(\tau) = \beta_0 + (\beta_1 + \beta_2 \tau) \, e^{-k\tau} \tag{29}$$

where $\tau = T - u$.



Then solving for w gives a closed form expression;

$$
\begin{aligned}
w &= \int_t^{T_1} \left(a_i(u,T_2) - a_i(u,T_1)\right)^2 \mathrm{d}u \\
&= \int_t^{T_1} \Big(\beta_0 + (\beta_1 + \beta_2 (T_2 - u))\, \mathrm{e}^{-k(T_2-u)} - \beta_0 - (\beta_1 + \beta_2 (T_1 - \\
&= \int_t^{T_1} \left(\begin{array}{c} (\beta_1 + \beta_2 (T_2 - u))^2\, \mathrm{e}^{-2k(T_2-u)} \\ + (\beta_1 + \beta_2 (T_1 - u))^2\, \mathrm{e}^{-2k(T_1-u)} \\ -2 (\beta_1 + \beta_2 (T_2 - u)) (\beta_1 + \beta_2 (T_1 - u))\, \mathrm{e}^{-k(T_1+T_2-2u)} \end{array} \right. \\
&= \int_t^{T_1} (\beta_1 + \beta_2 (T_2 - u))^2\, \mathrm{e}^{-2k(T_2-u)} \mathrm{d}u \\
&\quad + \int_t^{T_1} (\beta_1 + \beta_2 (T_1 - u))^2\, \mathrm{e}^{-2k(T_1-u)} \mathrm{d}u \\
&\quad -2 \int_t^{T_1} (\beta_1 + \beta_2 (T_2 - u)) (\beta_1 + \beta_2 (T_1 - u))\, \mathrm{e}^{-k(T_1+T_2-2u)} \mathrm{d}u
\end{aligned}
$$

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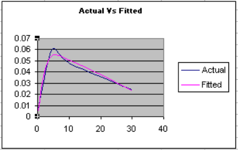
Microsoft Excel - caps price using estimated volatilties

price of a caplet	0.45820999	
p_2	0.9	
Strike price	0.93	
p_1	0.95	
lamda	0.193641	
beta1	-0.02285	
beta2	0.02228	
beta 0	0.021768	
t_1	0.25	
t_2	0.5	
u	0	



3m	2y	5y	10y	30y	maturity	actual volatility	fitted volatility using	errors
0.007094	0.031156	0.062490741	0.039109185	0.025934	0.25	0.007159407	0.005304681	3.44E-06
0.007159	0.031442	0.060833158	0.047878404	0.024205	2	0.031441921	0.036507125	2.57E-05
0.006937	0.030466	0.063111559	0.052283163	0.022601	5	0.060833158	0.055396298	2.96E-05
0.006755	0.029666	0.063691478	0.059616117	0.021113	10	0.047878404	0.050604917	7.43E-06
0.007021	0.030833	0.064463105	0.059207409	0.019734	30	0.024204968	0.023704191	2.51E-07
0.007102	0.031188	0.062559798	0.062696316	0.018456			Squared Error	6.63E-05
0.007039	0.030912	0.068615258	0.065681044	0.017273				

0.709423	3.11557	6.2490741	3.9109185	2.593407
0.715941	3.144192	6.0833158	4.7878404	2.420497
0.69373	3.046649	6.3111559	5.2283163	2.260063
0.6755	2.96659	6.3691478	5.9616117	2.111273
0.702086	3.083346	6.4463105	5.9207409	1.973351
0.710156	3.118789	6.2559798	6.2696316	1.845577
0.703864	3.091154	6.8615258	6.5681044	1.727284

Sheet1 / Calculations / Chart of rates / Chart of Volatilities / Macros

Conclusions

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3. We have also shown how we can use the instantaneous volatility to price caplets to hedge interest rate risk off a 5 minute HJM Yield Curve.