Green Coins


Croce


Guinez


Inzunza Mendez


Nguyen


Tebaldi

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Broad Motivation

Observation #1: Green regulation is becoming more stringent and it fosters new markets

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Question #2: Can MacroFinTech help? Yes, with Green Coins.
↪ **Model:** New benchmark brings us closer to the 'EU' first-best
↪ **Application:** Green coins in the US economy **[TBA]**.

Contribution (so far)

Novel Model:
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Novel Model:

$\hookrightarrow$ **Setting Emission Trading System (ETS):** ‘handy’ DSGE featuring:

1. both emission constraints & ESG-sensitive borrowing constraints;
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$\hookrightarrow$ **Setting Green Coins (GCCB):** ‘handy’ DSGE featuring Green Coin Central Bank.

1. Open Market Operations in the Green Coin market;
2. No more carbon credits.

TERMINOLOGY

Actions about emissions

Limiting: reduce emissions by reducing use of (or improving) brown assets

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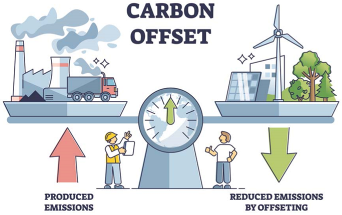
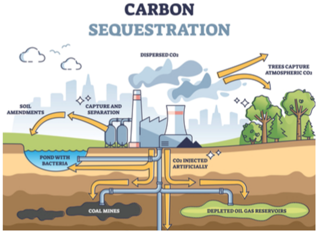
Offsetting and Sequestering:





Definition: Green assets are natural/technological assets (projects) that offset or sequester CO2

New Markets:

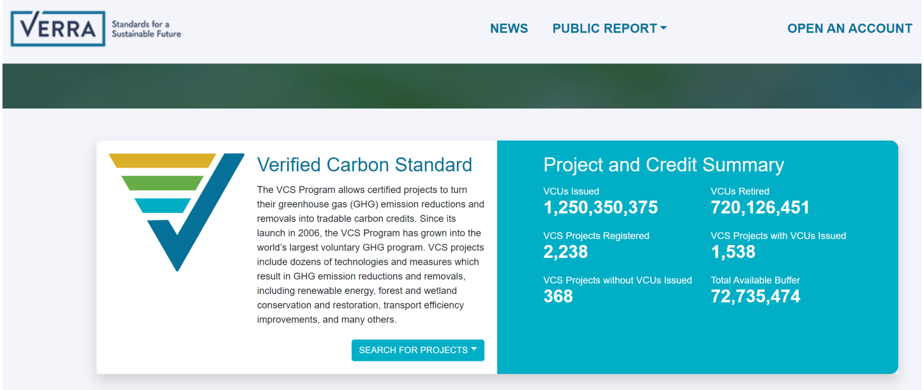
Voluntary/Verified Carbon Credits

Carbon Credits

Definition: Carbon Credits are measurable, verifiable emission reductions from certified projects that represents 1 ton of carbon dioxide removed from the atmosphere. These projects reduce, remove or avoid greenhouse gas (GHG) emissions. Verified by third-party agencies.

Verra I: the registry

Registry: The Verra ledger keeps track of both new and retired projects



VCU: Verified Carbon Unit = 1 CO2e with unique identifier [Example]

Verra II: the Trading Scheme

Registry: The Verra ledger keeps track of holdings:

[...] VCUs are issued to registry account holders listed on the Verra Registry. Registry account holders are companies or organizations which are interested in holding and transacting VCUs, and are typically project developers, carbon credit brokers, or other entities **involved in carbon markets.** Registry account holders must pass strict "Know-Your-Customer" **background checks** prior to opening an account. Individuals may not open a registry account, and **registry account holders are not authorized to hold VCUs on behalf of individuals.**
Ownership of VCUs can only be transferred between Verra Registry accounts. VCUs cannot be **transferred to other databases or traded as paper certificates.**

Who is buying CO2 certificates?

Brown companies have an incentive to freely buy Carbon Credits in order to:

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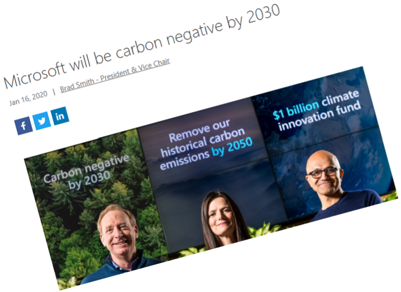

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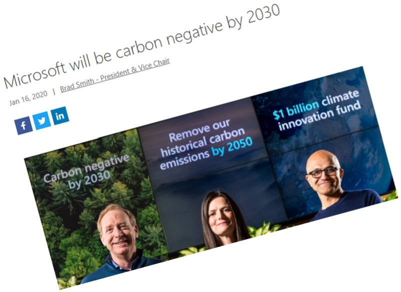

3. UK- , EU- and CA-based companies relax their need of Emission Allowances.

Is this ‘All Good’?

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Delta Air Lines faces lawsuit over $1bn carbon neutrality claim

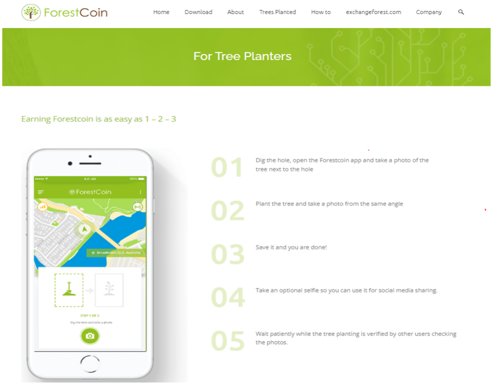
US airline pledged to go carbon neutral but plaintiffs say it is relying on offsets that do almost nothing to mitigate global heating

...

In January, a nine-month investigation by the Guardian, the German weekly Die Zeit and the investigative group SourceMaterial found Verra rainforest credits used by Disney, Shell, Gucci and other big corporations were largely worthless, often based on stopping the destruction of rainforests that were not threatened, according to independent studies. The lawsuit against Delta mentions the investigation. Verra strongly disputed the findings.

– Opaque market;
– EU ETS stricter requirements since 2021.

Can we do better? More on the Supply Side



Private coins: easy to trade, scalable, but opaque

Can we do better? More on the Supply Side (II)


PRESS RELEASE

World's First Commercial CO2 Sensor in Orbit

By GHGSat November 11 2023

- High-resolution instrument will monitor carbon-intensive industrial sites, worldwide
- Accurate, independent satellite CO2 data at the source to be available for the first time
- Supports efficient operations, better emissions reporting and Global Stocktake

Related articles

GHGSat to launch world's first commercial CO2 satellite

Potential solution: GreenTech = IoT + Blockchain (a central monitoring authority still required)

New Markets (II):

EU-ETS

Emissions Trading System

EU-ETS: visit Webpage

Key points:

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4. The registry is Here

A Model for ETS $+$ VCC

The Green Authority: Mission

- The green authority observes declared *Net Emissions*:

$$NE_t = \underbrace{\lambda_t Y_t}_{\text{gross emissions}} - \underbrace{G_t}_{\text{declared offsets}},$$

where λ_t measures CO2 per unit of real GDP.

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- A penalty is applied when net emissions exceed the *Emission Allowances*:

$$C_t^{EA} = \phi_0^{EA} \cdot \exp\left(\phi_1^{EA}\left\{\frac{\lambda_t Y_t - EA_t}{G_t} - 1\right\}\right) \cdot Z_{t-1},$$

which mimics the constraint $NE_t \leq EA_t$.

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which mimics the constraint $NE_t \leq EA_t$.

- The authority follows a rule for the supply of emission allowances (to be auctioned):

$$EA_t^s = \exp(a_t) \cdot Z_t > 0 \quad (1)$$

$$a_t = (1-\rho_a)(\mu_a - \phi_a \cdot (s_{t-1} - s_{ss})) + \rho_a a_{t-1} + \sigma_a \epsilon_{a,t-j} \quad (2)$$

$$s_t = (1-\delta_s)s_{t-1} + (\lambda_t Y_t - \underbrace{\tilde{G}_t}_{\text{true offsets}})/Z_{t-1}. \quad (3)$$

The Green Authority: Budget

- The green authority has a balanced budget:

$$p_t \cdot EA_t + T_t = \tau_g I_{gt} + \tau r_t^B B_{t-1},$$

- p_t price of one EA;
- T_t lump sum transfer;
- τ_g is a subsidy on green investment;
- τr_t^B is the tax shield on corporate debt.

Brown Firm: Problem

$$
\begin{aligned}
V_t &= \max_{B_t, I_t, L_t, G_t, EA_t} D_t + \mathbb{E}_t(M_{t+1} V_{t+1}) \\
D_t &= \underbrace{Y_t - W_t L_t - I_t}_{\text{neoclassical}} \underbrace{-(1-\tau) r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\
&\quad - \underbrace{(p_t \cdot EA_t + C_t^{EA})}_{\text{cost of EA}} - \underbrace{(p_{gt} G_t + C_t^C)}_{\text{cost of VCU}}
\end{aligned}
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Y_t &= (Z_t L_t)^{1-\alpha} K_{t-1}^{\alpha} \\
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C_t^E &= \phi_0 \cdot \exp\left(\phi_1 \left\{1 - \frac{\theta_t K_t}{B_t}\right\}\right) \cdot Z_{t-1} \quad (\text{mimics } B_t \le \theta_t \cdot K_t) \\
C_t^B &= \eta Z_{t-1} \left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}}\right)^2 \\
\theta_t &\equiv \theta\left(\frac{G_t}{\lambda_t Y_t}\right); \quad \theta'(\cdot) > 0
\end{aligned}
$$

Brown Firm: Optimality

- Brown Investment:

$$
\begin{aligned}
q_t &= \frac{1}{\Psi'_t} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_t}\right] - \frac{\partial C^E_t}{\partial K_t} \\
\frac{\partial V_t}{\partial K_{t-1}} &= \underbrace{\frac{\partial Y_t}{\partial K_{t-1}} + q_t\left(1 - \delta - \frac{\Psi'_t I_{bt}}{K_{t-1}} + \Psi_t\right)}_{\text{neoclassical}} - (\underbrace{\frac{\partial C^E_t}{\partial K_{t-1}} + \frac{\partial C^B_t}{\partial K_{t-1}}}_{\text{deviation from MM}}) - \underbrace{\frac{\partial C^{EA}_t}{\partial K_{t-1}}}_{\text{EA distress}}.
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- VCU versus EA prices:

$$p_t = \underbrace{\phi_1^{EA}\cdot\frac{C_t^{EA}}{G_t^{PD}}}_{\text{tightness of EA}}; \qquad p_{gt} = \underbrace{p_t \cdot \frac{\lambda_t Y_t - EA_t}{G_t}}_{\text{savings on EA}} \underbrace{-\frac{\partial C_t^E}{\partial G_t}}_{>0\text{, savings on borrowing}} \underbrace{-\zeta\cdot\frac{G_t-\tilde{G}_t}{Z_{t-1}}}_{<0}$$

Green Firm: Problem

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\begin{aligned}
V_{gt} &= \max_{I_{gt}, L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \\
D_{gt} &= p_{gt} G_t^s - (1 - \tau_{gt}) I_{gt} - W_{gt} L_{gt} \quad \rightarrow \text{Green investment is subsidized.} \\
K_{gt} &= (1 - \delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1}
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\begin{aligned}
\tilde{G}_t &= (Z_{gt} L_{gt})^{1-\kappa} K_{g,t-1}^{\kappa} \quad \rightarrow \text{Reduction/abatement of GHGs depends on green assets} \\
G_t^s &= \xi_t \tilde{G}_t \quad \rightarrow \text{Reported reduction/abatement of GHGs includes `cheating'} \\
\xi_t &= 1 + b_0 \exp(b_1 \epsilon_{\xi,t}) \quad \rightarrow \text{Exogenous process, a constant for now}
\end{aligned}
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Green Firm: Optimality

- Green Investment:

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q_{gt} \quad &:= \quad \frac{1-\tau_{gt}}{H_t'} = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \\
\frac{\partial V_{gt}}{\partial K_{g,t-1}} \quad &= \quad p_{g,t}\frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H_t' \cdot (I_{gt}/K_{g,t-1})\}
\end{aligned}
$$

The Representative Household

$$U_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t [U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{4}$$

where

$$C_t = \tilde{C}_t \cdot \chi(s_{t-1}) \tag{5}$$

and χ_t accounts for climate damages and it is a function of net emissions. In this setting, the SDF is:

$$M_{t+1} = \beta \frac{\chi_t}{\chi_{t-1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left[\mathbb{E}_t \, U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}. \tag{6}$$

A Model with Green Coins

The Green Blockchain

- The blockchain mines I_{gt} new coins & retires depreciated assets $\delta K_{g,t-1}$
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- The owner of a coin:
    1. receives a fraction $1/K_g$ of the green sector payout (like equity)
    2. claims a fraction $1/K_g$ of total offsets, $\tilde{G}$, paying the spot price p_g

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- The private sector is now subject to the following constraint:

$$NE_t = \underbrace{\lambda_t Y_t}_{\text{gross emissions}} - \underbrace{\tilde{G}_t(1 - S_{g,t-1})}_{\text{private offsets}} \leq \underbrace{\bar{A} \cdot Z_{g,t-1}}_{\text{emissions allowed}} .$$

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- The authority follows this rule:

$$S_{gt} = \frac{\exp(sg_t)}{1 + \exp(sg_t)} \in (0,\ 1)$$

where

$$sg_t = A_0 + A_s(s_{t-1} - s_{ss}) + A_x \epsilon_{x,t} + A_z \epsilon_{z,t} + A_\lambda \epsilon_{\lambda,t}.$$

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- The authority net payout is:

$$\Pi_t^{GB} = (V_{gt}^{ex} + D_{gt})S_{gt-1} - (V_{gt}^{ex} S_{gt} + p_{g,t} \cdot \tilde{G}_t \cdot S_{g,t-1} + \omega_g \cdot I_{gt}),$$

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$$V_t = \max_{B_t, I_t, L_t, G_t^{PD}, EA_t} D_t + \mathbb{E}_t(M_{t+1} V_{t+1})$$

$$D_t = \underbrace{Y_t - W_t L_t - I_t}_{\text{neoclassical}} - \underbrace{(1-\tau) r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}}$$

$$- \underbrace{C_t^{EA}}_{\text{Cost of Net Emissions}} - \underbrace{p_{gt} G_t^P}_{\text{cost of offsets}}$$

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$$V_t = \max_{B_t, I_t, L_t, G_t^{PD}, EA_t} D_t + \mathbb{E}_t(M_{t+1} V_{t+1})$$

$$D_t = \underbrace{Y_t - W_t L_t - I_t}_{\text{neoclassical}} - \underbrace{(1-\tau) r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}}$$

$$- \underbrace{C_t^{EA}}_{\text{Cost of Net Emissions}} - \underbrace{p_{gt} G_t^P}_{\text{cost of offsets}}$$

$$C_t^{EA} = \phi_0^{EA} \cdot \exp\left(\phi_1^{EA} \left\{ \frac{(\lambda_t Y_t - \bar{A} Z_t)}{G_t} - 1 \right\}\right) \cdot Z_{t-1}$$

Brown Firm: Problem

$$
\begin{aligned}
V_t &= \max_{B_t, I_t, L_t, G_t^{PD}, EA_t} D_t + \mathbb{E}_t(M_{t+1}V_{t+1}) \\
D_t &= \underbrace{Y_t - W_t L_t - I_t}_{\text{neoclassical}} - \underbrace{(1-\tau)r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\
& \quad - \underbrace{C_t^{EA}}_{\text{Cost of Net Emissions}} \quad - \underbrace{p_{gt} G_t^P}_{\text{cost of offsets}} \\
C_t^{EA} &= \phi_0^{EA} \cdot \exp\left(\phi_1^{EA}\left\{\frac{(\lambda_t Y_t - \bar{A} Z_t)}{G_t} - 1\right\}\right) \cdot Z_{t-1} \\
Y_t &= (Z_t L_t)^{1-\alpha} K_{t-1}^{\alpha} \\
K_t &= (1-\delta)K_{t-1} + \Psi(I_t / K_{t-1})K_{t-1} \\
C_t^E &= \phi_0 \cdot \exp\left(\phi_1\left\{1 - \frac{\theta_t K_t}{B_t}\right\}\right) \cdot Z_{t-1} \quad (\text{mimics } B_t \leq \theta_t \cdot K_t) \\
C_t^B &= \eta Z_{t-1}\left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}}\right)^2 \\
\theta_t &\equiv \theta\left(\frac{G_t}{\lambda_t Y_t}\right); \quad \theta'(\cdot) > 0
\end{aligned}
$$

Brown Firm: Optimality

- Brown Investment:

$$
\begin{aligned}
q_t &= \frac{1}{\Psi'_t} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_t}\right] - \frac{\partial C_t^E}{\partial K_t} \\
\frac{\partial V_t}{\partial K_{t-1}} &= \underbrace{\frac{\partial Y_t}{\partial K_{t-1}} + q_t\left(1-\delta-\frac{\Psi'_t I_{bt}}{K_{t-1}} + \Psi_t\right)}_{\text{neoclassical}} - (\underbrace{\frac{\partial C_t^E}{\partial K_{t-1}} + \frac{\partial C_t^B}{\partial K_{t-1}}}_{\text{deviation from MM}}) - \underbrace{\frac{\partial C_t^{EA}}{\partial K_{t-1}}}_{\text{EA distress}}.
\end{aligned}
$$

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\end{aligned}
$$

- Pricing brown debt:

$$
\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \frac{\tau \cdot r_{f,t}}{1 + r_{f,t}}.
$$

Brown Firm: Optimality

- Brown Investment:

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$$\frac{\partial V_t}{\partial K_{t-1}} = \underbrace{\frac{\partial Y_t}{\partial K_{t-1}} + q_t\left(1-\delta-\frac{\Psi'_t I_{bt}}{K_{t-1}}+\Psi_t\right)}_{\text{neoclassical}} - \underbrace{(\frac{\partial C_t^E}{\partial K_{t-1}} + \frac{\partial C_t^B}{\partial K_{t-1}})}_{\text{deviation from MM}} - \underbrace{\frac{\partial C_t^{EA}}{\partial K_{t-1}}}_{\text{EA distress}}.$$

- Pricing brown debt:

$$\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \frac{\tau \cdot r_{f,t}}{1+r_{f,t}}.$$

- GC versus EA prices:

$$p_{gt} = \underbrace{\phi_1^{EA} \cdot \frac{C_t^{EA}}{G_t^{PD}} \cdot \frac{(\lambda_t Y_t - \bar{A}Z_t)}{G_t^{PD}}}_{\text{tightness}} + \underbrace{\phi_1 \cdot \theta'_t \cdot \frac{C_t^E}{B_t} \cdot \frac{K_t}{\lambda_t Y_t}}_{\text{savings on borrowing}}$$

Green Firm: Problem

$$
\begin{aligned}
V_{gt} &= \max_{I_{gt}, L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \\
D_{gt} &= p_{gt} G_t^s - (1 - \tau_{gt}) I_{gt} - W_{gt} L_{gt} \quad \rightarrow \text{Green investment is subsidized.} \\
K_{gt} &= (1 - \delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1}
\end{aligned}
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K_{gt} &= (1 - \delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1}
\end{aligned}
$$

$$
\begin{aligned}
G_t^s &= \tilde{G}_t \quad \rightarrow \text{Verified reduction/abatement of GHGs} \\
\tilde{G}_t &= (Z_{gt} L_{gt})^{1-\kappa} K_{g,t-1}^{\kappa} \quad \rightarrow \text{Reduction/abatement of GHGs depends on green assets}
\end{aligned}
$$

Green Firm: Optimality

- Green Investment:

$$
\begin{aligned}
q_{gt} \quad &:= \quad \frac{1-\tau_{gt}}{H_t'} = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \\
\frac{\partial V_{gt}}{\partial K_{g,t-1}} \quad &= \quad p_{g,t}\frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H_t' \cdot (I_{gt}/K_{g,t-1})\}
\end{aligned}
$$

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- Green Investment:

$$
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\frac{\partial V_{gt}}{\partial K_{g,t-1}} \quad &= \quad p_{g,t}\frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H_t' \cdot (I_{gt}/K_{g,t-1})\}
\end{aligned}
$$

Key point: Green Coins are backed by green assets $\rightarrow$ their market value $q_{gt} =$ EPV(offsets flow)

The Representative Household & Government

$$U_t = \left[(1-\beta) \left(\tilde{C}_t \cdot \chi\left(s_{t-1}\right) \right)^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t [U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

The Representative Household & Government

$$U_t = \left[(1-\beta) \left(\tilde{C}_t \cdot \chi(s_{t-1}) \right)^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

The budget constraint of the household is:

$$\tilde{C}_t + \underbrace{V_{gt}^{ex} \cdot (1 - S_{g,t}) + V_t^{ex} + B_t}_{\text{privately owned green and brown assets}} = W_t L_t + W_{gt} L_{gt} + \underbrace{V_{gt} \cdot (1 - S_{g,t-1}) + V_t + B_{t-1}(1 + r_{ft-1})}_{\text{cum-dividend/interest brown \& green assets}} + \Pi_t^{GB} - T_t$$

The Representative Household & Government

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The fiscal authority budget is balanced,

$$T_t = \tau_{gt} I_{gt} + \tau r_t^B B_{t-1},$$

The Representative Household & Government

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The budget constraint of the household is:

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The fiscal authority budget is balanced,

$$T_t = \tau_{gt} I_{gt} + \tau r_t^B B_{t-1},$$

and the implied resource constraint is:

$$Y_t = \tilde{C}_t + (1 + \omega_g) I_{gt} + I_t + C_t^E + C_t^B + C_t^{EA}$$

Preliminary Results

Calibration

Table 1: Calibration

Parameter	Value	Parameter	Value
Preferences		***Climate***	
Discount factor (β)	0.97	GHG depletion rate % (δ_s)	0.84
Risk aversion (γ)	10.00	Mean of log emission rate (μ_λ)	-1.86
IES (ψ)	1.77	Persistent of log emission rate (ρ_λ)	0.28
Damage function parameters % (χ_0)	0.11	Log emission rate volatility (σ_λ)	0.02
Damage function parameters (χ_1)	0.69	***VCCs***	
		Reporting gap avg. (b_0)	2.20
		Reporting gap volatility (b_1)	0.50
		Reporting costs intensity (ζ)	15.44
Technology (Brown sector)		***Technology (Green sector)***	
Capital share (α)	0.35	Capital share (κ)	0.60
Capital depreciation rate % (δ)	8.40	Capital depreciation rate (δ_g)	0.05
Elasticity of investment adj. costs (ω)	1.94	Elasticity of investment adj. costs (ν)	247.00
Intensity of debt adjustment costs (η)	0.40	Intensity of emission violation costs (ϕ_1^{EA})	20.00
Debt-to-book avg (θ_0)	0.50	Emission violation costs parameter % (ϕ_0^{EA})	0.04
Productivity		***ETS Authority***	
Average productivity growth (μ_z)	0.01	Brown-sector debt interest tax deductible (τ)	0.20
Short-run productivity volatility (σ_z)	0.03	Green-sector investment subsidy (τ_g)	0.30
Long-run productivity persistence (ρ_x)	0.94	Mean emission allowances (μ_a)	-1.92
Long-run productivity volatility % (σ_x)	0.12	Persistent of emission allowances (ρ_a)	0.27
Long-run productivity exposure	0.99	Allowance shock volatility (σ_a)	0.12
to GHG emissions % (ϕ_s)		Allowance sensitivity to GHG emissions % (ϕ_a)	8.71
Cointegration parameter (ϕ_g)	0.80	Intensity of distress costs (ϕ_1)	20.00

Notes: This table reports our benchmark annual calibration. See section 3.2 for a detailed discussion.

Take away: RBC + IAM + AP = major calibration effort

Data Source

EuroStat: macrodata for EA19
Bloomberg: Global Emission Offsets futures; (NEF) green investment by country.
Haver Analytics: futures on EA emission allowances
UNECE: Emissions as percent of GDP by country
Clean Investment Monitor: additional data on green investment
Goldstandard Registry: data on VCC
IRENA and IEA: green investments in energy sector by International Renewable Energy Agency and the International Energy Agency

Moments

Table 2: Simulated Moments

	Data	SEs	Model
Standard Moments			
$\mathbb{E}(\Delta\tilde{c})$	0.010	0.004	0.010
$\sigma(\Delta\tilde{c})$	0.022	0.006	0.016
$ACF1(\Delta\tilde{c})$	0.026	0.225	-0.079
$\sigma(\Delta\tilde{c})/\sigma(\Delta gdp)$	0.729	0.118	0.885
$\sigma(\Delta i)/\sigma(\Delta gdp)$	1.983	0.279	1.752
$\mathbb{E}(\tilde{C})/E(GDP)$	0.715	0.004	0.692
$\mathbb{E}(r_f)$	1.500		1.111
$\mathbb{E}(r^B - r_f)$	5.000		6.311
$\mathbb{E}(r^G - r_f)$	2.000		2.017
Greenium	3.000		4.294
Emissions, ETS, and VCC prices			
$\mathbb{E}(p)/\mathbb{E}(p_g)$	4.148	1.871	4.685
$\mathbb{E}(\log(\lambda))$	-1.981	0.071	-1.864
$\sigma(\log(\lambda))$	0.022	0.002	0.021
$ACF1(\log(\lambda))$	0.287	0.166	0.221
$\mathbb{E}(a)$	-1.922	0.120	-1.922
$\sigma(a)$	0.121	0.021	0.117
$ACF1(a)$	0.268	0.197	0.200

Take away: good replication of the data.

Welfare (I)

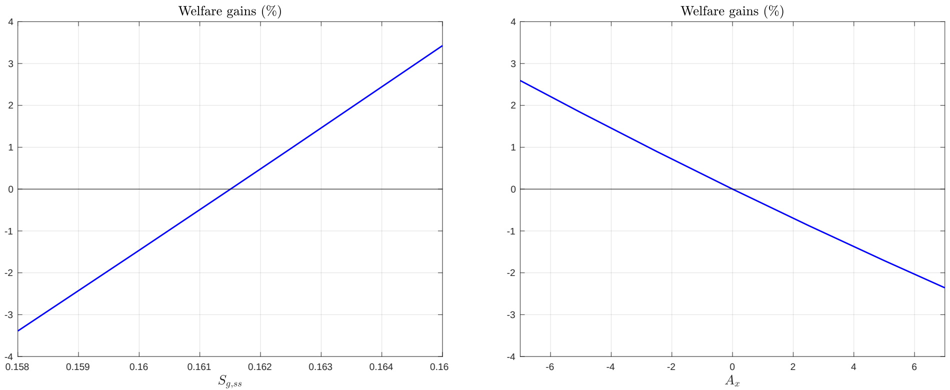

Figure: Welfare and Policy Parameters

Take away: commitment to tight policy ($S_{g,ss} \uparrow$)and accommodating long-run shocks ($A_x \downarrow$) helps!

Welfare (II)

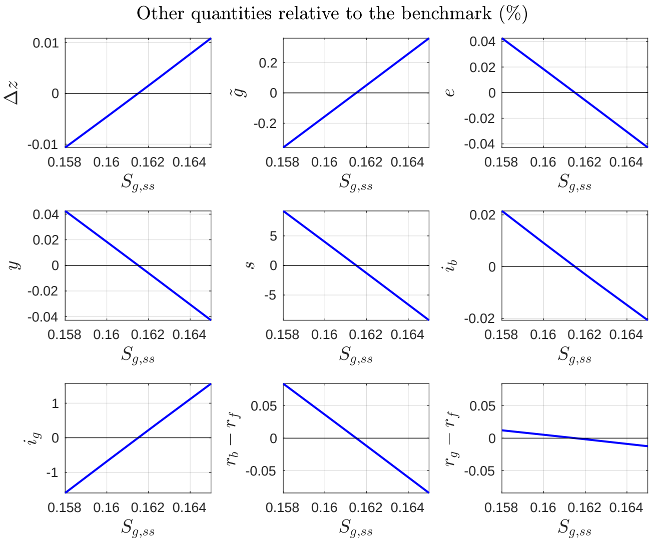

Figure: Steady State and Policy Parameter

Take away: commitment to tight policy ($S_{g,ss} \uparrow$) helps the green sector!

Conclusions

- We are scratching only the surface ...
  ... a lot of work to be done!

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- A new *MacroFinance* model to think of emissions management and ETS

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- We are scratching only the surface ...
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- A new *MacroFinance* model to think of emissions management and ETS

- A new *MacroFinTech* framework to improve welfare: green coins.