A Novel Portfolio Optimization Framework for Online Loan Investments and Interest Rate Determination

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Online Lending

- One of the most successful examples of fintech
- Alternative to banks in obtaining unsecured personal loans
    - Platforms allow individuals to lend and borrow from one another
    - Borrowers: Greater access to credit and lower interest rates
    - Lenders: A novel investment opportunity
- Rapid growth: $112.9 billion in 2021
    - Stock - $100 trillion; Bond - $119 trillion
- Challenges to continued growth: Risk of losing money, stringent government regulations, and low awareness of the benefits

Research Questions

- Demand side of loans
  - How to build a portfolio of online loans?
  - Do online loans offer attractive returns compared to traditional assets?
- A large literature focuses on credit risk prediction
  - Logistic regression, SVM (Serrano-Cinca et al., 2015), XGBoost (Fu et al., 2021)
  - Form a portfolio of loans with the lowest predicted risk
- A separate literature applies existing portfolio methods to loans
  - Guo et al. (2016) apply mean-variance optimization, ignores covariances
  - Byanjankar et al. (2021): Robust MV

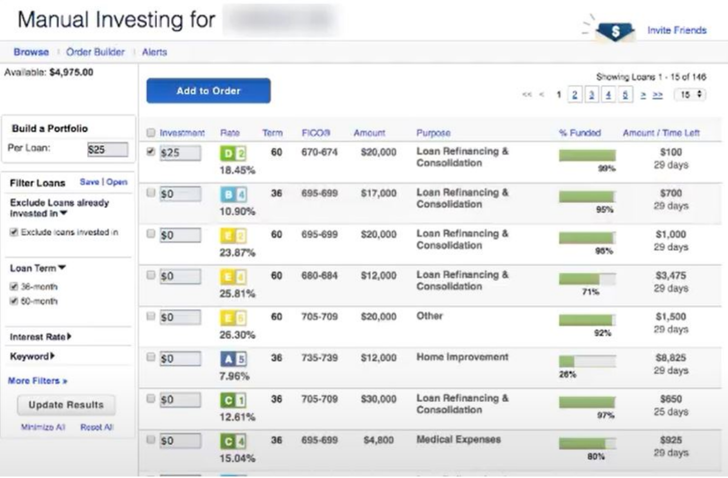
This Paper

- General characteristics-based portfolio policies (GCPP)
  - Portfolio weights directly from characteristics
  - Outperforms an equal-weight portfolio of all loans, annualize internal rate of return of 13.08% vs. 6.55%
  - Outperforms alternative methods from the literature including credit risk filtering and mean variance
- Online loans expand the investor's opportunity set
  - Competitive rates of return with stocks and bonds, with low correlation
- Additional implications for platforms
  - Restrictions on investment amount
  - Bias in interest rates
  - Credit access of high-risk borrowers

Data

- 1,158,476 loans from LendingClub between January 2013 and May 2020
- A total of 83 loan and borrower characteristics
  - Loan characteristics (20): Loan amount, interest rate, grade, status, total payment, last payment date, etc.
  - Borrower characteristics (63): Gross income, location, number of months since last delinquency, debt-to-income ratio, etc.
- After pre-processing, a total of 144 features
  - One-hot encoding, indicator variables for missing values, etc.

LendingClub Loans



Performance Measures

Consider a sequence of cash flows for a loan portfolio: $P_t, t \in \{1, \ldots, T\}$, initial investment P_0

- Internal rate of return (IRR)

$$0 = NPV = \sum_{t=1}^{T} \frac{P_t}{(1 + IRR)^t} - P_0$$

- Return on investment (ROI)

$$ROI = \frac{CDP - P_0}{P_0}, CDP = \frac{P_1}{(1 + \frac{d}{12})^1} + \cdots + \frac{P_T}{(1 + \frac{d}{12})^T}$$

where d is the discount rate, representing the time value of money

Building a Portfolio of Loans

- Key challenge to loan portfolios: Expected returns and covariance matrix are difficult to estimate
- GCPP: Portfolio weights as a function of characteristics
- N available loans, a direct mapping from characteristics to the *goodness* of a loan $g(\tilde{x};\theta)$
  - $\tilde{x}$ is the a vector of standardized characteristics
  - θ is a vector of parameters
- Portfolio weight for loan i

$$\omega_i = \frac{g(\tilde{x}_i;\theta)}{\sum_{j=1}^{N} g(\tilde{x}_j;\theta)}$$

- g could be linear or nonlinear

Portfolio Optimization

$$\sup_{\theta \in \Theta} \mathbb{E}\left[u(r_\omega)\right] = \sup_{\theta \in \Theta} \mathbb{E}\left[u\left(\sum_{i=1}^{N} \omega_i \cdot r_i\right)\right]$$

where r_i is the ROI of loan i

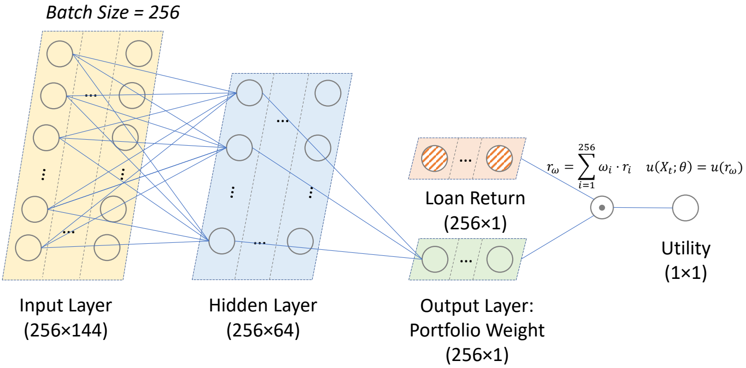
- The investor chooses portfolio weights to maximize expected utility under constant relative risk aversion (CRRA)

$$u(r_\omega) = \frac{(1 + r_\omega)^{1-\gamma}}{1-\gamma}$$

  where γ is the risk-aversion coefficient

Nonlinear Policy Specification

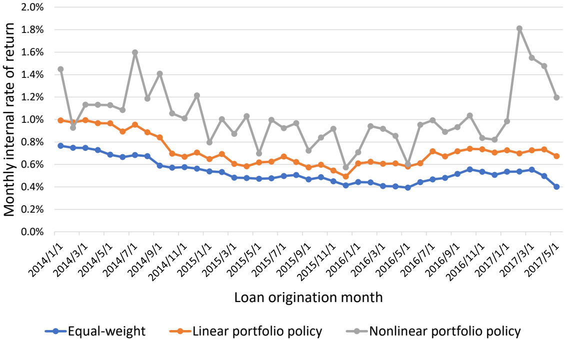
- A neural network with a single hidden layer of 64 nodes
- Activation function: Sigmoid function
- Mini-batch optimization with 256 loans
- Expanding window, out-of-sample testing



Performance vs. EW Benchmark

The nonlinear GCPP outperforms various benchmarks

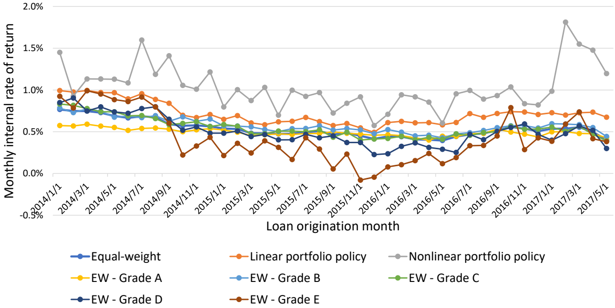
- An equal-weight portfolio is difficult to beat with portfolio optimization methods (DeMiguel et al., 2009)
- EW is the only allocation option available on LendingClub's automated investing tool



Performance vs. Automated Investing

The nonlinear GCPP outperforms various benchmarks

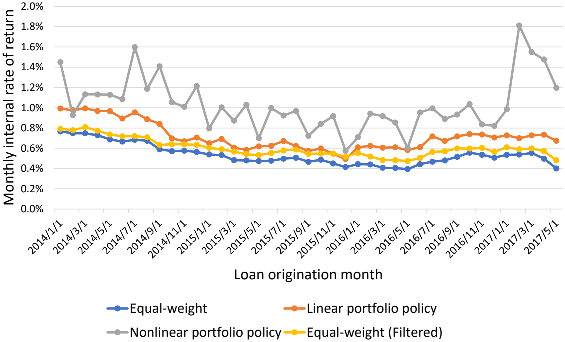
- LendingClub's automated investing tool is equal-weight
- Investors can select specific loan grades to invest in



Performance vs. Credit Risk Filtering

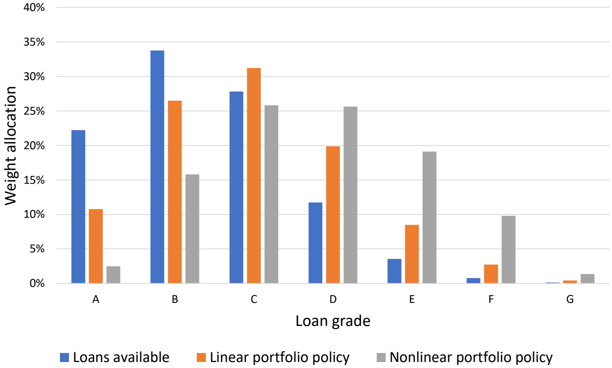
The nonlinear GCPP outperforms various benchmarks

- XGBoost (Chen and Guestrin, 2016) to predict charged-off probability, EW portfolio of loans with predicted charged-off rate < 0.15
- Nonlinear GCPP also outperforms mean-variance optimization



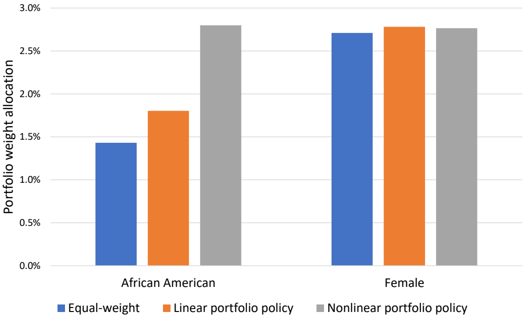
Portfolio Composition

- Mismatch of risk and interest rates
- Model advocates for greater allocation to credit-constrained borrowers



Bias in Interest Rate Assignment

- African-American borrowers tend to receive higher interest rates than their risk levels
- No evidence for bias against female borrowers



Investment Opportunity Set

- Comparison with stocks, bonds, and real estate
  - S&P 500 Index, Bloomberg U.S. Aggregate Bond Index, S&P 1-3/3-5/10-20 Year U.S. Treasury Bond Indexes, and MSCI U.S. REIT Index
- Public market equivalent (PME) based on portfolio cash flows

$$PME = \frac{IRR_{\text{loan}}}{IRR_{\text{benchmark}}}$$

Online Loans Versus Stocks, Bonds, and Real Estate

- High rates of return and low correlations with other asset classes

Loan strategy	PME	Corr.	Pr(win)	PME	Corr.	Pr(win)
	A: S&P 500 Index			D: 3-5 Year T-Bond Index		
Equal-weight	0.93	-0.32	44%	3.38	0.49	100%
Linear	1.49	-0.12	49%	4.64	0.26	100%
Nonlinear	1.93	-0.21	66%	6.00	0.13	100%
	B: Aggregate Bond Index			E: 10-20 Year T-Bond Index		
Equal-weight	2.41	0.44	100%	1.51	0.46	100%
Linear	3.56	0.31	100%	2.14	0.58	100%
Nonlinear	4.84	0.14	100%	2.81	0.24	100%
	C: 1-3 Year T-Bond Index			F: REIT Index		
Equal-weight	8.79	0.44	100%	2.18	0.48	76%
Linear	13.47	0.26	100%	2.49	0.46	88%
Nonlinear	17.99	0.08	100%	2.82	0.18	95%

Interest Rate Determination

- Portfolio construction: (interest rates, chars) $\rightarrow$ goodness
  - If loans are efficiently priced, the goodness of loans is equivalent
  - GCPP would prescribe an equal-weight portfolio
- Interest rate setting: (goodness, chars) $\rightarrow$ interest rates

$$r_i = g^{-1}(g^* | x_i)$$

where g^* is a fixed goodness value, same for all loans

Conclusion

- GCPP is a novel framework especially suitable for online loans
    - Difficult to apply traditional portfolio optimization techniques
    - The nonlinear GCPP outperforms various benchmarks and alternatives from the literature
- Online loans can expand the investor's opportunity set
    - Competitive rates of return compared to stocks, bonds, and real estate
    - Limited comovement offers diversification benefits

Appendix

The Goodness Function

- Truncated linear portfolio policy

$$g_L(\hat{x}_i; \theta) = \max(0, \bar{\omega}_i + \frac{1}{N}\theta^\top \hat{x}_i)$$

  where $\bar{w}_i$ is the portfolio weight of a benchmark portfolio, such as an equal-weight portfolio

- Nonlinear portfolio policy: Neural networks
  - Richer interactions and more complex transformations of features
  - Enlarged portfolio weight space

Binary Search Method

- With an initial amount \$M, the truncated portfolio weight is

$$\tilde{\omega}_i(M) = \begin{cases} 0 & \text{if } \omega_i * M < 25 \\ U_i/M & \text{if } \omega_i * M > U_i \\ w_i & \text{otherwise} \end{cases}$$

  where ω_i is the portfolio weight given by the nonlinear portfolio and U_i is the loan amount

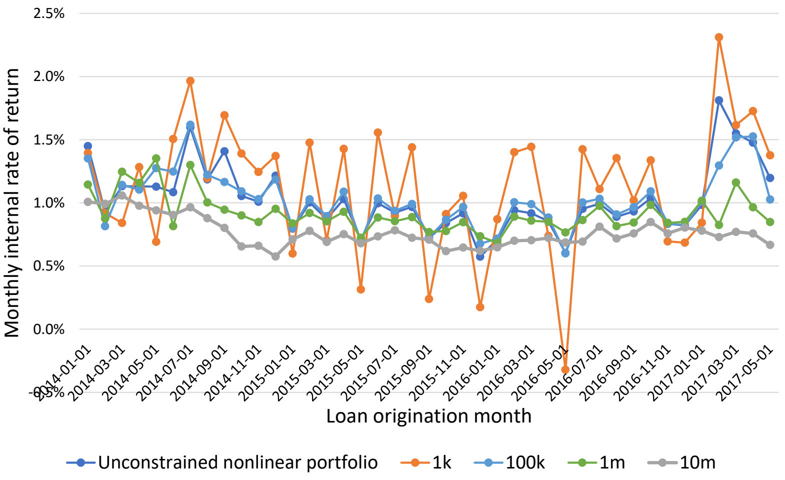
- The investment amount of the truncated portfolio

$$f(M) = \left(\sum_{i=1}^{N} \tilde{\omega}_i(M) \right) \times M$$

  - $f(M)$ is a monotone increasing function of M

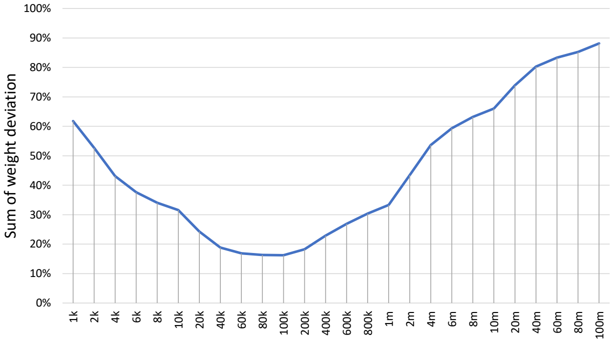
- If want to invest $\$F$, search for M such that $|f(M) - F| < 25$

Portfolio Performance with Constraints



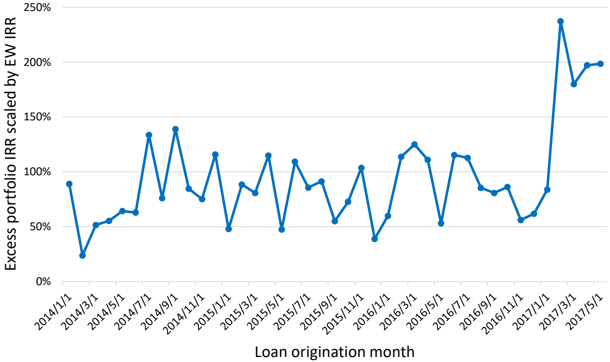
Sum of Weight Deviation

- Sum of Weight Deviation $= \frac{1}{2}\sum_{i=1}^{N}|\omega_i - \omega_{i,b}|$
- The smallest and largest investment amounts have the largest weight deviation from the unconstrained portfolio
- The \$100,000 portfolio deviates the least



Relative Portfolio Performance

Scaled excess portfolio return relative to the EW portfolio: $\hat{r}_\omega = \frac{r_\omega - r_{ew}}{r_{ew}}$



Features Importance

- Interest rate ($+$): Risk-return mismatch
  - Fundamentally different from the credit risk filtering approach
- Payback pressure ($-$): Ratio of loan payment to monthly income
- Borrowers who own their primary residence ($+$)
- Purposes suggesting poor record of personal finance ($-$), such as debt consolidation and credit card
- Purposes related to family, renewable energy, and small business ($+$)