Liquidity fragmentation on decentralized exchanges

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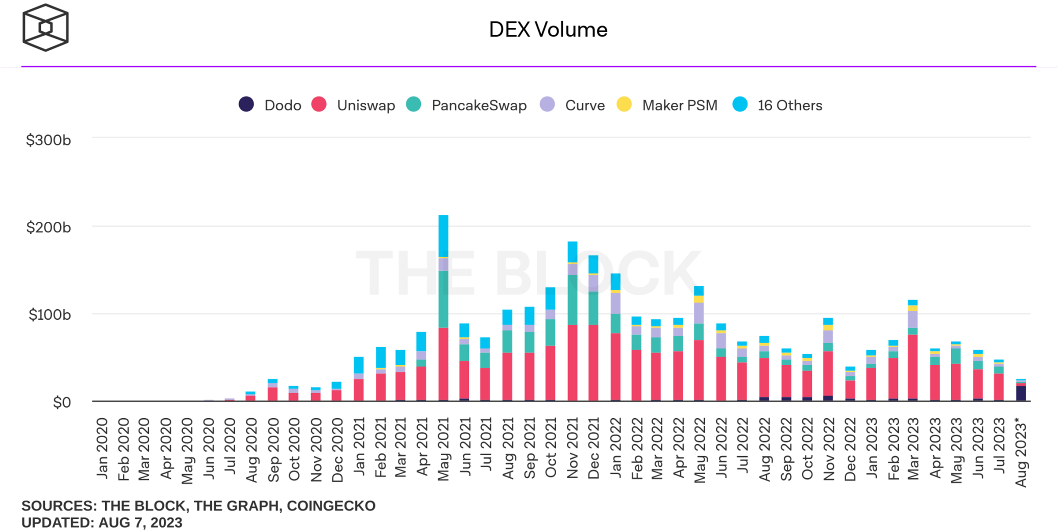
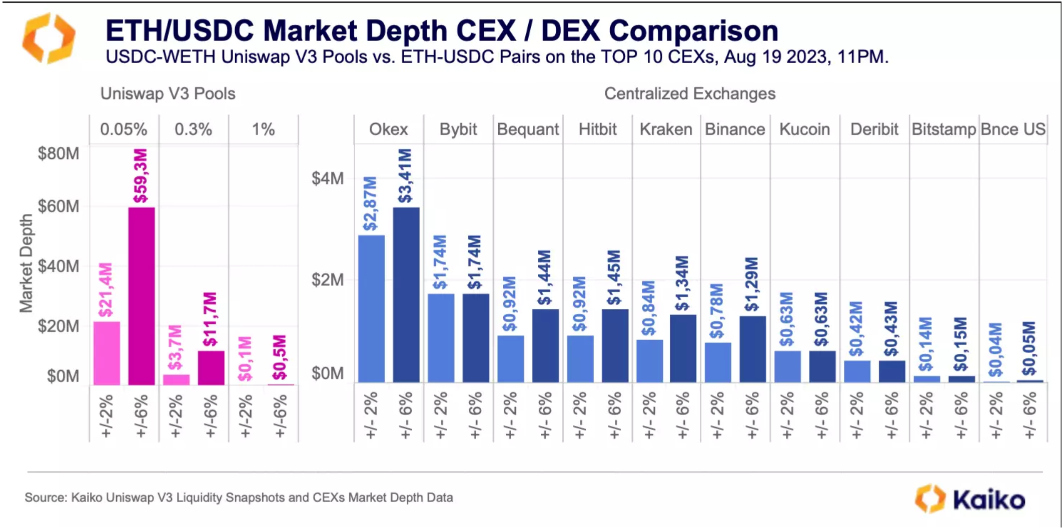
October 2023

Decentralized exchanges (DEX) trade over US$100bn each month



DEX have substantial liquidity (at least in major pairs)

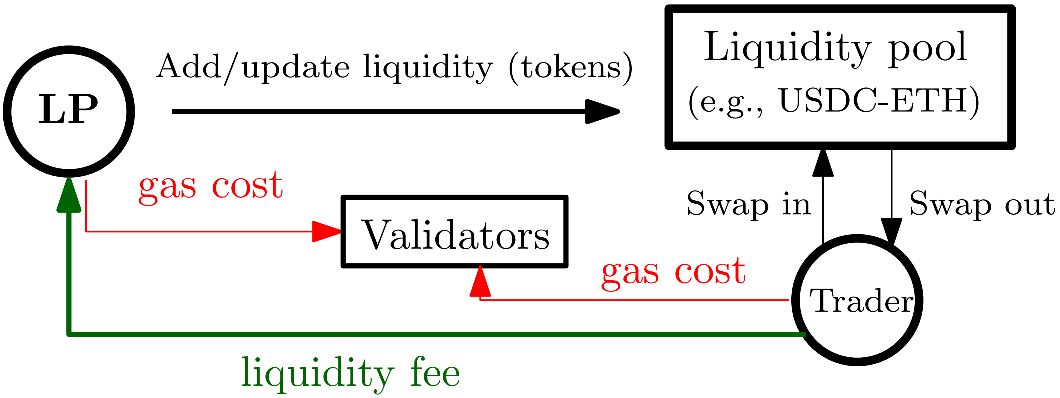


Why is this interesting?

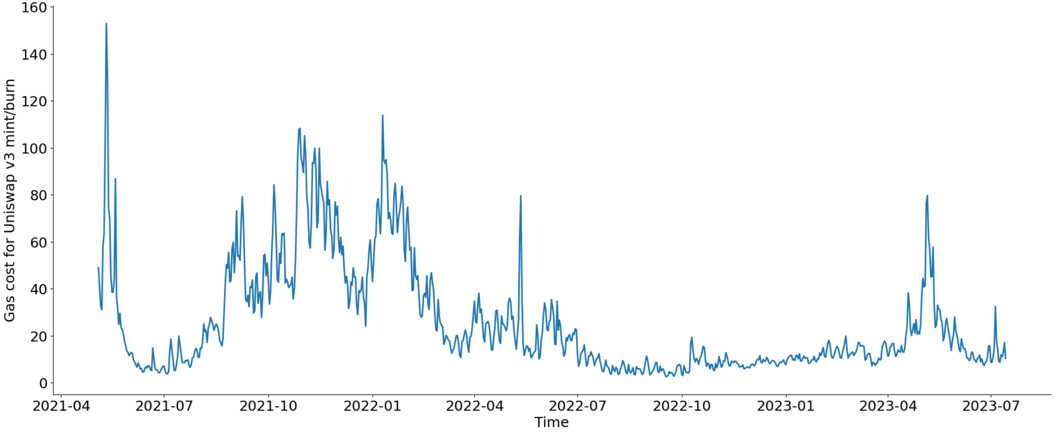
Demsetz (1968): "the question that is relevant for efficiency is whether or not the cost is appropriately economized."

1. Unique laboratory to study how transaction costs affect the market for liquidity.
2. DEX designed for **passive** liquidity provision.
3. On v3 actively managing liquidity is costly:
    3.1 *gas price* from interacting with Ethereum blockchain.
    3.2 *time/effort* to monitoring the position.

Managing liquidity on DEX is costly

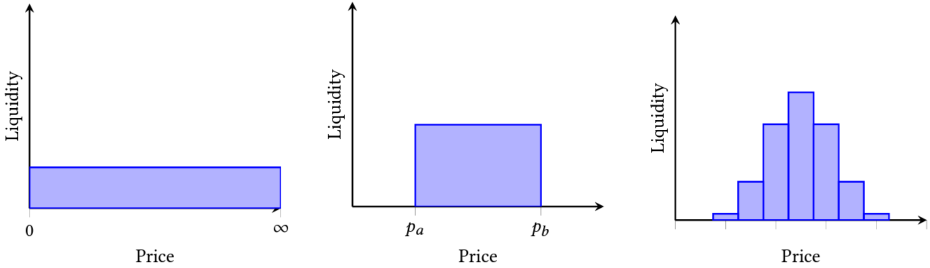


Fixed cost of supplying liquidity (gas fee) on Uniswap v3



Why actively manage liquidity?

- On Uniswap v3, liquidity providers can specify price limits on their positions.
- If the current pool price (e.g., “midpoint”) is outside the range, the position does not earn fees.
- $\rightarrow$ incentive to re-price the position to capture fees.



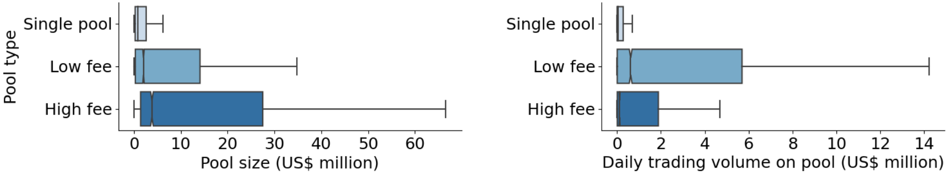
(I) Uniswap v2 **(II)** A single position on $[p_a, p_b]$ **(III)** A collection of custom positions

Uniswap v3 pairs can be traded in 1, 5, 30, or 100 bps fee pools

Finding The Right Pool Fee

We anticipate that certain types of assets will gravitate towards specific fee tiers, based on where the incentives for both swappers and liquidity providers come nearest to alignment.

1. Significant fragmentation across different-fee pools for the same pair.
2. Low-fee pools are more actively traded, but high-fee pools are deeper.



3. We show that fixed transaction costs partly drive this effect.

Results

We find evidence of LP "clienteles" based on their scale:

1. Small LPs are more passive and trade-off lower fill rates for lower fixed costs.
2. Small (large) LPs dominate high- (low-) fee pools for the same pair.

Related literature

We contribute to:

- a growing literature on decentralized exchanges (Lehar and Parlour, 2021; Caparros, Chaudhary, and Klein, 2023; Augustin, Chen-Zhang, and Shin, 2022; Malinova and Park, 2023; Capponi, Jia, and Yu, 2023; Capponi and Jia, 2021; Barbon and Ranaldo, 2021; Hasbrouck, Rivera, and Saleh, 2022).
- the literature on optimal routing for retail orders (Battalio, Corwin, and Jennings, 2016; Cimon, 2021; Foucault and Menkveld, 2008).
- the literature on the role of tick sizes on liquidity provision (Foucault, Kadan, and Kandel, 2013; Yao and Ye, 2018; Li, Wang, and Ye, 2021)

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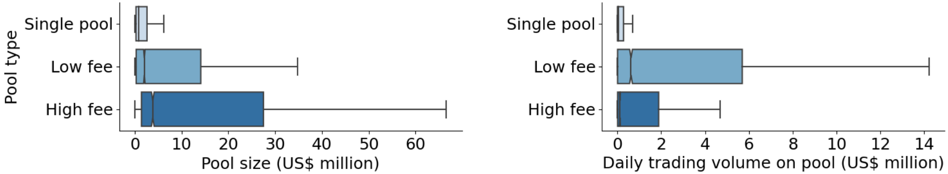


3. We show that fixed transaction costs partly drive this effect.

Model

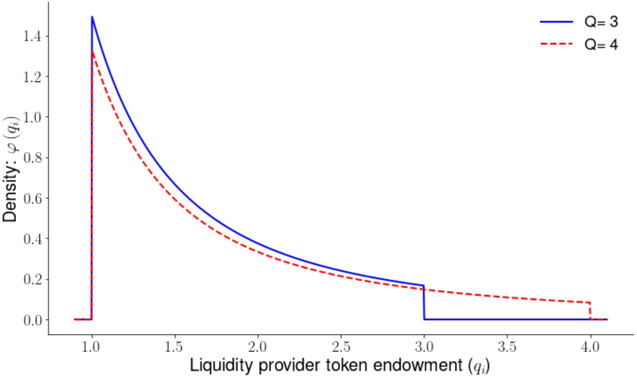
Asset and markets.

- Token with expected value v trades on two liquidity pools with fees $h > \ell > 0$.
- Fixed cost $\Gamma > 0$ of interacting with the pool (e.g., gas fee).

Liquidity providers (**LP**)

- Risk-neutral;
- Token endowments q_i;
- q_i follows a bounded Pareto distribution:

$$\varphi(q) = \frac{Q}{Q-1}\frac{1}{q^2}.$$

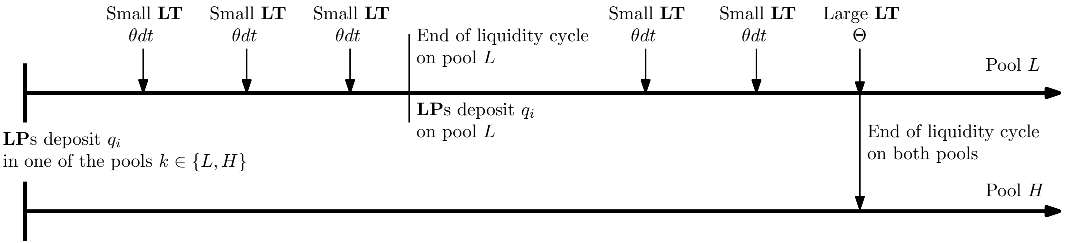


Model

Liquidity takers (**LT**).

Two types of **LT**:

1. *small* **LT** arrive at constant rate $\theta\,\mathrm{d}t$ and optimally go to the low-fee pool first (ℓ).
2. *large* **LT** demand Θ token units and arrive as Poisson process $J_t\ (\lambda)$.
   They are exogenously large enough to consume all liquidity on ℓ and h pools.



The liquidity provider problem

- Liquidity providers choose pool $k^\star$ to maximize expected profit per unit of time:

$$k^\star(q_i) = \arg \max_{\{\ell,h,\emptyset\}} \max\left[\frac{q_i\ell - \Gamma}{d_\ell}, \frac{q_i h - \Gamma}{d_h}, 0\right].$$

- d_k is the endogenous liquidity cycle duration, which $\nearrow$ in aggregate liquidity:

$$d_L = \frac{1}{\lambda} - \frac{1}{\lambda}\exp\left(-\frac{\mathcal{L}_\ell}{\theta}\lambda\right) \text{ and } d_H = \frac{1}{\lambda},$$

where $\mathcal{L}_\ell = \int_{i\in\Omega_L} q_i\varphi(q_i)$ is the aggregate liquidity on the low-fee pool.

- Trade-off between:
  1. higher liquidity fee per unit of time in low fee pools, and
  2. lower rebalancing cost in high-fee pools.

Equilibrium

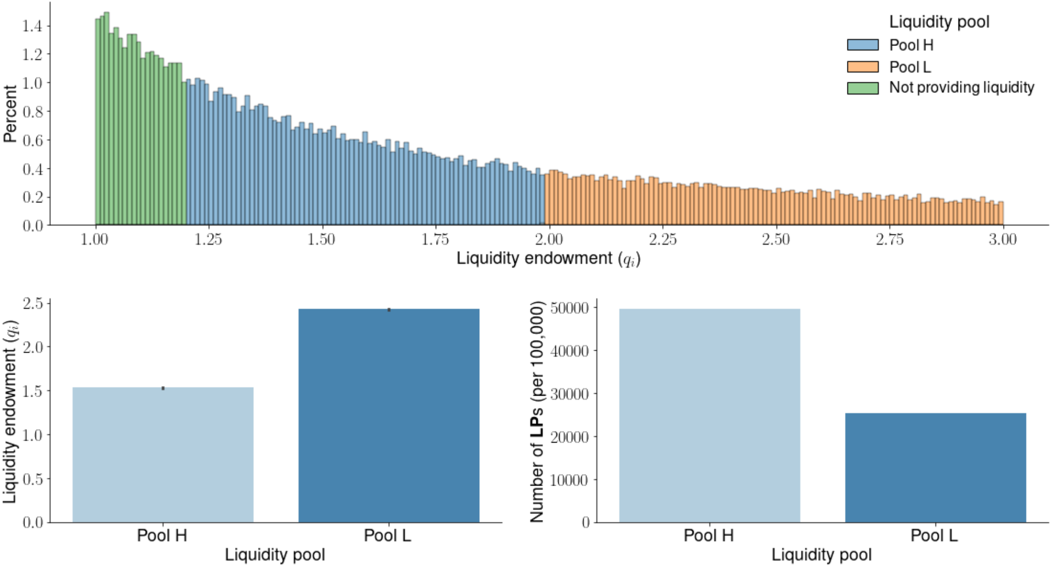
- We show there is a threshold endowment q_t such that all **LP**s with $q_i > q_t$ post liquidity on the low-fee pool and all **LP**s with $q_i \leq q_t$ choose the high-fee pool.

$$\mathcal{L}_\ell = \int_{q_t}^{Q} q_i \varphi(q_i)\,\mathrm{d}i = \frac{Q}{Q-1}\left(\log Q - \log q_t\right) \text{ and}$$
$$\mathcal{L}_h = \int_{\underline{q}}^{q_t} q_i \varphi(q_i)\,\mathrm{d}i = \frac{Q}{Q-1}\left(\log q_t - \log \underline{q}\right)$$

- The threshold **LP**'s endowment solves:

$$q_t - \Gamma \frac{q_t^{\frac{\lambda}{\theta}\frac{Q}{Q-1}} \times Q^{-\frac{\lambda}{\theta}\frac{Q}{Q-1}}}{h\left[q_t^{\frac{\lambda}{\theta}\frac{Q}{Q-1}} \times Q^{-\frac{\lambda}{\theta}\frac{Q}{Q-1}}\right] - (h-\ell)} = 0$$

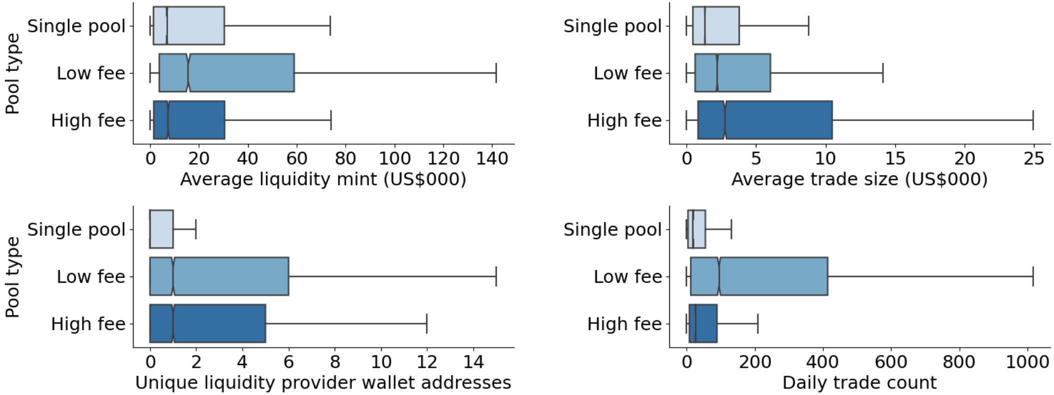
High-fee pools attract small liquidity providers



Data

- Data from Uniswap v3 Subgraph on all trades, liquidity deposits and withdrawals from May 4, 2021 until July 15, 2023, including traders' wallet addresses.
- Gas cost is the average of the lowest daily 1000 gas prices for mint events.
- Focus on economically sizeable pools:
  1. active in more than 100 days within the sample;
  2. 500+ liquidity events throughout the sample;
  3. average daily liquidity balance $>$US$100,000;
  4. $>$1% of volume for a traded pair.
- We obtain 274 pools in 242 asset pairs:
  1. aggregate daily volume of US$ 1.12bn;
  2. end-of-sample aggregate liquidity US$ 2.53bn.
  3. account for 86.04% of all Uniswap v3 interactions.

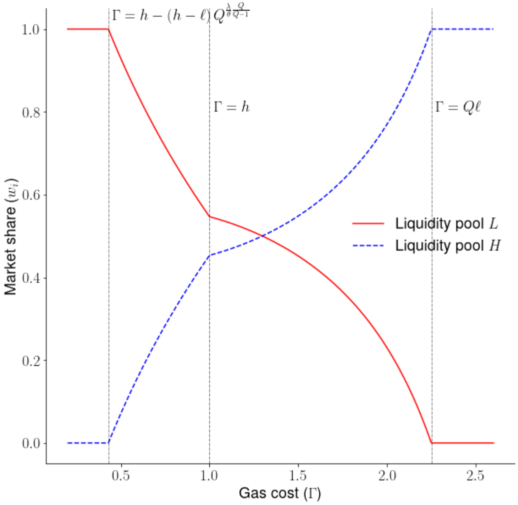
Liquidity clienteles: high fee pools feature many small **LP**s.

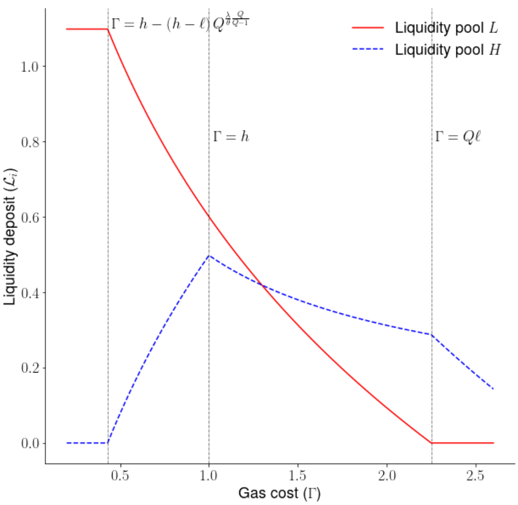


Low-fee pools: larger mints, fewer LP wallets, many small trades.

	Mint size	Trade size	Volume	# Trades	# Wallets	Liquidity yield	Price range
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$d_{\text{low-fee}}$	0.73***	-0.30***	0.89***	1.02***	-3.40***	2.03***	-0.18***
	(12.27)	(-10.05)	(14.23)	(32.95)	(-5.00)	(3.60)	(-41.84)
Gas price × $d_{\text{low-fee}}$	0.37***	0.08***	-0.03	-0.22***	-3.00***	3.57**	-0.00
	(4.96)	(3.75)	(-0.95)	(-7.29)	(-3.43)	(2.30)	(-0.47)
Gas price × $d_{\text{high-fee}}$	0.58***	0.17***	0.24***	0.07**	-2.89***	5.57***	-0.03***
	(7.52)	(8.81)	(5.95)	(2.46)	(-3.15)	(2.83)	(-4.65)
Volume	0.37***	0.16***	0.43***	0.20***	1.22***	1.01	-0.01**
	(8.68)	(21.38)	(15.27)	(13.85)	(6.56)	(0.81)	(-2.56)
Total value locked	-0.16	0.11***	0.23**	-0.01	-1.86	-13.42	-0.02
	(-1.30)	(3.54)	(1.99)	(-0.18)	(-0.99)	(-1.09)	(-0.99)
Volatility	-0.04	-0.01	-0.07	0.01	-0.09	1.18**	0.02***
	(-1.11)	(-1.34)	(-1.38)	(0.88)	(-1.03)	(2.21)	(3.98)
Constant	1.88***	1.64***	5.27***	3.26***	10.12***	10.01***	0.59***
	(58.27)	(111.47)	(168.58)	(209.84)	(28.65)	(26.04)	(184.91)
Observations	21,000	36,059	36,059	40,288	40,288	40,252	24,058
R-squared	0.26	0.53	0.55	0.52	0.37	0.09	0.42

Gas cost and liquidity market shares





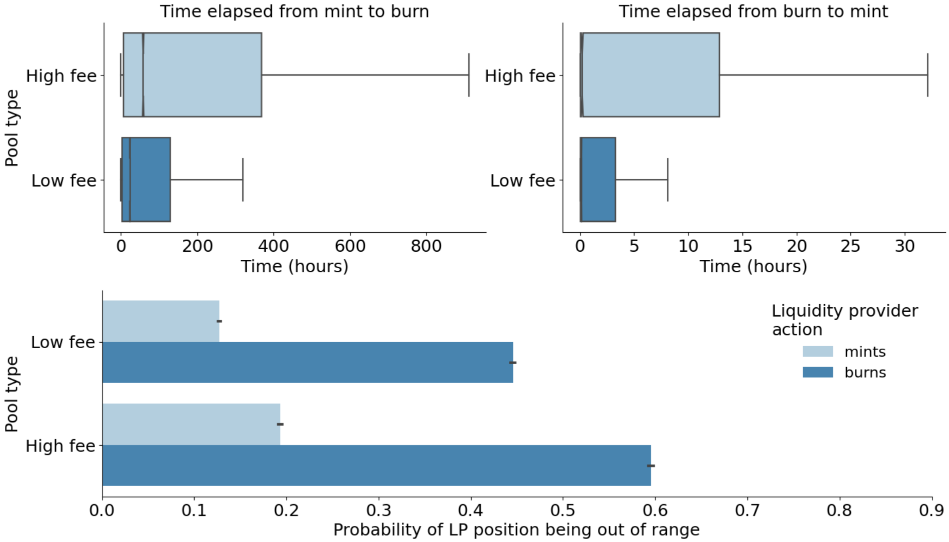
Do gas prices move market shares?

	Liquidity market share (%)			Volume market share (%)		
$d_{\text{low-fee}}$	-20.92***	-20.92***	-20.92***	24.62***	24.63***	24.62***
	(-27.42)	(-27.41)	(-27.42)	(20.55)	(20.56)	(20.55)
Gas price $\times$ $d_{\text{low-fee}}$	-4.63***	-4.62***	-4.63***	-6.52***	-6.52***	-6.52***
	(-7.32)	(-7.32)	(-7.32)	(-5.92)	(-5.92)	(-5.92)
Gas price	2.31***	2.31***	2.31***	3.63***	3.61***	3.61***
	(7.32)	(7.32)	(7.32)	(7.33)	(7.30)	(7.26)
Volume	0.00	0.00	0.00	-0.19**	-0.20**	-0.19**
	(0.65)	(1.33)	(0.65)	(-2.54)	(-2.61)	(-2.50)
Total value locked	-0.00	-0.00		0.58	0.58	
	(-0.58)	(-0.06)		(1.44)	(1.44)	
Volatility	-0.29		-0.29	-1.15***		-1.15***
	(-0.90)		(-0.90)	(-2.74)		(-2.74)
Constant	60.45***	60.46***	60.45***	41.96***	41.99***	41.96***
	(158.00)	(158.46)	(158.00)	(69.99)	(70.22)	(70.02)
Observations	40,288	40,288	40,288	36,059	36,059	36,059
R-squared	0.10	0.10	0.10	0.13	0.13	0.13

Liquidity flows and gas prices

	Daily mints (log US$)			Prob (at least one mint)		
$d_{\text{low-fee}}$	0.15*	0.16**	0.15*	1.33*	1.30*	1.33*
	(1.94)	(2.03)	(1.94)	(1.82)	(1.85)	(1.82)
Gas price × $d_{\text{low-fee}}$	-0.36***	-0.36***	-0.39***	-7.60***	-7.63***	-5.68***
	(-6.66)	(-6.43)	(-5.22)	(-9.36)	(-9.09)	(-8.22)
Gas price × $d_{\text{high-fee}}$	0.03	0.00		-1.92***	-2.14***	
	(0.33)	(0.00)		(-2.74)	(-2.85)	
Trade volume (pair)	0.45***	0.44***	0.45***	1.19	1.17	1.19
	(7.16)	(7.04)	(7.16)	(1.33)	(1.25)	(1.33)
Pool size (pair)	-0.45***	-0.52***	-0.45***	-5.31**	-5.56**	-5.31**
	(-2.75)	(-3.34)	(-2.75)	(-2.43)	(-2.52)	(-2.43)
Volatility	0.02		0.02	1.50*		1.50*
	(0.73)		(0.73)	(1.80)		(1.80)
Gas price			0.03			-1.92***
			(0.33)			(-2.74)
Constant	0.55	1.14	0.55	81.06***	82.73***	81.06***
	(0.60)	(1.36)	(0.60)	(6.12)	(5.72)	(6.12)
Observations	20,454	21,097	20,454	21,097	20,454	20,454
R-squared	0.51	0.51	0.51	0.61	0.62	0.62

Re-balancing cycles



Gas price $\uparrow \Rightarrow$ Liquidity supply on $L \downarrow \Rightarrow$ Re-balancing frequency $\uparrow$

	Mint-burn time			Burn-mint time		
$d_{\text{low-fee}}$	-99.74***	-100.17***	-104.09***	-157.95***	-159.71***	-159.69***
	(-8.86)	(-8.94)	(-9.22)	(-10.59)	(-10.81)	(-10.80)
Gas price $\times$ $d_{\text{low-fee}}$	-16.65**	-15.41*	-15.80**	-11.29	2.95	2.94
	(-2.13)	(-1.98)	(-2.02)	(-1.65)	(0.40)	(0.39)
Gas price $\times$ $d_{\text{high-fee}}$	-14.44**	-13.42*	-13.98*	-10.52*	1.96	1.95
	(-2.04)	(-1.89)	(-1.96)	(-1.69)	(0.32)	(0.32)
Volume		-5.87	-7.45		-24.84***	-24.82***
		(-1.15)	(-1.41)		(-4.10)	(-4.09)
Total value locked		-53.17*	-51.72*		-12.71	-12.71
		(-1.70)	(-1.66)		(-0.52)	(-0.52)
Volatility		-2.11***	-2.26***		-2.99***	-2.98***
		(-2.75)	(-2.86)		(-3.36)	(-3.36)
Position out-of-range			37.09***			-1.53
			(6.43)			(-0.22)
Constant	497.18***	497.00***	479.22***	248.00***	250.13***	250.47***
	(91.65)	(90.60)	(82.13)	(29.91)	(30.27)	(30.32)
Observations	405,586	405,584	405,584	265,848	265,848	265,848
R-squared	0.82	0.82	0.82	0.37	0.37	0.37

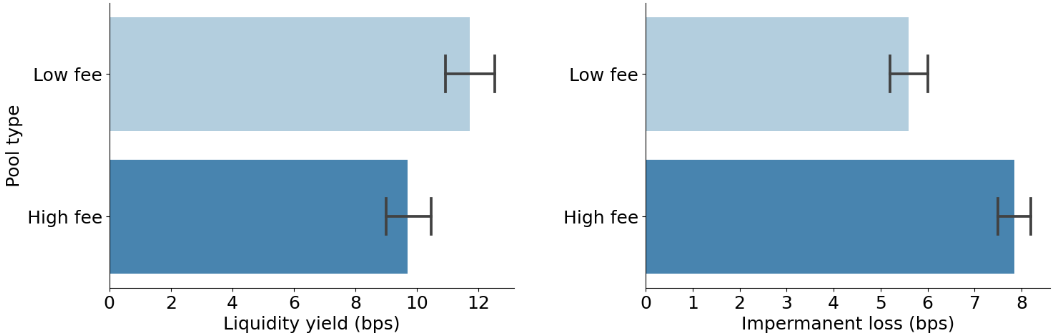
Is order flow on high-fee pools more toxic?

	Impermanent loss for a liquidity position with range $\left[\frac{p}{\alpha}, \alpha p\right]$ around price p							
	$\alpha = 1.01$		$\alpha = 1.05$		$\alpha = 1.10$		$\alpha = 1.25$	
$d_{\text{low-fee}}$	2.59***	-1.38	1.08***	-1.85**	0.71***	-1.56**	0.37**	-1.09*
	(11.26)	(-1.57)	(5.72)	(-2.28)	(4.28)	(-2.18)	(2.58)	(-1.98)
Gas price		4.75***		3.68***		2.72***		1.55***
		(3.97)		(3.96)		(3.86)		(3.42)
Trade count		4.82***		3.56***		2.78***		1.79***
		(4.59)		(3.71)		(3.30)		(2.83)
Volume		3.03***		1.19***		0.60**		0.22
		(7.00)		(3.87)		(2.45)		(1.25)
Total value locked		0.43		1.78		2.02		1.83
		(0.16)		(0.79)		(1.05)		(1.38)
Volatility		6.98***		6.65**		6.39**		6.06**
		(2.69)		(2.59)		(2.51)		(2.40)
Constant	15.52***	15.87***	7.37***	7.65***	4.63***	4.87***	2.45***	2.66***
	(134.72)	(103.02)	(77.84)	(60.07)	(55.47)	(43.73)	(34.58)	(29.33)
Observations	40,250	40,248	40,250	40,248	40,250	40,248	40,250	40,248
R-squared	0.17	0.23	0.09	0.15	0.06	0.11	0.03	0.08

Returns and costs from liquidity provision

Liquidity yield is computed as in Augustin, Chen-Zhang, and Shin (2022):

$$\text{Liquidity yield} = \text{liquidity fee}_i \times \frac{\text{Volume}_{i,t}}{\text{TVL}_{i,t-1}}, \tag{1}$$



Conclusion

- Decentralized exchanges encourage *passive* liquidity provision, both to reduce gas costs and encourage smaller traders to participate as market makers.
- However, fixed costs to participate in markets lead to different economies of scale for heterogeneous **LP**s.
- High-fee pools tend to have lower liquidity yields and higher adverse selection cost.
- Market-maker clienteles emerge if trading is fragmented across different-fee pools.

Low-fee pools	High-fee pools
High trading volume	Low trading volume
Low aggregate liquidity	High aggregate liquidity
Few, large **LP**s	Many, small **LP**s
Short liquidity cycles	Large liquidity cycles