Private vs public currency

B. Biais (HEC), J.C. Rochet (TSE), S. Villeneuve (TSE)

Preliminary !

October 26, 2023

Can private currencies such as bitcoin or ether be useful ?

With well functioning institutions (state, central bank, banks) traditional payment system, relying on public currency ($, €, £), likely to be more efficient than private currency: lower transaction costs, lower volatility

What if institutions dysfunctional ? Predatory government, hyperinflation, corrupt and risky banks...

Then private currency relying on distributed ledger can be useful because shielded from institutions' dysfunctionality:

- commitment to money growth rate
- government cannot directly tamper with blockchain accounts

Argentina, Turkey, Nigeria...

Hyperinflation, local currency volatile, depreciates // $ or bitcoin

Bank accounts can be frozen (Argentina late 1990s, Lebanon)

International transfers constrained

Cryptocurrency can be seen as lifeline, shield against depreciation of official currency (according to some estimates 50% people in Turkey own crypto)

Our model

Continuous time model with one government and continuum of agents operating iid risky technologies (government does not have the skills to operate these technologies)

Agents make portfolio choice: risky asset, safe asset = money

Government funds own consumption (public spendings) with taxes and seigneurage

Initial government somewhat benevolent: some weight on own consumption, some weight on agents' consumption

At first jump of Poisson process: political crisis $\rightarrow$ government becomes predatory: hyperinflation, high taxes; but government cannot seize private money

Results

Before shock: inflation to extract rents from agents but

- $\uparrow$ inflation $\rightarrow$ $\uparrow$ agent's holdings of private money
- $\downarrow$ agents' holdings of public money

Existence of private money $\implies$

- agents can hold asset shielding them from hyperinflation at time of shock and from inflation before the shock
- before the shock, government constrained by competition from private currency

Citizens better off with private money, but government worse off:

- private money competes with public money
- $\downarrow$ demand for public money $\rightarrow$ $\downarrow$ seigneurage before shock

Literature (1)

Related to the literature studying the coexistence between public and private money in OLG models à la Samuelson (1958):

- Garatt and Wallace (2018)
- Biais, Bisière, Bouvard, Casamatta and Menkveld (2023)

Also related to analysis of stablecoins by d'Avernas, Maurin, Vandemeyer (2023)

2 main differences:

- In these models the value of private money stems from exogenous transaction costs or convenience yield $\neq$ we offer a microfoundation for the usefulness or private money.
- In these models money is bubble $\neq$ in our analysis public money valuable because used to pay taxes

Literature (2)

Biais, Gersbach, Rochet, von Thadden, Villeneuve (2023)

- Without political risk and private money: our model = implementation of optimal mechanism with money and taxes in BGRTV
- With political risk and private money, we depart from the mechanism design approach, we directly assume the same policy instruments as in the implementation in BGRTV

Outline

1) Model

2) Equilibrium for a given fiscal and monetary policy → in the spirit of dynamic programing solve backward:

- After the shock
- Before the shock

3) Optimal fiscal and monetary policy

- When there is private money
- Without private money

→ Consequences of the possibility to use private money:

- Does it make citizens better off?
- Does it discipline governments?

Government

Partially benevolent:

- Weight α_t on agents' utility from consuming c_t
- Weight $1 - \alpha_t$ on own utility from consuming c_t^G

For simplicity, agents and government have log utility + discount rate $\rho \rightarrow$ government objective

$$E \int_0^\infty e^{-\rho t} \left[\alpha_t \log(c_t) + (1 - \alpha_t) \log(c_t^G) \right] dt$$

At first jump of Poisson process (intensity λ) political shock $\rightarrow$ government turns predatory

- Before shock $\alpha_t = \alpha$
- After shock $\alpha_t = 0$

Continuum of agents

Continuum agents, one good: both consumption and investment

Agents' objective

$$E \int_0^{\infty} e^{-\rho t} \log \left(c_t \right) dt$$

Can invest k_t good in constant return to scale technology with idiosyncratic risk

$$k_t \left(\mu dt + \sigma dB_t \right)$$

Can also hold public money $\left(m_t \right)$ and private money $\left(\hat{m}_t \right)$

Wealth and prices

Before shock:

- p_t price of good in public currency
- $\hat{p}_t$ price of good in private currency

Real wealth before shock:

$$e_t = k_t + \frac{m_t}{p_t} + \frac{\hat{m}_t}{\hat{p}_t} := e_t(x_t + b_t + \hat{b}_t)$$

After shock, value of public money inflated away ($p_t \to \infty$) and price of good in private currency goes to p_t^+ → real wealth:

$$e_t^+ = k_t + \frac{\hat{m}_t}{\hat{p}_t^+} := e_t^+(x_t^+ + b_t^+)$$

Economic environment

Government policy:

- Wealth tax rate: τ before shock, τ^+ after shock
- Choose monetary policy such that inflation on public money: π before shock (after shock hyperinflation)

Government revenue (= consumption):

- Before shock: revenue from tax on wealth (capital + public money) + seigneurage
- After shock: no seigneurage since no public money $\rightarrow$ only revenue = tax on capital (private money not taxable)

Private money: For simplicity constant supply $\rightarrow$ inflation ($\hat{\pi}$ before shock, π^+ after shock) = − real growth

Agents' program

Choose consumption c_t and portfolio structure $(x_t, b_t, \hat{b}_t)$ before shock, (x_t^+, b_t^+) after shock to maximize

$$E \int_0^\infty e^{-\rho t} \log (c_t) \, dt$$

subject to budget constraint before shock

$$e_t = e_t (x_t + b_t + \hat{b}_t)$$

and after shock

$$e_t^+ = e_t^+ (x_t^+ + b_t^+)$$

rationally anticipating that shock occurs at first jump of Poisson process with intensity λ, and government policy before and after shock

Wealth dynamics and Bellman equation after shock

No transaction costs $\longrightarrow$ continuously rebalance portfolio between capital (productive) & private money $\longrightarrow$ 1 state variable = wealth

$$de_t^+ = k_t(\mu dt + \sigma dB_t) - [c_t + \tau k_t + \pi^+(e_t^+ - k_t)]dt$$

Value function of agent after shock: u^+

Bellman equation:

$$\rho u^+ = \max_{c,k} logc + u^{+'}[\mu k - c - \tau^+ k - \pi^+(e^+ - k)] + \frac{\sigma^2 k^2 u^{+''}}{2}$$

Solution of Bellman equation after shock

We postulate (and check ex post) that $\rho u^+(e) = log(e) + \rho u^+(1)$

FOC c: Consumption = constant fraction of wealth

$$c_t = \rho e_t^+$$

more impatient → consume more

FOC k: Risky capital holdings = constant fraction of wealth

$$k_t = \frac{\mu - \tau^+ + \pi^+}{\sigma^2} e_t^+ := x^+ e_t^+$$

- increasing in productivity μ
- decreasing in tax τ^+
- increasing in inflation π^+

Growth & investment after shock

Growth = output − government's cons. − agent's cons.

Since $e^+ = k/x^+$ and government's consumption = tax on capital

$$g^+ = \mu - \tau^+ - \frac{\rho}{x^+}$$

Substituting FOC k into stationary equilibrium → equilibrium investment in risky capital after shock:

$$x^+ = \frac{\sqrt{\rho}}{\sigma}$$

More risk σ → less investment in risky asset

Wealth dynamics before shock

Wealth at time t until shock

$$e_t = e_t(x_t + b_t + \hat{b}_t)$$

At time of shock jumps to

$$e_t^+ = e_t(x_t + \hat{b}_t \frac{\hat{p}_t}{p_t^+})$$

Wealth dynamics until shock

$$\begin{aligned} de_t &= k_t(\mu dt + \sigma dB_t) + [e_t^+ - e_t]dN_t \\ &\quad - \left[c_t + \pi(e_t - \hat{b}_t - k_t) + \hat{\pi}\hat{b}_t + \tau(e_t - \hat{b}_t)\right] dt \end{aligned}$$

$dN_t = 1$ at time of shock $\rightarrow$ wealth jumps from e_t to e_t^+

Bellman equation before shock

Bellman equation:

$$\rho u(e) = \max_{c,x,\hat{b}} logc + \lambda(u^+(e^+)) - u(e))$$

$$+eu'(e)\left[(\mu+\pi)x - \frac{c}{e} - (\pi+\tau)(1-\hat{b}) - \hat{\pi}\hat{b}\right] + \frac{\sigma^2 x^2 e^2 u''(e)}{2}$$

Shock occurs with probability λdt and in this case value function switches from u to u^+, while wealth jumps from e to e^+

Solution of the Bellman equation before shock

Postulate and check later: $\rho u(e) = log(e) + \rho u(1)$

$$\text{FOC}c : c = \rho e$$

Interior equilibrium in which public & private money held $\longrightarrow$ indifference condition:

$$\text{FOC } \hat{b} : \pi + \tau = \hat{\pi} - \lambda \frac{x^{+}}{x} \frac{\hat{p}}{p^{+}}$$

LHS: cost of holding public money until shock
RHS: cost of holding private money − benefit of private money: hedge against political shock

$$\text{FOC } x : x = \frac{\mu + \pi + \lambda \frac{x^{+}}{x}}{\sigma^2}$$

Weight of risky capital in protfolio, decreasing in risk σ and increasing in

- value of capital as hedge against political shock
- inflation (since risky capital = real)

Stationary equilibrium

Optimality conditions above

+ market clearing/resource constraint

+ stationarity (constant growth rate until shock)

Equilibrium before shock

Fraction of wealth invested in capital $x \uparrow$ with inflation π

$$\pi = \sigma^2 x - \lambda \frac{x^+}{x} - \mu$$

Change in price of private currency at shock

$$\frac{\hat{p}}{p^+} = \frac{x(\lambda + \rho - \sigma^2 x^2)}{\lambda x^+}$$

Tax rate

$$\tau = \gamma + \frac{1 - x}{x}(\rho - \sigma^2 x^2) + \lambda \frac{x^+ - x}{x}$$

Fraction of wealth invested in private currency $\uparrow$ with π (or x)

$$\hat{b} = \frac{\lambda(1 - x^+)}{\lambda + \rho - \sigma^2 x^2},$$

only if $b = 1 - x - \hat{b} \geq 0$, otherwise corner: $b = 0$

Non negative public money holdings

We need

$$b = 1 - x - \hat{b} \geq 0$$

Now

$$\hat{b} = \frac{\lambda(1 - x^+)}{\lambda + \rho - \sigma^2 x^2}$$

So

$$b \geq 0 \iff x \leq x^+$$

$\implies$ fraction of wealth held as capital lower before shock (when public money also held) than after (when no public money held)

π increasing in $x \rightarrow x$ low when π low: agents willing to hold public money ($b \geq 0$) when inflation low i.e x low

Policy instruments

Government policy $= (\pi, \gamma)$

- π determines x by

$$\pi = \sigma^2 x - \lambda \frac{x^+}{x} - \mu$$

- π and γ determine all the other variables

$$\hat{b}, b, \frac{\hat{p}}{p^+}, \tau$$

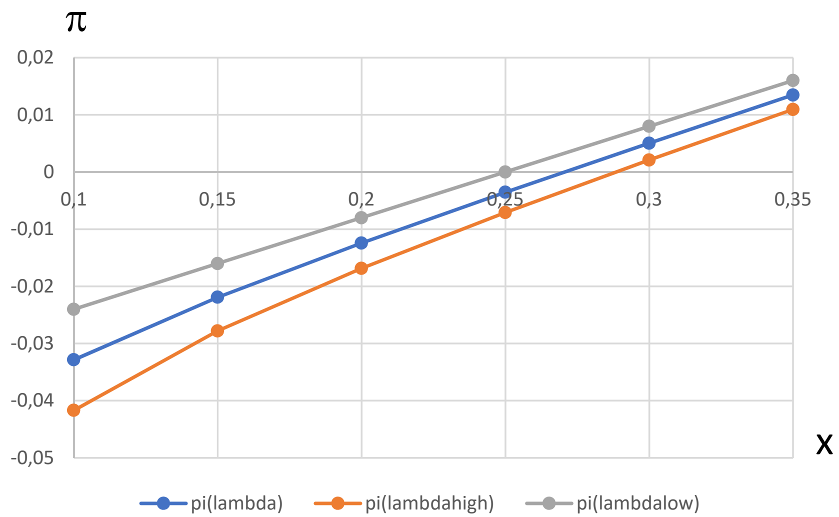
Equivalently, since x and π one to one,

$$(x, \gamma) \to \left(\pi, \hat{b}, b, \frac{\hat{p}}{p^+}, \tau\right)$$

π as a function of x

μ = 4%, σ =40%, ρ = 2%, λ = 0.25%, λ_{high} = 0.5%, λ_{low} = 0

x^{+}=0.35



Larger inflation π => more attractive to hold real assets x

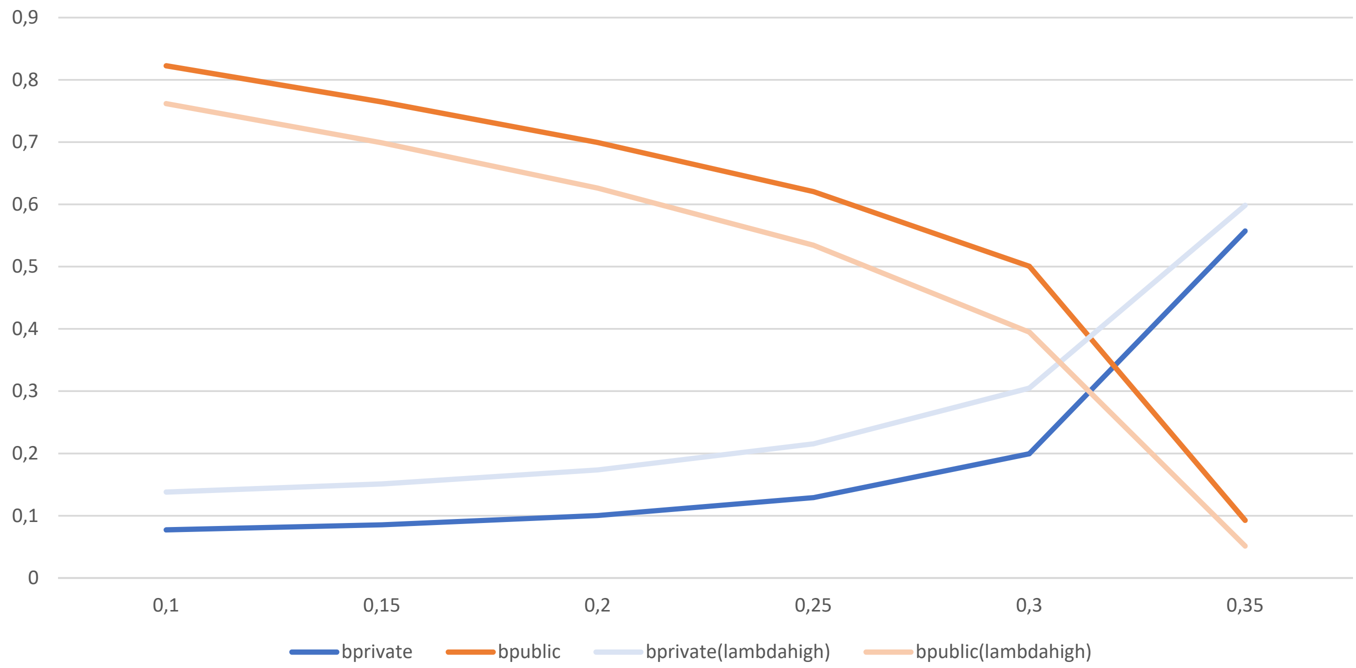
The larger the political risk λ, the lower the inflation it takes to convince agents to pick a given x because x not expropiated at time of political shock

Currency holdings as a function of x

$\mu = 4\%, \sigma = 40\%, \rho = 2\%, \lambda = 0.25\%, \lambda_{high} = 0.5\%, \lambda_{low} = 0$

$x^{+} = 0.35$

Currency holdings



x

Larger x ⇔ inflation on public currency π => less attractive to hold public currency

Largepolitical risk λ => more attractive to hold private currency

Government's utility from consumption if no political shock

If capital grows at rate g and government consumes $c_t^G = \gamma k_t$ then present value of utility of consumption (with $k_0 = 1$)

$$\int_0^\infty e^{-\rho t} \log(c_t^G) dt = \frac{1}{\rho}\left[\log\gamma + \frac{g}{\rho}\right]$$

Agents consume fraction ρ of wealth $e = k/x \rightarrow$

$$g = \mu - \gamma - \rho/x$$

Government's present value of utility of consumption

$$V(x,\gamma) = \frac{1}{\rho}\left[\log\gamma + \frac{\mu - \gamma - \frac{\rho}{x}}{\rho}\right]$$

Tradeoff: $\uparrow \gamma \rightarrow \uparrow$ fraction of output going to govt but $\downarrow$ growth

Agents' expected utility if no political shock

If government's policy $= (x, \gamma)$ forever

Agents' discounted expected utility of consuming fraction $\frac{\rho}{x}$ of capital growing on average at rate g − risk premium for volatility

$$\begin{aligned}
\omega(x,\gamma) &= \frac{1}{\rho}\left[\log\frac{\rho}{x} + \frac{g}{\rho} - \frac{\sigma^2 x^2}{2\rho}\right] \\
&= \frac{1}{\rho}\left[\log\frac{\rho}{x} + \frac{\mu - \gamma - \frac{\rho}{x}}{\rho} - \frac{\sigma^2 x^2}{2\rho}\right]
\end{aligned}$$

Utility when anticipating political shock

Government plays x before shock, x^+ after shock (occuring at first jump of Poisson with intensity λ)

Government's utility:

$$V = \int_0^\infty e^{-\rho t} \log(c_t^G) dt = \frac{1}{\rho}\left[\frac{\rho}{\rho+\lambda} V(x,\gamma) + \frac{\lambda}{\rho+\lambda} V(x^+,\gamma)\right]$$

Agents' utility:

$$\omega = E\left[\int_0^\infty e^{-\rho t} \log(c_t) dt\right] = \frac{1}{\rho}\left[\frac{\rho}{\rho+\lambda} \omega(x,\gamma) + \frac{\lambda}{\rho+\lambda} \omega(x^+,\gamma)\right]$$

Initial government policy

Initial government's program

$$\max_{x,\gamma} \alpha\omega + (1-\alpha)V$$

under constraint that policy s.t. $b \geq 0$, i.e., $x \leq x^+$

First order conditions

$$\gamma = \rho$$

$$x = x^* \text{ s.t } \frac{\sigma^2}{\rho} x^{*3} + x^* = \frac{1}{\alpha},$$

if $x^* < x^+$ (so that $b > 0$) optimal government policy is x^*, otherwise x^+(and $b = 0$)

When is non negative public money holdings constraint binding?

Constraint $b \geq 0$ binds when $x^* > x^+$

Since $\frac{\sigma^2}{\rho} x^{*3} + x^* = \frac{1}{\alpha}$, x^* decreasing in α

Non negative public money constraint $(x^* \leq x^+)$ binds when α low (and correspondingly x^* high)

More precisely $x^* \leq x^+$ binds when

$$\alpha < \alpha_{\text{min}} = \frac{1}{2} \frac{\sigma}{\rho^{\frac{1}{2}}}$$

$\alpha < \alpha_{\text{min}} \longrightarrow$ government not very benevolent
$\longrightarrow$ wants to set high x $\longrightarrow$ set high π
$\longrightarrow$ high inflation makes public money unattractive

Government policy & asset holdings with and without private money

Asset holdings after shock:

- Without private money: only asset agents can hold = risky
- With private money: agents can also hold private money

Government policy before shock:

- Without private money: government can set high inflation, to extract rents from agents
- With private money: government cannot set inflation too high, otherwise agents won't hold public money

$\Longrightarrow$ private money $\downarrow x$

Risky asset holdings with and without private money

Before shock $x = \min[x^*, x^+]$, where x^*, s.t.,

$$\frac{\sigma^2}{\rho} x^{*3} + x^* = \frac{1}{\alpha}$$

If $\alpha < \alpha_{\min}$	Before shock	After shock
with private money	x^+	x^+
without private money	$x^* > x^+$	1

If $\alpha \geq \alpha_{\min}$	Before shock	After shock
with private money	$x^* \leq x^+$	x^+
without private money	$x^* \leq x^+$	1

Consequences of private money

Consequences for agents:

- after shock: agents can still use (private) money, although value of public money annihilated by hyperfinflation
- before shock: if initial government not benevolent ($\alpha < \alpha_{\text{min}}$), option to use private money = competition for public money, reduces goverment ability to extract rents via inflation

$\implies$ citizens better off with private money than without

In contrast, government made worse off by private money

- after shock: less taxes (government can tax capital not private money)
- before shock: less seigneurage (lower demand for public money, since private money competes with public money)

First order condition after shock

We postulate (and check ex post) that $\rho u^+(e) = log(e) + \rho u^+(1)$

$$u^{+'}(e) = \frac{1}{\rho e}, u^{+''}(e) = -\frac{1}{\rho e^2}, -\frac{u^{+''}(e)}{u^{+'}(e)} = \frac{1}{e}$$

$$\text{FOC}c : \frac{1}{c} = u^{+'}(e) \iff c_t = \rho e_t^+$$

$$\text{FOC}k : k = \frac{\mu - \tau^+ + \pi^+}{-\frac{u^{+''}(e)}{u^{+'}(e)}\sigma^2} \iff k_t = \frac{\mu - \tau^+ + \pi^+}{\sigma^2} e_t^+ := x^+ e_t^+$$

Capital holdings after shock

Substituting FOC k:

$$x^{+} = \frac{\mu - \tau^{+} + \pi^{+}}{\sigma^{2}}$$

into stationary equilibrium

$$-\pi^{+} = \mu - \tau^{+} - \frac{\rho}{x^{+}}$$

$\longrightarrow$ equilibrium investment in risky capital after shock:

$$x^{+} = \frac{\sqrt{\rho}}{\sigma}$$

Bellman equation & optimality conditions before shock

Bellman equation:

$$\rho u(e) = \max_{c,x,\hat{b}} logc + \lambda(u^{+}(e_t(x_t + \hat{b}_t \frac{\hat{p}_t}{p_t^{+}})) - u(e))$$

$$+eu'(e)\left[(\mu+\pi)x - \frac{c}{e} - (\pi+\tau)(1-\hat{b}) - \hat{\pi}\hat{b}\right] + \frac{\sigma^2 x^2 e^2 u''(e)}{2}$$

Optimality conditions:

$$\text{FOC } c : \frac{1}{c} = u'(e)$$

$$\text{FOC } x : \mu + \pi = -\sigma^2 e \frac{u''(e)}{u'(e)} x - \lambda \frac{u^{+'}(e^{+})}{u'(e)}$$

$$\text{FOC } \hat{b} : \lambda \frac{\hat{p}_t}{p_t^{+}} \frac{u^{+'}(e^{+})}{u'(e)} + (\pi+\tau) = \hat{\pi}$$

With log utility

$$\text{FOC } x : x = \frac{\mu + \pi + \lambda \frac{e}{e^+}}{\sigma^2} = \frac{\mu + \pi + \lambda \frac{x^+}{x}}{\sigma^2}$$

(by conservation of capital: $xe = x^+ e^+$)

Consumption and growth before shock

Postulate (check later) government consumes fraction γ of capital

Agents consume fraction ρ of wealth $e = k/x \rightarrow$ growth:

$$g = \mu - \gamma - \rho/x$$

Government budget constraint:

$$\text{consumption} = \text{taxes} + \text{seigneurage}$$

$$\gamma x = \tau(1 - \hat{b}) + (g + \pi)(1 - x - \hat{b})$$

Equilibrium properties

Inflation π comoves with weight of risky asset x: risky asset generates real revenues, shielded from inflation

$\uparrow$ risk of shock $\lambda \implies$

- $\uparrow x$: capital shielded from political risk
- $\uparrow \hat{b}$: private money shielded from political risk