Private vs Public Currency

by Biais, Rochet, and Villeneuve

Discussion by Adrien d'Avernas (SSE)

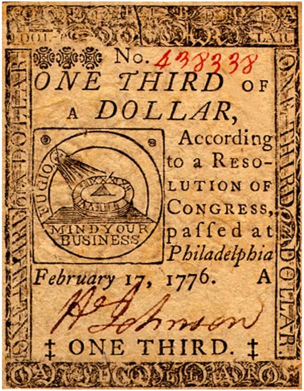
What is the Value of Crypto?

- Crytpocurrencies are here to stay
  → \$1.3 Trillion market cap, \$1.5 Trillion traded per day
  → 16% of Americans have used, invested in, or traded crypto

- First paper (that I know of) that takes seriously the ethos of crypto
  → algorithmic currency immune from poor/corrupt monetary policy

- Plenty past and current examples of bad monetary policy

Early American Currency



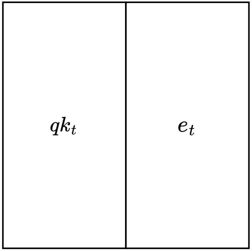
- In 1775, Continental Congress issued paper money known as Continentals
- Continentals depreciated badly during the war, giving rise to the famous phrase "not worth a continental"
- Monetary policy was not coordinated between Congress and the states
- Congress and the states lacked the will or the means to retire the bills from circulation through taxation
- By May 1781, Continentals had become worthless
- Franklin noted that the depreciation had, in effect, acted as a tax to pay for the war

Findings

- Private money competes with public money
- Constraints gov't misbehavior and increases welfare

▷ This discussion: the “balance sheet” view

Benchmark

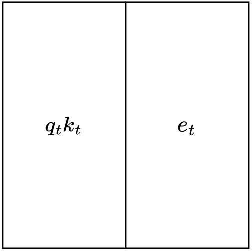


$$V(e_t) = \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(s-t)} \log(c_s) ds \right]$$

$$dk_t = (\mu - \iota_t) k_t dt + \sigma k_t dB_t$$

$$de_t = ((\mu - \iota_t) q k_t + \Phi(\iota_t) k_t - c_t) dt + \sigma q k_t dB_t$$

Benchmark



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$$dk_t = (\mu - \iota_t)dt + \sigma k_t dB_t$$

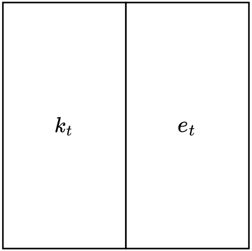
$$de_t = ((\mu - \iota_t)q_t k_t + \iota_t k_t - c_t)dt + \sigma q_t k_t dB_t$$

$$q_t = 1$$

$$c_t = \rho k_t = \iota_t k_t \qquad \mathbb{E}_t[dk_t/k_t] = \mu - \rho$$

- Investment function $\Phi(\iota) = \iota$ is such that $q_t = 1$

Benchmark



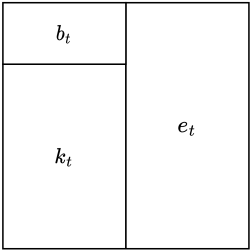
$$V(e_t) = \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(s-t)} \log(c_s) ds \right]$$

$$de_t = (\mu k_t - c_t)dt + \sigma k_t dB_t$$

$$k_t = \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(s-t)} \frac{u'(c_{t+s})}{u'(c_t)} c_{t+s} ds \right] = e_t$$

- $q_t k_t = k_t$ is still equal to the discounted sum of future real dividends c_t

Risk-free Capital



$$V(e_t) = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\log(c_s)ds\right]$$

$$de_t = (\mu k_t + rb_t - c_t)dt + \sigma k_t dB_t$$

$$\frac{\mu - r}{\sigma} = \sigma\frac{k_t}{e_t}$$

$$k_t + b_t = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{u'(c_{t+s})}{u'(c_t)}c_{t+s}ds\right] = e_t$$

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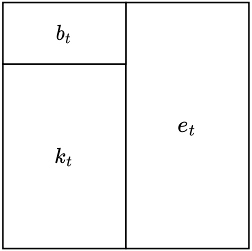
Risk-free Capital



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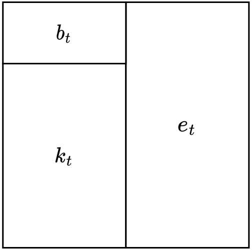
$$\frac{\mu - r}{\sigma} = \sigma\frac{k_t}{e_t}$$

$$r \geq \mu - \sigma^2 : \;\; b_t \geq 0$$

- Real consumption good is allocated to risk-free capital if $r \geq \mu - \sigma^2$

Storage Technology



$$V(e_t) = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \log(c_s) ds \right]$$

$$de_t = (\mu k_t + r b_t - c_t)dt + \sigma k_t dB_t$$

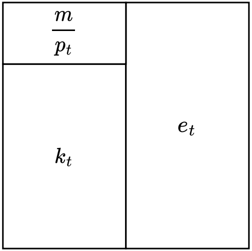
$$\frac{\mu - r}{\sigma} = \sigma \frac{k_t}{e_t}$$

$$\Delta V(e_t) = \frac{1}{2\rho} \frac{(\mu - r)^2}{\sigma^2} + \frac{r - \mu}{\rho} + \frac{\sigma^2}{2\rho}$$

$$r = 0 : \qquad \Delta V(e_t) > 0 \text{ iff } \sigma^2 > \mu$$

- Risk-free asset makes market complete and increases welfare
  $\rightarrow$ even if there is no return on investment

What is Money?



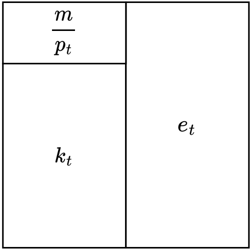
$$V(e_t) = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\log(c_s)ds\right]$$

$$de_t = \left(\mu k_t - \pi\frac{m}{p_t} - c_t\right)dt + \sigma k_t dB_t$$

$$\frac{\mu+\pi}{\sigma} = \sigma\frac{k_t}{e_t}$$

- Money is not a storage technology!

What is Money?



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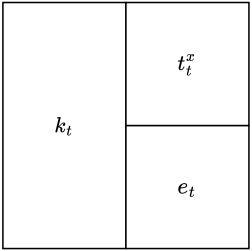
$$k_t = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{u'(c_{t+s})}{u'(c_t)}c_{t+s}ds\right] = e_t$$

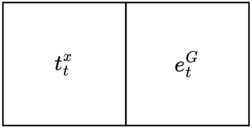
- Money is not a storage technology!
- Without OLG, another agent, or frictions, money cannot have value
  → Tirole (1982), Santos and Woodford (1997), Kamihigashi (2004)
  → Brunnermeier and Sannikov (2016): I-theory of Money

Government and Taxes



$$de_t = (\mu k_t - c_t - \tau k_t)dt + \sigma k_t dB_t$$

$$t_t^x = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \frac{u'(c_{t+s})}{u'(c_t)} \tau k_{t+s} ds \right]$$



$$\max_\tau \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \left(\alpha \log(c_s) + (1-\alpha) \log(\tau e_s) \right) ds \right]$$

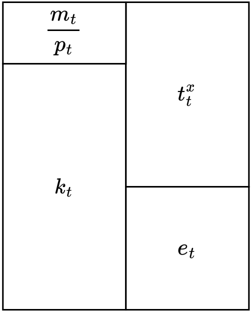
$$\alpha = 1: \quad \tau^{opt} = \tau^\star = 0 \qquad \alpha < 1: \quad 0 < \tau^{opt} < \tau^\star < 1$$

- Government uses taxes to extract ressources and consume

  $\rightarrow$ redistributive technology

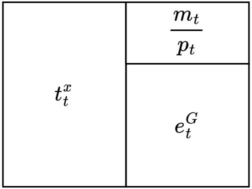
  $\rightarrow$ gov't does not steal everything $\triangleright$ gov't cannot own capital

Money as a Liability



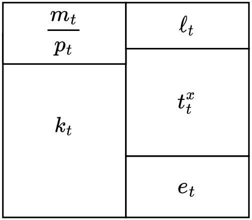
$$t_t^x = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \frac{u'(c_{t+s})}{u'(c_t)} \tau \left(k_{t+s} + \frac{m_{t+s}}{p_{t+s}} \right) ds \right]$$

$$\frac{m_t}{p_t} = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \frac{u'(c_{t+s})}{u'(c_t)} (-\pi) \frac{m_{t+s}}{p_{t+s}} ds \right]$$

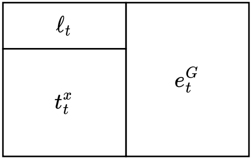


- Money can exist as a liability of the government
  - $\rightarrow$ need that gov't buys back or pays interest in equilibrium path
  - $\rightarrow$ closer to central bank reserves than outside money
- Deflation can implement this equilibrium
  - $\rightarrow$ gov't repurchases money with revenues from taxes
  - $\rightarrow$ but in that case money could be netted out, no added value

Liquidity Premium



$$\tau\left(k_t + \frac{m_t}{p_t}\right) \leq \kappa \frac{m_t}{p_t}$$ with shadow cost ζ_t



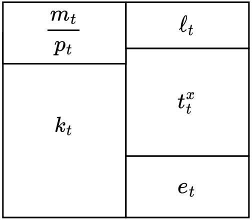
$$\frac{m_t}{p_t} = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{u'(c_{t+s})}{u'(c_t)}\zeta_{t+s}ds\right]$$

$$\ell_t = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{u'(c_{t+s})}{u'(c_t)}\pi\frac{m_{t+s}}{p_{t+s}}ds\right]$$

- The government can impose that you need money to pay taxes
  $\rightarrow$ cash-in-advance constraint not trivial in continuous time

▷ Inflation is now possible

Liquidity Premium



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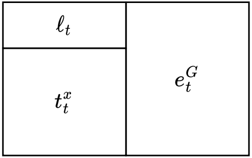

$$\frac{\mu + \pi - \zeta_t}{\sigma} = \sigma \frac{k_t}{e_t}$$

- Interest rate spread must contain a (liquidity) premium!
- The real value of money is pinned down in equilibrium
  $\rightarrow$ if not scarce enough, ζ_t is low and real value decreases
  $\rightarrow$ gov't can alter trade-off with inflation

Three Other (Quick) Comments

- Why would the gov't destroy immediately value of money?
  $\rightarrow$ unclear that $p_t = \infty$ is optimal when $\alpha_t = 0$ (gov't can't store c_t)
  $\rightarrow$ excessive inflation useful with nominal liabilities
  $\rightarrow$ maybe $\alpha_t = 0$ implies loss of commitment

- Observability/feasibility: only capital and public money can be taxed
  $\rightarrow$ but if private money can pay taxes, why it cannot be taxed?
  $\rightarrow$ if private money cannot pay taxes, where its value comes from?

- Cryptocurrencies are used to dodge capital control and taxation
  $\rightarrow$ Graf von Luckner, Reinhart, and Rogoff (2023)
  $\rightarrow$ transfer real consumption to offshore storage technology

Conclusion

- All my comments can (easily) be fixed
- Elegant framework, key question
- Paper of first-order importance