BIS

Discussion of "AMM Designs beyond Constant Functions"

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Disclaimer: The views expressed here are all mine and not necessarily of the Bank for International Settlements.

Summary

- An interesting paper on AMM trading mechanisms
- Key result: LP chooses *impact* functions $\rightarrow$ marginal rate/price discovery
- Arithmetic liquidity pool (ALP) vs geometric liquidity pool (GLP)
  - ALP: Marginal rate and LP inventory are additive
  - GLP: Marginal rate and LP inventory are multiplicative
  - No round-trip arbitrages (front-running/sandwich attacks)
- Second result: LP's optimal strategy in ALP and GLP
- Last-but-not-least result: constant functions (CFM) are a special case of ALP in which LP's strategy is sub-optimal

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- Key concepts:
  - Marginal rate Z $\Longrightarrow$ mid-quotes m
  - Shifts around marginal rate $Z \pm \delta \Longrightarrow$ bid and ask prices p (ie $\delta \Longrightarrow$ half spread s)

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- Consider a two-period model: Z_0, x_0, y_0, LT buys ζ unit of Y
  - **Cartea et al:** $dZ_t (= Z_1 - Z_0) = \eta(\cdot), \quad dy_t = -\zeta, \quad dx_t = \zeta(Z_0 + \delta)$

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  - **Translating to MM:** $m_1 = m_0 + \eta(\cdot)$
  - **Foucault, Pagano and Roell (2013) Chapter 3.5:**
    $m_1 = m_0 + \underbrace{(\mu_1 - \mu_0)}_{\Delta\text{Expected value}} + \underbrace{\rho\sigma q}_{\text{Inv cost}}$

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- CFMs as a special case
  - **Cartea et al:** level function $x = \varphi(y)$ and marginal rate $Z = -\varphi'(y)$
    - $\Rightarrow \eta = \varphi'(y_0) - \varphi'(y_1)$ and $\delta = \varphi'(y_0) - \frac{\varphi(y_1)-\varphi(y_0)}{y_1-y_0}$

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  - **Translating to MM:** $m_1 = m_0 + \underbrace{(\varphi'(y_0) - \varphi'(y_1))}_{\text{Change of } \frac{dx}{dy}}$
  - **Compared to Foucault et al (2013):** $m_1 = m_0 + \underbrace{(\mu_1 - \mu_0)}_{\Delta\text{Expected value}} + \underbrace{\rho\sigma q}_{\text{Inv cost}}$
  - **CFM is not optimal!**

Comment 1: A mapping from the paper to Econ/MM literature

- Optimal liquidity provision with price-sensitive LT
  - LT arrival intensity λ decreases in the spread δ.
  - LP maximizes expected wealth subject to inventory risk
  - Optimal spread

$$\delta^* = \text{Round-trip trade profit} + \text{adj for inv cost} + \underbrace{\text{impact component}}_{\text{price impact ?}}$$

  - What is η^* in $m_1 = m_0 + \eta$?
  - What is the size of the round-trip trade?

Comment 2: Informed trading

- The impact function η aims to allow LP adjusting mid-quote, enhancing price discovery
- But η is still a function of y only (?)
- The no-round-trip-arbitrage is achieved by widening the spread δ ...
- ... which is possible because LT arrival intensity is deterministic?
- In addition, what if LT are informed? How adverse selection is addressed?

Minor comments

- Exposition: the current draft is pretty cryptic
  - Consider lightening up notation
- I find the result that CFM is not optimal very interesting and relevant
  - Consider more numerical exercises to highlight the inefficiency

Concluding remarks

- This paper provides a rigorous analysis on AMM trading mechanism
- It is more of a Math- or CS-oriented paper, with an application in Economics
- Overall the paper provides a lot of interesting findings
- Entrepreneurs interested in building DeFi apps should definitely read this paper
- Economists can also learn a lot from the paper