Density Forecasting with BVAR Models under Macroeconomic Data Uncertainty

Michael P. Clements[*]
ICMA Centre
Henley Business School
University of Reading
m.p.clements@reading.ac.uk

Ana Beatriz Galvão
Warwick Business School
University of Warwick
ana.galvao@wbs.ac.uk

April, 2020

Abstract

Macroeconomic data are subject to data revisions as later vintages are released. Yet, the usual way of generating real-time density forecasts from BVAR models makes no allowance for this form of data uncertainty. We evaluate two methods that consider data uncertainty when forecasting with BVAR models with/without stochastic volatility. First, the BVAR forecasting model is estimated on real-time vintages. Second, a model of data revisions is included, so that the BVAR is estimated on, and the forecasts conditioned on, estimates of the revised values. We show that both these methods improve the accuracy of density forecasts for US and UK output growth and inflation. We also investigate how the characteristics of the underlying data and revisions processes affect forecasting performance, and provide guidance that may benefit professional forecasters.

Keywords: real-time forecasting, inflation and output growth predictive densities, real-time-vintages, stochastic volatility.

JEL code: C53.

[*]Helpful comments from three referees of this journal are gratefully acknowledged.

1 Introduction

Decision makers employ probabilistic forecasts of macroeconomic variables to compute the probability of future outcomes of interest as an aid to determining which course of action to take. For example, based on density forecasts, one can quantify the probability of sluggish growth (say lower than 1%) and/or of deflation to support a monetary policy decision. This paper considers whether it is important to make an allowance for data uncertainty when computing probabilistic forecasts in real-time, given that most macroeconomic variables are subject to data revisions.

Clements (2017) considers the impact of data revisions on the assessment of macroeconomic forecasting uncertainty. He shows that the standard real-time approach, which estimates the forecasting model on the vintage of data available at the forecast origin, will likely give an inaccurate assessment of the uncertainty surrounding future values of the variables, especially of the early-vintage estimates of those values. He considers autoregressive models with constant-variance disturbances, and his work follows on from the work on point forecasts of Koenig, Dolmas and Piger (2003) and Clements and Galvão (2013), *inter alia*. Yet the recent literature on macroeconomic forecasting suggests using multivariate models with time-varying conditional volatility to obtain accurate density forecasts in real time (see, for example, Clark (2011), Clark and Ravazzolo (2014), Diebold, Schorfheide and Shin (2016) and Carriero, Clark and Marcellino (2020)).

Although the literature on forecasting with multivariate models with time-varying volatility uses real-time data, that is, the vintages of macroeconomic time series that were actually available at the time the forecast was made, it does not explicitly consider the impact of data revisions on the measurement of forecasting uncertainty. The conventional approach to real-time forecasting that underlies this literature - using the vintage of data available at the forecast origin - fails to make an allowance for data uncertainty in the estimation of the model, or in the generation of forecasts from the model. It regards the data as given, and does not allow for the consequences of the data being revised over time. However, making an allowance for data uncertainty in the evaluation of the forecasts from the model is commonplace: forecasts are routinely compared to advance estimates of the actual data, or first-finals, or the latest vintage of data available when the study is undertaken.

This main purpose of the paper is to assess whether the conventional way of forecasting in real-time can be improved upon for Bayesian Vector Autoregressions (BVARs, see, e.g., Sims (1980) and Doan, Litterman and Sims (1984)) with stochastic volatility. These are multivariate models which are popular in probabilistic macro forecasting (Clark (2011), Clark and Ravazzolo (2014), and Carriero *et al.* (2020)). First and foremost, whether it is possible to obtain improvements in density forecasting performance from an appropriate treatment of data uncertainty in such models is regarded as an empirical issue. Obtaining analytical results in general settings that match the complexity of the actual forecasting environment is challenging, especially allowing for small-sample parameter estimation uncertainty. Instead we consider simplified settings which abstract from features such as parameter estimation uncertainty, non-gaussian disturbances, and multiple rounds of revisions. These analyses serve to illustrate some of the factors which shape the empirical findings, such as the properties of the revisions, but the analytical results are at best a rough guide to the empirics. The question as to whether the neglect of revision-driven data uncertainty in BVARs matters can ultimately only be answered by the establishment of a representative body of empirical work. Our paper is a first step in this direction.

To summarize our contribution, we focus on the impact of data uncertainty when computing one and four-step-ahead probabilistic forecasts in real-time, using BVAR models, allowing the disturbances to be characterized by time-varying volatility. We consider two ways of allowing for revision-driven data uncertainty, and these are compared to the conventional approach of using the latest vintage of the time series available at each point in time. This conventional approach makes no allowance for data uncertainty. It is sometimes known as end-of-sample, abbreviated to EOS. The first of the two approaches that allow for data uncertainty is the use of real-time-vintage (RTV) data, advocated by Koenig *et al.* (2003) and Clements and Galvão (2013) for point forecasting, and shown by Clements (2017) to be a simple and effective way of delivering more accurate assessments of forecasting uncertainty in univariate AR models when the error variance is homoscedastic. The second approach is based on Kishor and Koenig (2012) (KK), who propose estimating the VAR on the "true values" of the variables at the same as modelling the revisions. Instead of the estimation procedure proposed by Kishor and Koenig (2012), we develop a Bayesian estimation strategy. This allows priors (such as the Minnesota prior) to be used to counter problems of high dimensionality, and facilitates the modelling of the innovations (including the innovations to the revisions processes) as processes with stochastic volatility.

Because we are interested in potential improvements from allowing for data uncertainty, our focus is on relative measures of density forecasting performance (compared to a benchmark which ignores data uncertainty), rather than absolute tests for correct specification (such as those of Rossi and Sekhposyan (2013), for example). In simplified settings, we can show that RTV maximizes the out-of-sample real-time log score, and this motivates our interest in RTV as a method to deal with data uncertainty when forecasting with BVARs in real-time. Clearly, if the KK model provides an accurate description of the real-time forecasting environment, by explicitly modelling data revisions it will correctly account for data uncertainty, and so should also provide improved density forecasts.

The plan of the remainder of the paper is as follows. Section 2 describes the two ways of allowing for revision-driven data uncertainty. Section 3 sets out the empirical forecasting comparisons we undertake, and presents the findings. Section 4 explains some aspects of our findings using simplified settings, and section 5 reports the results of a Monte Carlo study. Section 6 offers some concluding remarks, and argues that there are occasions when revision-driven data uncertainty ought to be taken seriously.

Proofs of the main analytical results are available in an Appendix.

2 Methods to deal with Data Uncertainty in BVAR forecasting

Clements and Galvão (2013) show that if real-time data is reorganized into 'real-time vintages' (RTV) for model estimation, instead of employing the conventional end-of-sample approach, the real-time accuracy of point forecasts from autoregressive models may be improved. This is one of the methods we consider here to improve real-time forecasting with BVARs by taking into account data uncertainty. The other method is based on Kishor and Koenig (2012) (KK), who propose estimating the VAR using "true values" of the variables subject to revision, as part of a system that also includes equations to model the dynamics of data revisions. We extend KK to incorporate stochastic volatility (SV) to allow for time-variation in expected forecast uncertainty. This puts the KK approach on the same footing as the BVAR models with SV.

Below we consider the BVAR with RTV, followed by the KK approach, and finally the two models with SV.

2.1 BVAR with Real-Time Vintages

Consider the simple case of a forecaster using a vector autoregressive model of order p for forecasting in real time. If the forecaster employs the latest-available vintage, that is, EOS, she will estimate the following model:

$$\mathbf{y}_t^{T+1} = \boldsymbol{\beta}^{EOS}\mathbf{x}_{t-1}^{T+1} + \mathbf{e}_t^{EOS}, \quad \text{for } t = p+1, .., T \tag{1}$$

where $\mathbf{y}_t^{T+1}$ is a $N \times 1$ vector of the vintage $T+1$ estimate of each variable for the reference period t value, where t runs from $p+1$ up to T. The lags are also obtained from the latest vintage as $\mathbf{x}_{t-1}^{T+1} = (1, \mathbf{y}_{t-1}^{T+1\prime}, ..., \mathbf{y}_{t-p}^{T+1\prime})'$, implying that $\boldsymbol{\beta}^{EOS}$ is $N \times (Np+1)$ matrix. We assume the data are published with a one period delay.

If the forecaster has access to $T-1$ past vintages of the endogenous variables, that is, she has access to a real-time database, then RTV for the VAR(p) model is given by estimation of:

$$\mathbf{y}_t^{t+1} = \boldsymbol{\beta}^{RTV}\mathbf{x}_{t-1}^{t} + \mathbf{e}_t^{RTV}, \text{ for } t = p+1, .., T \tag{2}$$

where $\mathbf{y}_t^{t+1}$ is the first estimate of each endogenous variable for reference period t such that $\{\mathbf{y}_t^{t+1}\}_{t=p+1}^{t=T}$ is the time series of first releases for each variable. The $Np+1$ vector of right-hand side variables consist of:

$$\mathbf{x}_{t-1}^{t} = (1, \mathbf{y}_{t-1}^{t\prime}, \mathbf{y}_{t-2}^{t\prime}, ..., \mathbf{y}_{t-p}^{t\prime})', \text{ for } t = p+1, ..., T,$$

that is, all lags are taken from the vintage at t, so for more than one lag ($p > 1$), at least partially-revised data are used.

When forecasting in real-time, the forecasts are typically conditioned on initial and early releases, that is, on $\mathbf{x}_T^{T+1}$ (and $\mathbf{x}_{T-1}^{T+1}$ etc. depending on p).[1] As described by Clements and

[1] By 'real time' we mean feasible forecasts that could have been made at the time the forecasts are assumed

Galvão (2013), estimating the VAR with RTV data gives theoretically optimal forecasts, as opposed to using EOS. Hence the use of RTV is a promising way of dealing with variables subject to data revisions. Clements and Galvão (2013) apply RTV to univariate models and predictive regressions, but they do evaluate the forecasting performance of RTV applied to VAR models.

The literature on forecasting with VAR models typically employs Bayesian estimation, enabling the use of prior restrictions to mitigate dimensionality issues. We follow the literature and estimate the models in eqs. (1) and (2) using Bayesian methods. We estimate the BVAR as in Carriero, Clark and Marcellino (2015) with a Minnesota prior and an overall tightness prior chosen by maximizing the marginal data density. The restrictions from the imposition of prior information mean that the Clements and Galvão (2013) justification for RTV is no longer strictly applicable, even in principle, but the practical importance of RTV remains an empirical issue.

Draws from the predicted density are obtained using 10,000 draws from the posterior densities of the parameters (including the variance-covariance matrix of the disturbances) with multi-step forecasts obtained by iteration, including draws from the disturbances. We compute point forecasts and their forecasting uncertainty using the mean and the variance of the predicted density draws for each horizon.

2.2 The KK BVAR Approach

An alternative to RTV to deal with data uncertainty, when forecasting variables subject to data revisions, is the approach proposed by Kishor and Koenig (2012), henceforth KK. KK propose estimating the VAR on the 'true values' of the endogenous variables, that is:

$$\mathbf{y}_t = \mathbf{c}^{KK} + \boldsymbol{\beta}^{KK}\mathbf{x}_{t-1} + \mathbf{e}_t^{KK}, \text{ for } t = p+1, .., T, \tag{3}$$

where $\mathbf{x}_{t-1} = (\mathbf{y}'_{t-1}, ..., \mathbf{y}'_{t-p})'$. The problem of course is that we do not observe true (or even the revised) values of all the observations at time $T+1$, the forecast origin. The solution is to simultaneously model data revisions to provide 'true values' up to T. The main assumption

to have been made.

is that the true values, or an efficient estimate of these, is available l quarters after the reference quarter, that is, $\mathbf{y}_t = \mathbf{y}_t^{t+l}$ for $t = p+1, ..., T-l+1$. For the remaining observations, $\mathbf{y}_{T-l+2}^{T+1}, \ldots, \mathbf{y}_T^{T+1}$, KK suggest using a system of equations and the Kalman Filter. Let $\mathbf{rev}_t$ denote the $N(l-1) \times 1$ vector of revisions given by

$$\mathbf{rev}_t = ((\mathbf{y}_t^{t+1} - \mathbf{y}_t)', ..., (\mathbf{y}_t^{t+l-1} - \mathbf{y}_t)')'. \tag{4}$$

Then we assume the equations for revisions are given by a VAR(1):

$$\mathbf{rev}_t = \mathbf{k}_0 + \mathbf{K}\mathbf{rev}_{t-1} + \mathbf{w}_t. \tag{5}$$

Were we to estimate (3) and (5) separately, then data revisions would be treated as being serially-correlated measurement errors. This ignores the possibility that revisions may add new information (see, e.g., Mankiw and Shapiro (1986), and Clements and Galvão (2019) for a review). If data revisions are news, then $cov(\mathbf{e}_t^{KK}, \mathbf{w}_t)$ would not be the null matrix. KK propose estimating both equations jointly using the seemingly unrelated regression estimator (SURE) with observations up to $T-l+1$, and then using the Kalman filter and observations on initial releases up to T to obtain filtered values for $\mathbf{y}$ up to T. Finally, forecasts of the revised values of future observations, $\mathbf{y}_{T+1}, ..., \mathbf{y}_{T+h}$ are obtained by iteration using eq. (3) conditional on $\mathbf{y}_T, \mathbf{y}_{T-1}, ...$ Forecasts of future values of first releases, $\mathbf{y}_{T+1}^{T+2}, ..., \mathbf{y}_{T+h}^{T+h+1}$, require forecasts of the revision vector too, obtained via iteration using eq. (5).

A concern is that the number of coefficients in eq. (5), and the number of states in the state-space representation of the model, increase rapidly in l. At a rate N^2 for the coefficients, and N for the number of states and observations equations. We follow KK and set $l = 2$. This supposes that the second quarterly estimate can be taken to be an efficient estimate of the true value. Setting l to a low value facilitates the inclusion of stochastic volatility in the innovations in eqs. (3) and (5), as explained in the next section.

We estimate eqs. (3) and (5) by Bayesian methods. Assuming $l = 2$, the model is cast in state-space form, assuming no errors in the measurement equations, as:

$$\begin{bmatrix} \mathbf{y}_t^{t+1} \\ \mathbf{y}_{t-1}^{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{c}^{KK} + \mathbf{k}_0 \\ \mathbf{c}^{KK} \end{bmatrix} + \begin{bmatrix} I_N & 0_N & \cdots & 0_N & I_N \\ 0_N & I_N & \cdots & 0_N & 0_N \end{bmatrix} \begin{bmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \vdots \\ \mathbf{y}_{t-p+1} \\ \mathbf{rev}_t^{(1)} \end{bmatrix}. \tag{6}$$

This implies we use time series of first releases, $\left\{\mathbf{y}_t^{t+1}\right\}_{t=p+1}^{t=T}$, and of second releases, $\left\{\mathbf{y}_{t-1}^{t+1}\right\}_{t=p+1}^{t=T}$, in estimation. Note that $\mathbf{y}_{t-1} = \mathbf{y}_{t-1}^{t+1} - \mathbf{c}^{KK}$, because the second release is equal to the true values, and we include the VAR intercepts in the measurement equations. By way of contrast, the first release is not equal to the true values, but instead $\mathbf{y}_t^{t+1} - \mathbf{c}^{KK} - \mathbf{k}_0 = \mathbf{y}_t + \mathbf{rev}_t^{(1)}$. The state equations are then:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \vdots \\ \mathbf{y}_{t-p+1} \\ \mathbf{rev}_t^{(1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\beta}_{p=1}^{KK} & \boldsymbol{\beta}_{p=2}^{KK} & \cdots & \boldsymbol{\beta}_{p=p}^{KK} & 0_N \\ I_N & & & & \vdots \\ & \ddots & & & \vdots \\ & & I_N & & 0_N \\ & & & 0_N & \mathbf{K} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{y}_{t-2} \\ \vdots \\ \mathbf{y}_{t-p} \\ \mathbf{rev}_{t-1}^{(1)} \end{bmatrix} + \begin{bmatrix} I_N & 0_N \\ 0_N & \vdots \\ \vdots & \vdots \\ \vdots & 0_N \\ 0_N & I_N \end{bmatrix} \begin{bmatrix} \mathbf{e}_t^{KK} \\ \mathbf{w}_t \end{bmatrix}. \tag{7}$$

Define the $2N \times 1$ vector of disturbances in the state equation by $\zeta_t = (\mathbf{e}_t^{KK\prime}, \mathbf{w}_t^\prime)^\prime$. When $var(\zeta_t) = \mathbf{Q}$ is a full matrix, the revisions disturbances are correlated with the disturbances to the true values, allowing for revisions to be news.

If we further assume that the disturbances of the state equation are $\zeta_t \sim N(0, \mathbf{Q})$, then we can use the Kalman filter and a smoother to obtain estimates for the state variables $\mathbf{y}_t$ and $\mathbf{rev}_t^{(1)}$ if the parameters $\mathbf{c}^{KK}$, $\boldsymbol{\beta}^{KK}$, $\mathbf{k}_0$, $\mathbf{K}$ and $\mathbf{Q}$ in (6) and (7) are known. To be able to jointly estimate state-equation parameters and the unobserved components, we obtain draws for the time-series of state variables ($\alpha_t = (\mathbf{y}_t^\prime, ..., \mathbf{y}_{t-p+1}^\prime, \mathbf{rev}_t^{(1)\prime})^\prime$) using that $\alpha_t \sim N(\alpha_{t|T}, P_{t|T})$. We use the Carter and Kohn (1994) filtering/smoothing approach to obtain $\alpha_{t|T}$ and $P_{t|T}$ for $t = 1, ..., T$, with smoothing step as described in Galvão (2017). To obtain posterior distribution draws for all the parameters, we employ a Gibbs sampler, so the first step is to obtain draws for the unobserved components as just described, and the second step is to obtain draws for the VAR parameters assuming $\mathbf{y}_t$ and $\mathbf{rev}_t^{(1)}$ are fully observed (up to T). We assume normal-Wishart conjugate priors, that is, we apply the SUR model strategy described by Greenberg (2013, ch. 10.1) to draw from normal-Wishart conditional distributions. The SUR model approach is required when $\mathbf{Q}$ is full. We use Minnesota-type priors for the parameters of eqs. (3) with the overall prior tightness fixed at the same value as in a standard BVAR model with only the observed values.

We run the Gibbs sampler over 10,000 draws, remove the first 20% as burn-in, and compute multi-step forecasts for each one of the kept draws by iteration including draws from the disturbances. We compute point forecasts and forecast uncertainty as described earlier for the BVAR specifications.

2.3 Adding Stochastic Volatility

In the macroeconomic forecasting literature stochastic volatility has been found to play a key role in density forecasting (see in particular Clark and Ravazzolo (2014)). Hence we allow for time-variation in the volatility of the disturbances in eqs. (1) and (2) by allowing for a random walk process for the conditional variances. We choose the BVAR-SV specification and estimation algorithm used by Carriero, Clark and Marcellino (2019).[2] The specification is such that the variance of each disturbance in the VAR may change slowly over time, but the covariances are fixed.

For the KK BVAR model, we add stochastic volatility in two ways. The first one assumes that

$$var \begin{bmatrix} \mathbf{e}_t^{KK} \\ \mathbf{w}_t \end{bmatrix} = var(\zeta_t) = \mathbf{Q}_t = \mathbf{A}^{-1}\mathbf{\Lambda}_t\mathbf{A}^{-1}$$

where $\mathbf{\Lambda}_t$ is a diagonal matrix and $\mathbf{A}^{-1}$ is lower triangular with ones on its main diagonal, as in Carriero *et al.* (2019). This specification permits time-varying volatilities for the disturbances in the equations for both the true values and the revisions. This also has the advantage of allowing revisions to be news (i.e., that data revisions may be incorporated into true values),

[2] We use the code made available on M. Marcellino's webpage.

since $cov(\mathbf{e}_t^{KK}, \mathbf{w}_t)$ is allowed to be non-zero. Allowing for SV in this form implies that:

$$\zeta_t = \mathbf{A}^{-1}\mathbf{\Lambda}_t^{0.5}\eta_t,\ \eta_t \sim N(0, \mathbf{I}_{2N}).$$

The fact that $\mathbf{\Lambda}_t$ is diagonal implies that the j^{th} element of the rescaled disturbances, $\tilde{\zeta}_t = \mathbf{A}\zeta_t$, can be written as $\tilde{\zeta}_t = \sqrt{\lambda_{j,t}}\eta_{j,t}$. The observational link between the disturbances of the KK model (eq. (7)) and the unobserved volatility processes is:

$$\ln \tilde{\zeta}_t^2 = \ln \lambda_{j,t} + \ln \eta_{j,t}^2. \tag{8}$$

And the process for the time-varying volatility is given by:

$$\ln \lambda_{j,t} = \ln \lambda_{j,t-1} + \epsilon_{j,t},\ \epsilon_{j,t} \sim N(0, \Phi), \tag{9}$$

where Φ is full. We estimate the KK BVAR-SV model by adding additional steps to the Gibbs sampler described earlier to implement the Kim, Shephard and Chib (1998) algorithm to draw values for the time-varying volatilities using the state-space form implied by eqs. (8) and (9). We also include an additional step to draw the values in $\mathbf{A}$ using a Gaussian density.

Although the empirical literature provides evidence of changes in volatility for true values of macroeconomic series (see, e.g., McConnell and Perez-Quiros (2000) and Sensier and van Dijk (2004)), it is not clear that the variability of revisions has also been changing. This motivates our second way of adding stochastic volatility. We suppose that only $var(\mathbf{e}_t^{KK})$ changes over time, with $var(\mathbf{w}_t)$ assumed constant. To do so, we need to assume that $\mathbf{Q}_t$ is a block diagonal matrix, implying that $cov(\mathbf{e}_t^{KK}, \mathbf{w}_t)$ is constrained to zero. The block diagonality assumption allows us to treat the errors of eq. (3) as described in the previous paragraph, while drawing the values for the revision block (eq. (5)) from a Wishart distribution. From a practical perspective, this constrained specification leads to fewer issues with the sampler algorithm when the estimation sample is small. But has the disadvantage of not accommodating news revisions.

As before, we use a Gibbs sampler to obtain the posterior distribution of the parameters and the unobserved components (including the stochastic volatility). We compute forecasts by iteration for each set of the posterior parameters. Forecasts are obtained as in Clark and

Ravazzolo (2014), so draws from the disturbances are included at each horizon. These draws use $\zeta_t \sim N(0, Q_{T+h})$, that is, the variance covariance matrix may change with the horizon, as we compute $\ln\lambda_{j,T+1}, ..., \ln\lambda_{j,T+h}$ using equation (9) with draws from the variance equation disturbances. We apply a similar approach to calculate forecasts from the BVAR-SV models.

3 Applications to US and UK real-time density forecasting

In this section, we use BVAR models estimated by both RTV and KK to forecast US and UK macroeconomic variables. Our aim is to assess the relevance of accounting for data uncertainty when making probabilistic forecasts of macro-variables using BVARs, and to determine whether one of the two approaches we consider is superior to the other.

3.1 Forecasting Exercise Design

The BVAR model estimated with EOS is the benchmark against which we assess forecasting performance, as this is the approach typically used in the literature (as in, for example, Clark and Ravazzolo (2014)). We are interested in real-time forecasting, and EOS is a real-time approach, in that it makes use only of data (including vintages of data) available at the time the forecasts are made. But as argued by Koenig *et al.* (2003) and Clements and Galvão (2013), amongst others (and see the review by Clements and Galvão (2019)), other real-time approaches may give superior forecasts. RTV and KK are considered as alternatives which account for data uncertainty. These approaches are implemented both with, and without, stochastic volatility. This results in the following set of forecasting models: BVAR RTV, BVAR-SV EOS, KK-BVAR, KK-BVAR-SV and KK-BVAR-SVT. The last specification (KK-BVAR-SVT) allows for SV in the innovations in eq. (3) but not in eq. (5).

The VAR comprises four key quarterly macroeconomic variables: the first differences of the logs of real GDP (GNP prior to 1991) and the GDP deflator (so that these variables are effectively growth rates), the unemployment rate and an interest rate. These are the variables considered by Clark and Ravazzolo (2014) for the US. They establish the benefits of allowing for SV when forecasting with BVAR models. They use our benchmark method, EOS.

For the US, the real-time data for real GDP, the GDP deflator and unemployment are all obtained from the Philadelphia Fed Real-Time Database,[3] and we consider quarterly vintages. Prior to 1991, GDP is GNP. The interest rate is the Treasury Bill rate. For the UK, a similar set of variables is used. We obtain monthly real-time vintages from the Office of National Statistics (ONS) website on real GDP (GDP in chained volume measures) and on nominal GDP (GDP at current prices).[4] We compute the implied GDP deflator using the ratio between the nominal and real GDP values, and use the monthly vintages that include first releases as the quarterly vintages. UK data on the unemployment rate and the 3-month interbank rate are taken from the St Louis FRED dataset.[5]

There are some differences between the availability of real-time vintages of data between the two countries, which might be expected to have an impact on the results. For the US, data vintages are available for growth and inflation from 1965Q4, but only from 1990Q1 for the UK (limited by the availability of nominal GDP data). We set the US out-of-sample period to the forecast origins (vintages) of 2000Q3 to 2017Q4 (a total of 70), while for the UK it is 2004Q4 to 2019Q2 (59 origins). For both countries we use increasing estimation windows (a 'recursive scheme'), because larger sample sizes are helpful when forecasting with models with SV. Nonetheless, estimation periods are clearly markedly shorter for the UK.

The description of the KK approach in section 2.2 assumed all variables in the VAR were subject to revision. However, this is only true of GDP growth and the GDP deflator, because revisions to US and UK unemployment are negligible, and the short-rate is not subject to revision. Given $l = 2$, as explained earlier, the vector $\mathbf{rev}_t$ is therefore 2×1.

We evaluate the forecasting accuracy of each BVAR specification for the four variables, for two measurement of actual values. We use the first release such that the target is Y_{T+1}^{T+2} when evaluating one-step-ahead forecasts and Y_{T+4}^{T+5} for four-step-ahead forecasts, and also the second quarterly release (equivalent to the Bureau of Economic Analysis third estimate, and the Office for National Statistics Quarterly National Accounts (ONS QNA) end of quarter release), that is, Y_{T+1}^{T+3} and Y_{T+4}^{T+6}, respectively.

[3] https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/

[4] https://www.ons.gov.uk/economy/grossdomesticproductgdp/datasets/realtimedatabaseforukgdpabmi and:
https://www.ons.gov.uk/economy/grossdomesticproductgdp/datasets/realtimedatabaseforukgdpybha

[5] https://fred.stlouisfed.org/

Two measures of forecast performance are reported. For point forecasts, we use the root mean squared forecast error (RMSFE). For density forecast evaluation, we calculate minus the log of the predictive density score (logscore), such that a smaller (more negative) value is preferred. If $p_{T+1|T}(.)$ is the one-step ahead density of Y_{T+1} made at time T, the logscore is $-\ln(p_{T+1|T}(y_{T+1})$ for realization y_{T+1}. We compute the logscore using its closed-form solution for Gaussian densities (as in Clark and Ravazzolo (2014)), using the mean and the variance obtained using the predictive density draws. When the mean and variance are given by $\mu_{T+1|T}$ and $\sigma^2_{T+1|T}$, the (negative) score is:

$$-\ln(p_{T+1|T}(y_{T+1})) = \frac{(y_{T+1} - \mu_{T+1|T})^2}{2\sigma^2_{T+1|T}} + \frac{1}{2}\ln(\sigma^2_{T+1|T}) + \frac{1}{2}\ln(2\pi) \tag{10}$$

The negative of the logscore computed analytically for a normal predictive density is equivalent to the Dawid-Sebastiani score function. It is a proper score function, meaning that the optimal forecast is to deliver the true density function - there is no incentive to gameplay (as surveyed in Gneiting and Katzfuss (2014)).

We test whether the differences in forecast performance between the models are statistically significant using the Diebold and Mariano (1995) (DM) test statistic. Values of the DM statistic in bold in the tables signify rejection of the null hypothesis of equal accuracy in favour of the alternative (to the benchmark, BVAR EOS) at the 5% level (with critical values from a Gaussian distribution).

3.2 Empirical Results

Tables 1 to 4 present the results of the forecasting exercise. Tables 1 and 2 evaluate one-step-ahead forecasts, and Tables 3 and 4 one-year-ahead forecasts. Tables 1 and 3 are for the US, and Tables 2 and 4 for the UK. Each table presents the two measures of accuracy. Entries for the BVAR EOS model are either RMSFE or the logscore. The remaining entries are either ratios of the RMSFE with respect to the BVAR EOS benchmark, or logscore differences with respect to the benchmark. Ratios smaller than 1 suggest that the alternative forecasting model is more accurate than the BVAR EOS in terms of RMSFE. Negative differences in logscore also suggest improvements in accuracy in comparison with the BVAR EOS.

The tables also present the DM statistics computed with respect to the BVAR EOS forecast performance. The results in Panel A use the first release, y_{T+H}^{T+H+1} as the target variable, and Panel B presents the results for the second quarterly release y_{T+H}^{T+H+2} as the target variable. Note that accuracy only differs across releases for variables subject to revision (that is, for GDP growth and GDP deflator inflation).

Firstly, does SV enhance forecast accuracy? The comparisons between specifications with and without SV suggest that SV improves density forecasts, and even point forecasts, for the US. To illustrate, consider the comparison of the BVAR_SV EOS against the benchmark. The RMSFE ratio of 0.765 (table 1A) suggests a reduction of nearly 25% from SV, which is statistically significant. Similarly, there is a statistically significant improvement in the log score. For the UK the gains are more limited and are usually confined to the deflator and the short-rate. The shorter sample period for the UK data may explain why SV is less beneficial.

At the one-year horizon, whether or not there are improvements from including SV depend in part on whether we look at the point forecasts or density forecasts. The main reason is that the predictive variance increases with the horizon, given the form of the stochastic process assumed for the SV. This can result in improved logscores when the point forecasts are relatively inaccurate, but conversely may be detrimental in other cases.

More importantly for our purposes, the improvements from modelling data uncertainty are also clear. At $h = 1$ the BVAR_SV RTV yields significant improvements for GDP growth for the US, and for inflation for the UK. The qualitative results for forecasting one-quarter ahead do not change if instead we use the second release values. For the US, there is evidence that taking into account data uncertainty also improves the forecasts of variables not subject to data revision - unemployment and the short rate. The improvements for forecasting the short rate as measured by the logscore are mainly at the one-quarter-ahead horizon, while improvements for the unemployment are at the one-year-horizon using the the KK-BVAR-SV model.

The tables also indicate (in blue) the best forecasting model for each variable, horizon and loss function. If we consider only the forecasts for output growth and inflation, we have 32 competitions. Of these, only 4 (1-in-8) are won by a forecasting model that disregards data uncertainty (BVAR-SV EOS). In contrast, a half are won by one of the three KK BVAR specifications.

The findings suggest the KK BVAR models perform well for UK output growth and inflation at the one-year horizon, for first releases. There is some evidence (e.g., point forecasts of output growth) the KK models work well at the longer horizon for the US too. There is less evidence that the performance of models using RTV depends on the forecast horizon.

In the next section, we explain some key aspects of these empirical results. In particular, we relate the density-forecasting performance of RTV to the characteristics of data revisions, and we discuss why the KK approach may perform relatively better at longer horizons.

4 Explaining the Empirical Evidence

The complexity of the forecasting environment once we allow for multivariate models estimated by Bayesian methods, for data revisions, time-varying conditional variances, small-sample parameter estimation uncertainty, etc., means that clearcut analytical expressions that explain the empirical outcomes in their entirety will be hard to come by. In this section, we consider a number of aspects which will influence the outcomes of the empirical forecast comparisons.

4.1 News and Noise Revisions and Forecasting Uncertainty

We start by considering the roles of news and noise revisions in determining the importance of allowing for data uncertainty. That is, the role played by the characteristics of data revisions. Our statistical framework assumes an autoregressive model (AR(1)) for the true (i.e., fully-revised) values y_t:

$$y_t = \phi y_{t-1} + \eta_t + v_t, \ |\phi| < 1 \tag{11}$$

where η_t is the underlying disturbance. Here v_t is a news revision, and the first estimate is given by:

$$y_t^{t+1} = y_t - v_t + \varepsilon_t \tag{12}$$

with $y_t^{t+l} = y_t$ for $l = 2, 3, \ldots$, when we assume the second estimate reveals the true value. ε_t is a noise revision. Then the revision $y_t^{t+2} - y_t^{t+1} \equiv y_t - y_t^{t+1} = v_t - \varepsilon_t$ consists of a noise component (when $\sigma_\varepsilon^2 = E\left(\varepsilon_t^2\right) \neq 0$) and a news component (when $\sigma_v^2 = E\left(v_t^2\right) \neq 0$), see e.g., Mankiw and Shapiro (1986). We assume η_t, v_t and ε_t are mutually uncorrelated, zero-mean random variables.

This setup is a simplified version of the statistical model of Jacobs and van Norden (2011): they allow for l-revisions ($l > 1$); the possibility that the truth is not eventually revealed; and for more general processes for the true data. The statistical framework described in this section is general enough to bring out the impact of certain characteristics of data revisions on the calculation of forecast uncertainty, using both RTV and the KK approaches, compared to the traditional approach.

Pure news revisions (with $\sigma_\varepsilon^2 = 0$) are characterized by the revision being uncorrelated with the first estimate:

$$Cov\left(y_t^{t+2} - y_t^{t+1}, y_t^{t+1}\right) = Cov\left(v_t, \alpha y_{t-1} + \eta_t\right) = 0,$$

and the revised estimate - the fully-revised estimate here - adds the news v_t. Later estimates are more accurate estimates of the true value than earlier estimates (here, $y_t^{t+2} = y_t$) and have larger unconditional variance than earlier estimates. This implies that if data revisions are pure news, we expect that $var(y_t^{t+2}) > var(y_t^{t+1})$.

Conversely, noise revisions remove measurement error: the revisions are predictable (based on period $t-1$ information) but are not correlated with the true value, i.e.:

$$Cov\left(y_t^{t+2} - y_t^{t+1}, y_t^{t+1}\right) = Cov\left(-\varepsilon_t, y_t + \varepsilon_t\right) = -\sigma_\varepsilon^2,$$

but:

$$Cov\left(y_t^{t+2} - y_t^{t+1}, y_t\right) = Cov\left(-\varepsilon_t, y_t\right) = 0.$$

Hence later estimates have smaller unconditional variances, that is, under pure noise revisions $var(y_t^{t+2}) > var(y_t^{t+1})$.

In Tables 1 to 4 we have evaluated forecast performance using both the first and the second releases of output growth and inflation. The forecasts themselves from the BVAR models do not depend on the target, except for the KK BVAR. We can clearly see that the RMSFEs are larger for the second release than for the first release for forecasting output growth. But the opposite is true for inflation. This suggests that data revisions to output growth and inflation are mainly news and noise, respectively, because the RMSFEs reflect the magnitudes of the unconditional variances of the target variables.

Another way of looking at the effects of data revisions is via their effects on forecast uncertainty, $\hat{\sigma}^2_{T+1}$. We graph the time series of estimates of the conditional predictive variances using the KK BVAR-SV model, for the first release $\hat{\sigma}^{2,KK,T+2}_{T+1|T}$, and for the second releases $\hat{\sigma}^{2,KK,T+3}_{T+1|T}$. We do this for both output growth and inflation for the US: see Figure 1. As expected, for output growth $\hat{\sigma}^{2,KK,T+2}_{T+1|T} < \hat{\sigma}^{2,KK,T+3}_{T+1|T}$ for all quarters, because data revisions are news. Although differences are small in some periods, in others they are large, such as in 2009. For inflation the differences between the two series of predicted variances are generally small, and sometimes change sign. At least for output growth, though, the KK BVAR-SV suggests that data revisions increase the predicted variability of 'true' US output growth, relative to that of the first estimate. And this is consistent with revisions to output growth by the statistical agency adding news.

The characteristics of data revision also affect the calculation of forecast uncertainty by EOS and RTV. To understand why, first define δ as the relative size of the data revision process, that is:

$$\delta = \frac{var(rev_t)}{\sigma^2_\eta},$$

implying that if data revisions are news, $\delta = \sigma^2_v/\sigma^2_\eta$, and if data revisions are noise, $\delta = \sigma^2_\varepsilon/\sigma^2_\eta$. Suppose eqs. (11) and (12) are supplemented with eq. (13):

$$\begin{aligned} \eta_t &= \sigma_\eta \xi_{1t};\ v_t = \sigma_v \xi_{2t};\ \varepsilon_t = \sigma_\varepsilon \xi_{3t} \\ \text{where } \xi_{it} &\sim iidN(0,1) \text{ for } i = 1,2,3, \text{ and are mutually uncorrelated,} \end{aligned} \tag{13}$$

to emphasize the independence of the three disturbances, as well as that they are assumed to be homoscedastic. Using the statistical framework given by the data generation process, eqs. (11), (12) and (13), and the assumption that the forecasting model is an AR(1) with an intercept, Clements (2017) provides expressions for the predictive variances for EOS and RTV. In our empirical exercise, the model is a VAR(p), so that the results here are indicative. The predicted variances (forecast uncertainty), under EOS and RTV are denoted by $\sigma^{2,EOS}_{T+1|T}$ and

$\sigma^{2,EOS}_{T+1|T}$, respectively, and for the case of pure news revisions are given by:

$$\begin{aligned}\sigma^{2,EOS}_{T+1|T} &= \sigma^2_\eta + \sigma^2_v = \sigma^2_\eta(1+\delta)\\ \sigma^{2,RTV}_{T+1|T} &= \sigma^2_\eta + \phi^2\sigma^2_v = \sigma^2_\eta(1+\phi^2\delta).\end{aligned}$$

Under stationarity $\phi^2 < 1$, so that $\sigma^{2,EOS}_{T+1|T} > \sigma^{2,RTV}_{T+1|T}$. Thus, for news revisions the EOS forecast-error variance exceeds that of RTV.

Next, consider noise revisions. Because all but the last observation used to compute $\sigma^{2,EOS}_{T+1|T}$ are revised data, for large T the EOS forecast of the variance is simply:

$$\sigma^{2,EOS}_{T+1|T} = \sigma^2_\eta \tag{14}$$

The one-step-ahead variance with RTV is:

$$\sigma^{2,RTV}_{T+1|T} = \sigma^2_\eta(1+\delta+\varrho), \tag{15}$$

where $\varrho = [\phi^2(B-1)^2/(1-\phi^2) + \delta B^2\phi^2]$. This expression is derived in the Appendix, where we show that $B < 1$ and so $\varrho > 0$. Consequently, for pure noise revisions, we expect that $\sigma^{2,EOS}_{T+1|T} < \sigma^{2,RTV}_{T+1|T}$, the opposite to the finding for news.

Because RTV accurately reflects one-step ahead forecast uncertainty for the first release value, i.e., for y^{T+2}_{T+1} (as shown by Clements (2017)), in this simple setup EOS under-estimates future uncertainty when there are noise revisions, but over-estimates uncertainty for news revisions.

These analytical results abstract from stochastic volatility, and assume the forecasting model is an AR(1). Given that the empirical exercise shows SV improves the BVAR density forecasts (in particular for the longer time series available for the US data), we check whether these findings carry over to SV models. Figure 2 presents the empirical ratio of the time series $\sigma^{2,EOS}_{T+1|T}/\sigma^{2,RTV}_{T+1|T}$ for US output growth and inflation. The results are as expected, in that the ratio is usually above one for output growth, for which revisions are news. For inflation the ratio is generally lower, and for the most part is less than one, which is consistent with noise revisions.

From Figure 2, the largest differences between the EOS and RTV predicted variances are for output growth forecasts after 2011 (with ratios from 1.5 up to in excess of 3). This is consistent with the large forecasting improvement on logscore from using RTV (instead of EOS) for the BVAR-SV for output growth: the improvement in accuracy is close to 25%. For inflation the improvement is much more muted at around 5%. (See the last panel of Table 1).

In summary, we have shown how the characteristics of data revisions (news/noise) can affect predicted forecast uncertainty in a relatively simple setup, and have shown that these effects carry over to the empirical comparisons involving the BVAR-SV forecasting models.

4.2 Why does RTV improve logscore performance relative to EOS?

The findings for predictive variances - when the model is estimated by RTV or EOS - directly affect log score performances, as described below. We provide some analytical results consistent with the empirical findings that RTV often delivers superior densities on logscore at the one-quarter-horizon for both output growth and inflation, and for the US and the UK data.

For the DGP given by (11), (12) and (13), propositions 1 and 2 provide the analytical values of the expected differences between EOS and RTV logscores for an AR(1) forecasting model. From (10), the logscore for a Gaussian forecast density is:

$$E[-\ln(p_{T+1|T}(y_{T+1}))] = E\left[\frac{(y_{T+1}-\mu_{T+1|T})^2}{2\sigma^2_{T+1|T}} + \frac{1}{2}\ln(\sigma^2_{T+1|T}) + 0.5\ln(2\pi)\right] \quad (16)$$

where $p_{T+1|T}(.)$ is the one-step-ahead density, $\mu_{T+1|T}$ is the mean of the predictive density, $\sigma^2_{T+1|T}$ is the predicted variance, and y_{T+1} is the realization.

Proposition 1 *The difference between the EOS and RTV logscores,* $\Delta score^{News}$*, for an* $AR(1)$ *model, when the target is the initial release* y^{T+2}_{T+1}*, data revisions are pure news, and the DGP is given by (11), (12) and (13), is given by:*

$$\Delta score^{News} = \frac{1}{2}\left[\frac{\delta(\phi^2-1)}{1+\delta} + \ln[(1+\delta)/(1+\phi^2\delta)]\right]. \quad (17)$$

The Proof is given in the Appendix, where we show that $\Delta score^{News} \geq 0$*, that is, the loss for EOS density forecasts exceeds that of the RTV density forecasts.*

Proposition 2 *The difference between the EOS and RTV log scores, $\Delta score^{Noise}$, for an $AR(1)$ model, when the target is the initial release y_{T+1}^{T+2}, data revisions are pure noise, and the DGP is given by (11), (12) and (13), is given by*

$$\Delta score^{Noise} = \frac{1}{2}\left[(\delta(1+\phi^2)) - \ln(1+\delta+\varrho)\right]. \tag{18}$$

The Proof is given in the Appendix, where we show that $\Delta score^{Noise} \geq 0$, that is, the loss for EOS density forecasts exceeds that of the RTV density forecasts..

Figure 3 shows how $\Delta score^{News}$ and $\Delta score^{Noise}$ vary with the revision size δ, and the persistence of the underlying process ϕ, for some illustrative values. For a given revision size δ, it is apparent that differences in logscore are larger when when data revisions are noise (compare the scales of the vertical axes). Moreover, logscore differences are increasing in ϕ for noise revisions, but declining in the persistence of the process for news revisions.

Figure 3 suggests we might expect to see larger differences between EOS and RTV one-step-ahead logscores for inflation than output growth, if revisions to inflation are noise, and those to output news. If we look at the results in table 2 for the UK, we do indeed find larger gains from RTV over EOS for inflation, than for output. For US data, we find the gains are smaller. A close examination of figures 1 and 2 suggests that although the broad categorization of early US inflation revisions as being noise may be correct, there are periods where the binary news/noise dichotomy may not hold. If US inflation revisions are not pure noise, then the lack of corroboration between the analytical results and empirical evidence would not be surprising.

An additional implication of Figure 3 is that if data revisions are news, then larger gains should result for series that are a) less persistent and b) have larger revisions. The first-order serial correlation of the second release of UK GDP growth is approximately 0.6, compared to 0.5 for the US, while revision are similar in size. Comparing the logscore differences for the UK growth in Table 2 to the those for the US in Table 1, we find that RTV gains are indeed smaller for the UK, as expected.

The analytical results presented here provide insight into why larger improvements in real-time forecasting with BVARs are obtained from using RTV (instead of EOS) for some variables as opposed to others.

4.3 Why the KK model performs better at longer horizons

We consider why the KK model performs relatively better at longer horizons ($h = 4$), especially for the first-release actuals. Compare, for example, the KK model US GDP growth forecasts in tables 1 ($h = 1$) and 4 ($h = 4$). Intuitively, RTV might be expected to beat KK for forecasting the first release because in essence it exploits the correlation between adjacent first-release data estimates. It uses only observed data, and the forecast is conditioned on $\mathbf{y}_T^{T+1}$. This simple approach might be more robust in some circumstances than KK. Forecasts from the KK BVAR model are conditioned on filtered estimates of $\mathbf{y}_T$, since we only observe $\mathbf{y}_T^{T+1}$, and not $\mathbf{y}_T^{T+2}$. The KK approach generates a forecast of the true value, and then adjusts this using the forecast revision to obtain $\mathbf{y}_{T+1|T}^{T+2}$, a forecast of the first release. The relative advantage of RTV over KK is likely to diminish at longer horizons if the KK approach accurately captures the dynamics of revisions and true values.

We might expect the KK BVAR approach to produce predicted densities which are relatively more accurate than RTV for $\mathbf{y}_{T+1}^{T+3}$, than for $\mathbf{y}_{T+1}^{T+2}$, in principle, because the RTV estimates of the two densities are the same. In practice, any gains will be tempered by the similarity between the KK predictive variances for the two targets for many time periods (as shown in figure 3).

5 A Monte Carlo Exercise

In this section, we report the results of a Monte Carlo exercise designed to further explore when we should expect gains from considering data uncertainty when computing forecasts with BVAR models in real time, including the effects of forecast horizon and forecast target on the relative performance of the RTV and the KK approaches. As in the empirical exercise, we measure forecast performance using the RMSFE and the logscore, and we evaluate relative forecasting performance using the Diebold and Mariano test for equal forecasting accuracy using the BVAR EOS (the traditional way to deal with real-time data when forecasting) as the benchmark forecasting method.

The data generation process (DGP) is the KK model. We assume that y_t^{t+14} reveals the true values, so as to encompass not only the initial revisions but also the subsequent annual rounds of revisions.[6] The model is estimated on US data for output growth and GDP deflator inflation. An advantage of having the KK model as the DGP is that we are able to accommodate data revisions being a combination of news and noise revisions. We consider a VAR(1) for the $(l-1) = 13$ data revisions processes for each variable (output growth and inflation), and we allow revisions across variables and maturity to be contemporaneously and dynamically correlated. For the true values, we consider a VAR (4), and we estimate the KK VAR model with data from 1985, that is, for the period after the Great Moderation. This lends support to the assumption that the volatility of the disturbances is constant, that is, we do not allow for SV in the DGP. As $l > p$, the extended measurement equation for this case can be written as:

$$\begin{bmatrix} \mathbf{y}_t^{t+1} \\ \vdots \\ \mathbf{y}_{t-p+1}^{t+1} \\ \vdots \\ \mathbf{y}_{t-l+1}^{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{c}^{KK} + \mathbf{k}_{0,1} \\ \vdots \\ \mathbf{c}^{KK} + \mathbf{k}_{0,p} \\ \vdots \\ \mathbf{c}^{KK} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_N & & & & \mathbf{I}_N & & & & \\ & \ddots & & & & \ddots & & & \\ & & \mathbf{I}_N & & & & \mathbf{I}_N & & \\ & & & \ddots & & & & \ddots & \\ & & & & \mathbf{I}_N & & & & \mathbf{0}_N \end{bmatrix} \begin{bmatrix} \mathbf{y}_t \\ \vdots \\ \mathbf{y}_{t-p+1} \\ \vdots \\ \mathbf{y}_{t-l+1} \\ \mathbf{rev}_t^{(1)} \\ \vdots \\ \mathbf{rev}_t^{(p)} \\ \vdots \\ \mathbf{rev}_t^{(l-1)} \end{bmatrix}.$$

And for the state equation, $\mathbf{rev}_t$ is as defined in (4), implying that $\mathbf{K}$ is a $(l-1)N$ square

[6] See e.g., Fixler, Greenaway-McGrevy and Grimm (2014) on US data releases.

matrix, and we have:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \vdots \\ \mathbf{y}_{t-p+1} \\ \vdots \\ \mathbf{y}_{t-l+1} \\ \mathbf{rev}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\beta}^{KK}_{p=1} & \boldsymbol{\beta}^{KK}_{p=2} & \cdots & \boldsymbol{\beta}^{KK}_{p=p} & & & \\ \mathbf{I}_N & & & & & & \\ & \ddots & & & & & \\ & & \mathbf{I}_N & & & & \\ & & & \ddots & & & \\ & & & & \mathbf{I}_N & & \\ & & & & & & \mathbf{K} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{y}_{t-2} \\ \vdots \\ \mathbf{y}_{t-p} \\ \vdots \\ \mathbf{y}_{t-l+2} \\ \mathbf{rev}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_N & \mathbf{0}_N \\ \mathbf{0}_N & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{I}_N \end{bmatrix} \begin{bmatrix} \mathbf{e}^{KK}_t \\ \mathbf{w}_t \end{bmatrix}.$$

Using data generated from the KK model, the end-of-sample vintage available at the forecasting origin $T+1$ $\left\{\mathbf{y}^{T+1}_t\right\}_{t=1}^{t=T}$ is $\mathbf{y}^{T+1}_t = \mathbf{y}_t$ for $t = 1, ..., T-l+1$, then $\mathbf{y}^{T+1}_{T-l+2} = \mathbf{y}_{T-l+1} + \mathbf{rev}^{(l-1)}_t$ and so on up to $\mathbf{y}^{T+1}_T = \mathbf{y}_T + \mathbf{rev}^{(1)}_t$. This implies that the variability of the most recent $l-1$ observations which are still subject to revision will differ from that of the fully-revised earlier values. As a consequence, it may be that allowing for stochastic volatility in the BVAR estimated on $\left\{\mathbf{y}^{T+1}_t\right\}_{t=1}^{t=T}$ will result in better forecast. The main reason is that SV may help to accommodate the changing data variability due to the different maturities of the observations. For comparison purposes, we consider both the BVAR-SV EOS and BVAR-SV RTV, in addition to the BVAR RTV.

Although we generate data from a KK BVAR for $l = 14$, we use a KK BVAR with $l = 2$ as one of the alternative models included in the Monte Carlo. This is so for two reasons. First because it mimics the empirical exercise in section 3. Output growth and inflation data are subject to annual revisions, but we only model the initial revisions as the real-time target is either first estimates or the 'first final' (or second estimates). This is supported by the fact that in real time, one takes additional 3 years to observe y^{t+2}_t in contrast to y^{t+14}_t, making it hard to update forecasting strategies in real-time. Second, the estimation of the KK model with large l is computationally intensive because of the large number of state variables. This challenge can be overcome if the model is used as the DGP and estimated just once, but if the plan is to estimate the model P times per Monte Carlo replication, then the exercise is not feasible in a reasonable time frame.

We consider a relatively short sample period, matching that of the empirical exercise. The in-sample period $T = 150$, and the out-of-sample period is set to $P = 50$.[7] We evaluate forecasts for both first and second releases, and for two horizons: one-step-ahead and four-steps-ahead. We set the number of draws to approximate the posterior distribution and the predictive density to 10,000 and the number of Monte Carlo replications is 288. The number of replications is small because of the computational time required to re-estimate each forecasting model with increasing samples over the out-of-sample period, P. Our estimates for accuracy measures (RMSFE and logscore) are actually over 50×288 realizations. The number of replications is used to calculate the rejection rates for the DM statistic. On each replication, DM is computed using $P-h$ observations, as a 5% level one-sided test against the BVAR EOS. The percentage of rejections across replications estimates the rejection frequency against equality with the BVAR EOS, the same benchmark as in the empirical exercises.

Table 5 presents the results of the Monte Carlo exercise. As in the case of the empirical exercises, the values for the BVAR EOS column are either the RMSFE or the logscore. For the remaining models we report ratios to these values (RMSFE) or differences with (Logscore). We summarize out findings as follows.

First, gains from dealing with data uncertainty are reduced as we increase the horizon from $h = 1$ to $h = 4$. Second, the KK BVAR modelling is a better option than BVAR RTV if the target is the second instead of the first release. The BVAR RTV delivers the most accurate point and density forecasts for nowcasting ($h = 1$) first releases. This confirms our empirical results that suggest that the KK BVAR is not as good as the BVAR RTV for forecasting current-quarter first-estimates of output growth and inflation. Third, we are more likely to find statistically significant improvements relative to the benchmark (BVAR EOS) for densities on logscore, as opposed to point forecasts using RMSFE. Gains from accounting for data uncertainty are more readily apparent for density forecasting.

Finally, BVAR-SV EOS improves on BVAR EOS for logscore when predicting the second release. A point of note is that the second-release unconditional variance exceeds that for the first-release, as evidenced by the benchmark model RMSFEs and logscores for the second-release being larger than for the first release. As explained earlier, although $var(\zeta_t)$ is homoscedastic in our DGP, the fact that the variance changes across maturities in the last 13 observations

[7] At each replication, we simulate $150 + 50 + 100 = 300$ observations and discard the first 100, so as to remove the effects of the initial values on our results.

may explain why the stochastic volatility specification may do better than assuming constant volatility. Looking back at our empirical exercises in section 3, the evidence that SV improves forecasting performance may reflect not just changes in the volatility of shocks, but also its ability to accommodate varying data uncertainty of the end of the sample due to the different data maturities.

Using a KK model as a DGP ought to favour the KK forecasting model. That we nevertheless find a pattern of results that broadly matches our empirical findings suggests that the DGP captures some of the salient features of the data and does not unduly bias the findings in favour of the KK model.

6 Conclusions

In this paper we consider whether it is possible to improve on the standard practice of effectively ignoring data uncertainty when generating density forecasts from Bayesian VAR models. By 'data uncertainty' we mean that the recent observations at the time a forecast is to be made will be subject to future data revisions. Such observations are therefore uncertain. Two methods are suggested as offering potential improvements - the use of real-time-vintage data (RTV), and simultaneously modelling data revision along with the true or fully-revised values of the data. We explore these two possibilities, relative to the standard approach, in empirical exercises using typical small VARs for output growth and inflation for the US and the UK.

We then consider what can be learnt from analytical expressions for differences in forecast accuracy between the various models and methods. We consider simplified settings so that analytical results can be derived, and these serve to highlight certain features of the forecasting environmental which are influential in determining rankings across models and approaches. Notwithstanding the complexity of the empirical forecasting environment, including the use of multivariate models estimated by Bayesian methods, allowing time-varying conditional variances, etc., we find the analytical results do illuminate the empirical findings.

Lastly, we resort to Monte Carlo to dig deeper, and are able to obtain a reasonable match between the simulation findings and the empirical comparisons.

In summary, our findings suggest the following. Making an allowance for data uncertainty can lead to improvements in forecast accuracy for (small) BVAR models. Of the 32 empirical 'forecasting competitions' we consider, comprising combinations of variable, forecast horizon, and loss function, in only 1 in 8 of these is the winner a model which disregards data uncertainty. Beyond that, the picture is more nuanced. Modelling data revisions appears to be relatively better than RTV at longer horizons when the aim is to forecast revised values of the observations.

We confirm the finding in the literature that allowing for stochastic volatility tends to improve forecast accuracy. But whilst the improvement in density forecasting is usually attributed to modelling changes in the variances of the underlying shocks in the system, we suggest that the improvements may also reflect the SV process capturing time-varying variances at the end of the estimation sample due to the different maturities of the data. That is, including SV in the model may have the unintended but beneficial effect of dealing with data-revision data uncertainty.

References

Carriero, A., Clark, T. E., and Marcellino, M. (2019). Large Vector Autoregressions with stochastic volatility and non-conjugate priors. *Journal of Econometrics*, **212**, 137–154.

Carriero, A., Clark, T. E., and Marcellino, M. (2015). Bayesian VARs: Specification Choices and Forecast Accuracy. *Journal of Applied Econometrics*, *30*(1), 46–73.

Carriero, A., Clark, T. E., and Marcellino, M. (2020). Capturing Macroeconomic Tail Risks with Bayesian Vector Autoregressions. Discussion paper. Federal Reserve Bank of Cleveland, Working Paper No. 20-02.

Carter, C. K., and Kohn, R. (1994). On Gibbs sampling for state space models. *Biometrika*, *81*(3), 541–553.

Clark, T. E. (2011). Real-time density forecasts from Bayesian Vector Autoregressions with Stochastic Volatility. *Journal of Business and Economic Statistics*, **29**, 327–341.

Clark, T. E., and Ravazzolo, F. (2014). Macroeconomic forecasting performance under alternative specifications of time-varying volatility. *Journal of Applied Econometrics*, *30*(4), 551–575.

Clements, M. P. (2017). Assessing macro uncertainty in real-time when data are subject to revision. *Journal of Business & Economic Statistics*, **35:3**, 420 – 433.

Clements, M. P., and Galvão, A. B. (2013). Real-time forecasting of inflation and output growth with autoregressive models in the presence of data revisions. *Journal of Applied Econometrics*, **28(3)**, 458–477. DOI: 10.1002/jae.2274.

Clements, M. P., and Galvão, A. B. (2019). Data revisions and real-time forecasting. *The Oxford Research Encyclopedia of Economics and Finance.* Oxford University Press. doi:10.1093/acrefore/9780190625979.013.248.

Diebold, F. X., and Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics*, **13**, 253–263.

Diebold, F. X., Schorfheide, F., and Shin, M. (2016). Real-time forecast evaluation of DSGE models with Stochastic Volatility. Mimeo, University of Pennsylvania.

Doan, T., Litterman, R., and Sims, C. A. (1984). Forecasting and conditional projection using realistic prior distributions. *Econometric Reviews*, **3**, 1–100.

Fixler, D. J., Greenaway-McGrevy, R., and Grimm, B. T. (2014). The revisions to GDP, GDI, and their major components. *Survey of Current Business*, **August**, 1–23.

Galvão, A. B. (2017). Data revisions and DSGE models. *Journal of Econometrics*, *196*(1), 215–232.

Gneiting, T., and Katzfuss, M. (2014). Probabilistic forecasting. *Annual Review of Statistics and Its Application*, **1**, 125–51.

Greenberg, E. (2013). *Introduction to Bayesian Econometrics, 2nd Edition*: Cambridge University Press.

Jacobs, J. P. A. M., and van Norden, S. (2011). Modeling data revisions: Measurement error and dynamics of 'true' values. *Journal of Econometrics*, **161**, 101–109.

Kim, S., Shephard, N., and Chib, S. (1998). Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models. *Review of Economic Studies*, *65*(3), 361–393.

Kishor, N. K., and Koenig, E. F. (2012). VAR estimation and forecasting when data are subject to revision. *Journal of Business and Economic Statistics*, **30(2)**, 181–190.

Koenig, E. F., Dolmas, S., and Piger, J. (2003). The use and abuse of real-time data in economic forecasting. *The Review of Economics and Statistics*, **85(3)**, 618–628.

Mankiw, N. G., and Shapiro, M. D. (1986). News or noise: An analysis of GNP revisions. *Survey of Current Business (May 1986), US Department of Commerce, Bureau of Economic Analysis*, 20–25.

McConnell, M. M., and Perez-Quiros, G. (2000). Output fluctuations in the United States: What has changed since the early 1980s?. *American Economic Review*, **90**, 1464–1476.

Rossi, B., and Sekhposyan, T. (2013). Conditional predictive density evaluation in the presence of instabilities. *Journal of Econometrics*, *177*(2), 199–212.

Sensier, M., and van Dijk, D. (2004). Testing for volatility changes in U.S. macroeconomic time series. *The Review of Economics and Statistics*, *86*(3), 833–839.

Sims, C. A. (1980). Macroeconomics and reality. *Econometrica*, **48**, 1–48.

A Proofs

A.1 Preliminaries

We begin by summarising the statistical setup, and repeat for convenience the key equations in the main text.

The negative of the expected log score is given by:

$$E[-\ln(p_{T+1|T}(y_{T+1}))] = E\left[\frac{(y_{T+1} - \mu_{T+1|T})^2}{2\sigma^2_{T+1|T}} + \frac{1}{2}\ln(\sigma^2_{T+1|T}) + 0.5\ln(2\pi)\right] \tag{19}$$

where $p_{T+1|T}(.)$ is the density.

The DGP is as follows. The true values y_t follow an AR(1):

$$y_t = \phi y_{t-1} + \eta_t + v_t,\ |\phi| < 1 \tag{20}$$

where η_t is the underlying disturbance, and v_t is a news revision, and the first estimate is given by:

$$y_t^{t+1} = y_t - v_t + \varepsilon_t \tag{21}$$

with $y_t^{t+n} = y_t$ for $n = 2, 3, \ldots$. Further:

$$\begin{aligned} \eta_t &= \sigma_\eta \xi_{1t};\ v_t = \sigma_v \xi_{2t};\ \varepsilon_t = \sigma_\varepsilon \xi_{3t} \\ \xi_{it} &\sim iidN(0,1) \text{ for } i = 1,2,3. \end{aligned} \tag{22}$$

Let $y_t^{t+1} = y_t + rev_t$, and define δ as the relative size of the data revision process, that is:

$$\delta = \frac{var(rev_t)}{\sigma^2_\eta},$$

implying that if data revisions are news, $\delta = \sigma^2_v/\sigma^2_\eta$, and if data revisions are noise, $\delta = \sigma^2_\varepsilon/\sigma^2_\eta$.

A.1.1 News revisions

The EOS and RTV forecasts of the mean and the variance are given by:

$$\begin{aligned}\mu_{T+1|T}^{EOS} &= \phi y_T^{T+1} = \phi(y_T - \sigma_v \xi_{2T}) \\ \sigma_{T+1|T}^{2,EOS} &= \sigma_\eta^2 + \sigma_v^2 = \sigma_\eta^2(1+\delta) \\ \\ \mu_{T+1|T}^{RTV} &= \phi y_T^{T+1} = \phi(y_T - \sigma_v \xi_{2T}) \\ \sigma_{T+1|T}^{2,RTV} &= \sigma_\eta^2 + \phi^2\sigma_v^2 = \sigma_\eta^2(1+\phi^2\delta)\end{aligned} \tag{23}$$

using results in Clements and Galvão (2013) and Clements (2017).

A.1.2 Noise revisions

The EOS and RTV forecasts of the mean are given by:

$$\begin{aligned}\mu_{T+1|T}^{EOS} &= \phi y_T^{T+1} = \phi(y_T + \sigma_\varepsilon \xi_{3T}) \\ \mu_{T+1|T}^{RTV} &= B\phi y_T^{T+1} = B\phi(y_T + \sigma_\varepsilon \xi_{3T}).\end{aligned}$$

where:

$$B = \frac{\sigma_y^2}{\sigma_y^2 + \sigma_\varepsilon^2} = \frac{\sigma_\eta^2/(1-\phi^2)}{\sigma_\eta^2/(1-\phi^2) + \sigma_\eta^2\delta} = \frac{(1-\phi^2)^{-1}}{((1-\phi^2)^{-1} + \delta)}$$

(see Clements and Galvão (2013)).

The EOS and RTV forecasts of the variances are given by:

$$\sigma_{T+1|T}^{2,EOS} = \sigma_\eta^2 \tag{24}$$

$$\sigma_{T+1|T}^{2,RTV} = \sigma_\eta^2(1+\delta+\varrho), \tag{25}$$

with $\varrho = [\phi^2(B-1)^2/(1-\phi^2) + \delta B^2\phi^2]$. Eq. (25) is derived as:

$$\begin{aligned}
\sigma^{2,RTV}_{T+1|T} &= var(y_{T+1} + \sigma_\varepsilon \xi_{3T+1} - B\phi y_T - B\phi\sigma_\varepsilon\xi_{3T}) \\
&= var(y_t) + B^2\phi^2 var(y_t) + (1 + B^2\phi^2)\sigma^2_\varepsilon - 2B\phi Cov\,(y_t y_{t-1}) \\
&= \sigma^2_y(1 + B^2\phi^2 - 2B\phi^2) + (1 + B^2\phi^2)\sigma^2_\varepsilon \\
&= \sigma^2_y[(1-\phi^2) + \phi^2(B-1)^2] + (1 + B^2\phi^2)\sigma^2_\varepsilon \\
&= \sigma^2_\eta/(1-\phi^2)[(1-\phi^2) + \phi^2(B-1)^2] + \sigma^2_\eta\delta(1 + B^2\phi^2) \\
&= \sigma^2_\eta + \sigma^2_\eta(\phi^2(B-1)^2/(1-\phi^2)) + \sigma^2_\eta\delta + \sigma^2_\eta\delta B^2\phi^2 \\
&= \sigma^2_\eta + \delta\sigma^2_\eta + \sigma^2_\eta(\phi^2(B-1)^2/(1-\phi^2)) + \sigma^2_\eta\delta B^2\phi^2 \\
&= \sigma^2_\eta(1 + \delta + \varrho),
\end{aligned}$$

Because $B < 1$ and $|\phi| < 1$, then $\varrho > 0$, implying that for the same δ and ϕ, $\sigma^{2,RTV}_{T+1|T}$ for noise is greater than $\sigma^{2,EOS}_{T+1|T}$ for news. If there are no revisions ($\delta = 0$), then $\varrho = 0$ since $B = 1$.

A.2 Proof of Proposition 1

The difference between EOS and RTV log score when revisions are news:

$$\begin{aligned}
\Delta score^{News} &= E[-\ln(p^{EOS}_{T+1|T}(y^{T+2}_{T+1}))] - E[-\ln(p^{RTV}_{T+1|T}(y^{T+2}_{T+1}))] \\
&= E[-\ln(p^{EOS}_{T+1|T}(y_{T+1} - \sigma_v\xi_{2T+1}))] - E[-\ln(p^{RTV}_{T+1|T}(y_{T+1} - \sigma_v\xi_{2T+1}))] \\
&= \left[\frac{1+\phi^2\delta}{2(1+\delta)} + \frac{1}{2}\ln(\sigma^2_\eta(1+\delta))\right] - \left[\frac{1}{2} + \frac{1}{2}\ln\sigma^2_\eta(1+\phi^2\delta)\right] \\
&= \frac{1}{2}\left[\frac{1+\phi^2\delta}{1+\delta} + \ln(1+\delta) - 1 - \ln(1+\phi^2\delta)\right] \\
&= \frac{1}{2}\left[\frac{\delta(\phi^2-1)}{1+\delta} + \ln[(1+\delta)/(1+\phi^2\delta)]\right],
\end{aligned}$$

and we need to show that $\Delta score^{News} \geq 0$ in order to establish the dominance of RTV over EOS on log score. If we take the derivative of the expression in brackets with respect to ϕ^2, we find $\delta/(1+\delta) - \delta/(1+\phi^2\delta)$. Because $(1+\delta) > (1+\phi^2\delta)$, since $\phi^2 < 1$ and $\delta \geq 0$, the derivative is always negative. This means that the minimum value of $\Delta score^{News}$ will be at the maximum value of ϕ^2, that is, $\phi^2 \approx 1$. Based on the expression above, it is clear that if

$\phi^2 = 1$, $\Delta score^{News} = 0$. If $\Delta score^{News}$ is equal to zero at its minimum, then for values such that $0 \leq \phi^2 < 1$, we have $\Delta score^{News} \geq 0$.

A.3 Proof of Proposition 2

The difference between EOS and RTV log score when revisions are noise:

$$\begin{aligned}
\Delta score^{Noise} &= E[-\ln(p^{EOS}_{T+1|T}(y^{T+2}_{T+1}))] - E[-\ln(p^{RTV}_{T+1|T}(y^{T+2}_{T+1}))] \\
&= E[-\ln(p^{EOS}_{T+1|T}(y_{T+1} + \sigma_\varepsilon \xi_{3T+1}))] - E[-\ln(p^{RTV}_{T+1|T}(y_{T+1} + \sigma_\varepsilon \xi_{3T+1}))] \\
&= \left[\frac{1}{2}(1+\delta(1+\phi^2)) + \frac{1}{2}\ln(\sigma^2_\eta)\right] - \left[\frac{1}{2} + \frac{1}{2}\ln(\sigma^2_\eta(1+\delta+\varrho))\right] \\
&= \frac{1}{2}\left[(1+\delta(1+\phi^2)) - 1 - \ln(1+\delta+\varrho)\right] \\
&= \frac{1}{2}\left[(\delta(1+\phi^2)) - \ln(1+\delta+\varrho)\right].
\end{aligned}$$

To show that $\Delta score^{Noise} \geq 0$, we use the concavity of the logarithm function. But first note that we can rewrite ϱ as:

$$\begin{aligned}
\varrho &= [\phi^2(B-1)^2/(1-\phi^2) + \delta B^2 \phi^2] \\
&= \phi^2(1-\phi^2)^{-1}\left[\frac{\delta^2}{((1-\phi^2)^{-1}+\delta)^2}\right] + \delta\phi^2\left[\frac{(1-\phi^2)^{-2}}{((1-\phi^2)^{-1}+\delta)^2}\right] \\
&= \frac{\phi^2(1-\phi^2)^{-1}\delta(\delta+(1-\phi^2)^{-1})}{((1-\phi^2)^{-1}+\delta)^2} \\
&= \frac{\phi^2(1-\phi^2)^{-1}\delta}{((1-\phi^2)^{-1}+\delta)} = \phi^2\delta B
\end{aligned}$$

Recall that $x \geq \ln(1+x)$ if $x \geq 0$. In the case that $B = 1$, $\varrho = \delta\phi^2$, and then $\delta(1+\phi^2) > \ln(1+\delta(1+\phi^2))$. When $\sigma^2_\varepsilon > 0$, then $B < 1$, and we have:

$$\Delta score^{Noise} = \frac{1}{2}(\delta(1+\phi^2)) - \ln(1+\delta(1+\phi^2 B)).$$

Since $\phi^2 B < \phi^2$, then it must be the case that $\Delta score^{Noise} \geq 0$, establishing the dominance of RTV over EOS on log score for noise revisions.

Tables 1- Forecast Performance at a one-quarter-ahead horizon for the US. Out-of-sample origins: 2000Q3-2017Q4 (70 observations).

Table 1A: Forecasting the first-release values

	BVAR EOS	BVAR RTV	BVAR_SV EOS	BVAR_SV RTV	KK	KK_SVT	KK_SV
	RMSFE	Ratios to BVAR EOS RMSFE					
GDP growth	0.403	0.868	0.765	0.716	0.833	0.881	0.849
Deflator Inf.	0.076	0.902	0.821	0.836	0.844	0.689	0.688
Unemp.	0.084	1.163	0.950	1.024	1.085	1.186	1.159
3-month rate	0.203	0.828	0.718	0.685	0.836	0.636	0.621
		DM t-stat:					
GDP growth		-1.439	**-2.933**	**-2.967**	**-1.674**	-0.966	-0.928
Deflator Inf.		**-2.227**	**-2.853**	**-2.720**	**-2.176**	**-2.091**	**-2.159**
Unemp.		1.625	**-1.863**	0.409	1.186	1.219	1.106
3-month rate		-1.254	**-2.228**	**-1.780**	-0.964	**-1.986**	**-2.107**
	Logscore	Differences to BVAR EOS Logscore					
GDP growth	0.984	-0.091	-0.203	-0.235	-0.109	-0.084	-0.169
Deflator Inf.	0.116	-0.016	-0.033	-0.044	-0.028	-0.039	-0.070
Unemp.	0.188	0.061	-0.124	-0.069	0.022	-0.053	-0.048
3-month rate	0.785	-0.062	-0.570	-0.570	-0.084	-0.553	-0.572
		DM t-stat:					
GDP growth		-1.517	**-3.753**	**-3.229**	**-2.072**	-1.621	**-2.546**
Deflator Inf.		-0.470	-0.654	-0.765	-0.613	-0.513	-0.915
Unemp.		1.325	-0.987	-0.646	0.417	-0.481	-0.401
3-month rate		**-2.612**	**-3.860**	**-3.690**	**-2.733**	**-3.996**	**-4.154**

Table 1B: Forecasting the second-release (equivalent to BEA third) values

	BVAR EOS	BVAR RTV	BVAR_SV EOS	BVAR_SV RTV	KK	KK_SVT	KK_SV
	RMSFE	Ratios to BVAR EOS RMSFE					
GDP growth	0.464	0.920	0.826	0.776	0.927	0.969	0.908
Deflator Inf.	0.067	0.892	0.812	0.836	0.851	0.723	0.721
		DM t-stat:					
GDP growth		-1.074	**-2.429**	**-2.509**	-0.871	-0.275	-0.645
Deflator Inf.		**-2.311**	**-3.071**	**-2.615**	**-2.018**	**-2.224**	**-2.306**
	Logscore	Differences to BVAR EOS Logscore					
GDP growth	1.040	-0.051	-0.136	-0.094	-0.036	-0.027	-0.080
Deflator Inf.	0.060	-0.004	-0.031	-0.017	-0.001	-0.038	-0.039
		DM t-stat:					
GDP growth		-0.757	**-1.741**	-0.685	-0.939	-0.374	-1.303
Deflator Inf.		-0.143	-0.922	-0.397	-0.033	-0.612	-0.656

Notes: Entries are RMSFE or logscore in the first column. Remaining entries are RMSFE ratios to BVAR EOS or differences in logscore. Improvements over BVAR EOS are indicated by ratios smaller than 1 and negative logscore differences and t-statistics. Models are re-estimated at each new quarterly forecasting origin by extending the sample period that starts in 1965Q3. The values in blue indicate the best forecasting model for each variable using the respective accuracy measure. The t-statistics in bold indicate that the null of equal accuracy with BVAR EOS is rejected at the 5% level in favour of the alternative model (indicated in the column headings).

Table 2 – Forecast Performance at a one-quarter-ahead horizon for the UK. Out-of-sample origins: 2004Q4 -2019Q2 (59 observations).

Table 2A: Forecasting the first-release values

	BVAR EOS	BVAR RTV	BVAR_SV EOS	BVAR_SV RTV	KK	KK_SVT
	RMSFE	Ratios to BVAR EOS RMSFE				
GDP growth	0.285	0.894	1.136	1.049	0.890	1.389
Deflator Inf.	0.515	0.553	0.986	0.463	0.548	0.514
Unemp.	0.048	1.032	1.000	1.064	1.067	1.134
3-month rate	0.214	0.795	0.744	0.736	0.801	0.912
		DM t-stat:				
GDP growth		-1.391	1.021	0.490	-1.173	1.135
Deflator Inf.		**-2.422**	-0.076	**-2.818**	**-2.069**	**-2.375**
Unemp.		0.454	-0.005	0.923	0.908	0.944
3-month rate		-1.080	-1.310	-1.293	-1.181	-0.322
	Logscore	Differences to BVAR EOS Logscore				
GDP growth	0.850	-0.054	-0.184	-0.182	-0.025	0.007
Deflator Inf.	1.112	-0.381	0.132	-0.423	-0.375	-0.401
Unemp.	-0.120	0.059	-0.095	0.019	0.085	0.017
3-month rate	0.605	-0.071	-0.406	-0.411	-0.047	-0.152
		DM t-stat:				
GDP growth		-0.765	-1.302	-1.225	-0.029	0.126
Deflator Inf.		**-2.573**	1.289	**-2.656**	**-2.245**	**-3.079**
Unemp.		1.248	-1.762	0.227	1.648	0.310
3-month rate		-1.320	**-2.474**	**-2.662**	-0.365	-1.292

Table 2B: Forecasting the second-release (ONS QNA) values

	BVAR_EOS	BVAR_RTV	BVAR_SV_EOS	BVAR_SV_RTV	KK	KK_SVT
	RMSFE	Ratios to BVAR EOS RMSFE				
GDP growth	0.329	0.863	1.086	0.962	0.873	1.279
Deflator Inf.	0.448	0.564	1.228	0.509	0.676	0.587
		DM t-stat:				
GDP growth		**-1.934**	0.690	-0.529	-1.489	0.927
Deflator Inf.		**-2.346**	0.949	**-2.789**	-1.418	**-2.025**
	Logscore	Differences to BVAR EOS Logscore				
GDP growth	0.971	-0.073	-0.236	-0.193	-0.046	-0.023
Deflator Inf.	1.059	-0.377	0.220	-0.344	-0.297	-0.368
		DM t-stat:				
GDP growth		-0.862	-1.319	-1.050	-0.202	-0.150
Deflator Inf.		**-2.680**	2.047	**-2.420**	-1.629	**-2.730**

Notes: Entries are RMSFE or logscore in the first column. Remaining entries are RMSFE ratios to BVAR EOS or differences in logscore. Improvements over BVAR EOS are indicated by ratios smaller than 1 and negative logscore differences and t-statistics. Models are re-estimated at each new quarterly forecasting origin by extending the sample period that starts in 1989Q4. The values in blue indicate the best forecasting model for each variable using the respective accuracy measure. The t-statistics in bold indicate that the null of equal accuracy with BVAR EOS is rejected at the 5% level in favour of the alternative model (indicated in the column headings).

Table 3 - Forecast Performance at four-quarters-ahead for the US. Out-of-sample origins: 2000Q3-2017Q1 (67 observations).

Table 3.1: Forecasting the first-release values

	BVAR_EOS	BVAR_RTV	BVAR_SV_EOS	BVAR_SV_RTV	KK	KK_SVT	KK_SV
	RMSFE	Ratios to BVAR EOS RMSFE					
GDP growth	0.541	0.889	0.738	0.713	0.805	0.811	0.628
Deflator Inf.	0.093	0.835	0.759	0.758	0.810	0.797	0.860
Unemp.	1.390	1.086	0.859	0.868	1.018	0.912	0.867
3-month rate	1.890	1.002	0.982	1.058	1.035	0.936	0.904
		DM t-stat:					
GDP growth		**-2.514**	**-3.320**	**-2.973**	**-1.937**	**-2.501**	**-2.687**
Deflator Inf.		**-2.324**	**-2.184**	**-2.394**	**-1.998**	-1.275	-0.933
Unemp.		2.472	**-2.276**	**-1.938**	0.473	**-1.880**	**-2.200**
3-month rate		0.036	-0.326	0.604	0.433	-0.868	-1.604
	Logscore	Differences to BVAR EOS Logscore					
GDP growth	1.128	-0.062	-0.084	-0.120	-0.089	-0.008	-0.039
Deflator Inf.	0.279	-0.018	0.265	0.267	-0.008	0.322	0.341
Unemp.	2.176	0.090	-0.675	-0.624	-0.082	-0.538	-0.551
3-month rate	1.771	-0.003	0.015	0.039	0.008	0.023	0.010
		DM t-stat:					
GDP growth		**-1.674**	**-2.271**	**-3.515**	**-1.958**	-0.307	-1.149
Deflator Inf.		-0.797	4.193	4.105	-0.292	4.319	4.383
Unemp.		2.397	-1.361	-1.298	-0.913	-1.212	-1.217
3-month rate		-0.147	0.331	0.830	0.290	0.560	0.253

Table 3.2: Forecasting the second-release (BEA third) values

	BVAR_EOS	BVAR_RTV	BVAR_SV_EOS	BVAR_SV_RTV	KK	KK_SVT	KK_SV
	RMSFE	Ratios to BVAR EOS RMSFE					
GDP growth	0.612	0.916	0.774	0.759	0.883	0.886	0.711
Deflator Inf.	0.087	0.807	0.757	0.771	0.825	0.806	0.867
		DM t-stat:					
GDP growth		**-1.960**	**-3.191**	**-2.725**	-1.231	**-1.858**	**-2.313**
Deflator Inf.		**-2.494**	**-2.069**	**-2.172**	**-1.752**	-1.160	-0.849
	Logscore	Differences to BVAR EOS Logscore					
GDP growth	1.182	-0.040	-0.084	-0.106	-0.039	-0.018	-0.055
Deflator Inf.	0.254	-0.017	0.282	0.284	0.010	0.326	0.325
		DM t-stat:					
GDP growth		-0.870	**-2.007**	**-3.105**	-1.042	-0.685	-1.510
Deflator Inf.		-0.813	4.540	4.446	0.377	4.603	4.575

Notes: See notes to Table 1.

Table 4 – Forecast Performance at four-quarters-ahead for the UK. Out-of-sample origins: 2004Q4 -2018Q3 (56 observations).

Table 4A: Forecasting the first-release values

	BVAR_EOS	BVAR_RTV	BVAR_SV_EOS	BVAR_SV_RTV	KK	KK_SVT
	RMSFE	Ratios to BVAR EOS RMSFE				
GDP growth	0.463	0.962	1.128	1.307	0.884	1.092
Deflator Inf.	0.549	0.597	1.122	0.459	0.497	0.389
Unemp.	0.599	0.969	0.955	1.043	1.086	1.140
3-month rate	2.109	0.883	0.722	1.125	1.216	1.302
		DM t-stat:				
GDP growth		-0.598	2.450	1.983	-1.153	0.962
Deflator Inf.		**-2.425**	0.646	**-2.958**	**-3.109**	**-3.058**
Unemp.		-0.392	-0.359	0.302	1.073	0.927
3-month rate		-1.154	-1.354	0.467	0.747	1.106
	Logscore	Differences to BVAR EOS Logscore				
GDP growth	1.758	-0.233	-0.448	-0.440	-0.155	-0.596
Deflator Inf.	1.109	-0.294	0.144	-0.221	-0.406	-0.169
Unemp.	1.432	0.103	-0.201	-0.133	0.053	-0.126
3-month rate	1.753	0.009	-0.210	-0.019	0.059	0.094
		DM t-stat:				
GDP growth		-1.415	-0.950	-0.914	**-2.627**	-0.962
Deflator Inf.		**-1.926**	2.601	**-2.700**	**-3.514**	**-2.345**
Unemp.		1.301	-0.644	-0.420	0.933	-0.396
3-month rate		0.178	-1.779	-0.156	0.837	0.749

Table 4.2: Forecasting the second-release (ONS QNA) values

	BVAR_EOS	BVAR_RTV	BVAR_SV_EOS	BVAR_SV_RTV	KK	KK_SVT
	RMSFE	Ratios to BVAR EOS RMSFE				
GDP growth	0.500	0.945	1.104	1.232	0.911	1.072
Deflator Inf.	0.585	0.514	1.155	0.406	0.459	0.375
		DM t-stat:				
GDP growth		-0.848	2.097	1.904	-0.942	0.887
Deflator Inf.		**-2.887**	1.018	**-3.162**	**-3.033**	**-3.047**
	Logscore	Differences to BVAR EOS Logscore				
GDP growth	1.963	-0.278	-0.552	-0.552	-0.042	-0.734
Deflator Inf.	1.218	-0.434	0.090	-0.328	-0.510	-0.307
		DM t-stat:				
GDP growth		-1.498	-0.956	-0.937	-0.506	-0.992
Deflator Inf.		**-2.862**	0.990	**-2.524**	**-3.498**	**-2.360**

Notes: See notes to Table 2

Table 5: Simulation Exercise for T=150 and P=50 with KK BVAR(4) l=14 as DGP

Table 5.1: One-step-ahead Forecasts

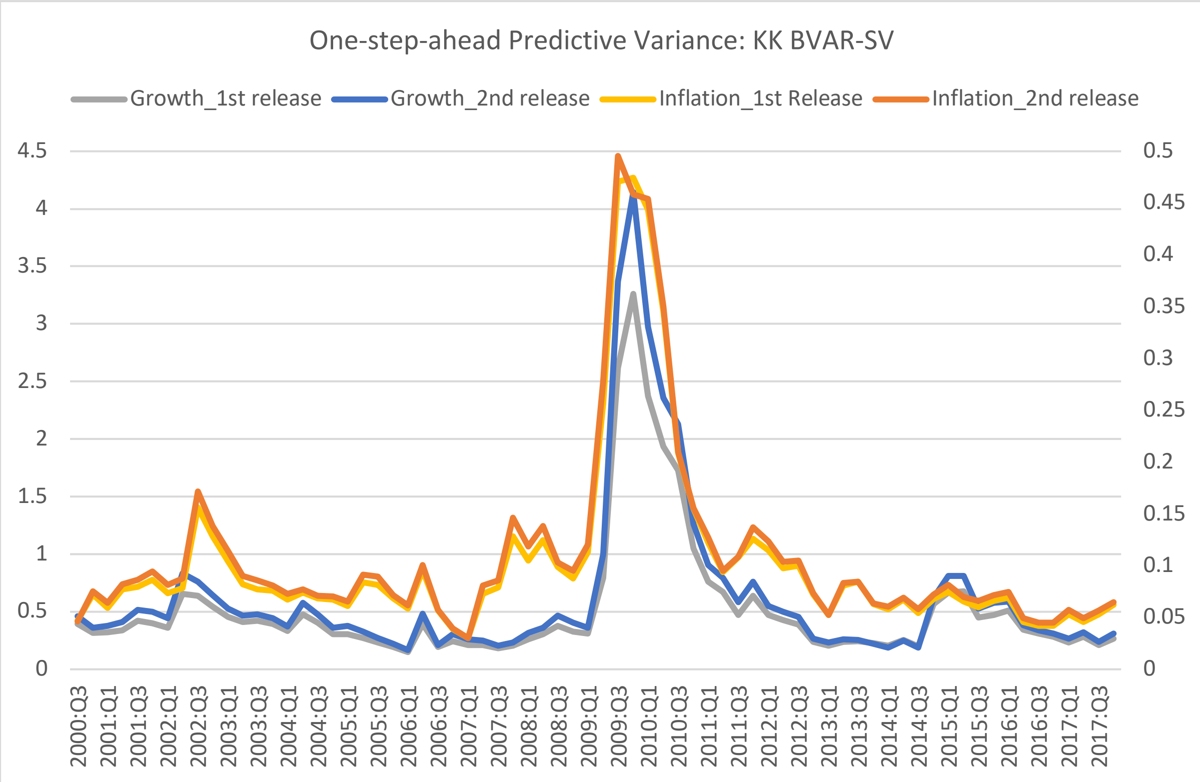
	First Release					Second Release				
	BVAR EOS	BVAR RTV	BVAR-SV EOS	BVAR-SV RTV	KK	BVAR EOS	BVAR RTV	BVAR-SV EOS	BVAR-SV RTV	KK
	RMSFE	Ratios to BVAR EOS RMSFE				RMSFE	Ratios to BVAR EOS RMSFE			
growth	0.703	0.896	1.023	0.903	0.940	1.223	0.958	1.007	0.960	0.981
inflation	0.338	0.984	1.012	0.987	0.976	0.486	0.987	1.008	0.990	0.989
		Rejection Rate DM test:					Rejection Rate DM test:			
growth		**0.484**	0.005	**0.401**	0.292		**0.328**	0.031	0.292	0.146
inflation		0.125	0.010	0.125	0.188		0.151	0.016	0.120	0.094
	Logscore	Differences to BVAR EOS Logscore				Logscore	Differences to BVAR EOS Logscore			
growth	1.073	-0.107	0.177	-0.062	-0.058	2.019	0.240	-0.321	0.173	-0.408
inflation	0.382	-0.054	0.064	-0.014	-0.063	1.158	-0.258	-0.376	-0.284	-0.462
		Rejection Rate DM test:					Rejection Rate DM test:			
growth		**0.563**	0.000	0.234	**0.359**		0.000	**0.677**	0.078	**0.708**
inflation		0.177	0.000	0.042	0.219		**0.724**	**0.641**	**0.594**	**0.698**

Table 5.2: Four-quarter-ahead Forecasts

	First Release					Second Release				
	BVAR EOS	BVAR RTV	BVAR-SV EOS	BVAR-SV RTV	KK	BVAR EOS	BVAR RTV	BVAR-SV EOS	BVAR-SV RTV	KK
	RMSFE	Ratios to BVAR EOS RMSFE				RMSFE	Ratios to BVAR EOS RMSFE			
growth	0.684	0.986	0.998	0.988	0.984	1.217	0.993	0.999	0.993	0.992
inflation	0.358	1.012	1.000	1.002	0.980	0.490	1.016	0.999	1.009	0.994
		Rejection Rate DM test:					Rejection Rate DM test:			
growth		0.115	0.021	0.063	0.083		0.063	0.057	0.078	0.115
inflation		0.104	0.083	0.104	0.115		0.047	0.047	0.068	0.104
	Logscore	Differences to BVAR EOS Logscore				Logscore	Differences to BVAR EOS Logscore			
growth	1.070	-0.031	0.225	0.009	-0.031	1.879	0.292	-0.213	0.203	-0.262
inflation	0.416	0.010	0.127	0.041	-0.023	0.946	-0.085	-0.180	-0.125	-0.232
		Rejection Rate DM test:					Rejection Rate DM test:			
growth		**0.318**	0.000	0.068	**0.307**		0.000	**0.333**	0.036	**0.438**
inflation		0.036	0.000	0.000	0.057		0.297	0.208	0.240	**0.333**

Note: The entries for BVAR EOS are the average (across replications and out-of-sample period *P*) RMSFE or logscore. The entries for the other models are RMSFE ratios or differences in logscore. DM t-statistics are computed for each replication using *P-h* recursive forecasts (computing by re-estimating the forecasting models with increasing samples from *T+1* up to *T+P-h*), and entries are rejection rates of a 5% sized (one-sided) test in favour of the alternative model. The DGP is a KK-VAR(4) for output growth and inflation estimated using US data from 1985 and assuming *l=14*. We compute the posterior distribution for each model using 10.000 MCMC draws for the conditional distributions. Number of Monte Carlo replications is 288.

Figure 1: Predicted variances obtained with the KK BVAR-SV Model with US Data



Note: Inflation values in the right axis and output growth values in the left axis.

Figure 2: Ratio of EOS to RTV Predicted Variances obtained for the BVAR-SV forecasting model with US data

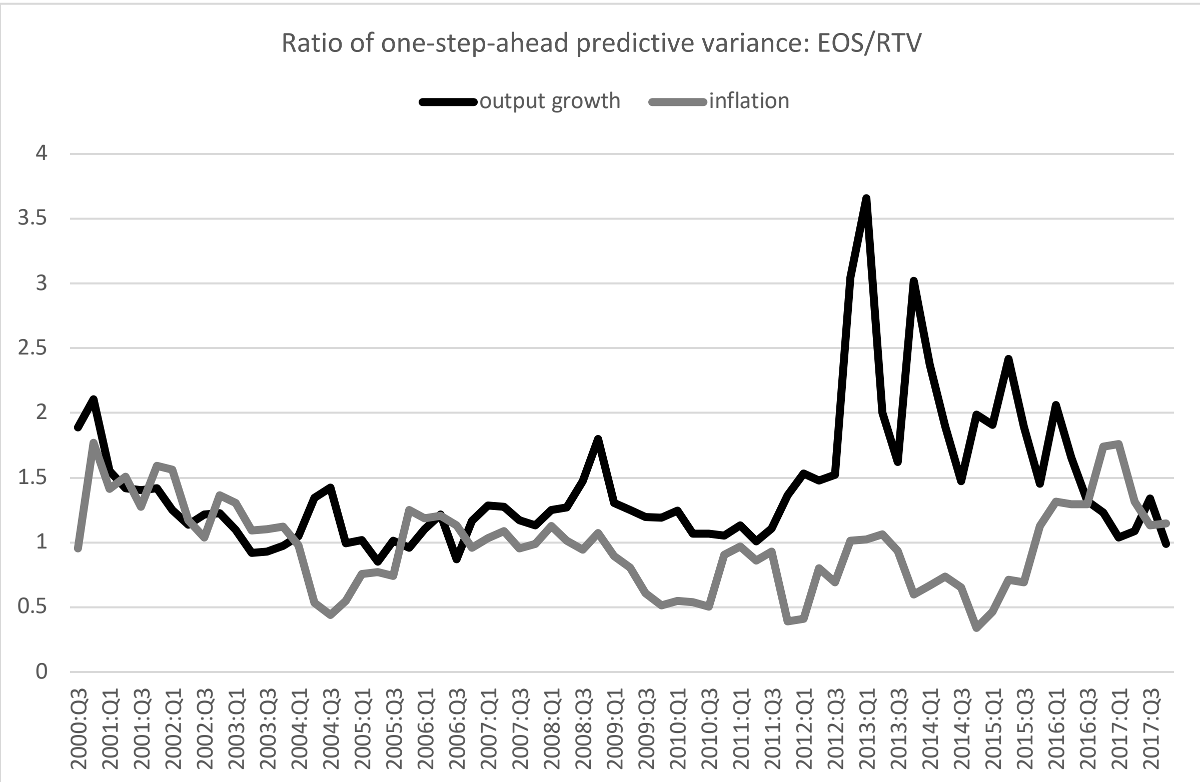
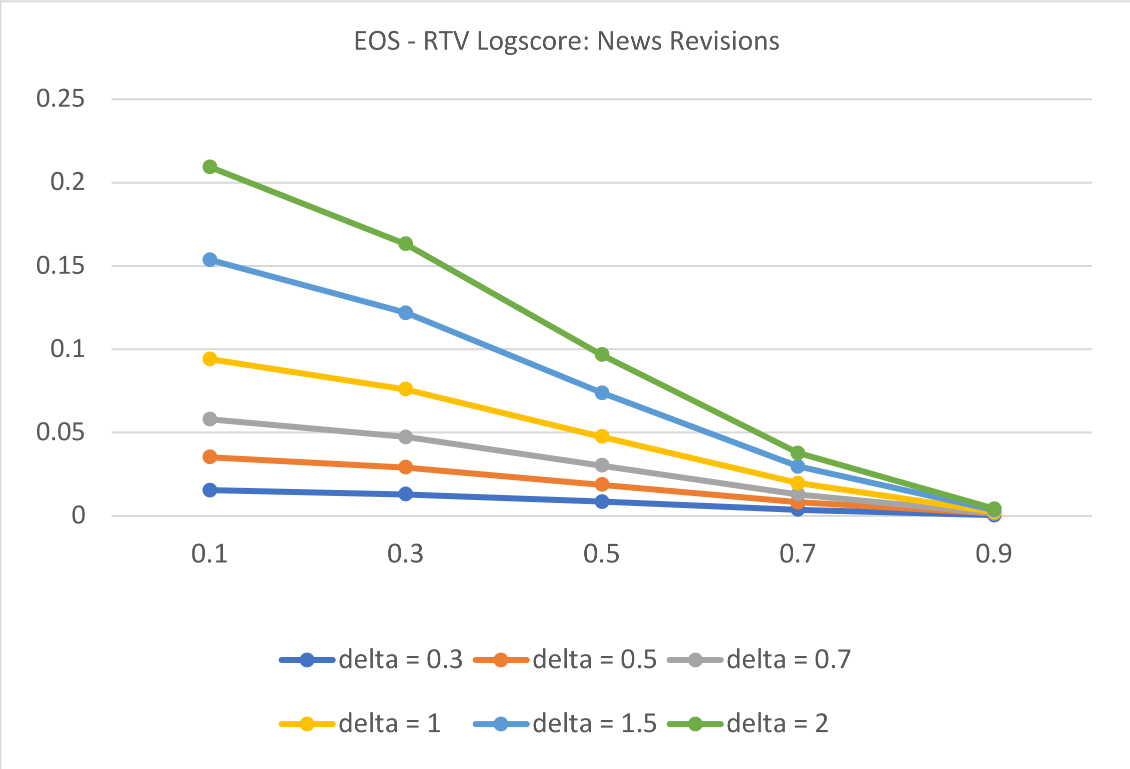
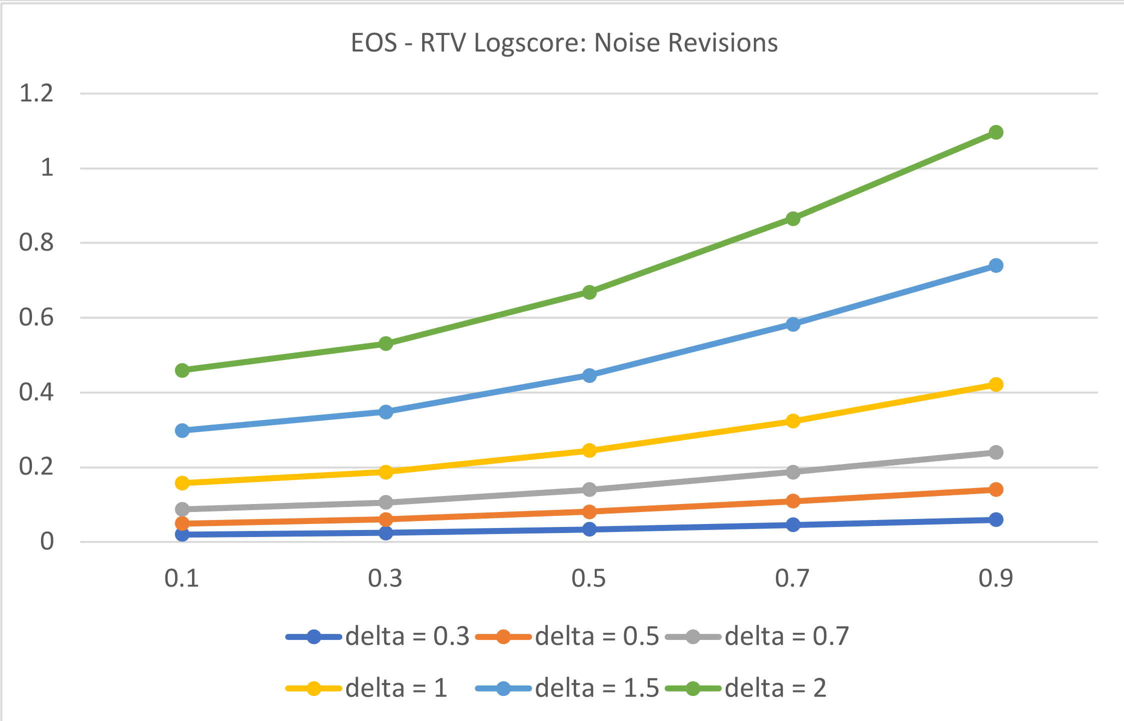


Figure 3: Analytical Results for $\Delta score^{News}$ and $\Delta score^{Noise}$

comp





Note: Values for the AR(1) parameter ϕ on the horizontal axis.
[For news revisions when ϕ=0.9, the lines are all above the axis].