Financial Options:

New Markets and New Valuation Techniques

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August 1988

Chapter 1 in *Options: Recent Advances in Theory and Practice*, Ed S D Hodges, Manchester University Press, 1990, pp3-9

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FORC Preprint: 88/4

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I Introduction

We are in the middle of a quiet revolution in the way financial risk is managed by companies, investment funds, and particularly financial institutions. In this paper I wish to concentrate on one of the most important aspects of this revolution, the development of markets in new securities called financial options, and the use of these securities. I shall not concern myself with the other major development, that of futures, for, while they are important, they have been highlighted elsewhere — for example, in a recent paper by Miller (1986). Academic research has played an important role in facilitating many of the current developments; indeed, this is a field in which there is increasing direct collaboration between the financial and academic communities. I shall describe, first, what has been happening to financial markets; second, what has been happening in terms of academic research; and finally, an agenda for further research.

II The new financial markets

In 1973 the Chicago Board Options Exchange (CBOE) established the first market in call options on 16 stocks. These provided the right, but not the obligation, to buy stock at a fixed price on or before an agreed date. Before this, options contracts could be negotiated between parties, but were not standardised or traded on the market.

The CBOE was an immediate success. Within the year 1.1 million contracts were traded and another 16 stocks were added. This rapid growth continued, and by 1977, with the addition of put options — providing the right, but not the obligation, sell at a fixed price on or before an agreed date — CBOE volume reached 24.8 million contracts.

Other exchanges began trading options, including in 1978 the European Options Exchange (in Amsterdam) and the London Stock Exchange. Options on foreign currency, interest rates and bonds first appeared in 1982, and on stock market indices in 1986. These options have particular importance for risk management; in 1986 turnover reached 17.3 million contracts on the Chicago Board of Trade Treasury bond option, 2.2 million contracts on the Chicago Mercantile Exchange Deutschemark option and 1.9 million contracts on the same exchange's option on the S&P 500 stock market index. By now options are traded on at least 14 separate exchanges world-wide, and volume continues to increase on virtually all exchanges.

Options differ from all other financial instruments in the patterns of risk they produce, and this is what makes them attractive. A recent Bank for International Settlements report (1986), for example, describes how active trading in foreign exchange and interest rate options surged in the early 1980s, 'spurred by growth in customer demand'.

III The new valuation methods

Not surprisingly, a key problem for academic research has been how options should be valued. Obviously, this is a problem that also interests participants in the markets themselves who need to decide at what prices they are prepared to buy and sell them. The conventional approach to valuing securities with uncertain payoffs consists of the following four steps: first, working out the probability of future payoffs; second, finding their expected value; third, deciding what rate of return investors require (on average); and finally, using this to calculate the value of the security today. For many years this approach could not be satisfactorily applied to valuing options because we had insufficient knowledge to enable us to describe the probability distributions of future payoffs, or to determine the return expected by investors in options. We will look at these two issues in turn.

What do we know about the behaviour of share prices? Share prices have always been something of a problem. There is a story told of the great American banker, J. P. Morgan. It was around the turn of the century, and Morgan was at the height of his power. A rather junior investment analyst had the temerity to ask him one day what he thought stocks would do next. He drew himself up, turned to the analyst and, without a moment's hesitation, said: 'They will fluctuate, young man, they will fluctuate.'

Well, that more or less sums up what we know about how stock prices behave, even today, except that today we know it rather more precisely. A stream of research during the 1950s and 1960s established that share prices fluctuate randomly through time in what statisticians describe as a 'random walk'. This research was initially mainly empirical. It included important contributions by the British statistician, Maurice Kendall (1953), and by Sidney Alexander (1961) and Paul Cootner (1962) at the Massachusetts Institute of Technology (MIT). A key theoretical cornerstone was provided by the MIT economist, Paul Samuelson (1965), who showed that in rational markets properly anticipated prices ought to fluctuate randomly, in the same sort of way as they had already been found to fluctuate empirically. Share prices are virtually impossible to forecast, and thus their movements appear random, precisely because markets are competitive and prices reflect all information available from which to try to make forecasts.

At the same time, other research was taking place to provide a framework for understanding the rates of return required by investors. This research was rooted in the work of Harry Markowitz (1952) which first provided a coherent theory for how and why investors should hold diversified portfolios. The economist James Tobin provided a crucial insight into the role of the interest rate in equity portfolios in 1958. This stream of research culminated in the theory of required returns we call the 'capital asset pricing model' developed by Sharpe (1964), and almost simultaneously by other researchers, notably John Lintner (1965) at Harvard, and by another rare European, Jan Mossin (1966).

In principle then, by about 1965, we knew enough about the distributions of share prices and about expected rates of return to be able to fill in the missing steps and evaluate options properly. However, there was a snag. As share prices fluctuate through time, the risk of an option, and hence its required rate of return, also fluctuate. This problem proved so difficult that its solution took another seven or eight years to find.

I imagine that every subject has its important breakthroughs which change, once and for all, the way in which certain problems are tackled. I am reminded of the late Gerard Hoffnung's mythical German professor of music who declared: 'Music began when Arnold Schoenberg had invented the twelve tone row. Before this, was chaos absolute!' Options pricing may not have quite been 'chaos absolute' in 1965, but the breakthrough by Black and Scholes in 1973 certainly heralded the beginning of a new era, and the use of an entirely new and powerful methodology.

The concept behind their approach is stunningly simple. By continuously adjusting our holdings of existing securities, we can manufacture entirely new ones. If we know how to manufacture a security, then we know how much it costs to manufacture and how much it is worth. By way of illustration, let us consider an analogy drawn from a betting casino. Suppose I take \$80 to the casino and proceed to make four 50–50 bets (for simplicity, we assume no bias in favour of the casino). One obvious strategy might be to bet \$20 each time. The outcomes from this range between ending up with \$160 and ending up with nothing. If we denote by $P(x)$ the probability of ending up with x dollars, then it is easy to establish that $P(0) = P(160) = \frac{1}{16}$, $P(40) = P(120) = \frac{1}{4}$ and $P(80) = \frac{3}{8}$. A quite different probability distribution is obtained if I decide to follow a second, more subtle, strategy of betting an extra \$5 every time I win, and betting \$5 less every time I lose. This time I end up with a distribution which, instead of being symmetric, is skewed to the right. If I am unlucky, and lose repeatedly, I will only have lost \$20 + \$15 + \$10 + \$5 = \$50, so I will still have \$30 remaining. If I win repeatedly, I will have won \$20 + \$25 + \$30 + \$35 = \$110, so I will end up with \$190. So $P(30) = P(190) = \frac{1}{16}$. Calculating the other probabilities is also straightforward: $P(40) = P(120) = \frac{1}{4}$, $P(70) = \frac{3}{8}$. The individual probabilities are the same as in the simpler strategy of four \$20 bets because all the bets are 50–50. Note also that my expected outcome is still to end up with \$80:

$$(\tfrac{1}{16} \times 0) + (\tfrac{1}{4} \times 40) + (\tfrac{3}{8} \times 80) + (\tfrac{1}{4} \times 120) + (\tfrac{1}{16} \times 160) = 80$$

$$(\tfrac{1}{16} \times 30) + (\tfrac{1}{4} \times 40) + (\tfrac{3}{8} \times 70) + (\tfrac{1}{4} \times 120) + (\tfrac{1}{16} \times 190) = 80$$

This illustrates the concept of how new payoff profiles can be constructed by allowing the investments made (bets placed) to vary through time in a way which depends on what has already occurred. What Black and Scholes (1973) showed was that, by borrowing an amount B and buying H shares (with B and H managed continuously through time) we can precisely manufacture the outcomes of a call option. The value of a call is therefore the cost of the net investment: H times the share price, minus the amount of borrowing, B. While the concept behind this is simple, the mathematics involved is not. Some quite complicated partial differential equations have to be solved of a kind which also occur in mathematical physics in the context of heat conduction.

The approach adopted by Black and Scholes has subsequently spurred a rapid growth in related theoretical work, the analysis of new applications of the technique, and empirical studies of security pricing. A recent textbook by Cox and Rubinstein (1983) lists nearly 300 references, most of them subsequent to Black and Scholes and many of them both important and difficult. Considerable work has been done to study the traded options markets, and to deal with the problems posed by dividends and the possibility of options being exercised prior to their expiry date. A range of other securities have been studied using the 'continuous-time' methodology of option valuation. These include warrants, convertible bonds, bonds carrying early redemption provisions, corporate debt with a risk of default (and with or without protective convenants), and finally even straightforward government bonds. Aside from traded securities, the exploitation of physical assets (such as a gold mine or oil well), the value of underwriting and loan guarantees, corporate tax and aspects, the management of investment portfolios through time (portfolio insurance) have all been studied within this new framework.

IV Future research

I now turn to future research. I shall first describe a research project I am currently embarking on with funding from the ESRC. I shall then talk about two further areas where I expect to see particularly fruitful developments.

The Warwick ESRC project
The ESRC has provided funds for two researchers to work with me for two years on the applications of option valuation models. The project has four parts to it. First, we will provide a survey and taxonomy of options pricing models according to the nature of each application. Some earlier surveys exist, but there is no up-to-date survey, and no clear taxonomy has yet been developed. The issue here is the assumptions on which models are based. Second, we shall identify the formal mathematical structure of each model, and create a second (related) taxonomy of these. At present, except for the simplest models, there is a great deal of confusion about numerical methods. Methods in use include finite difference methods, numerical integration and approximation by a binary process. Rather little work has been done to compare the relative efficiency of different approaches for solving different problems. Our third task will be to provide this. Finally, our systematic approach will enable us to perform valuations for a wide variety of instruments quoted on UK and international markets. We will undertake sufficient empirical work to demonstrate the viability of the modelling approaches to make some comparisons between models. We expect this project to interest financial institutions and for it to be the focus of seminars and other forms of collaboration. The emphasis is mainly developmental and aims to capitalise on substantial pre-existing knowledge and literature, both in finance and in mathematics.

Government bonds
An area of research I shall follow with particular interest (and possibly contribute to) is the study of government bonds in a continuous time framework. Even 'straight' bonds involve elements of options. If we form a portfolio of a one year and a 20-year bond it will go up a lot if interest rates fall, but only fall slightly if rates increase. It is possible to construct such a portfolio so that it appears to dominate a single five-year bond. Of course, it will not really dominate it for the yield of the five-year bond will be bid up to compensate for its different risk profile. The literature here is developing rapidly and shows great promise for the future, that will be of practical interest to participants in the gilts market.

Models based on weaker assumptions
The final area of research I want to mention is the development of models based on weaker than normal assumptions. Again, this work is motivated by the practical needs to value securities, hedge risk and manage portfolios in a world which does not exactly correspond to our idealised assumptions.

In options pricing theory the following assumptions are usually made:

1. Trading is continuous (and so is the time path of security prices).
2. There are no transactions costs (or buy–sell spreads).
3. Asset variation is known and constant.
4. Asset value follows a random walk. This raises questions such as what processes should we assume for interest rates, and are there speculative bubbles (or other sources of mean reversion) in financial markets.

Work has begun to confront each of these assumptions head on, and to explore the implications of using weaker assumptions. This is an area of tremendous importance to anyone using the new instruments for risk management and one where important new results are bound to emerge.

V Concluding remarks

We have seen some of the dramatic changes that are occurring in options markets and in the management of financial risk. We have seen, too, something of the parallel development of academic work and some of the outstanding issues for future research. The tools of option pricing theory have enabled important insights to be developed into the valuation and use of a wide range of securities. Applications run from traded options through warrants to convertible bonds, corporate debt, and government debt, and include issues as wide-ranging as decisions to exploit oil wells, the analysis of tax liabilities and the management of investment portfolios through time. Much work remains to be done. My ESRC project will undertake a survey and synthesis of application areas and model structures, it will research numerical methods for evaluating various models, and it will provide some empirical analysis comparing models. Other key areas for future research are the government bond market, and the relaxation of the usual assumptions, particularly with discrete time hedging in mind. Another important concern is whether security prices are adequately modelled by a random walk, or whether perhaps models which allow for mean reversion (as, for example, if there are fads or 'bubbles' in markets) may be a more appropriate representation. The conventional roles are for the academics to claim allegance to the random walk, and for the practitioner to point to examples of non-random behaviour.

I would like to conclude with a story which redresses the balance slightly. It will serve as a talisman to further future co-operation between the two communities. It is a story I tell myself whenever I am tempted to believe too strongly in the academic dogma of the unpredictability of share prices. It concerns Charles Merrill, who founded what eventually became the 'thundering herd' of Merrill Lynch. Apparently, some time in 1928 when Wall Street was booming, Charles Merrill underwent a crisis of confidence.

He found that he could no longer understand the market, it no longer made any sense to him. Fearing he was beginning to lose his sanity, he consulted a psychiatrist. To cut a long story short, after only a couple of sessions of therapy, both Merrill and his psychiatrist began to sell their shares, and both survived the Wall Street crash unscathed. We ignore tales like these at our peril.

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