Current Research on Derivative Products

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DERIVATIVE PRODUCTS have undergone intensive development and have been the subject of a great deal of research in the last few years. This is an area in which current academic research lies particularly close to the needs and interests of participants in the financial markets. The foundation in 1989 of the Financial Options Research Centre, which I direct, sponsored by a group of leading City institutions, reflected these common interests. This article sketches the main themes of current research in the derivatives area that are already affecting the products available to the corporate treasurer, or which are likely to do so in the near future.

Whereas swaps and futures contracts are in some respects quite static instruments, derivatives which bear option features owe most of their recent growth to the ideas of dynamic hedging introduced by Black and Scholes. Instruments with such characteristics can be manufactured (or hedged) by means of 'delta hedging' strategies, whereby the risk exposure is continuously adjusted in response to market movements. The success of a dynamic hedging strategy depends on how closely the process of asset returns approximates to that of the underlying hedging model. In the case of the Black-Scholes option valuation model the assumption is that (log) returns have a normal distribution with known variance, and that returns in periods which do not overlap are independent of one another.

For many kinds of assets, there is a greater possibility of sudden jumps or falls in price than would be suggested by the probabilities given by a log-normal distribution. This leads to options which are well into- or out-of-the-money often being priced at higher volatilities than at-the-money options. Implied volatilities plotted against exercise price show a U-shaped relationship, often called the volatility 'smile'. One theme of current research is to understand more about these relationships. Researchers have developed more general option pricing models which embody jumps or allow the asset volatility to vary through time, and research is also being done to quantify the regularities in the empirical behaviour of implied volatilities. The pay-off from these kinds of research promises to be more precise ways of pricing and hedging a variety of different types of options.

Foreign exchange options

In foreign exchange markets, options contracts have conventionally been priced and hedged using the Black (1976) model for options on futures as modified for currency by Garman and Kohlhagen (1983) for European options, and by Barone-Adesi and Whaley (1987) for American ones. Adjustments for options which are well out-of- or into-the-money are usually made on an *ad hoc* basis. This is particularly problematic for EMS currencies, characterized by a band around a central parity figure which may itself jump from time to time. Current research by Clifford Ball and Antonio Roma deals with modelling exchange rate processes of this kind. They use two variables, one to represent the level of the central parity rate (subject to jumps) and the second to represent the position of the exchange rate within its band. They find empirical evidence for mean reversion in EMS exchange rates, but not for non-EMS rates which are freely floating. Another area of important practical relevance is in integrating models of exchange rates with models of interest rate behaviour.

Models of interest rates

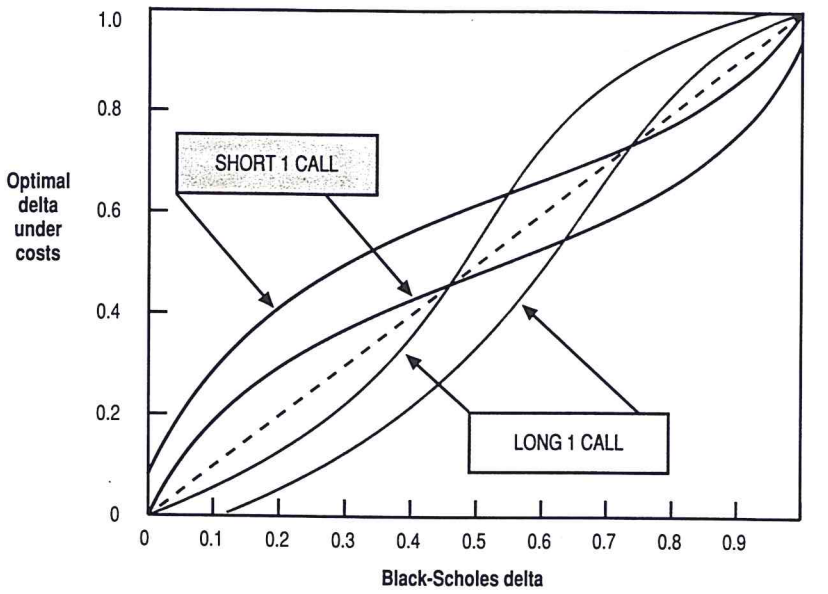
Modelling the dynamic of interest rate movements has become a big subject in itself. The last few years have seen a rapid growth in the literature concerning modelling the behaviour of interest rates in order to obtain better methods for valuing and hedging the wide range of interest rate derivatives now available. The challenge has been to derive and estimate models that are internally consistent, that are consistent with observed term structures of interest rates, that embody the richness of shapes and movements that we observe, and yet that retain sufficient simplicity to be appropriate for practical application.

The first published model to start from a fitted term structure, and to model subsequent changes to it, was the binomial model introduced by Ho and Lee in 1986. Unfortunately, this model had some other disadvantages: primarily that it could only characterize essentially flat shifts in the term structure, it permitted negative interest rates and, in the long run, rates could drift off to infinity. Since then, several other models have been developed which also take an initial fitted term structure as their starting point. Among the best known of these are models by Black, Derman and Toy (1990) and by Hull and White (1990). Andrew Carverhill at Warwick has also contributed in this area. These models allow flexibility in both the initial term structure of interest rates and the term structure of volatility. However, they are not without their problems. It is all to easy to choose parameter values for the models which give unsatisfactory time series behaviour, and lead to future term structures or volatility functions which would be unlikely to be experienced in a real economy.

A related potential problem with these models is that they may throw away information which ought to be important. There is evidence (for example, a statistical study by Brown and Dybvig (1986)) that the shape of the current term structure contains information about the prospective uncertainty of interest rate movements. The models I have just described ignore this feature. A recent paper by Longstaff and Schwartz (1991) succeeds in developing a closed-form analytic model for term structures where both the short-term interest rate and volatility are random processes. This provides a breakthrough which seems extremely promising for future term structure research.

As companies demand increasingly sophisticated ways of hedging their interest rate and foreign exchange rate exposure, these kinds of research become more important than ever.

CHART 1
Control Regions for Delta to Hedge Long/Short Call Positions



Exotic options

The last few years have also seen a great deal of interest in the development of the so-called 'exotic ' options. The interest has come from both the financial community and the academic one. A paper by Mark Rubinstein (1990) summarizes many of the known valuation results. Analytic formulae are available for quite a number of these options. They include:

- Look-back options, giving the right to buy at the cheapest price over a period, or to sell at the dearest price.
- 'Knock-out' options, such as down-and-out calls which are extinguished if the underlying asset price ever drops below the knock-out boundary, and up-and-out puts which are extinguished if the asset rises above its knock-out boundary.
- 'Knock-in' options, which only become effective after crossing a boundary (eg down-and-in-calls).

For other kinds of options such as Asian or average rate options (where the option is on the arithmetic average of the asset price over some period) there is no exact formula, but research has produced a variety of approximations and numerical schemes.

Besides meeting real hedging needs, part of the attraction of these instruments is that they are often cheaper than conventional options. For example, where a conventional at-the-money 12-month call option may cost £12, it is possible that an average rate one may be only £8, and a call with a knock-out barrier at 85 per cent of the current price may cost about £10.

Hedging under transactions costs

One problem with delta hedging under a Black-Scholes kind of model is that the more frequently the hedge is rebalanced the more turnover is

involved. In fact the expected level of turnover is proportional to the square root of the number of revisions, becoming unbounded in the limit. Even with the quite low levels of costs involved in transacting futures contracts, this is clearly a problem.

Recent research has considered optimal hedging strategies under transactions costs. A paper by myself and Anthony Neuberger at the London Business School characterizes these strategies. Rather than maintain the portfolio delta at a single theoretical value, under costs it becomes optimal only to transact to maintain the delta within a computed region. This is illustrated in Chart 1, which shows the regions within which delta should be maintained (as a function of the Black-Scholes delta) to hedge call options with one year remaining to expiry. Interestingly, the regions differ depending on whether you wish to hedge a short position or a long position. Furthermore, in some cases it can be advantageous to make transactions to move the delta away from the Black-Scholes delta, which would be correct if no transactions costs were involved.

Fund management

The use of derivatives is also having an increasing impact on fund management. There are two aspects to this. First, funds are able to use derivatives directly as part of their normal investment programme. Second, many of the issues involved with managing funds through time are closely related to those of the dynamic replicating strategies used to manufacture the various kinds of options. Fund managers can now tailor the distribution of return to the objectives of a given fund. The hardest part of all this is to determine those objectives. In the US, pension fund trustees and their advisers have become quite sophisticated and the UK and Europe seem set to follow their lead.

Recent academic research (eg Dybvig (1989)) has pointed out that funds need to consider not only whether they are well diversified across securities, but also whether the asset allocation strategy leads to consistent risk return trade-offs being made through time. This idea is sometimes referred to as the idea of diversification through time. It can be shown that strategies such as 'stop-loss' strategies, and strategies of rolling over portfolio insurance are seriously inefficient. Current research at Warwick is concerned with a number of practical aspects of this problem. ■