The Dynamics of Stochastic Volatility

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ABSTRACT

This paper presents a theoretical and empirical study of the dynamical behaviour of the volatility of the Chicago Mercantile Exchange Standard and Poor 500 index futures. The time series properties of the underlying instrument, its realised volatility and volatilities implied from options prices are examined. We study a range of estimators of both realised volatility and implied volatility and draw some conclusions concerning their utility. We show how B-splines can be used to fit smooth curves to the biases in the implied volatilities across strike prices. The properties of volatility which are revealed are compared with those assumed in some recent models for pricing options under stochastic volatility. We then examine the ability of a particular, fairly general yet tractable, diffusion model (Hull and White (1988)) to explain the biases observed in the S&P 500 index futures option market prices.

1 Introduction

The Black and Scholes (1973) no-arbitrage, risk neutral valuation option pricing model is the standard used in today's derivatives markets. However, none of the assumptions of this model hold in real markets. Costless trading in continuous time is not possible, the riskless interest rate is not constant, the underlying security often pays dividends and the underlying security does not follow geometric Brownian motion with constant volatility. It is with this latter fact that we are concerned in this study.

The assumption of geometric Brownian motion for the underlying security implies that it is lognormally distributed or that returns are normally distributed. Since daily price changes are the sum of many intra day price changes it may be supposed that the central limit theorem would imply they are normally distributed. However, the conditions for the central limit theorem almost certainly do not hold intra day. For example if the number of intra day price changes is random this leads to a subordinated process.

There is now a considerable body of evidence both from empirical studies of asset returns and implied volatilities that the volatility of asset prices is stochastic. Mandelbrot (1963), Fama (1965) and Blattberg and Gonedes (1974) found that the normal distribution was not a good descriptor of stock returns. Many authors provide evidence from prices that the distributions are fat-tailed, skewed and that the variance is not constant, for example; Press (1967), Praetz (1972), Hsu, Miller and Wichern (1974), Westerfield (1977), Christie (1982), Kon (1984). Evidence from implied volatilities is provided by Latane and Rendleman (1976), Schmalensee and Trippi (1978), MacBeth and Merville (1979), Poterba and Summers (1986), Stein (1989), Franks and Schwartz (1991), Heynen, Kemna and Vorst (1991) and Xu and Taylor (1992) and from ARCH models (see Bollerslev, Chou and Kroner (1992)). This evidence has motivated extensive research into option pricing models which incorporate stochastic volatility (see the review by Clewlow and Xu (1992)).

Given that volatility is stochastic it is rational for expectations of average volatility over different horizons to be different. In other words we expect a term structure of implied volatility. Furthermore, it is also rational for implied volatilities to vary with the strike price of the option (Hull and White (1988), Stein and Stein (1991)) although other factors such as transaction costs, bid/ask spreads and nonsynchronous data can also lead to implied volatility biases.

The observation that volatility is stochastic leads to the need for an extension to the Black-Scholes model which takes account of this source of risk. Recently there has been a great deal of work in this area. The papers by Hull and White (1987), Johnson and Shanno (1987), Scott (1987) and Wiggins (1987) established that option prices could be computed if the risk premium for the volatility could be identified. This tells us what options prices should be in equilibrium in an economy in which investors have certain preferences. However it does not indicate how arbitrage profits can be locked in if market prices deviate from the model prices.

This paper studies the dynamic behaviour of volatility by analysing a range of statistics computed from the underlying price series and options prices and relating these to the theoretical models of option pricing with stochastic volatility. The data we study is the Chicago Mercantile Exchange Standard and Poor's 500 index futures and options from 1985 to 1992.

Firstly we examine the time series properties of the underlying futures prices. We then compare and contrast the historical volatility, realised volatility and the implied volatility. Garman and Klass (1980) described a range of estimators of historical/realised volatility, we study these together with a range of estimators of implied volatility which attempt to utilise the strike price cross sectional information. The mean, standard deviation and autocorrelation characteristics of a subset of these allow us to draw conclusions about the stochastic process followed by volatility.

We then study the strike price biases in the implied volatilities. We show how B-splines can be used to obtain smooth estimates of the linear biases (skews) and non-linearities (smiles/frowns) in implied volatilities. We compute the correlation between the changes in the shortest underlying futures contract and the implied volatility of the respective option contract. This gives us information on the correlation between the processes driving the underlying security and its volatility.

In the light of these empirical results we briefly review the recent option pricing models with stochastic volatility. We select the Hull and White (1988) model as a fairly general yet tractable diffusion based model. The ability of this model to explain the observed biases is studied by inferring reasonable parameter estimates from our empirical analysis. From this benchmark we examine the behaviour of the model for extreme values of the parameters and compare it with the fitted B-splines.

The paper is organised as follows. Section 2 describes the data we use for the empirical analysis. Section 3 contains the study of historical/realised volatilities and implied volatilities. The study of the empirical implied volatility biases is contained in section 4, and in section 5 we examine the theoretical biases obtained from the Hull and White (1988) model. Section 6 contains the conclusions.

2 The Data

The Chicago Mercantile Exchange (CME) introduced futures on the Standard and Poor 500 index (S&P 500) in 1982. The S&P 500 is a capitalisation-weighted index of the 500 listed firms. Each component stock's price is multiplied by the number of common shares outstanding and the resulting market values are totalled. The total value is then compared with the baseline period (1941-1943 ≡ 10) to derive the index value. The "Stats Database" we obtained from the CME contains daily trading summary data from 1985 to 1992, this period was selected as a recent period having reasonable liquidity. The information includes opening, high, low, closing and settlement prices and trading volume of the underlying futures contracts and the options on the futures. In addition we use London euro-currency interest rates collected from Datastream to approximate the riskless rate in our option pricing model.

The futures contracts have maturities every March, June, September and December and are cash settled on the Thursday prior to the third Friday of the contract month. The options have the same maturities as the futures. In addition, in July 1987 the other two nearest months were introduced with the nearest future as the underlying instrument. Therefore, after July 1987, at any time five maturities are traded up to nine months to maturity. They are American style options and exercise results in a position in the underlying futures contract except at expiry when cash settlement takes place.

Certain exclusion criteria are applied to the data to remove potentially biased option data. Options with seven or less days to maturity are removed because of well-documented expiration effects. We also eliminate options with prices smaller than 10 cents as transaction costs including the bid-ask spread and liquidity premia are large relative to the option price.

3 The Dynamics of Historical and Implied Volatilities

The classical estimator of historical volatility, the sample standard deviation of returns over a chosen period, is widely used to provide an estimate of the volatility of asset prices. However, there is typically other useful information available for security prices. In particular opening, high, low and closing prices and trading volume are usually quoted. The first paper to consider alternative estimators which utilise this information was Parkinson (1977). Garman and Klass (1980) (GK) reported an extensive study of estimators. These estimators are based on a model of security prices such that under some transformation the price changes are normally distributed with zero mean and a variance proportional to the length of the interval and the assumption that prices can be observed continuously. We select a subset of these in order to study their behaviour on a real price series. Our benchmark estimator is the standard deviation of returns ($\hat{\sigma}_0$) and we compare this with $\hat{\sigma}_1$ ($\hat{\sigma}_0$ in GK) which uses closing prices only, $\hat{\sigma}_2$ ($\hat{\sigma}_1$ in GK) which uses opening and closing prices, $\hat{\sigma}_3$ ($\hat{\sigma}_2$ in GK) which uses the daily high and low only, $\hat{\sigma}_4$ ($\hat{\sigma}_3$ in GK) which uses open, close, high and low prices and finally $\hat{\sigma}_5$ ($\hat{\sigma}_6$ in GK) which uses the open, close, high and low and takes account of the fraction of the day within which trading takes place. The mathematical definitions can be found in appendix A.

Implied volatilities are calculated from the market option settlement prices using the Barone-Adesi and Whaley (1987) model which has been shown to be very accurate for maturities shorter than twelve months. We compute various weighted averages of the implied volatilities over the set of traded strike prices for each contract on each day. We compute the at-the-money implied volatility (IV_1) where we define the at-the-money option as the option which minimises the absolute difference between the strike price and the futures price. The arithmetic average across strikes (IV_2) gives equal weight to all option prices. The average of the implied volatilities weighted by the vega (the partial differential of the option price with respect to the volatility) (IV_3) gives proportionally more weight to the at-the-money option with the proportion increasing as the time-to-maturity decreases. The average of the implied volatilities weighted by the vega elasticity (vega divided by the ratio of the option price to the volatility) (IV_4) gives proportionally more weight to the out-of-the-money options. The reasoning behind IV_4 being that rational investors are more concerned with the relative percentage change in the option price with respect to its volatility. Finally the average of the implied volatilities weighted by the volume of trade in relevant option (IV_5) is computed on the basis that the volume of trade is a proxy for the reliability of the information in the option price. The mathematical definitions of these can be found in appendix A.

The autocorrelation characteristics of the futures daily returns are summarised in Table 1. Autocorrelations are tabulated for the daily returns on the S&P 500 futures contract. Those for the two sub-samples, from January 1985 to September 1987 and from January 1988 to December 1992, excluding the crash period of 1987 tell a familiar tale; the autocorrelations of returns are very small but the autocorrelations of absolute and squared returns are all positive and several are statistically significant at low significance levels under the null hypothesis of an i.i.d process (in particular the first order autocorrelations of absolute and squared returns of sub-sample 2). For the full sample, January 1985 to December 1992, the autocorrelations of returns, absolute returns and squared returns are significant indicating the extreme behaviour in the crash period.

We also compute the sample moments of the daily returns. The mean, variance, skewness and excess kurtosis are 0.00032, 0.000188, -6.7219 and 197.840 including the crash period and 0.00048, 0.00686, -0.8977 and 8.6836 excluding the crash period. These are of similar order or magnitude as those computed for the index by Turner and Weigel (1992).

Tables 2 and 3 present summary statistics of monthly historical volatilities and implied and realised volatilities respectively. Table 4 presents the autocorrelation analysis of the implied volatilities and realised volatilities.

Figures 1 and 2 contain plots of the March, June, September and December futures information used in the realised volatility estimators (opening, high, low and closing prices) for the years 1987 and 1989. These years were chosen because of the extreme market behaviour in the October of those years. Figures 3 to 6 contain plots of the realised and implied volatilities for the same contracts.

It is clear from the summary statistics and the plots that the estimators using the full information available have the smallest variation. Their means are marginally higher than the other estimators although not significantly different from the classical estimator. The estimator which uses the high and low prices only has the lowest mean and standard deviation.

The plots of the implied volatility estimators are more interesting and reveal evidence of strike price biases. This is most evident from the difference between the mean implied and the ATM implied. This difference does not necessarily indicate the presence of a non-linear bias (the "smile" effect). The ATM option is not generally at the centre of the range of traded strikes and so a linear skew in implied volatilities can give the same effect. The vega weighted implied actually follows the mean very closely. This is because vega only becomes close to zero for the typical extremal traded strikes when the time to maturity becomes less than one month. Therefore at longer times to maturity the vega weighted implied volatility is close to the mean and as the time to maturity decreases it moves away from the mean. The vega elasticity weighted implied is generally a reasonable estimator and generally follows the ATM estimator quite closely. However, it does suffer at times from giving more weight to out-of-the-money options which are the most susceptible to biases. The volume of trade weighted estimator seems to be a good alternative to the ATM estimator adding reasonable weight to important away from the money implied volatility information without distorting the estimates.

The autocorrelation analysis of the weighted implied volatilities and realised volatilities in Table 4 show clearly that both the implied and realised volatility is mean reverting. For most samples (contracts), only the first order autocorrelations and partial autocorrelations are significant and consequently an AR(1) model is adequate. It is noticeable that the different estimators of implied and realised volatility often give very different autocorrelation coefficients.

It is interesting to examine whether the implied volatilities are unbiased estimates of future realised volatilities. We run simple regressions of realised volatility against weighted implied volatility, the results are presented in Table 5. Only in some cases do implied volatilities have limited power in predicting future realised volatility, but in all cases we can reject the null hypothesis that implied volatilities are an unbiased estimate of realised volatility. More importantly, the predictive power is significantly related to which estimators are used. The results in Table 5 clearly illustrate this point.

4 Skews, Smiles and Frowns in Implied Volatilities

We have already noted that the implied volatility estimators exhibit evidence of strike price biases. However, the relative small number of liquidly traded strikes and the presence of a multitude of factors affecting the biases, makes it difficult to study the implied volatility data directly. We therefore fit B-splines (see Powell (1981)) to implied volatilities as a function of strike price on a weekly basis, that is using all implied volatilities within each week. The results for the March, June, September and December contracts in 1987 and 1989 are plotted in figures 7 and 8. The most striking facet is the persistent negative skew (that is the slope is negative with respect to increasing strike price) in the 1989 data which is not present to the same degree in 1987. Also noticeable is the increase in the non-linearities as the option contract approaches maturity, which is to be expected and is predicted by Taylor and Xu (1993).

An important facet of the behaviour of volatility which is known to influence the skew is its correlation with the security price. This is very difficult to estimate. We compute the correlation between these changes for all the March, June, September and December contracts from 1985 to 1992, contract by contract. This gives us information on the correlation between the processes driving the underlying security and its volatility. The results are plotted in figure 9. It is clear that the correlation is not constant and indeed appears to vary randomly. The mean of the samples is slightly positive (0.066072) but not significant. The general consensus from empirical work has been that the correlation coefficient is negative for indices, with the story being that if the market falls the volatility increases. However one can generalise the story to say that if the market rises or falls then the volatility increases. This would lead to a U shaped correlation function and indeed practitioners generally talk of a non-linear correlation function. The scatter plots show some evidence of this although it is not strong. We also estimate the correlation coefficient between monthly returns and the monthly historical volatility (following French, Schwert, and Stambaugh (1987)) using the estimators $\hat{\sigma}_0$ and $\hat{\sigma}_5$. We obtain correlation coefficients of -0.52242 and -0.52110 respectively with the crash period included and -0.17067 and -0.12185 excluding the crash period.

These results suggest it is important for any model of stochastic volatility to allow a non zero correlation between the security price and its volatility. This would at least allow practitioners to back out implied correlations from option prices in an analogous way to obtaining implied volatilities.

5 Stochastic Volatility and Option Pricing Models

Recently there has been a significant amount of theoretical work on option pricing with stochastic volatility. The first papers to consider the problem; Hull and White (1987), Johnson and Shanno (1987), Scott (1987) and Wiggins (1987) established that option prices could be computed if the risk premium on volatility could be identified. This tells us what options prices should be in equilibrium in an economy in which investors have certain preferences. However it does not indicate how arbitrage profits can be locked in if market prices deviate from the model prices. Furthermore, only in the Hull and White paper is any form of analytical solution obtained and this has a very restrictive set of assumptions. Namely that the variance follows geometric Brownian motion and is independent of the security price.

Some authors have considered alternatives to specifying a diffusion process for volatility. Bookstaber and McDonald(1988) suggest fitting a generalised beta distribution of the second kind (GB2) to security prices. Madan and Senata (1990) model security price returns as normally distributed conditional on a variance which is gamma distributed.

Recently Dupire (1992) has described a new approach to pricing and hedging volatility. The aim being to develop continuous-time no-arbitrage pricing with stochastic volatility without the need to specify a volatility risk premium.

In this section we wish to examine whether a diffusion process for the volatility can explain the empirical biases we observe in option prices. Based on the empirical analysis in section 3 we require the diffusion process to be mean reverting. The stochastic process should exclude the possibility of negative volatilities. Finally the specification should allow the volatility or variance to be correlated with the security price process.

A reasonable diffusion process for the variance V is therefore

$$dV = \alpha(V_m - V)dt + \xi V^{\beta} dw \qquad (1)$$

where V_m is the level to which the variance mean reverts at rate α and dw is a Wiener process with correlation ρ to that driving the security price. We also define; T the time to maturity, S the security price, σ the volatility, r the riskless rate, and d the dividend yield.

Stein and Stein (1991) study the problem when $\beta = 0$, V is the volatility and the correlation between the security price and the volatility is zero. They are able to derive a quasi-closed form formula involving a numerical integration for the probability density of the security price. Option prices can then be obtained by evaluating the risk neutral expectation of the option payoff which involves a further numerical integration. This model has three serious drawbacks; the volatility can become negative, the correlation is zero and the option pricing formula is not analytical.

Hull and White (1988) assume $\beta = 0.5$ and are able to obtain an option pricing formula of the form

$$C = c + f_0 + f_1\xi + f_2\xi^2 + \ldots \qquad (2)$$

where C is the option price under stochastic volatility, c is the Black-Scholes price and the f_i are functions of S, V and t. Their model also allows a constant correlation ρ between the security price and the variance. However, there is are restrictions on the functional forms allowed for the volatility risk premium. Hull and White show that only the first three terms in the power series are needed to obtain accurate prices.

In a recent paper Scott (1992) essentially assumes the same processes together with a stochastic interest rate. The model specification is consistent with the general equilibrium model of Cox, Ingersoll, and Ross (1985). Again there are no restrictions on the correlation between the stock return and volatility. The process for the short term interest rate is a linear combination of the variance and another stochastic variable following the same form of process as the variance. This is not a very realistic model of the interest rate and its correlation with volatility. However, it is straightforward to assume that the short term interest rate is constant and the second stochastic factor does not exist. A quasi closed form solution for option prices can be derived by using the Fourier inversion formula for probability distribution functions. To price options only two univariate numerical integrations are needed which is very efficient to compute.

Since this model has an analytical option pricing formula it opens the possibility of fitting the model cross-sectionally to option prices. That is we can estimate the risk neutral parameters of the variance process by minimising the difference between the model option prices and market prices for the range of traded strike prices. However, we found it difficult to get the estimation procedure to converge, the main problem being the inability of the model to fit the empirical skews. We therefore chose to study the models properties for a range of parameters.

As a benchmark we infer reasonable parameters values from our empirical analyses. For the long term mean (V_m) we use the mean of the monthly historical volatilities excluding the October 1987 crash period (approximately 0.0225) (see Table 2, Panel B). For the mean reversion rate (α) we examine the implied volatility time series. These show that large shocks tend to decay over a period of approximately one month for the preferred estimators. This corresponds to a value of α of approximately 10. The volatility of the volatility is $\xi/2$, for this we take the standard deviation of the monthly historical volatilities (approximately 0.05). Scott (1992) has estimated equivalent parameters for the S&P 500 index, he obtains; $\alpha = 5.0$, V_m = 0.019 and $\xi = 0.38$.

The coefficients f_0, f_1 and f_2 are given by

$$f_0 = c(\overline{V}) - c(V) \qquad (3)$$

where $\overline{V} = \frac{1}{T}\int_0^T \mathrm{E}[\mathrm{V(t)}]dt$,

$$f_1 = k_1 \frac{\partial^2 c}{\partial S \partial V} \qquad (4)$$

$$f_2 = k_2 \frac{\partial^2 c}{\partial S \partial V} + k_3 \frac{\partial^2 c}{\partial V^2} + k_4 \frac{\partial^3 c}{\partial S \partial V^2} + k_5 \frac{\partial^3 c}{\partial V^3} \qquad (5)$$

Thus the option price is the Black-Scholes price evaluated with $\overline{V}$ plus the sum of the terms involving the partial differentials. Therefore if we compute these terms for the a range of strike prices we can determine which of the terms are important and what kinds of biases they cause. Figures 10 to 13 show this information together with the Black-Scholes implied volatility (the terms involving k_1 through to k_5 are labelled d2cSV, d2cSV2, d2cVV, d3cSVV and d3cVVV). The figures show that only two of the terms are important for determining the biases and their effect is clear. The term $k_1 \frac{\partial^2 c}{\partial S \partial V}\xi$ (d2cSV) determines the skew and the term $k_3 \frac{\partial^2 c}{\partial V^2}\xi^2$ (d2cVV) determines the smile.

The figures 10 and 11 which show the behaviour for the benchmark parameters and extremal correlation coefficients of 1 and -1, demonstrate that the model does not give the observed skews for these parameter values. Figure 12 shows the behaviour for a correlation of 0 and for a larger value of $\xi = 0.3$. Here only the smile term (d2cVV) is non zero. As ξ is increased the smile term, which is proportional to ξ^2 becomes more important relative to the skew term which is proportional to ξ. Also shown are implied volatilities for different maturities showing how the smile "tightens" as maturity approaches. Figure 13 shows that larger skews can be obtained by decreasing the mean reversion rate α. The limit is reached for the value of α used in figure 13 of 0.1. The skew in this figure is still not large enough to explain the largest observed skews. Furthermore, this value of α corresponds to a mean reversion half-life of approximately seven years, which does not seem realistic.

6 Conclusions

We have studied the time series properties of Chicago Mercantile Exchange's Standard and Poor 500 index futures, its realised volatility and volatilities implied from options prices. We examined a range of estimators of both historical/realised volatility and implied volatility. The results show that the choice of estimator can have important consequences for the results. Estimators which utilise the full range of daily price information are significantly more efficient. The correlation properties of both the realised and implied volatilities indicate mean reversion is present.

The implied volatilities exhibit substantial skews in addition to the smiles/frowns. This indicates that changes in volatility are negatively correlated with the returns as we demonstrate with the properties of the Hull and White (1988) model.

Finally, we also demonstrate, using the properties of this model, that diffusion processes for the volatility probably cannot fully account for the observed biases in implied volatilities. Alternatives are that the volatility process contains a jump component and/or there are transaction cost effects which are important.

References

Barone-Adesi, G., R.E. Whaley, "Efficient Analytical Approximation of American Option Values", Journal of Finance 42, pp 301 - 320, 1987.

Black, F., M. Scholes, "The Pricing of Options and Corporate Liabilities", Journal of Political Economy 81, pp 637 - 659, 1973.

Blattberg, R.C., N.J. Gonedes, "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices", Journal of Business 47, pp 244 - 280, 1974.

Bookstaber, R.M., J.B. McDonald, "A General Distribution for Describing Security Price Returns", Journal of Business 60(3), pp 401 - 424, 1987.

Boyle, P., T. Vorst, "Option Replication in Discrete Time with Transactions Costs", Journal of Finance, 47, pp 271-293.

Christie, A.A., "The Stochastic Behaviour of Common Stock Variances: Value, Leverage and Interest Rate Effects", Journal of Financial Economics 10, pp 407 - 432, 1982.

Clewlow, L.J., X. Xu, "A Review of Option Pricing with Stochastic Volatility", FORC Preprint 92/35, Warwick Business School, University of Warwick, 1992.

Cox, J.C., J.E. Ingersoll, S.A. Ross, "An Intertemporal General Equilibrium Model of Asset Prices", Econometrica 53, pp 363 - 384, 1985.

Dupire, B., "Arbitrage Pricing with Stochastic Volatility", Working Paper, Société Générale Division Options, Paris, May, 1992.

Fama, E.F., "The Behaviour of Stock Market Prices", Journal of Business 38, pp 34 - 105, 1965.

Franks, J.R., E.S. Schwartz, "The Stochastic Behaviour of Market Variance Implied in the Prices of Index Options", The Economic Journal 101, pp 1460 - 1475, 1991.

French, K.R., G.W. Schwert, R.F. Stambaugh, "Expected Stock Returns and Volatility", Journal of Financial Economics 19, pp 3 - 29, 1987.

Garman, M., Klass, M.J., "On the Estimation of Security Price Volatilities from Historical Data", Journal of Business 53, pp 67 - 78, 1980.

Heynen, R., A.G.Z. Kemna, T. Vorst, "Analysis of the Term Structure of Implied Volatilities", Working Paper, Erasmus University, Netherlands, 1992.

Hsu, D.A., R.B. Miller, D.W. Wichern, "On the Stable Paretian Behaviour of Stock Market Prices", Journal of the American Statistical Association 69, pp 108 - 113, 1974.

Hull, J., A. White, "The Pricing of Options on Assets with Stochastic Volatilities", Journal of Finance 42(2), pp 281 - 300, June 1987.

Hull, J., A. White, "An Analysis of the Bias in Option Pricing Caused by A Stochastic Volatility", Advances in Futures and Options Research 3, pp 29 - 61, 1988.

Johnson, H., D. Shanno, "Option Pricing when the Variance is Changing", Journal of Financial and Quantitative Analysis 22(2), pp 143 - 151, June 1987.

Kon, S.J., "Models of Stock Returns - a Comparison", Journal of Finance 39, pp 147 - 166, 1984.

Latané, H.A., R.J. Rendleman, Jr., "Standard Deviations of Stock Price Ratios Implied in Option Prices", Journal of Finance 31, pp 369 - 381, 1976.

MacBeth, J., L. Merville, "An Empirical Examination of the Black-Scholes Call Option Pricing Formula", Journal of Finance 34, pp 1173 - 1186, 1979.

Madan, D.B., E. Senata, "The Variance Gamma (V.G.) Model for Share Market Returns", Journal of Business 63(4), pp 511 - 524, 1990.

Mandelbrot, B.B., "New Methods in Statistical Economics", Journal of Political Economy 71, pp 421 - 440, 1963.

Poterba, J.M., L.H. Summers, "The Persistence of Volatility and Stock Market Fluctuations", American Economic Review 76, pp 1142 - 1151, 1986.

Powell, M.J.D., "Approximation Theory and Methods", Cambridge University Press, 1981.

Praetz, P.D., "The Distribution of Share Price Changes", Journal of Business 45, pp 49 - 55, 1972.

Press, S.J., "A Compound Events Model for Security Prices", Journal of Business 40, pp 317 - 335, 1967.

Rosenberg, B., "The Behaviour of Random Variables with Nonstationary Variance and the Distribution of Security Prices", Working Paper, University of California at Berkeley, 1972.

Schmalensee, R., R.R. Trippi, "Common Stock Volatility Expectations Implied by Option Premia", pp 129 - 147, 1978.

Scott, L., "Option Pricing when the Variance Changes Randomly: Theory, Estimation and an Application", Journal of Financial and Quantitative Analysis 22(4), pp 419 - 438, December 1987.

Scott, L., "Stock Market Volatility and the Pricing of Index Options: An Analysis of Implied Volatilities and the Volatility Risk Premium in a Model with Stochastic Interest Rates and Volatility", Working Paper, Department of Finance, University of Georgia, 1992.

Stein, J., "Overreactions in the Options Market", Journal of Finance 44, pp 1011 - 1023, 1989.

Stein, E.M., J.C. Stein, "Stock Price Distributions with Stochastic Volatility: An Analytical Approach", The Review of Financial Studies 4(4), pp 727 - 752, 1991.

Taylor, S.J., Xu X., "The Magnitude of Implied Volatility Smiles: Theory and Empirical Evidence for Exchange Rates", Working Paper, The Management School, Lancaster University, 1993.

Turner, A.L., E.J. Weigel, "Daily Stock Market Volatility: 1928 - 1989", Management Science 38, pp 1587 - 1609, 1992.

Westerfield, R., "The Distribution of Common Stock Price Changes: An Application of Transaction Time and Subordinated Stochastic Models", Journal of Financial and Quantitative Analysis 12, pp 743 - 765, 1977.

Wiggins, J.B., "Option Values under Stochastic Volatility: Theory and Empirical Estimates", Journal of Financial Economics 19, pp 351 - 372, 1987.

Xu, X., S.J. Taylor, "The Term Structure of Volatility Implied by Foreign Exchange Options", FORC Preprint 92/32, Warwick Business School, University of Warwick, 1992.

Appendix A

Historical volatility estimators

$R_i \equiv$ today's return.

$\overline{R} \equiv$ mean return over sample.

$f \equiv$ fraction of the day that trading is closed.

$C_{i-1} \equiv$ previous day's closing price.

$O_i \equiv$ today's opening price.

$H_i \equiv$ today's high.

$L_i \equiv$ today's low.

$C_i \equiv$ today's closing price.

$u = H_i - O_i$

$d = L_i - O_i$

$c = C_i - O_i$

$$\hat{\sigma}_0^2 \equiv (R_i - \overline{R})^2 \tag{1}$$

$$\hat{\sigma}_1^2 \equiv (C_i - C_{i-1})^2 \tag{2}$$

$$\hat{\sigma}_2^2 \equiv \frac{(O_i - C_{i-1})^2}{2f} + \frac{(C_i - O_i)^2}{2(1-f)} \tag{3}$$

$$\hat{\sigma}_3^2 \equiv \frac{(H_i - L_i)^2}{4\ln(2)} \tag{4}$$

$$\hat{\sigma}_4^2 \equiv 0.17\frac{(O_i - C_{i-1})^2}{f} + 0.83\frac{(u_i - d_i)^2}{4\ln(2)(1-f)} \tag{5}$$

$$\hat{\sigma}_5^2 \equiv 0.12\frac{(O_i - C_{i-1})^2}{f} + 0.88\frac{0.511(u_i - d_i)^2 - 0.019(c_i(u_i + d_i) - 2u_i d_i) - 0.383c_i^2}{(1-f)} \tag{6}$$

Implied volatility estimators

$\sigma_i \equiv$ is the implied volatility at strike price i .

$n \equiv$ is the number of strike prices.

$c_i \equiv$ is the model option price for strike price i .

$v_i \equiv$ is the volume of trade in strike price i .

$$IV_1 = \sigma_{AT-THE-MONEY} \tag{7}$$

$$IV_2 = \frac{1}{n}\sum \sigma_i \tag{8}$$

$$IV_3 = \frac{\sum \sigma_i \frac{\partial c_i}{\partial \sigma_i}}{\sum \frac{\partial c_i}{\partial \sigma_i}} \tag{9}$$

$$IV_4 = \frac{\sum \sigma_i \frac{\partial c_i}{\partial \sigma_i} \Big/ \frac{c_i}{\sigma_i}}{\sum \frac{\partial c_i}{\partial \sigma_i} \Big/ \frac{c_i}{\sigma_i}} \tag{10}$$

$$IV_5 = \frac{\sum \sigma_i v_i}{\sum v_i} \tag{11}$$

Table 1: Autocorrelations and Partial Autocorrelations of Daily Returns on S&P500 Futures

Panel A: Returns

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Std.Error
Full Sample	-0.01	-0.17	-0.03	-0.04	0.08	0.01	0.02
	-0.01	-0.17	-0.03	-0.07	0.07	0.00	0.02
Sub-sample 1	0.01	0.00	-0.01	-0.06	-0.03	-0.02	0.04
	0.01	0.00	-0.01	-0.06	-0.03	-0.02	0.04
Sub-sample 2	-0.06	-0.04	-0.02	-0.03	0.03	-0.03	0.03
	-0.06	-0.04	-0.02	-0.03	0.02	-0.03	0.03

Panel B: Absolute Returns

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Std.Error
Full Sample	0.27	0.31	0.24	0.15	0.19	0.16	0.02
	0.27	0.26	0.12	0.00	0.09	0.06	0.02
Sub-sample 1	0.00	0.06	0.02	0.12	0.05	0.11	0.04
	0.00	0.06	0.02	0.12	0.05	0.10	0.04
Sub-sample 2	0.12	0.04	0.05	0.12	0.11	0.08	0.03
	0.12	0.03	0.04	0.11	0.08	0.05	0.03

Panel C: Squared Returns

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Std.Error
Full Sample	0.08	0.26	0.06	0.01	0.07	0.02	0.02
	0.08	0.25	0.02	-0.06	0.05	0.03	0.02
Sub-sample 1	0.04	0.04	0.00	0.10	0.04	0.10	0.04
	0.04	0.04	0.00	0.10	0.04	0.09	0.04
Sub-sample 2	0.12	0.00	0.00	0.09	0.03	0.04	0.03
	0.14	-0.02	0.00	0.10	0.00	0.03	0.03

Notes: The full sample is from January 1985 to December 1992, the first sub-sample is from January 1985 to September 1987 and the second from January 1988 to December 1992.

Table 2: Summary Statistics of Monthly Non-overlapping Historical Volatility

Panel A: Full Sample

	Mean	Std.Dev.	Minimum	Maximum
$\hat{\sigma}_0$	0.16492	0.15087	0.0712	1.4793
$\hat{\sigma}_1$	0.16472	0.4793	0.0719	1.4539
$\hat{\sigma}_2$	0.16377	0.18258	0.0610	1.8023
$\hat{\sigma}_3$	0.15464	0.12727	0.0790	1.2785
$\hat{\sigma}_4$	0.16775	0.14708	0.0850	1.4792
$\hat{\sigma}_5$	0.16597	0.13702	0.0903	1.3842

Panel B: Sample excluding October 1987 contract

	Mean	Std.Dev	Minimum	Maximum
$\hat{\sigma}_0$	0.14752	0.06137	0.0712	0.4347
$\hat{\sigma}_1$	0.14757	0.05985	0.0719	0.4245
$\hat{\sigma}_2$	0.14286	0.06338	0.0610	0.4688
$\hat{\sigma}_3$	0.13952	0.04682	0.0790	0.3462
$\hat{\sigma}_4$	0.15058	0.05219	0.0850	0.3917
$\hat{\sigma}_5$	0.14987	0.04810	0.0903	0.3660

Table 3: Summary Statistics of
Call and Put Options Weighted Implied Volatilities and Realised Volatilities

Panel A: Call Options

Full Sample including October 1987 to December 1987

	Mean	Std.Dev	Minimum	Maximum
IV_1	0.17565	0.061674	0.0931	0.9520
IV_2	0.18355	0.058698	0.0941	0.8885
IV_3	0.18570	0.058977	0.0948	0.8834
IV_4	0.16412	0.054114	0.0969	0.8600
IV_5	0.17018	0.055045	0.0980	0.8354

Full Sample excluding October 1987 to December 1987

	Mean	Std.Dev	Minimum	Maximum
IV_1	0.16883	0.041878	0.0931	0.3560
IV_2	0.17703	0.039907	0.0941	0.3670
IV_3	0.17927	0.040744	0.0948	0.3620
IV_4	0.15783	0.035329	0.0969	0.3289
IV_5	0.16406	0.037956	0.0980	0.3290

Panel B: Put Options

Full Sample including October 1987 to December 1987

	Mean	Std.Dev	Minimum	Maximum
IV_1	0.17559	0.061712	0.0940	0.9519
IV_2	0.19327	0.059632	0.0915	0.9185
IV_3	0.18850	0.057714	0.0913	0.9085
IV_4	0.22476	0.070810	0.0978	0.9350
IV_5	0.19687	0.066447	0.0948	0.8851

Full Sample excluding October 1987 to December 1987

	Mean	Std.Dev	Minimum	Maximum
IV_1	0.16881	0.041922	0.0940	0.3562
IV_2	0.18683	0.041275	0.0915	0.3680
IV_3	0.18228	0.039849	0.0913	0.3618
IV_4	0.21812	0.054752	0.0978	0.4219
IV_5	0.18928	0.047821	0.0948	0.4170

Panel C: Realised Volatility

Full Sample including October 1987 to December 1987

	Mean	Std.Dev	Minimum	Maximum
$\hat{\sigma}_0$	0.15809	0.11068	0.0660	1.0443
$\hat{\sigma}_1$	0.15791	0.10994	0.0666	1.0336
$\hat{\sigma}_2$	0.15521	0.13966	0.0534	1.2791
$\hat{\sigma}_3$	0.14863	0.09604	0.0713	0.9140
$\hat{\sigma}_4$	0.16170	0.11216	0.0773	1.0580
$\hat{\sigma}_5$	0.16023	0.10467	0.0830	0.9948

Full Sample excluding October 1987 to December 1987

	Mean	Std.Dev	Minimum	Maximum
$\hat{\sigma}_0$	0.14893	0.088256	0.0660	0.9269
$\hat{\sigma}_1$	0.14889	0.087843	0.0666	0.9222
$\hat{\sigma}_2$	0.14426	0.109030	0.0534	1.1307
$\hat{\sigma}_3$	0.14072	0.075795	0.0713	0.8135
$\hat{\sigma}_4$	0.15260	0.088043	0.0773	0.9395
$\hat{\sigma}_5$	0.15173	0.082283	0.0830	0.8831

Table 4: First-order Autocorrelation and the Number of Significant Lags

Panel A: Call Options Weighted Implied Volatilities

	IV_1		IV_2		IV_3		IV_4		IV_5	
8503	0.86	1	0.15	0	0.24	1	0.76	1	0.85	1
8506	0.91	1	0.81	2	0.77	2	0.82	2	0.92	2
8509	0.84	1	0.57	1	0.57	1	0.77	1	0.80	1
8512	0.89	1	0.78	2	0.78	3	0.83	1	0.83	1
8603	0.94	1	0.91	1	0.93	1	0.94	1	0.94	2
8606	0.94	1	0.61	2	0.76	2	0.92	1	0.93	1
8609	0.91	1	0.74	2	0.78	2	0.89	1	0.91	1
8612	0.94	1	0.92	1	0.89	1	0.94	1	0.94	1
8703	0.96	1	0.94	1	0.94	1	0.96	1	0.97	1
8706	0.94	1	0.90	2	0.93	2	0.93	1	0.94	1
8709	0.94	2	0.87	1	0.91	1	0.95	2	0.94	1
8803	0.85	1	0.81	1	0.66	1	0.85	1	0.87	1
8806	0.79	1	0.83	1	0.77	1	0.86	1	0.78	1
8809	0.64	1	0.78	1	0.61	1	0.68	2	0.61	1
8812	0.84	1	0.69	2	0.59	2	0.80	1	0.72	1
8903	0.82	1	0.60	2	0.70	1	0.76	1	0.57	1
8906	0.75	1	0.13	0	0.17	0	0.79	1	0.69	1
8909	0.81	1	0.75	1	0.80	3	0.88	2	0.70	1
8912	0.88	1	0.93	1	0.86	2	0.80	1	0.52	2
9003	0.90	1	0.93	1	0.83	2	0.88	1	0.78	2
9006	0.84	1	0.80	1	0.69	1	0.78	1	0.44	1
9009	0.92	1	0.94	1	0.88	1	0.90	1	0.88	1
9012	0.90	1	0.89	1	0.74	2	0.83	1	0.81	1
9103	0.89	1	0.81	1	0.73	1	0.82	1	0.58	1
9106	0.81	1	0.18	2	0.24	2	0.75	1	0.57	1
9109	0.86	1	0.72	1	0.47	1	0.69	1	0.68	1
9112	0.78	1	0.86	1	0.74	1	0.84	1	0.67	1
9203	0.89	1	0.83	1	0.67	2	0.80	1	0.57	1
9206	0.90	1	0.32	1	0.28	1	0.92	1	0.81	2
9209	0.81	1	0.43	1	0.39	2	0.78	1	0.51	2
9212	0.91	1	0.89	1	0.54	3	0.79	1	0.86	1

Panel B: Put Options Weighted Implied Volatilities

	IV_1		IV_2		IV_3		IV_4		IV_5	
8503	0.85	1	0.86	1	0.86	1	0.80	1	0.83	1
8506	0.91	1	0.89	1	0.90	1	0.88	1	0.90	1
8509	0.80	1	0.56	1	0.55	1	0.76	1	0.58	1
8512	0.85	1	0.85	1	0.84	1	0.76	2	0.83	1
8603	0.93	1	0.94	1	0.94	1	0.92	1	0.93	1
8606	0.94	1	0.94	1	0.94	1	0.90	1	0.93	1
8609	0.90	1	0.89	1	0.89	1	0.85	1	0.88	1
8612	0.93	1	0.90	1	0.84	1	0.91	1	0.90	1
8703	0.96	1	0.97	1	0.97	1	0.97	1	0.98	1
8706	0.94	1	0.94	1	0.93	1	0.92	1	0.93	1
8709	0.94	1	0.94	1	0.94	2	0.92	1	0.93	1
8803	0.85	1	0.79	1	0.76	1	0.78	1	0.76	1
8806	0.80	1	0.78	1	0.81	1	0.70	2	0.62	2
8809	0.63	1	0.73	1	0.76	2	0.70	1	0.47	2
8812	0.83	1	0.85	1	0.79	1	0.93	2	0.72	1
8903	0.82	1	0.74	1	0.64	1	0.91	1	0.60	1
8906	0.76	1	0.75	1	0.68	1	0.81	1	0.64	1
8909	0.80	1	0.95	1	0.93	1	0.95	1	0.70	1
8912	0.87	1	0.93	1	0.82	2	0.94	1	0.84	1
9003	0.90	1	0.73	2	0.62	2	0.93	1	0.53	3
9006	0.83	1	0.68	2	0.50	3	0.94	1	0.51	3
9009	0.92	1	0.90	2	0.92	1	0.95	1	0.89	1
9012	0.90	1	0.52	2	0.52	2	0.87	1	0.71	1
9103	0.89	1	0.62	1	0.36	1	0.83	1	0.65	1
9106	0.81	1	0.51	1	0.23	2	0.76	2	0.60	2
9109	0.86	1	0.79	1	0.46	1	0.89	1	0.45	1
9112	0.78	1	0.81	1	0.67	2	0.92	2	0.62	1
9203	0.89	1	0.87	2	0.52	2	0.91	2	0.38	1
9206	0.90	2	0.81	1	0.57	1	0.87	1	0.46	3
9209	0.82	1	0.80	1	0.28	1	0.91	1	0.39	2
9212	0.91	1	0.88	1	0.33	2	0.94	1	0.63	2

Panel C: Realised Futures Volatilities

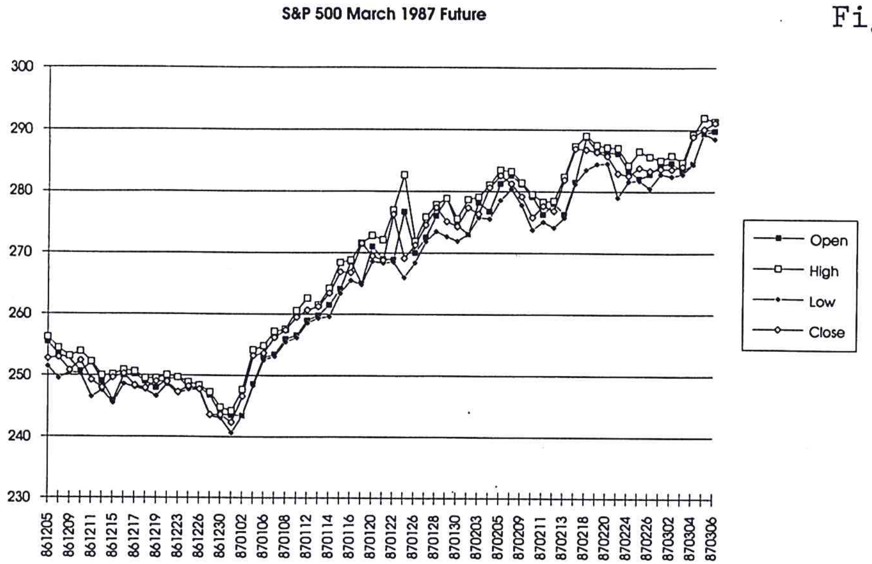
	$\hat{\sigma}_0$		$\hat{\sigma}_1$		$\hat{\sigma}_2$		$\hat{\sigma}_3$		$\hat{\sigma}_4$		$\hat{\sigma}_5$	
8503	0.89	1	0.90	1	0.92	1	0.89	1	0.89	1	0.86	1
8506	0.83	1	0.81	1	0.83	1	0.86	1	0.84	1	0.91	1
8509	0.83	1	0.82	1	0.78	1	0.78	1	0.77	1	0.80	1
8512	0.84	1	0.69	1	0.84	1	0.91	1	0.92	1	0.93	2
8603	0.88	1	0.87	1	0.87	1	0.90	1	0.91	1	0.90	1
8606	0.77	1	0.77	1	0.87	1	0.92	1	0.92	1	0.89	1
8609	0.94	1	0.92	1	0.93	1	0.92	1	0.92	1	0.91	1
8612	0.91	1	0.92	1	0.90	1	0.95	1	0.94	1	0.96	1
8703	0.96	1	0.95	1	0.96	1	0.96	1	0.96	1	0.95	1
8706	0.92	1	0.91	1	0.94	1	0.95	1	0.96	1	0.96	1
8709	0.95	1	0.95	1	0.96	1	0.98	3	0.98	3	0.97	2
8803	0.97	1	0.97	1	0.97	1	0.97	1	0.97	1	0.97	1
8806	0.72	1	0.71	1	0.80	1	0.78	1	0.79	1	0.87	1
8809	0.83	1	0.86	1	0.90	1	0.94	1	0.95	1	0.97	1
8812	0.88	1	0.89	1	0.81	1	0.91	1	0.90	1	0.87	1
8903	0.82	1	0.82	1	0.82	1	0.75	1	0.74	1	0.78	1
8906	0.79	1	0.76	1	0.87	1	0.85	1	0.86	1	0.89	1
8909	0.84	1	0.85	1	0.89	1	0.88	1	0.90	1	0.90	1
8912	0.97	1	0.97	1	0.92	2	0.98	2	0.98	2	0.98	2
9003	0.96	1	0.96	1	0.94	1	0.96	1	0.96	1	0.95	1
9006	0.69	1	0.70	1	0.54	1	0.66	1	0.54	1	0.69	1
9009	0.91	1	0.92	1	0.93	1	0.93	1	0.93	1	0.93	1
9012	0.95	1	0.94	1	0.90	1	0.95	1	0.94	1	0.95	1
9103	0.90	1	0.92	1	0.92	1	0.91	1	0.91	1	0.91	1
9106	0.95	1	0.95	1	0.96	1	0.96	1	0.95	1	0.95	1
9109	0.90	1	0.89	1	0.90	1	0.91	1	0.91	1	0.91	1
9112	0.88	1	0.88	1	0.83	1	0.91	1	0.90	1	0.93	1
9203	0.93	1	0.93	1	0.93	1	0.94	1	0.94	1	0.94	1
9206	0.96	1	0.97	1	0.90	1	0.97	1	0.97	1	0.94	1
9209	0.80	1	0.83	1	0.84	1	0.86	1	0.87	1	0.81	3
9212	0.98	1	0.97	1	0.97	1	0.97	1	0.97	1	0.97	1

Table 5: Regression Coefficients and Adjusted R^2
of Realised Volatility on Implied Volatility of Call Options

	$\hat{\sigma}_0$ vs IV_1			$\hat{\sigma}_0$ vs IV_2			$\hat{\sigma}_0$ vs IV_3		
1985	0.077	0.251	0.110	0.079	0.227	0.116	0.076	0.250	0.117
	(14.4)	(5.26)		(15.3)	(5.06)		(14.2)	(5.32)	
1986	0.161	0.060	-0.002	0.177	-0.025	-0.003	0.152	0.112	0.001
	(12.9)	(0.84)		(13.2)	(-0.45)		(12.8)	(1.67)	
1987	0.195	0.393	0.026	0.198	0.377	0.022	0.176	0.490	0.030
	(8.65)	(7.03)		(8.47)	(6.64)		(7.55)	(7.60)	
1988	0.017	0.657	0.509	0.025	0.600	0.393	0.008	0.725	0.525
	(1.63)	(13.7)		(2.03)	(10.7)		(0.77)	(15.0)	
1989	0.170	-0.253	0.009	0.182	-0.298	0.023	0.113	0.116	-0.001
	(8.11)	(-1.91)		(13.0)	(-3.80)		(4.63)	(0.73)	
1990	0.138	0.118	0.018	0.171	-0.051	-0.001	0.148	0.072	0.002
	(16.2)	(2.69)		(14.5)	(-0.85)		(13.6)	(1.22)	
1991	0.050	0.515	0.489	0.047	0.494	0.326	0.046	0.560	0.494
	(9.50)	(17.6)		(6.49)	(12.3)		(8.12)	(17.1)	
1992	0.055	0.283	0.136	0.098	-0.016	-0.003	0.062	0.251	0.090
	(8.49)	(6.14)		(16.2)	(-0.43)		(10.0)	(5.42)	

	$\hat{\sigma}_5$ vs IV_1			$\hat{\sigma}_5$ vs IV_2			$\hat{\sigma}_5$ vs IV_5		
1985	0.032	0.650	0.499	0.039	0.576	0.505	0.034	0.624	0.491
	(9.10)	(21.2)		(10.6)	(18.9)		(9.46)	(20.9)	
1986	0.173	-0.046	-0.001	0.180	-0.088	0.006	0.170	-0.031	-0.002
	(25.2)	(-1.22)		(24.7)	(-2.18)		(25.7)	(-0.86)	
1987	0.196	0.422	0.036	0.198	0.409	0.031	0.174	0.533	0.042
	(7.55)	(4.51)		(7.28)	(4.20)		(6.21)	(4.92)	
1988	0.018	0.640	0.756	0.016	0.626	0.669	0.009	0.706	0.779
	(3.59)	(27.7)		(2.54)	(21.7)		(1.96)	(32.4)	
1989	0.121	0.025	-0.003	0.126	-0.010	-0.003	0.095	0.189	0.027
	(12.3)	(0.41)		(17.2)	(-0.24)		(9.08)	(2.77)	
1990	0.138	0.171	0.036	0.158	0.068	0.000	0.145	0.147	0.014
	(15.0)	(3.99)		(12.7)	(1.14)		(12.6)	(2.49)	
1991	0.052	0.533	0.588	0.043	0.541	0.439	0.053	0.547	0.529
	(9.22)	(16.7)		(6.55)	(15.3)		(9.23)	(16.2)	
1992	0.047	0.407	0.282	0.075	0.187	0.046	0.046	0.437	0.275
	(9.23)	(11.0)		(10.3)	(3.95)		(9.43)	(11.5)	

Figure 1



S&P 500 March 1987 Future
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250
240
230
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High
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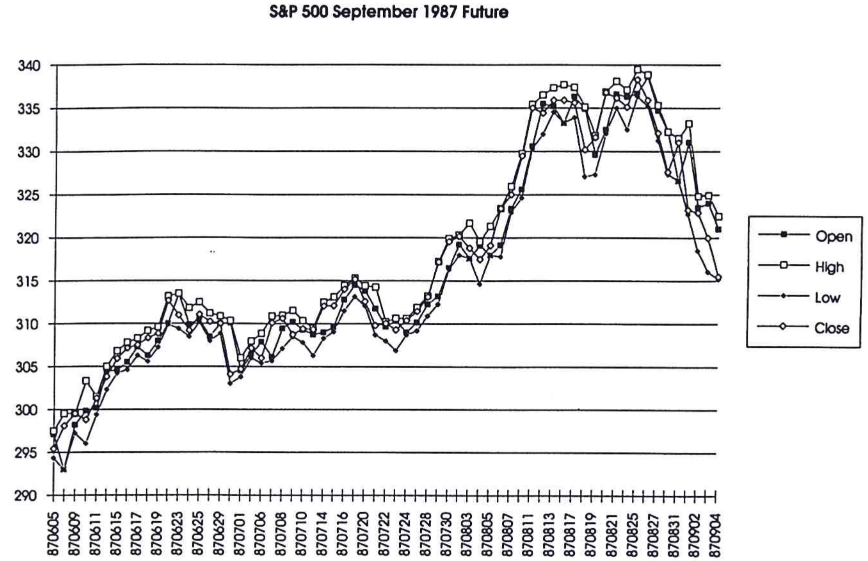

S&P 500 September 1987 Future
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330
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305
300
295
290
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High
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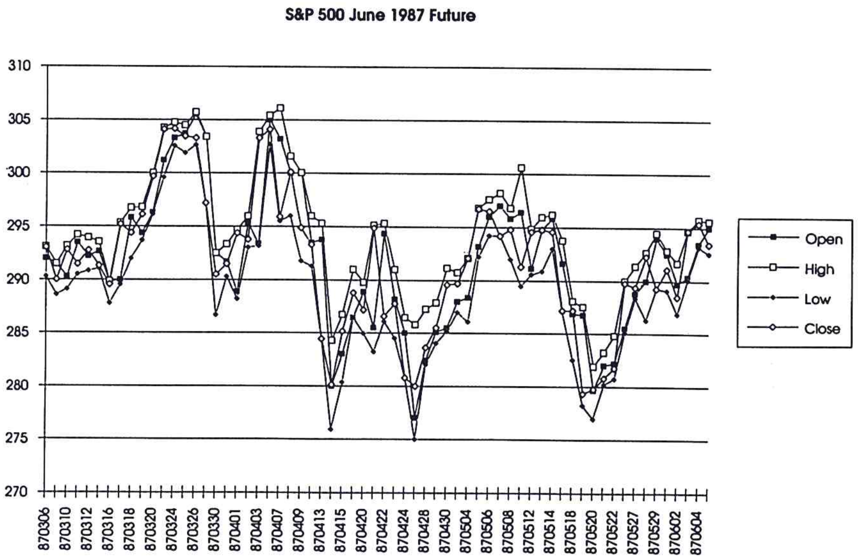

S&P 500 June 1987 Future
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280
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270
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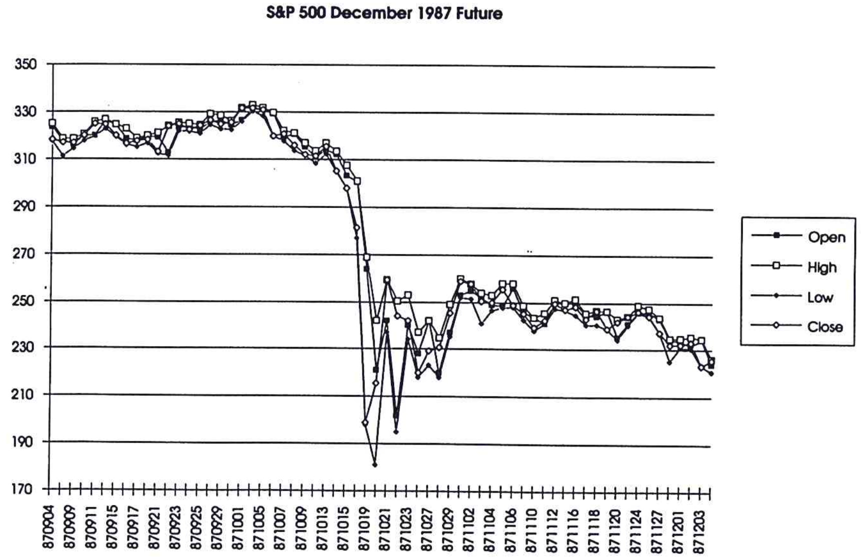
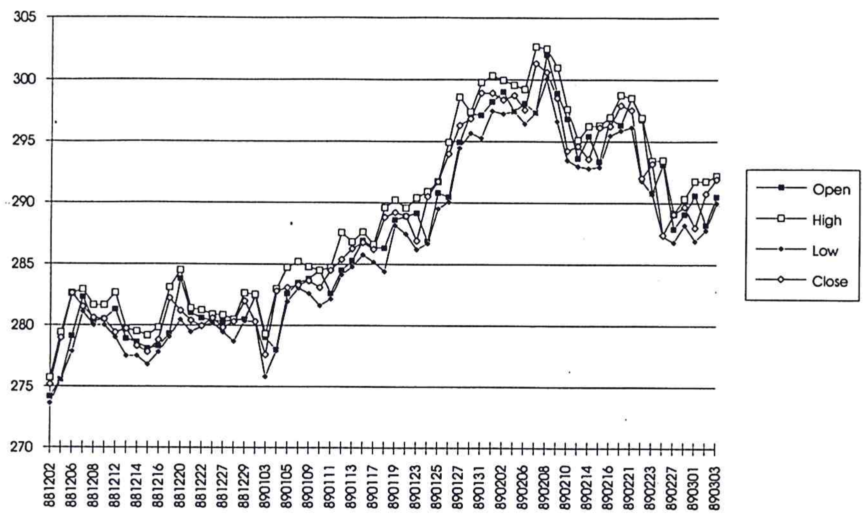
Open
High
Low
Close



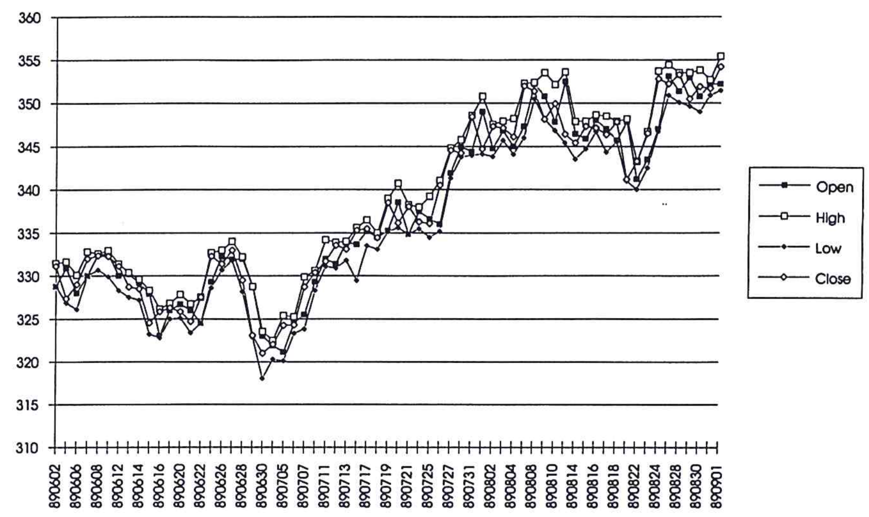
S&P 500 December 1987 Future
350
330
310
290
270
250
230
210
190
170
870904 870909 870911 870915 870917 870921 870923 870925 870929 871001 871005 871007 871009 871013 871015 871019 871021 871023 871027 871029 871102 871104 871106 871110 871112 871116 871118 871120 871124 871127 871201 871203
Open
High
Low
Close

Figure 2

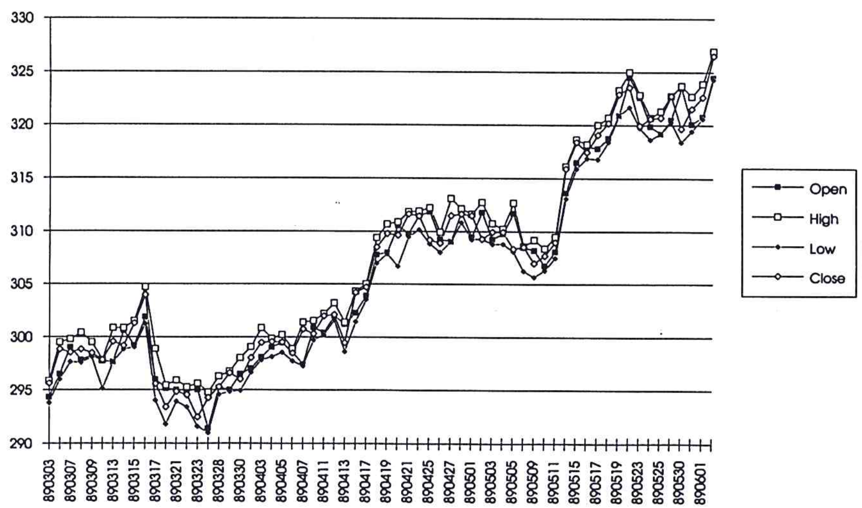
S&P 500 March 1989 Future



S&P 500 September 1989 Future



S&P 500 June 1989 Future



S&P 500 December 1989 Future

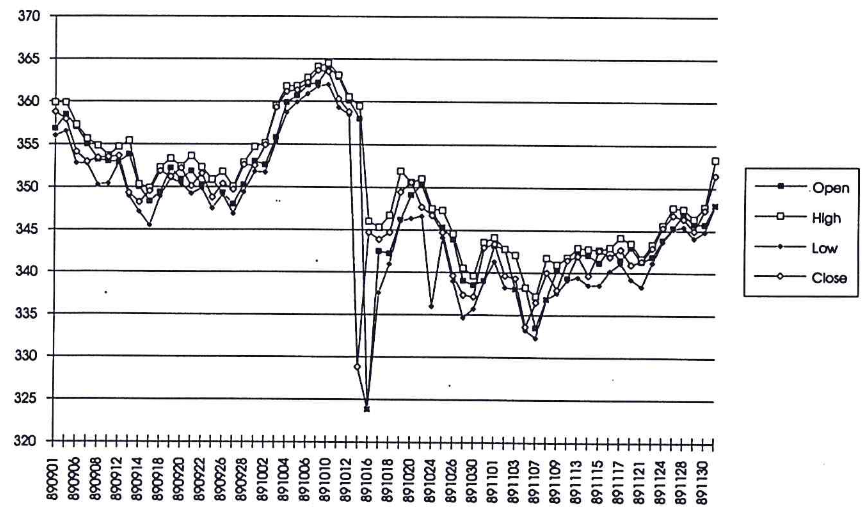


Figure 3

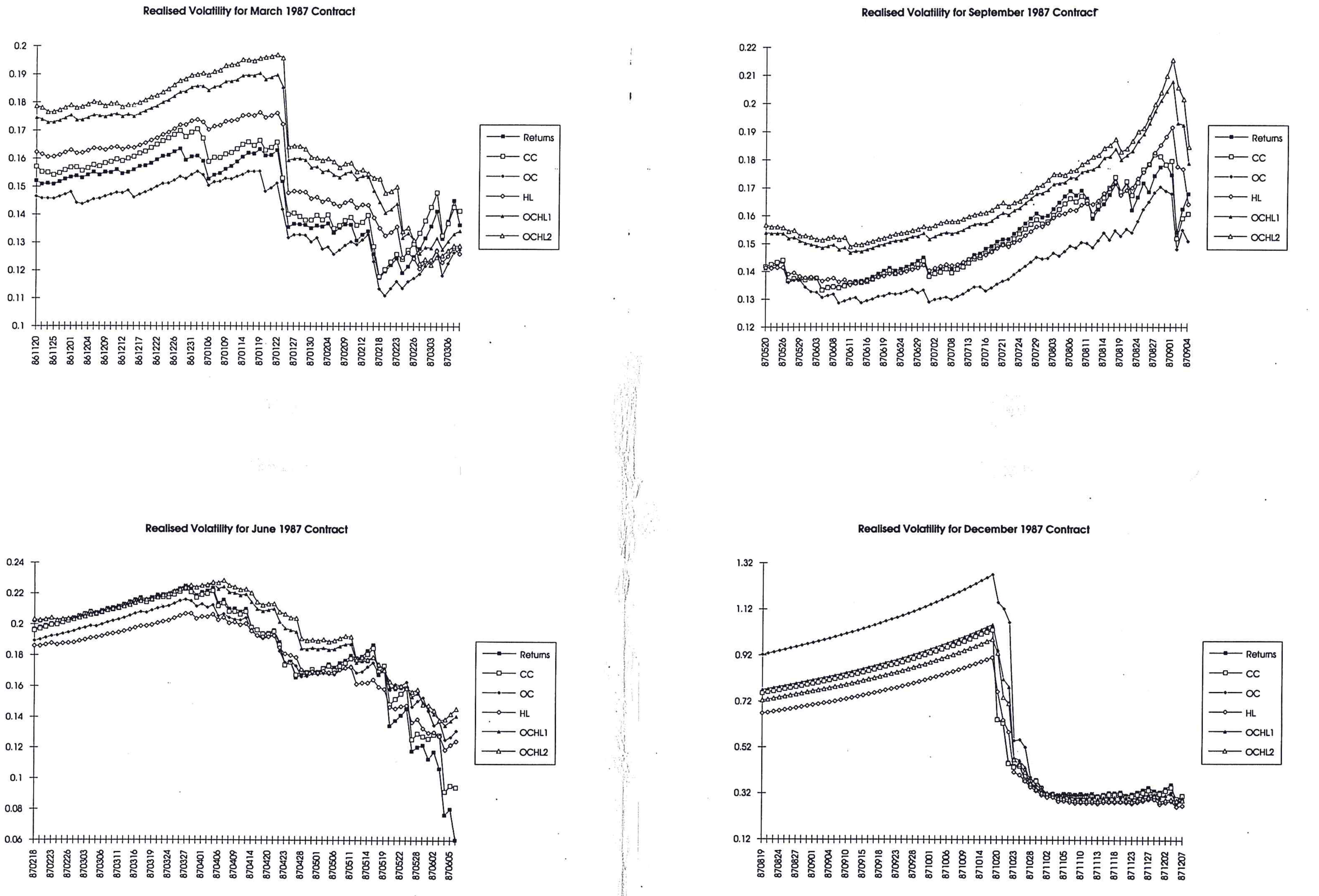
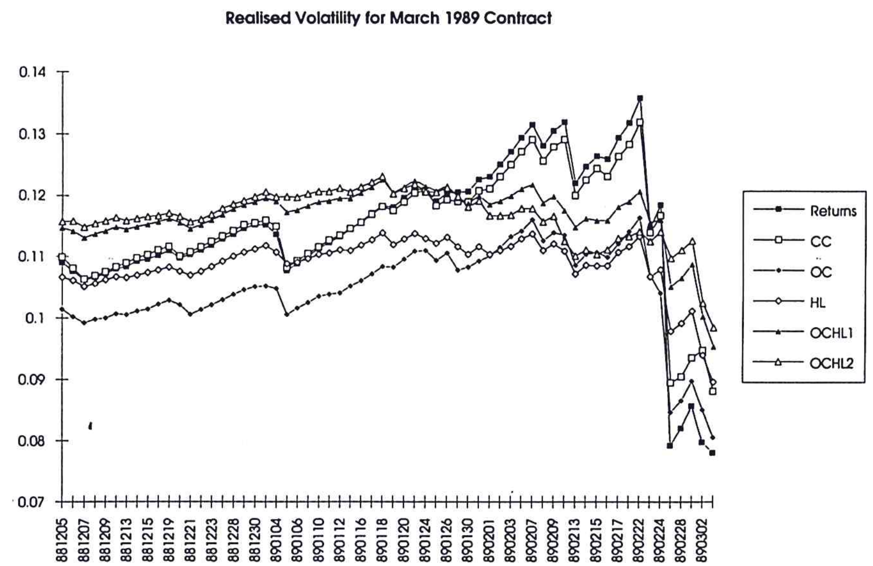
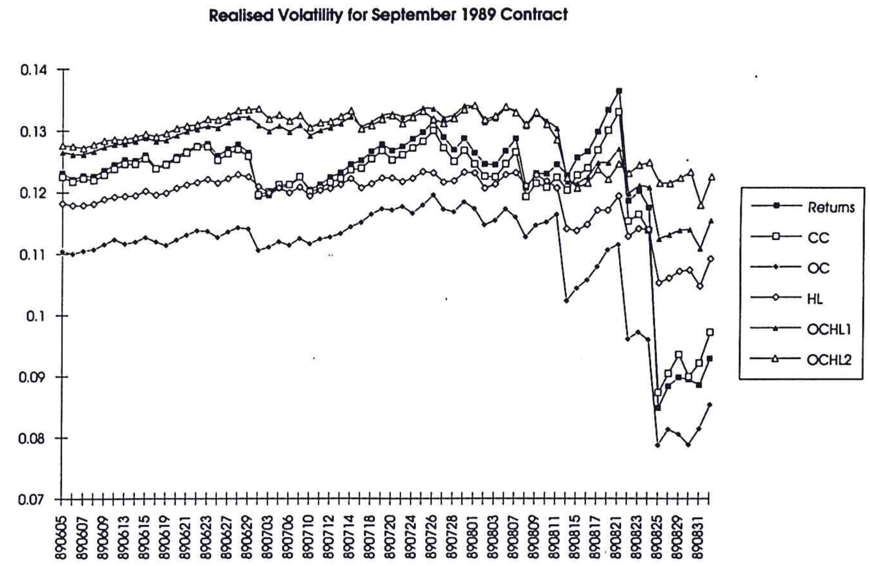
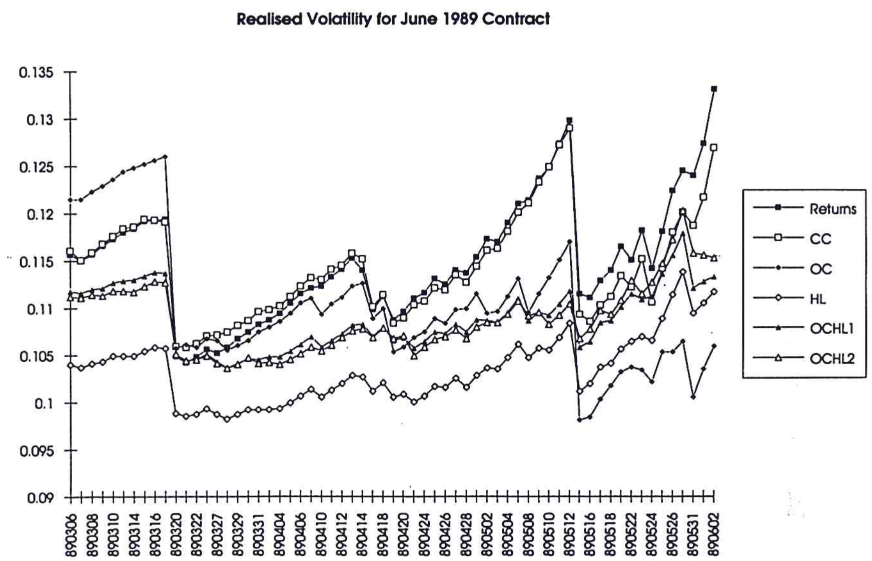

Realised Volatility for March 1987 Contract
Returns
CC
OC
HL
OCHL1
OCHL2
Realised Volatility for September 1987 Contract
Returns
CC
OC
HL
OCHL1
OCHL2
Realised Volatility for June 1987 Contract
Returns
CC
OC
HL
OCHL1
OCHL2
Realised Volatility for December 1987 Contract
Returns
CC
OC
HL
OCHL1
OCHL2

Figure 4


Realised Volatility for March 1989 Contract
Returns
CC
OC
HL
OCHL1
OCHL2


Realised Volatility for September 1989 Contract
Returns
CC
OC
HL
OCHL1
OCHL2


Realised Volatility for June 1989 Contract
Returns
CC
OC
HL
OCHL1
OCHL2

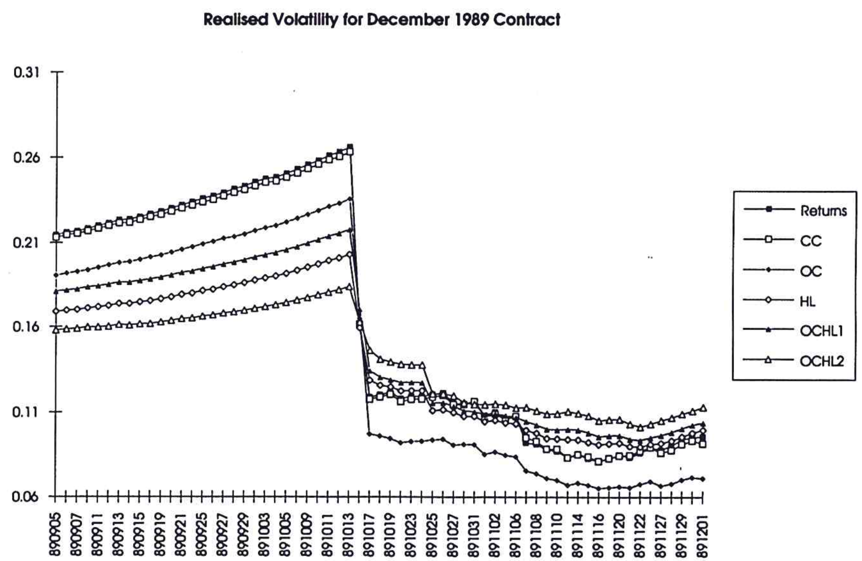
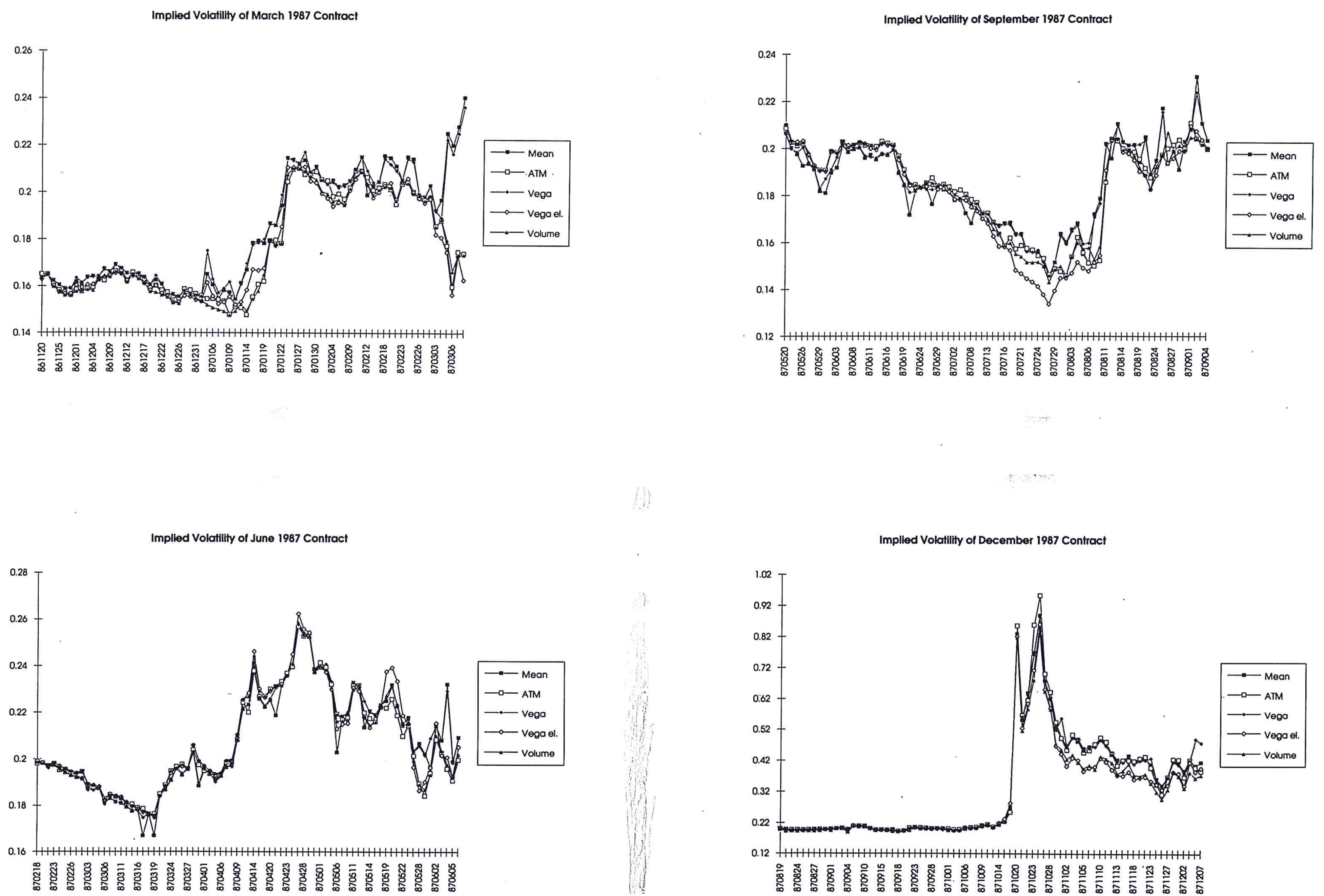

Realised Volatility for December 1989 Contract
Returns
CC
OC
HL
OCHL1
OCHL2

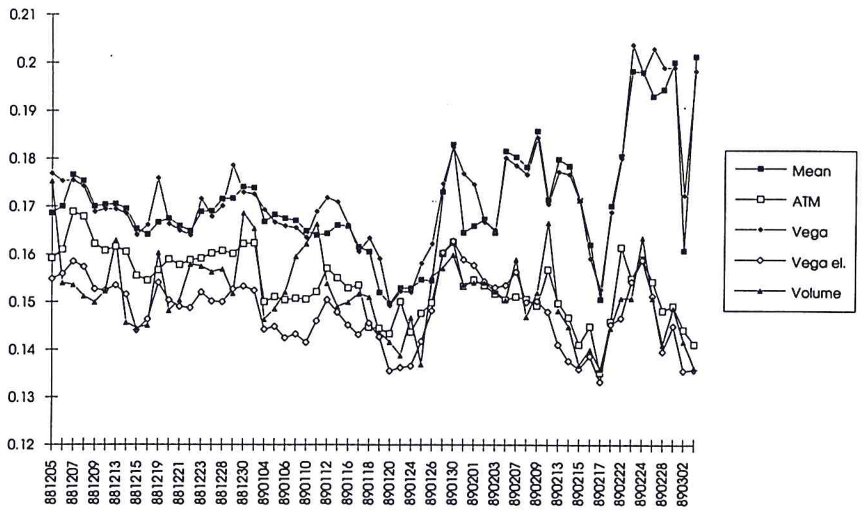
Figure 5



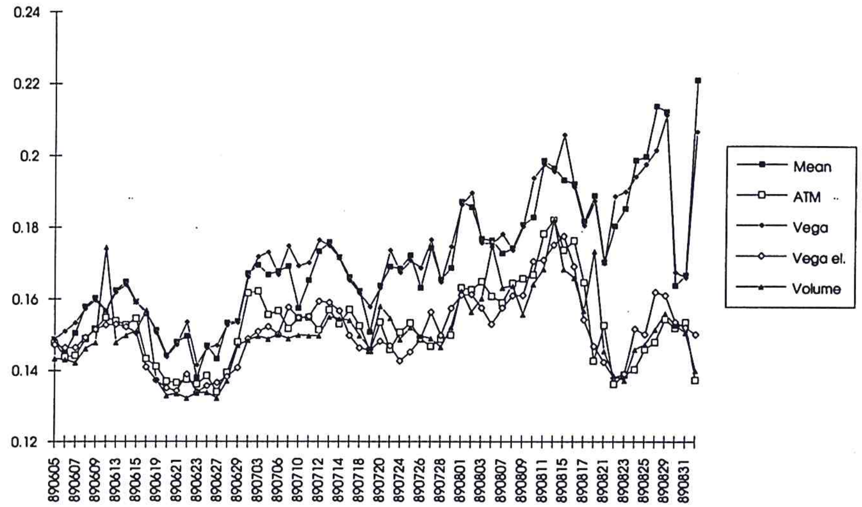
Implied Volatility of March 1987 Contract
Mean
ATM
Vega
Vega el.
Volume
Implied Volatility of September 1987 Contract
Mean
ATM
Vega
Vega el.
Volume
Implied Volatility of June 1987 Contract
Mean
ATM
Vega
Vega el.
Volume
Implied Volatility of December 1987 Contract
Mean
ATM
Vega
Vega el.
Volume

Figure 6

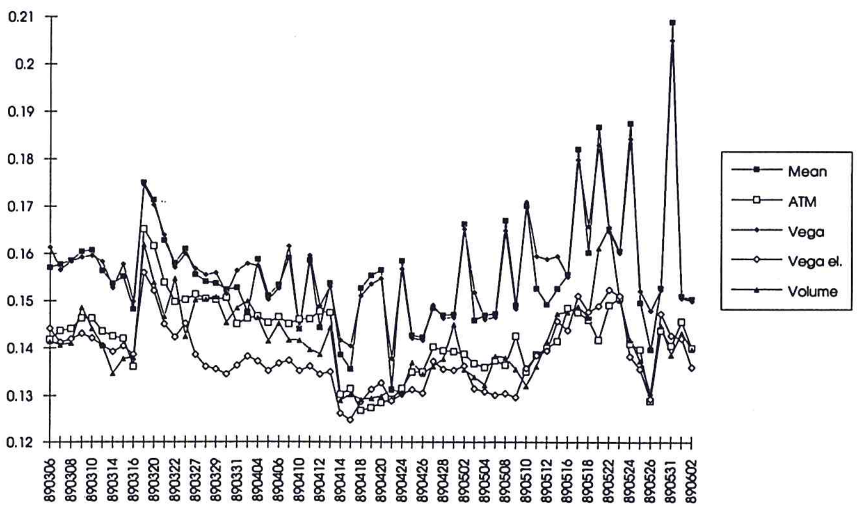
Implied Volatility of March 1989 Contract



Implied Volatility of September 1989 Contract



Implied Volatility of June 1989 Contract



Implied Volatility of December 1989 Contract

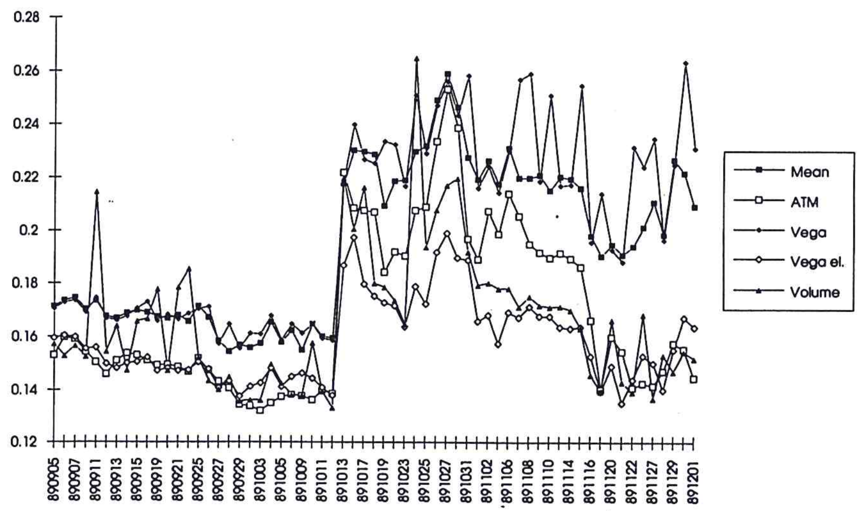
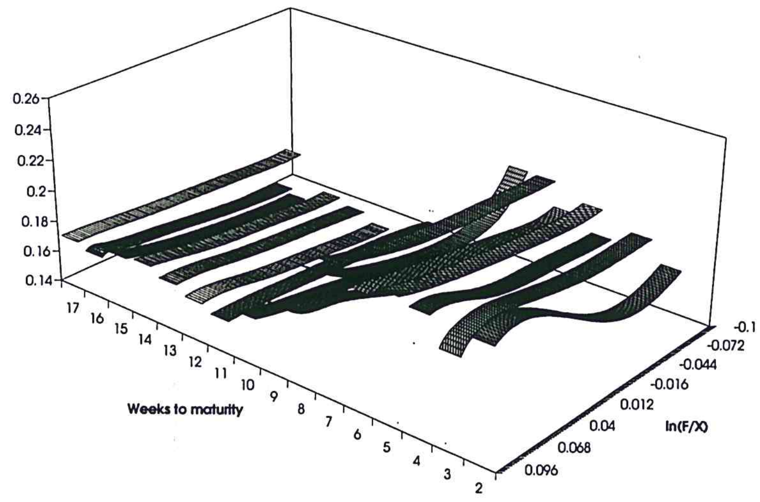
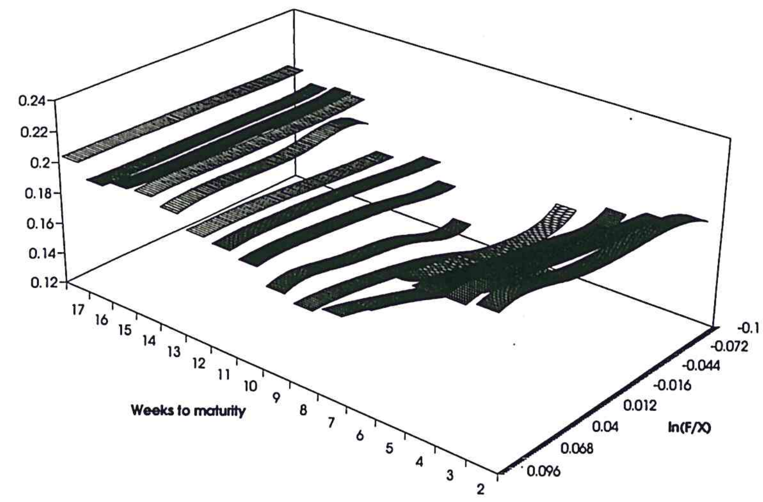


Figure 7

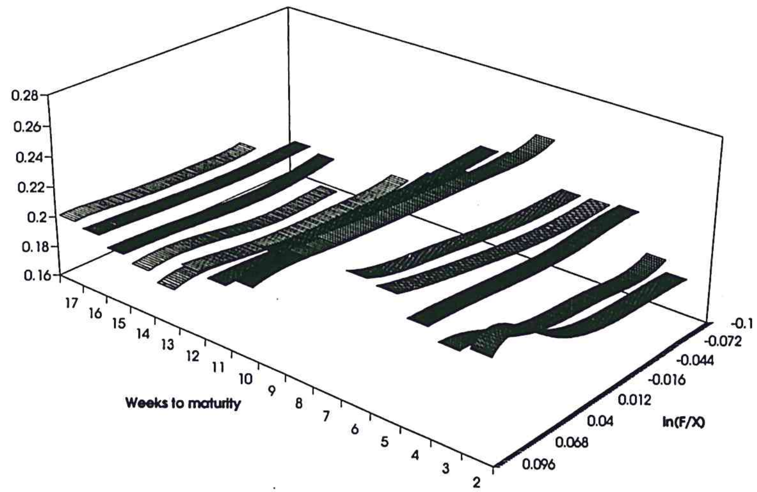
Implied Volatility Biases of March 1987 Contract



Implied Volatility Biases of September 1987 Contract



Implied Volatility Biases of June 1987 Contract



Implied Volatility Biases of December 1987 Contract

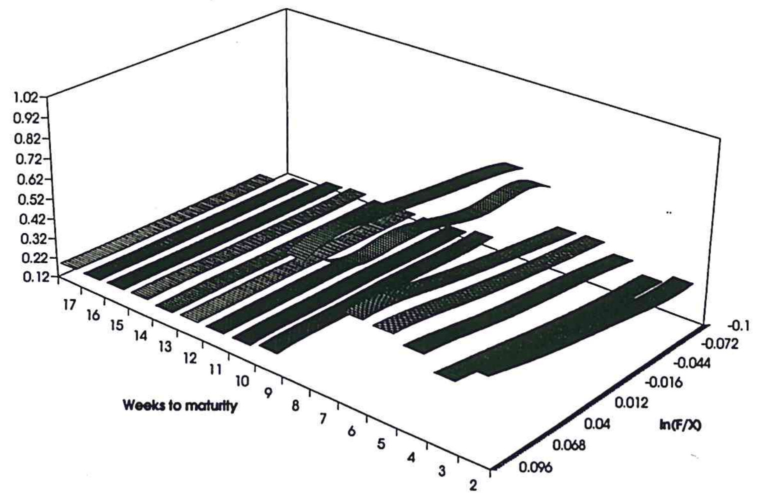
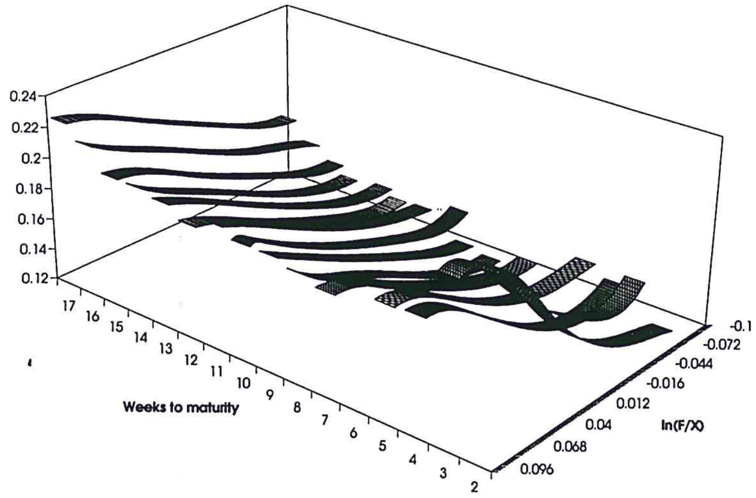


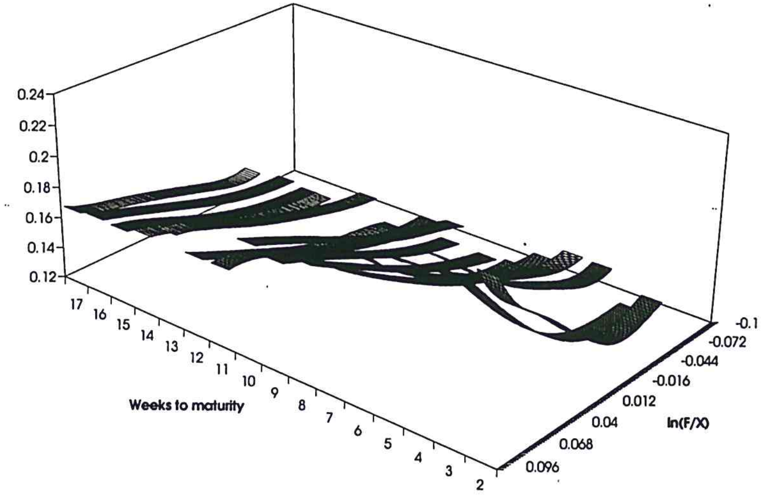
Figure 8

Implied Volatility Biases of March 1989 Contract



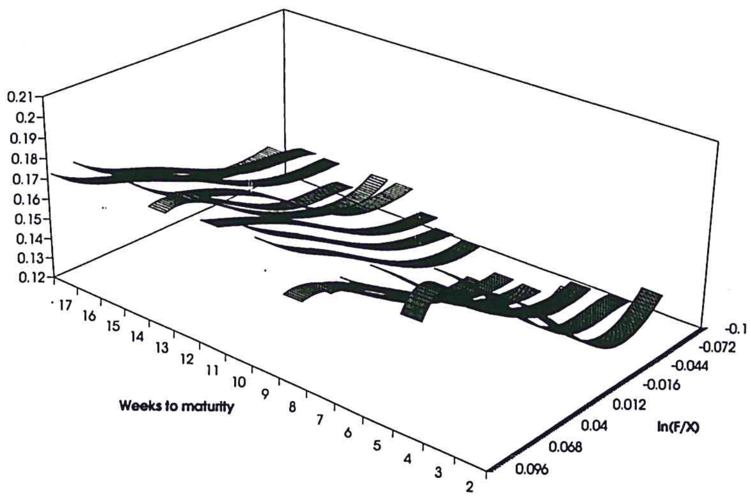
Page 1

Implied Volatility Biases of September 1989 Contract



Page 1

Implied Volatility Biases of June 1989 Contract



Implied Volatility Biases of December 1989 Contract

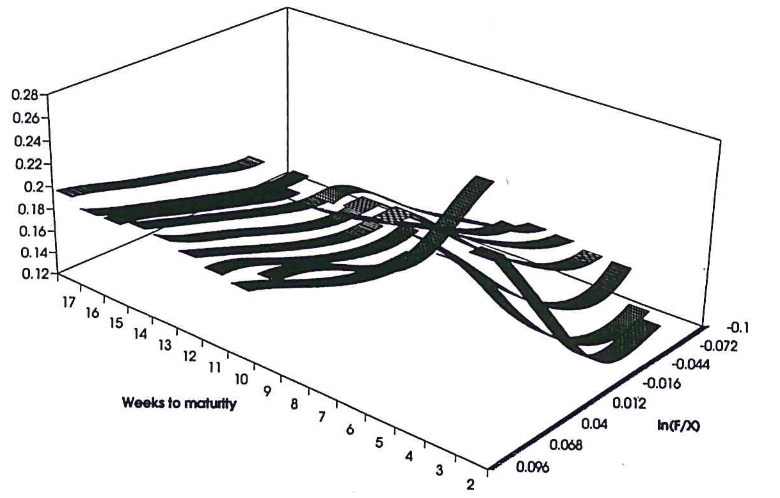
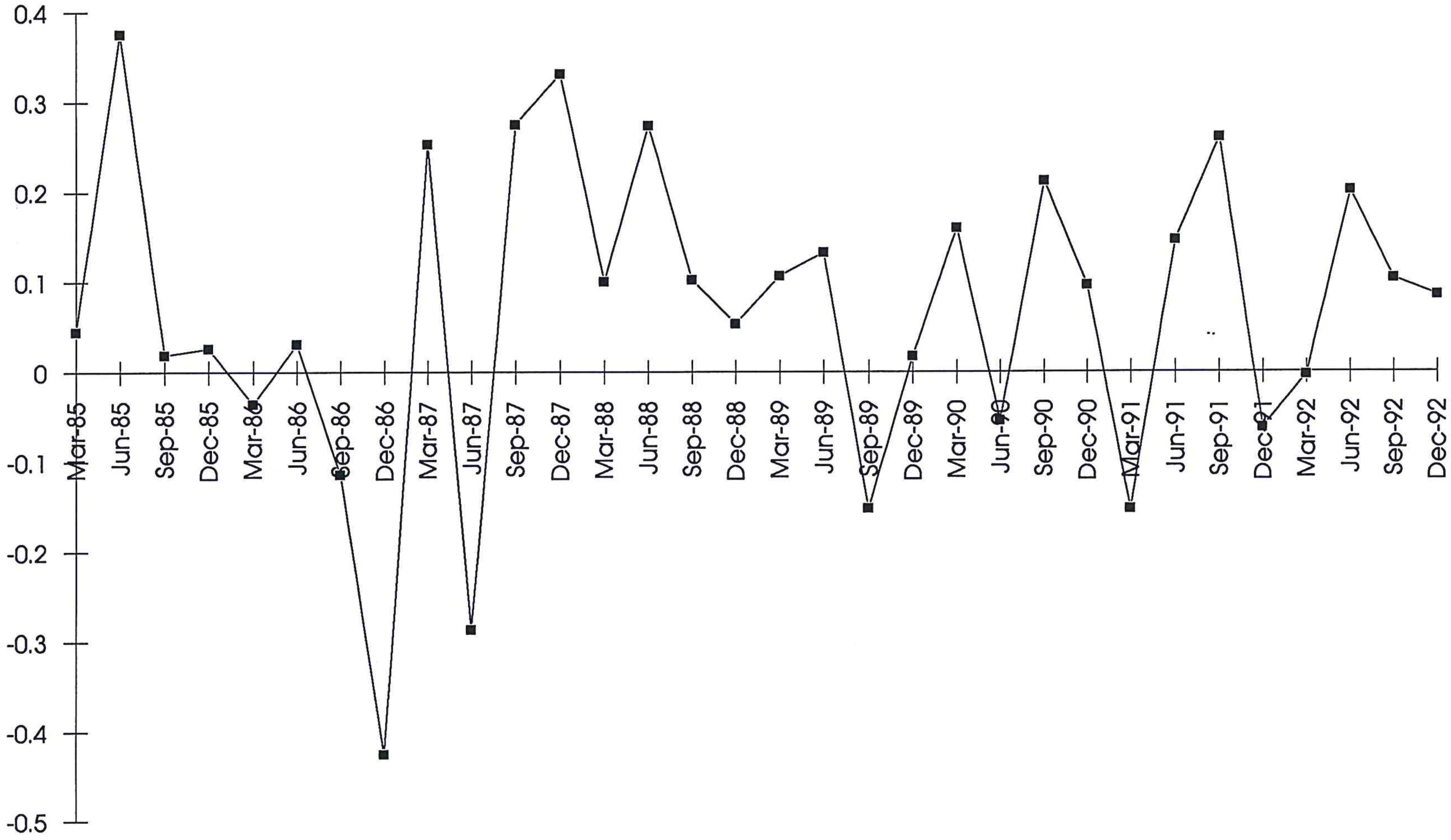


Figure 9

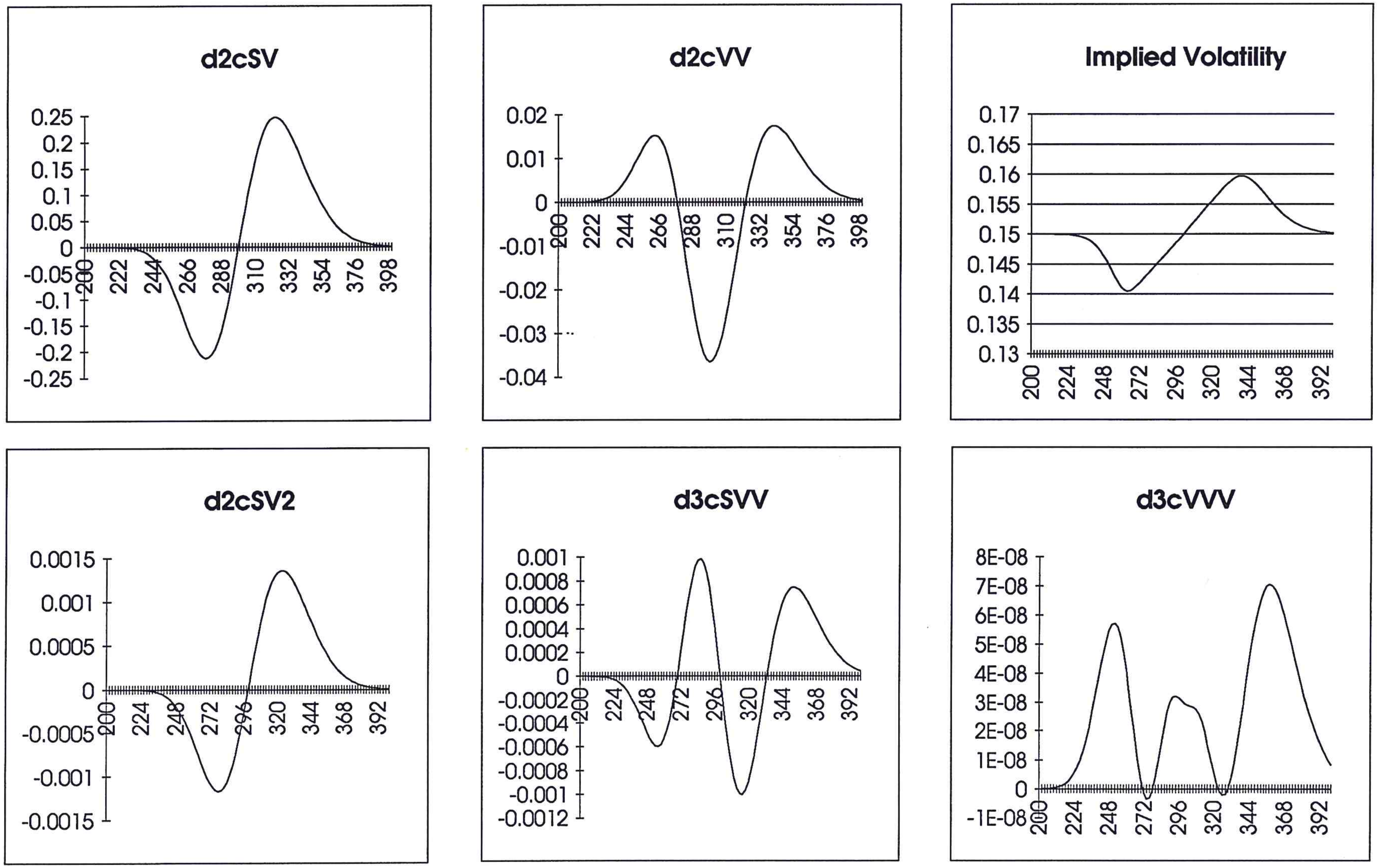
Correlation of changes in nearest future with changes in nearest implied volatility



Hull and White (1988) Stochastic Volatility Model

Figure 10

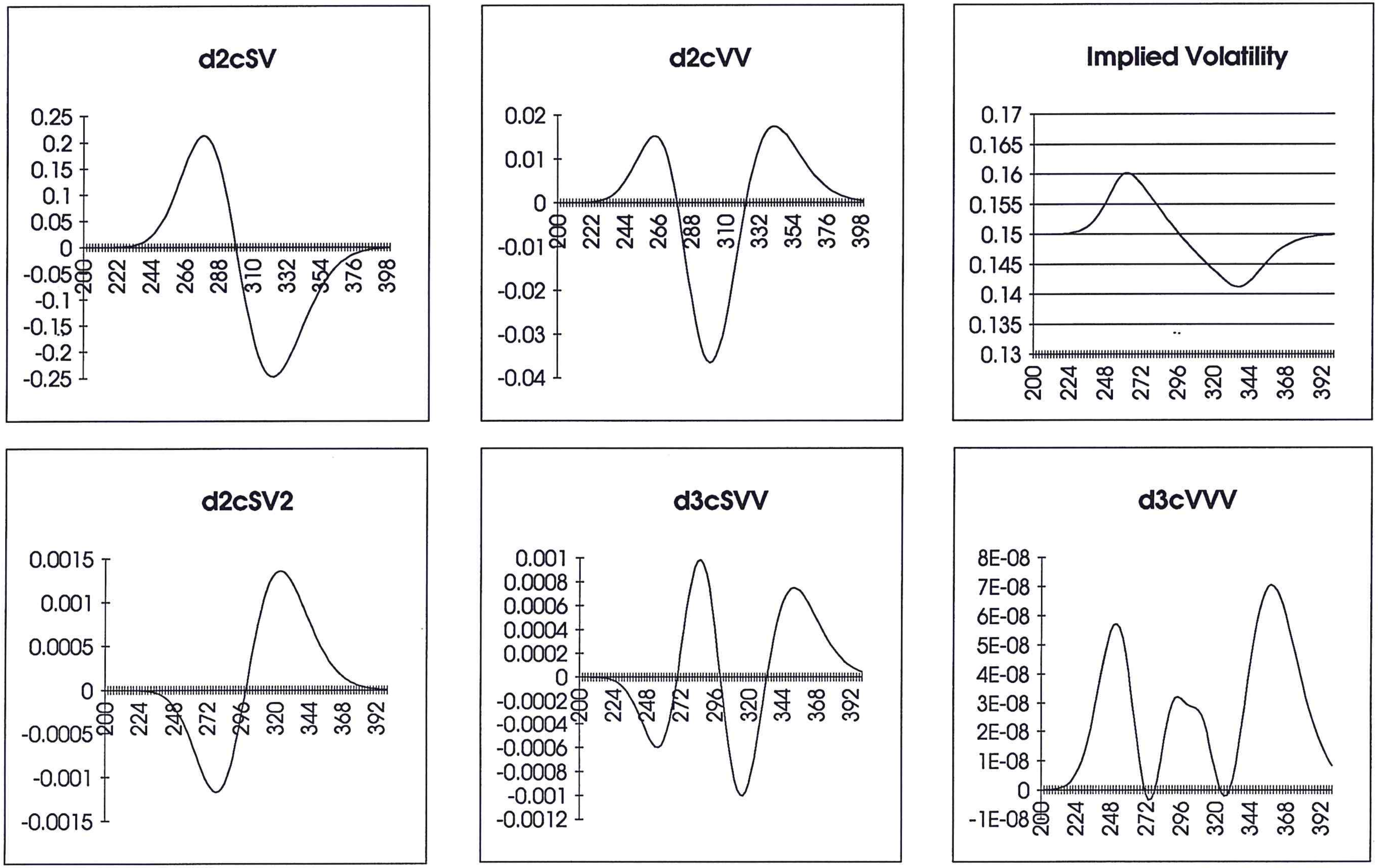
T	S	sigma	r	d	alpha	Vm	xi	rho
0.25	300	0.15	0.05	0.05	10	0.0225	0.1	1



Hull and White (1988) Stochastic Volatility Model

Figure 11

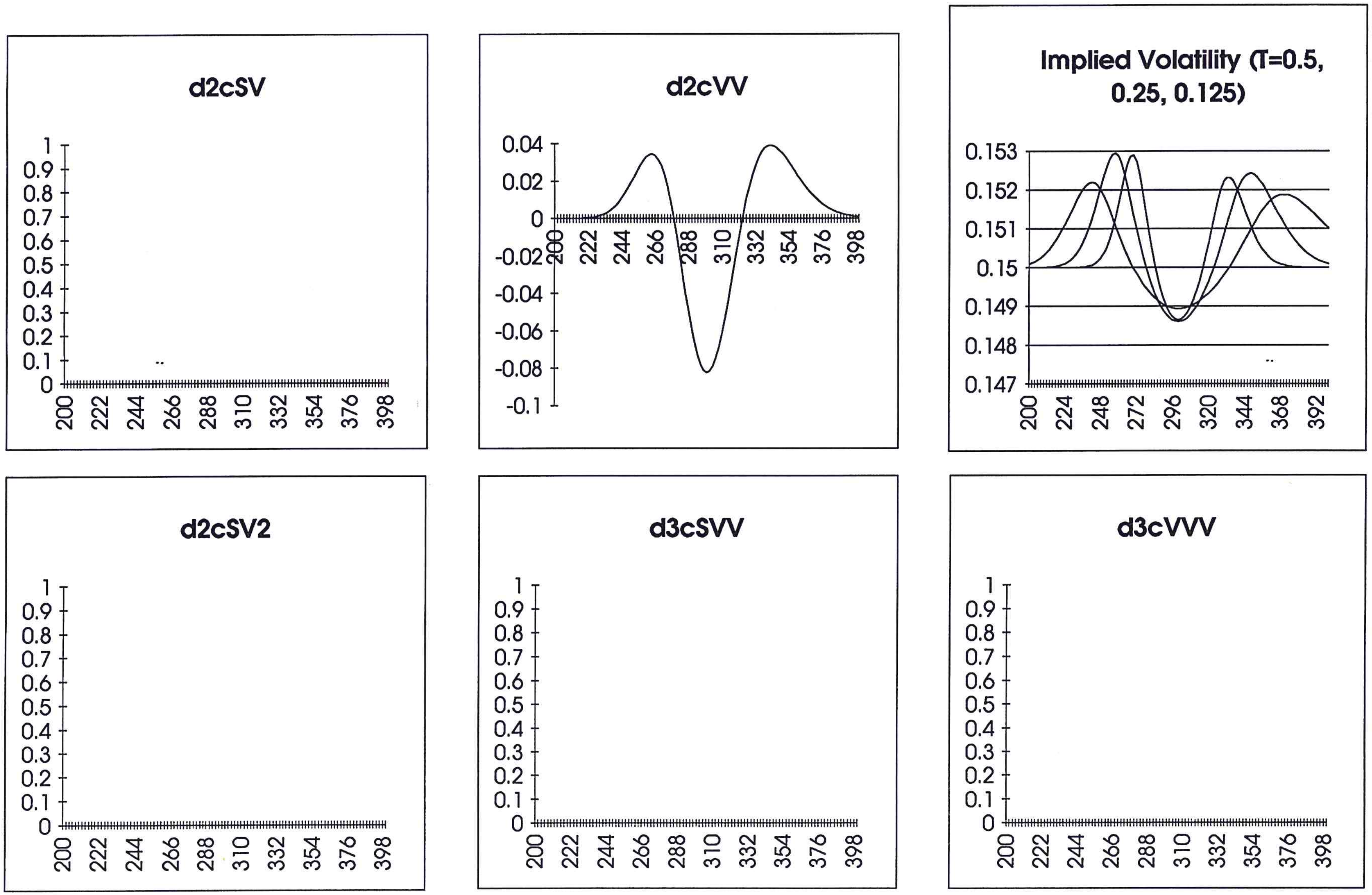
T	S	sigma	r	d	alpha	Vm	xi	rho
0.25	300	0.15	0.05	0.05	10	0.0225	0.1	-1



Hull and White (1988) Stochastic Volatility Model

Figure 12

T	S	sigma	r	d	alpha	Vm	xi	rho
0.25	300	0.15	0.05	0.05	10	0.0225	0.3	0



Hull and White (1988) Stochastic Volatility Model

Figure 13

T	S	sigma	r	d	alpha	Vm	xi	rho
0.25	300	0.15	0.05	0.05	0.1	0.0225	0.1	-1