Martingale Restriction Tests of Option Pricing Models and Their Interpretation

Silio David Aparicio[1] and Stewart Hodges[2]

[1]Financial Options Research Centre
Warwick Business School
University of Warwick
s.a.aparicio@warwick.ac.uk

[2]Financial Options Research Centre
Warwick Business School
University of Warwick
forcsh@wbs.warwick.ac.uk

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Please address any correspondence or request to Professor Stewart Hodges, Financial Options Research Centre, University of Warwick, Coventry, CV4 7AL, England.

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Financial Options Research Centre,

University of Warwick

Abstract

Most, if not all, option pricing models imply some parameterization of the risk neutral density function of the price of the underlying asset. Longstaff (1995) has proposed that the specification of options models may be tested by estimating the risk neutral density function from options data, and then checking whether the mean differs from the forward price of the asset. The purpose of our paper is to investigate why this test seems to be a sensitive one, and to clarify correct and incorrect interpretations of the approach. Our conclusions are reinforced by means of both simulations, and the empirical analysis of an extensive and different data set.

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Introduction

Most, if not all, option pricing models imply some parameterization of the risk neutral density function of the price of the underlying asset. A paper by Longstaff (1995) proposes a new method for testing the specification of option pricing models. Using data on call options, he estimates the risk-neutral density function implied by a particular model and tests whether its mean equals the market forward price. He concludes that this "martingale restriction test" rejects the Black-Scholes (1973) model, and other common parametric option pricing models on a sample of stock index options.

Our paper investigates why this test seems to be a sensitive one, and we also discuss correct and incorrect interpretations of the approach. For example, if the mean of the distribution is above the forward price, can we say that the call options (from which the risk neutral density function is estimated) are in any sense too expensive? The answer to this latter question is, of course, that it does not mean this. We conclude that the main reason why the test is in practice quite sensitive is that changing the mean of the distribution provides a great deal of (really quite illegitimate) flexibility to fit the kind of implied volatility skews which are often found in equity option prices. This flexibility disappears if we fit to both puts and calls. The put-call parity relationship ensures in this case that the mean of the risk neutral distribution is very close to the forward price.

The structure of the paper is as follows. We first review the risk neutral pricing ideas on which the test is based and then summarize Longstaff's work. Next we present our analysis to show how the range of option smiles and skews which can be represented depends on whether the mean is constrained to equal to forward price, on the effect of random pricing errors, and on whether data on puts are used as well as on calls. Finally we describe the results of simulations and empirical analysis of a different data set.

1. Martingale Restriction Test

Since the seminal work of Cox and Ross (1976) and subsequently elaborated in other papers[1], it has been understood that under the assumption of no-arbitrage, contingent claims can be valued as expectations under the risk neutral density function, with discounting at the risk free rate. In other words, in a market without frictions, the absence of arbitrage implies the existence of a linear valuation operator which may be interpreted in terms of a change of probability measure.

In a market which admits no arbitrage, using any non dividend paying asset Z as a numeraire, there is a probability measure Q_Z such that the price of any non dividend paying asset X is a martingale under Q_Z. Therefore, if X_Z is the price of X using Z as numeraire, we have that:

$$X_Z(0) = E^{Q_Z}\left[X_Z(T)\right] \qquad (1)$$

alternatively, in terms of dollars, the price of X could be expressed as follows:

$$X(0) = Z(0)E^{Q_Z}\left[\frac{X(T)}{Z(T)}\right] \tag{2}$$

Using the balance accumulated in a bank account as a numeraire, $\beta(T) = e^{\int_0^T r(t)dt}$, we have that any asset X in the no-arbitrage economy with payoff function X(T) may be valued by simply taking the risk neutral expectations of its payoffs discounted at the risk free rate:

$$X(0) = E^{Q_\beta}\left[e^{-\int_0^T r(t)dt} X(T)\right] \tag{3}$$

In the case of call and put prices their values are given as expectations respect to the risk-neutral density function:

$$C(K) = e^{-rT}\int_K^{+\infty}(S_T - K)dQ_\beta(S_T) \tag{4}$$

$$P(K) = e^{-rT}\int_0^K(K - S_T)dQ_\beta(S_T) \tag{5}$$

where S_T is the underlying asset price at the time T, Q_β is the risk-neutral distribution, K is the strike price, T is the maturity of the options and r is the constant risk-free interest rate.

These general formulae in the no-arbitrage approach lead to different option pricing models depending on the characteristics of the risk neutral density function. For example, under the known Black-Scholes (1973) assumptions, the underlying stock price follows a geometric Brownian motion process where drift and volatility are assumed constant implying that the underlying asset price has a lognormal distribution.

There are many empirical studies of published option pricing models which reveal evidence of the inconsistency (bias) of these theoretical models respect to the option market prices. Explanations of these biases generally fall into the following three categories: parameter estimation, model specification and market efficiency. Latane-Rendleman (1976), Boyle and Ananthanarayan (1977), Chiras-Manaster (1978), Manaster and Rendleman (1982) and Lo (1986) are good examples of studies in parametric estimation. Merton (1976), Galai (1978), Macbeth-Merville (1979), Whaley (1982) and Harvey-Whaley (1991) have worked on model specification biases and several studies have considered testing the efficiency of the market, e.g., Black-Scholes (1972), Galai (1978) and Finnerty (1978).

Longstaff (1995) presents a new method based on the no-arbitrage principle for testing the specification of option pricing models. In the absence of arbitrage, the first moment of the risk-neutral density must be equal to the forward price of the underlying asset. Longstaff, defines this condition as the martingale restriction and uses it as a test of whether a particular model is correctly specified:

$$E\left(S_T\right) = F_T \tag{6}$$

He proposes a simple approach for implementing the martingale restriction test. First, he identifies the risk-neutral density implied by a particular option pricing model and uses observed option prices to recover estimates of the parameters that specify the function. Then, the first moment is discounted and compared to the present value of the asset forward price.

For the Black-Scholes model, the underlying asset follows a lognormal distribution under the risk-neutral measure:

$$f(S_T|\alpha,\beta)=\frac{1}{\sqrt{2\pi}\alpha S_T}\exp\left\{-\frac{(\ln S_T-\beta)^2}{2\alpha^2}\right\} \tag{7}$$

where

$\beta=\ln S_0+(r-\frac{1}{2}\sigma^2)T,$
$\alpha^2=\sigma^2 T$
$E(S_T)=\exp\{\beta+\frac{1}{2}\alpha^2\}$

and the Longstaff's statistic for the martingale restriction test is given by:

$$\text{Spot price bias } =\frac{e^{-rT}e^{(\alpha+\beta^2/2)}-S_0}{S_0}. \tag{8}$$

Longstaff uses call option prices from the S&P 100 index option traded at the Chicago Board Options Exchange (CBOE) during 1988 and 1989. He analyses a sample of 444 daily observations containing the following information: a set of four or more call options very close to the money, the S&P 100 index value, the present value of the dividends, the Treasury-bill yields, the average bid-ask spread of the options and the total trading volume.

He finds that in the 99.5% of the samples the estimates of the statistic are greater than zero and on average 0.5% higher than the current index price. He uses the z-statistic to test the hypothesis that the proportion of positive estimates is 0.50 and the t-statistic to test the hypothesis that the mean estimate is zero (but he ignores the presence of serial correlation in the spot price bias). He rejects both of these hypotheses and concludes that the martingale restriction is not satisfied by the Black-Scholes model indicating that it is more expensive to buy a stock in the options market that in the stock market. By way of comment, while this does suggest that the model is missing something systematic, it is not clear that we ought to be much happier with large deviations whose mean equalled zero.

Longstaff regresses these biases on variables reflecting transaction costs and option market liquidity and concludes that the Black-Scholes model biases are closely related to the bid-ask spread and the daily trading volume. He also proposes an unrestricted version of the Black-Scholes model by allowing the mean of the estimated lognormal distribution to differ from the actual index market price. He claims that this translation in the mean incorporates the effects of market frictions and improves the pricing performance of the model. Unfortunately, this is only true for the valuation of calls. As we will see in the following section if puts and calls were to be priced according to Longstaff's unrestricted model, these prices would violate put-call parity and admit very simple arbitrages. Finally, he repeats the above procedure with a general no-arbitrage option pricing model using Edgeworth Series approximation (Jarrow and Rudd, 1982). The results are very similar to the Black-Scholes case and the model fails the martingale restriction.

2. Impact of Model Specification Error on the Estimated Mean

If the Black-Scholes assumptions hold, i.e., markets are perfect, costless trading takes place continuously and the stock price follows a geometric Brownian motion with a constant volatility then the implied volatility will be equal to the true volatility and the model will be perfectly consistent with the market prices. However, the empirical evidence suggests that the Black-Scholes assumptions do not hold exactly.[2] Black-Scholes prices differ from market prices and implied volatility curves can be obtained by inverting the option pricing formulae for the market prices of the options. It is well known that this implied volatility function depends on the maturity and the strike of the options and exhibits well-defined patterns (smiles and skews) which are acknowledged by market participants.

The main purpose of this section is to illustrate how a misspecification of the model can affect the estimation of the implied parameters. We focus our analysis on the Black-Scholes implied volatility curves generated when the underlying asset's distribution is different from the lognormal. As already mentioned, if the model fits the underlying process the implied volatility is a flat curve equal to the true volatility of the underlying asset. On the contrary, any misspecification in the model would produce a volatility smile whose shape will be determined by the first four central moments of the risk-neutral general distribution.

Positive (negative) skewness generates a straight line volatility curve with positive (negative) slope. Excess kurtosis is an indicator of fat tails in the asset's price distribution and produces the well known smile effect in the volatility curve. We use the Jarrow-Rudd (1982) model to generate different sets of option prices showing how it is possible to span different smiles by varying the skewness and the kurtosis of the risk neutral distribution (Figures 1 and 2). Changing the kurtosis imposes almost a constant curvature to the implied volatility structure, while changing the skewness rotates the curve giving it a skew. Similarly, Figures 3 and 4 show how changing the mean of the risk neutral distribution away from the known forward price affects the volatility structure. We now get different effects for calls and puts and the effect has changing curvature as a function of the strike.

To summarize, our concern is addressed to the flexibility of the theoretical models to create smiles consistent with the markets. In the Black-Scholes case distorting the mean allows us to originate a very rich set of smiles using samples of call market prices or put market prices only. But certainly these smiles are not consistent with the put-call parity relation (given the fact that the means have been shifted) and it will produce bias and arbitrage opportunities.

This analysis explains why for realistic market skews and smiles Longstaff obtains a bias between implied forward prices and market values. With the Black-Scholes model it is possible to make a parametric estimation of the true risk-neutral distribution with the correct mean if we have a flat volatility curve in the market. However, any smile or skew in the volatility function may lead to shift the mean. We shall now demonstrate that these biases do not imply any suggestion of inefficiency in the options markets.

Given the boundaries restrictions and general arbitrage relationships that options must satisfy if there are to be riskless arbitrage opportunities, we can always derive a risk-neutral density function for any convex function C(K) provided it has the right asymptotes (Breeden and Litzenberger, 1978).

Proposition 1: If call prices satisfy the following restrictions related to the strike price:

a) $S_0 \geq C(K) \geq MAX\left[0, S_0 - e^{-rT}K\right]$

b) $\frac{\partial C(K)}{\partial K} \geq -e^{-rT}$ and $\lim_{K \to +\infty} \frac{\partial C(K)}{\partial K} = 0$

c) $\frac{\partial^2 C(K)}{\partial K^2} \geq 0$

then there exists a risk neutral density function whose mean is equal to the underlying forward market price.

Proof.

Restriction (a) set the boundaries for the call prices with $C = S_0$ for $K = 0$, (b) determine the asymptotes and (c) the convex shape of the function.

From (4) we have that:

$$\frac{\partial C}{\partial K} = -e^{-rT} \int_K^{+\infty} dQ_\beta (S_T)$$

$$\frac{\partial^2 C}{\partial K^2} = e^{-rT} dQ_\beta (S_T)$$

then, given that $\frac{\partial^2 C}{\partial K^2} > 0$ and

$$\int_0^{+\infty} \frac{\partial^2 C}{\partial K^2} dK = \lim_{K \to 0} \frac{\partial C(K)}{\partial K} - \lim_{K \to \infty} \frac{\partial C(K)}{\partial K}$$

$$= e^{-rT}$$

we can consider the function:

$$f(K) = \frac{1}{e^{-rT}} \frac{\partial^2 C}{\partial K^2}$$

as a good candidate for the risk-neutral density function whose mean is given by:

$$E_f = \int_0^{\infty} K \frac{1}{e^{-rT}} \frac{\partial^2 C}{\partial K^2} dK$$

$$= -\frac{1}{e^{-rT}} \int_0^\infty \frac{\partial C}{\partial K} dK$$

$$= \frac{1}{e^{-rT}} S \qquad \text{underlying forward price.}$$

end of proof.

Proposition 1[3] establishes that the rejection of the martingale restriction undoubtedly implies a model specification error in any empirical application of contingent claim analysis. However, if we have a set of option prices which do not contain any risk free arbitrages, then a more general model would be capable of providing a risk neutral pricing function which gave the correct forward price and also satisfied put call parity. This idea, of course, lies behind much of the current work on the estimation of the risk-neutral density function from options data. Since Breeden and Litzenberger (1978), many attempts have been done to recover information about the underlying asset distribution from option prices, for example, Shimko (1993). Recently, Derman (1994), Rubinstein (1994) and Dupire (1994) have suggested methods to recover an entire risk-neutral process as a recombining binomial tree that is consistent with observed options prices.

At this point it is worth providing some further clarification of the nature of the put-call parity relationship within a general no-arbitrage risk neutral pricing framework. For a European option on a stock, that pays dividends during the life of the option with present value of the dividends equals to D, is easily shown that:

$$C = P + S - D - Xe^{-r(T-t)} \tag{10}$$

This well known put-call parity relation, may be considered as a conversion mechanism whereby a call option value can be obtained from the value of a put with identical exercise price and maturity. Violation of put-call parity provides the simplest of arbitrage opportunities by taking opposite positions in the call plus a bond against the put plus the underlying..

Market participants are very aware of this put-call parity arbitrage, so whenever $E[S_T]$ estimated from call option prices is different from the forward price in the market, the market values of puts *would not* be explained by the estimated risk-neutral density function. Consequently, two different implied volatility curves would be obtained for calls and puts and a different p.d.f. would be estimated from put option prices. Conversely, if puts and calls were to be priced according to the general formulae (4) and (5), but from a risk neutral distribution whose mean differed from the forward asset price, these prices would violate put-call parity and admit very simple arbitrage.

Figure 5 plots the volatility smiles for puts and calls resulting from a given shift in the mean of the risk neutral density function. This shows very clearly the inconsistency in the way they are priced, whenever the risk neutral density function has the wrong mean. Normally we would expect (at least for European options) that put-call parity should ensure that, for any given strike price, puts and calls are priced at the same implied volatility whatever smile effects are present. We shall now explore this a bit further. Figure 6 confirms that if we simultaneously fit to (a Black-Scholes model) put and call option prices, where the underlying risk neutral distribution has varying amounts of skewness, the size of the bias in the mean becomes very small, and much smaller than for using only calls (or only puts).

3. Simulation and Empirical Analysis

We were concerned that the procedure used for fitting the risk neutral density function might itself in some way induce part of the bias observed. To investigate this we conducted a small simulation experiment. We created simulated options data from the Black-Scholes model plus some uniformly distributed random noise (to give an effect of pricing errors and/or tick-size rounding). We eliminated samples where the noise had created arbitrage opportunities (samples that violates the upper or lower boundary conditions and the convexity relation creating a butterfly spread arbitrage), and we then used our procedure for fitting the distribution. We found no evidence of bias in the mean of the distribution. As a result we are fairly confident that noise in option prices will not by itself introduce any significant bias, and it is probably the shape of the empirical smiles which the technique is picking up.

We also conducted our own empirical analysis using Longstaff's technique on a much larger sample of the Chicago Mercantile Exchange (CME) options on the Standard & Poor's 500 (S&P 500) index future. The Stats Database provided by the CME contains daily trading data that includes: opening, high, low and closing prices and trading volume of the options and futures contracts. Riskless interest rates have been calculated from London euro-currency interest rates collected from Datastream. The futures contracts have maturities every March, June, September and December and are cash settled on the Thursday prior to the third Friday of the contract month. Options and futures have the same maturities and since July 1987 maturities up to 60 days were introduced with the nearest future as the underlying instrument.

We have examined a subset of eight years of daily data from 11/02/1985 to 31/12/1992, and have used every contract-day for which at least eight distinct strikes are available for call and put options with deltas between 1% and 99%. The combined sample was divided in three groups: 5098 daily sets of call options, 5102 daily sets of put options and 4625 daily sets of call and put options. Each set has on average 12.73 closing prices with the same maturity date. This range ensures a reasonable degree of precision in the pricing. We divided all data sets in two periods - before and after October 1987 - in order to take in account the well documented changes of volatility patterns after the crash. As we did for the simulations, we eliminated sets that violate the upper or lower boundary conditions or the convexity relation. To solve for the parameters we minimize the sum of the squared differences between the model and observed prices (Formulas 4 and 5). A Marquardt non-linear least-squares routine is used for simplicity.

Tables 1-6 summarize our results showing the statistics for the percentage differences between the index value implied by the Black-Scholes model and the current index value. In both sub-samples (pre-crash and post-crash periods) we have estimated the biases for each data set: only calls, only puts and the combined group of calls and puts. Tables 1, 2 and 3 contain the results for the pre-crash period. Using a t-statistic to test the hypothesis that the mean estimate is equal to zero we reject the martingale restriction when only calls or puts are used in the sample, but we do not find a significant bias when both, puts and calls are included. Also, the biases follow a different pattern. For the calls data sets (Table 1) there is a predominant downward bias with a mean for all sets of (-0.03) and a t-statistic of (-2.02). The martingale restriction is rejected across different maturities with the exception of the categories between 8 and 16 weeks where there is no evidence of significant bias. For the puts data sets (Table 2) the biases are positive and very significant across maturities, and the overall mean of (0.36) is much higher than the one obtained for the calls. Table 3 shows the reduction in the average bias when puts and calls are used in the sample. The overall mean is (-0.01) and there is no evidence of significant bias across maturities so the test can not be rejected. It is worthwhile stressing the following point: because the patterns for the biases depend on the composition of the data sets and the maturity of the options it would not be appropriated to reach any conclusion about the signs of the biases. Figures 7, 8 and 9 provide a plot of the estimates biases against contract expiry for the pre-crash period.

Tables 4, 5 and 6 shows the results for the post-crash period. The Black-Scholes model is strongly rejected with overall means of (0.87), (3.03) and (-0.65) for only calls, only puts and the combined calls and puts samples. For all groups there is a very significant bias that is increasing across maturities. Once again, there is a different pattern across the data observing positive biases for the only calls or puts data and negative one for the combined calls and puts group. Figures 10, 11 and 12 provide a plot of the estimates biases against contract expiry for the post-crash period.

We have also examined the serial correlation in the biases. This is not entirely straightforward as there are some gaps in the time series through the criteria for inclusion in the sample not being met. The overall first order serial correlation is estimated at 0.35 in the calls sample taking values from 0.24 to 0.60 across maturities. In the puts sample the overall correlation is estimated at 0.42 with values from 0.26 to 0.72 for different maturities. The formal tests will still appear to be significant so these results agree with those found by Rubinstein (1994) in the sense that the Black-Scholes model provides more accurate values for the pre-crash period.

Qualitatively, our results are very similar to Longstaff's, and we can interpret our findings in terms of a distinctive change in the general shape of the volatility skew occurring after the crash (deeply pronounced smiles).

4. Conclusions

This paper has clarified the meaning of the Martingale Restriction Test proposed by Longstaff (1995). We have shown that it is not surprising to get biases between the mean of the implied distribution and the forward asset price when fitting just to call option data, because varying this parameter provides additional flexibility to fit arbitrary volatility smiles. The technique provides a test of the specification of particular parametric models, but it would be a very poor way of estimating implied distributions.

As a part of our analysis we have simulated the effect of adding random errors to a correctly specified model. We found that when we did this (even after screening out situations which violate no-arbitrage conditions) our means were slightly noisy but generally unbiased.

Since practitioners are very aware of put-call parity, it holds in all markets to within the costs of executing trades. We therefore conclude that using a model with translated mean distribution does not fit the market volatility structure. This suggests a different interpretation of Longstaff's results. When the expected value of the underlying asset differs from the market forward price it does not mean that the model overvalues (undervalue) the underlying asset but it does reveal an inconsistency between the model and the market. The martingale restriction should not be considered as a market efficiency test because it would imply the false premise that the theoretical models fit the market prices.

Analysis of a long data set on CME options on an index future confirmed Longstaff's qualitative findings, but the biases from our database were significantly smaller than his for the pre-crash period. Changes in the patterns of volatility smiles after October 1987 provided a reason as to why this may be the case.

[1]e.g. Harrison and Kreps (1979), Breeden and Litzenberger (1978), Banz and Miller (1978), Ross (1978).

[2] For examples: Merton (1973), Black (1975), Macbeth and Merville (1979,1980), Rubinstein (1985).

[3] Proposition 1 could be extended to consider density functions with a positive mass at zero under appropriate convexity conditions, see Kemp and Smith (1996).

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Table 1
Pre-crash Black-Scholes Model Statistics
Call option prices

Maturity (weeks)	Mean	Standard Deviation	Min Value	Max Value	No. of sets	t-statistic
1 _ 4	0.11	0.21	-0.66	0.78	103	5.29
4 _ 8	0.09	0.35	-1.19	2.01	166	3.20
8 _ 12	0.04	0.33	-1.28	1.83	160	1.57
12 _ 16	0.01	0.47	-2.57	1.44	147	0.35
16 _ 20	-0.16	0.41	-1.18	1.54	146	-4.65
20 _ 24	-0.19	0.48	-1.82	2.73	193	-5.47
Total	-0.03	0.41	-2.57	2.73	915	-2.02

Summary statistics for the percentage differences between the index value implied by the Black-Scholes model and the current index value.

Table 2
Pre-crash Black-Scholes Model Statistics
Put option prices

Maturity (weeks)	Mean	Standard Deviation	Min Value	Max Value	No. of sets	t-statistic
1 _ 4	0.02	0.47	-2.45	1.79	86	0.46
4 _ 8	0.19	0.64	-1.75	2.65	135	3.45
8 _ 12	0.23	0.49	-1.01	2.47	138	5.53
12 _ 16	0.40	0.55	-1.31	2.54	141	8.63
16 _ 20	0.50	0.71	-1.60	4.71	145	8.44
20 _ 24	0.64	0.74	-1.59	6.03	159	10.87
Total	0.36	0.65	-2.45	6.03	804	15.63

Summary statistics for the percentage differences between the index value implied by the Black-Scholes model and the current index value.

Table 3
Pre-crash Black-Scholes Model Statistics
Call and put option prices

Maturity (weeks)	Mean	Standard Deviation	Min Value	Max Value	No. of sets	t-statistic
1 _ 4	-0.11	0.12	-0.56	0.10	72	-7.16
4 _ 8	-0.05	0.47	-0.62	3.93	151	-1.21
8 _ 12	-0.02	0.39	-0.40	3.70	97	-0.44
12 _ 16	-0.04	0.15	-0.57	0.35	106	-2.76
16 _ 20	0.07	1.00	-1.16	7.02	144	0.82
20 _ 24	0.01	0.66	-0.92	6.34	116	0.22
Total	-0.01	0.60	-1.16	7.02	686	-0.50

Summary statistics for the percentage differences between the index value implied by the Black-Scholes model and the current index value.

Table 4
Post-crash Black-Scholes Model Statistics
Call option prices

Maturity (weeks)	Mean	Standard Deviation	Min Value	Max Value	No. of sets	t-statistic
1 _ 4	0.44	0.51	-1.11	5.69	951	26.05
4 _ 8	0.66	0.64	-1.78	4.74	1078	34.14
8 _ 12	0.77	0.60	-0.73	3.88	517	29.14
12 _ 16	0.99	0.82	-0.61	5.36	423	24.84
16 _ 20	1.30	1.02	-0.85	5.23	421	26.12
20 _ 24	1.42	1.08	-2.72	9.90	793	36.74
Total	0.87	0.85	-2.72	9.90	4183	65.75

Summary statistics for the percentage differences between the index value implied by the Black-Scholes model and the current index value.

Table 5
Post-crash Black-Scholes Model Statistics
Put option prices

Maturity (weeks)	Mean	Standard Deviation	Min Value	Max Value	No. of sets	t-statistic
1 _ 4	0.42	1.17	-3.35	11.07	852	10.55
4 _ 8	2.12	2.34	-3.78	13.90	1001	28.67
8 _ 12	3.22	3.24	-3.10	25.67	613	24.60
12 _ 16	2.70	2.30	-2.73	12.50	402	23.52
16 _ 20	4.10	2.84	-1.21	16.50	394	28.72
20 _ 24	5.64	3.79	-1.56	31.25	1036	47.89
Total	3.03	3.32	-3.78	31.25	4298	59.72

Summary statistics for the percentage differences between the index value implied by the Black-Scholes model and the current index value.

Table 6
Post-crash Black-Scholes Model Statistics
Call and put option prices

Maturity (weeks)	Mean	Standard Deviation	Min Value	Max Value	No. of sets	t-statistic
1 _ 4	-0.29	0.22	-1.42	1.47	901	-39.90
4 _ 8	-0.54	0.41	-2.35	5.97	981	-40.93
8 _ 12	-0.67	0.59	-2.54	6.14	568	-27.04
12 _ 16	-0.82	0.56	-2.53	5.64	332	-26.81
16 _ 20	-0.88	0.49	-2.70	0.46	310	-31.31
20 _ 24	-1.01	0.69	-3.29	6.53	847	-42.81
Total	-0.65	0.56	-3.29	6.53	3939	-72.69

Summary statistics for the percentage differences between the index value implied by the Black-Scholes model and the current index value.

Figure 1: Kurtosis and the Smile
Black-Scholes implied volatility

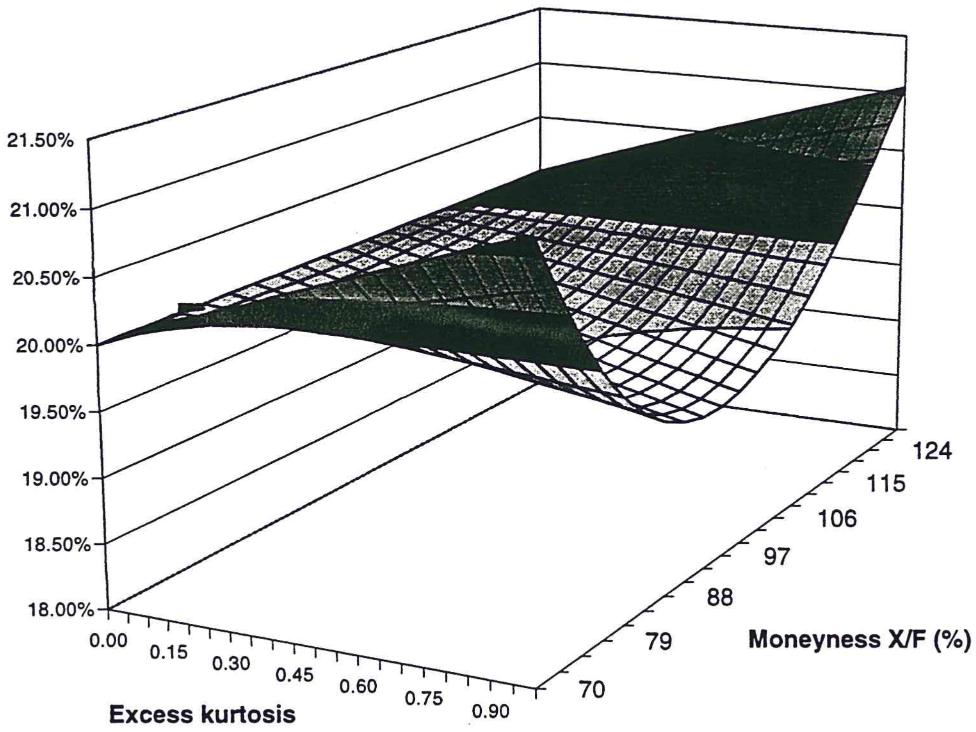

Figure 2: Skewness and the Smile
Black-Scholes implied volatility

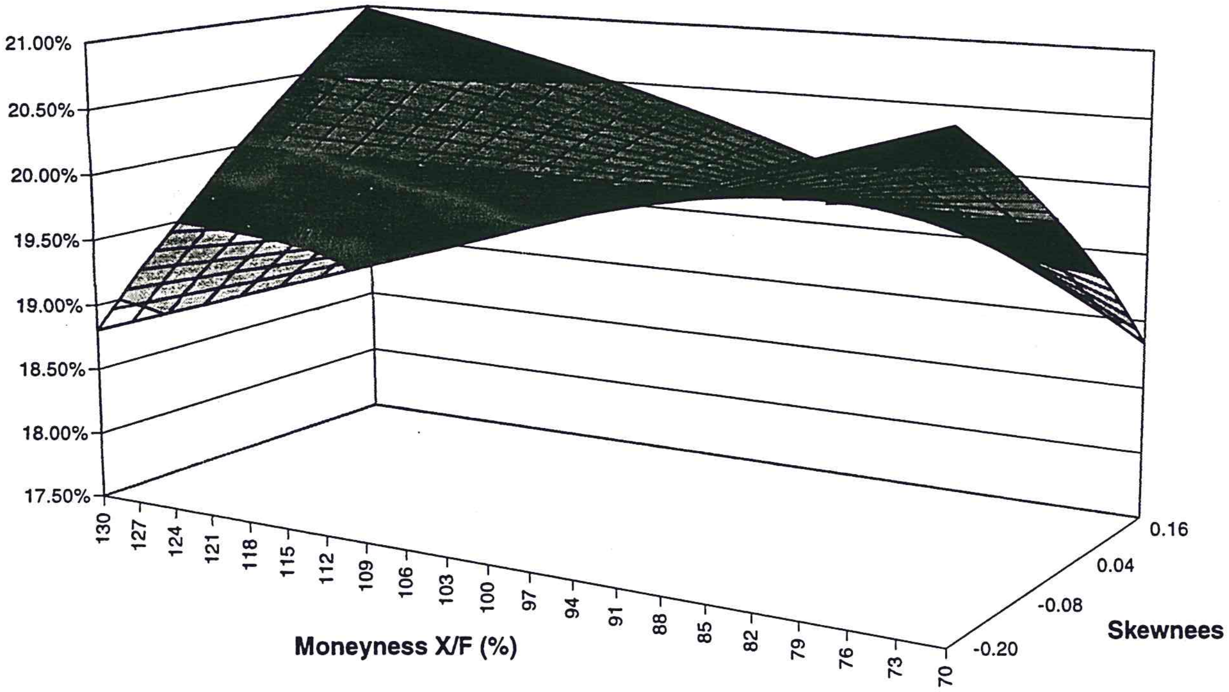

Figure 3: The Mean and the Smile
Black-Scholes implied volatility
Call option prices

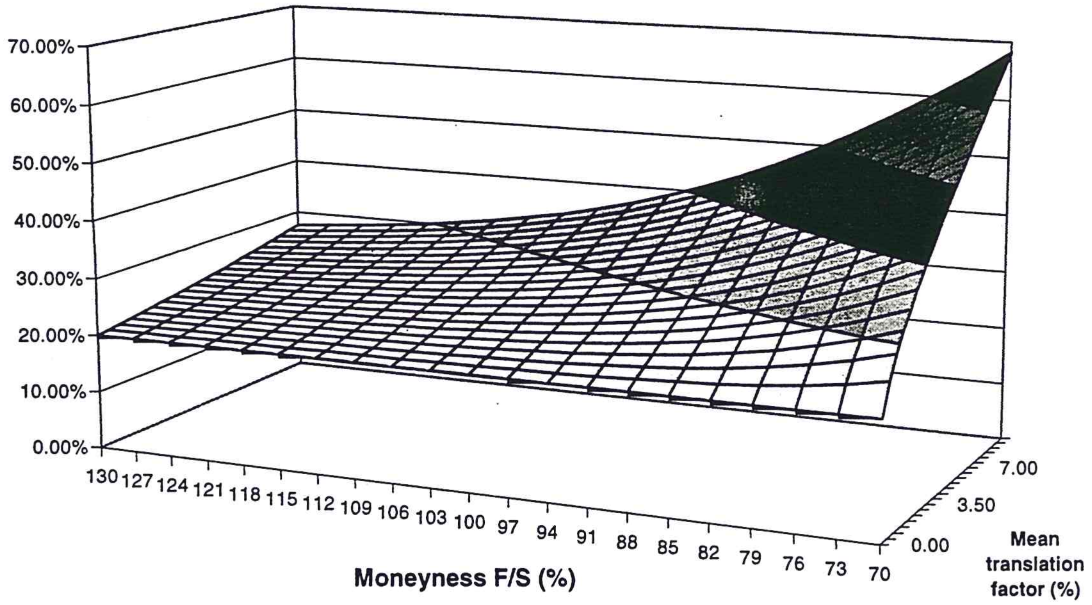

Figure 4: The Mean and the Smile
Black-Scholes implied volatility
Put options prices

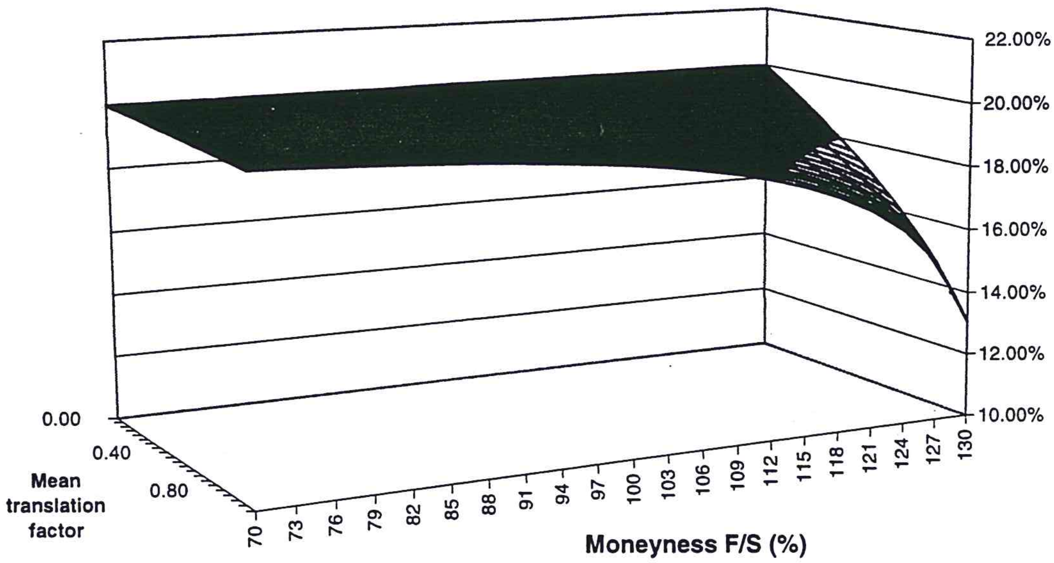

Figure 5: Black-Scholes Implied volatility
Mean translated distribution

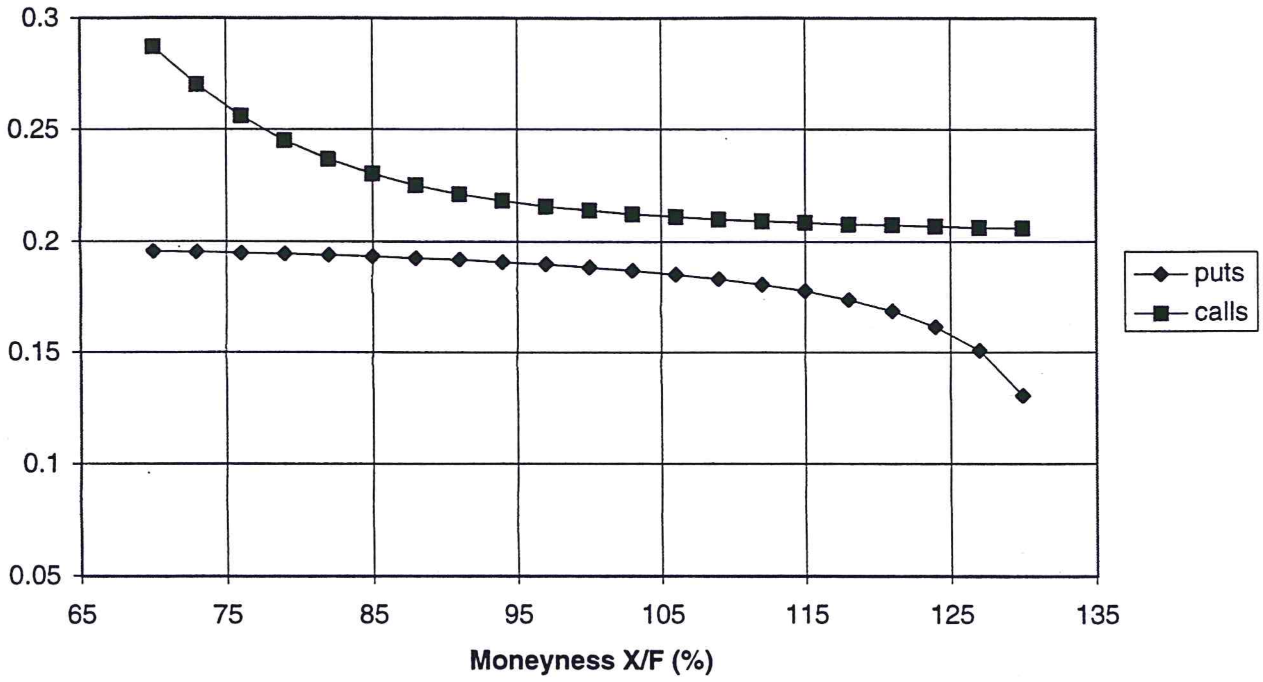

Figure 6:
Percentage differences between the asset value implied by the Black_Scholes model and the true asset market value.

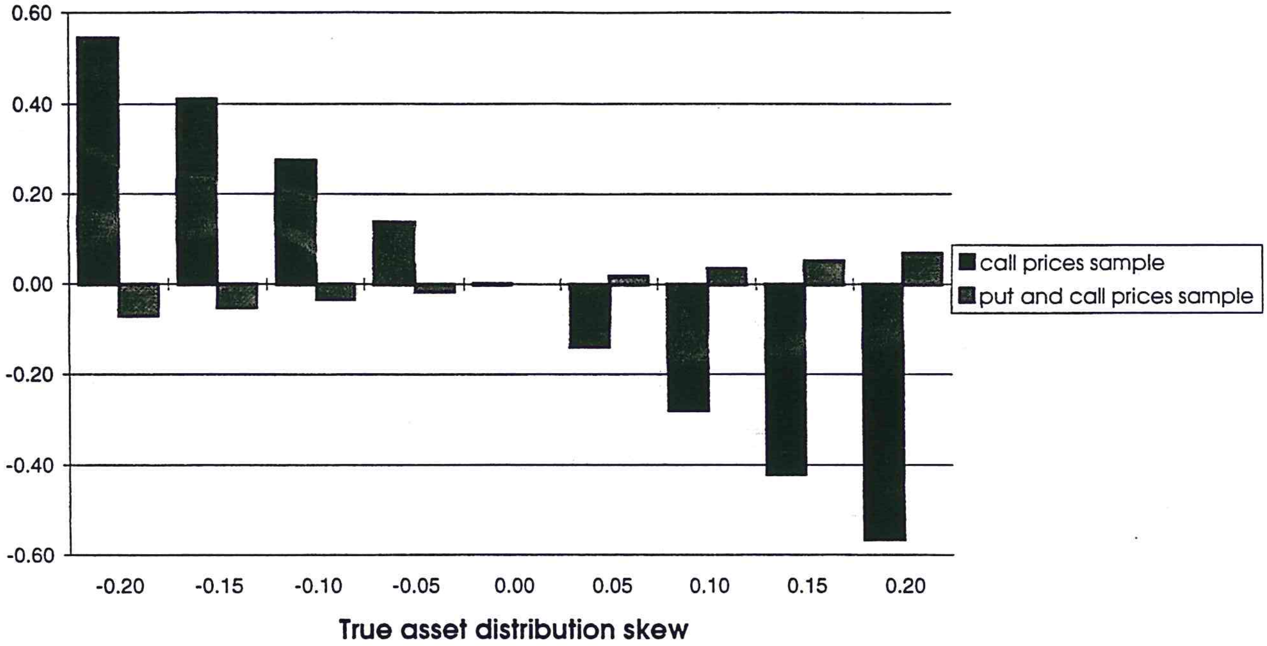

Figure 7:
Biases in the estimated mean distribution respect to the current index
Pre-crash period
Call option prices

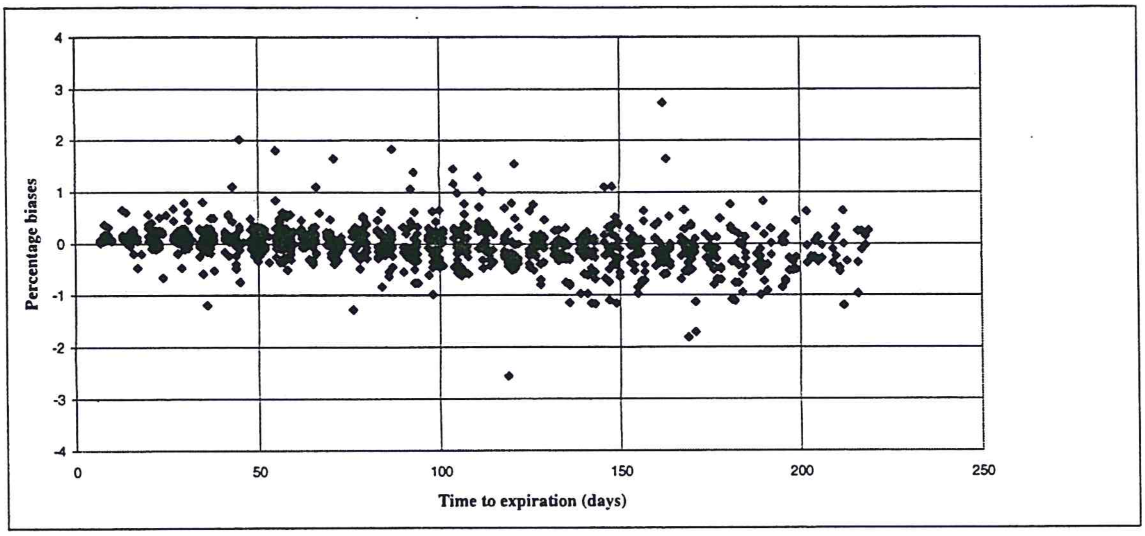

Figure 8:
Biases in the estimated mean distribution respect to the current index
Pre-crash period
Put option prices

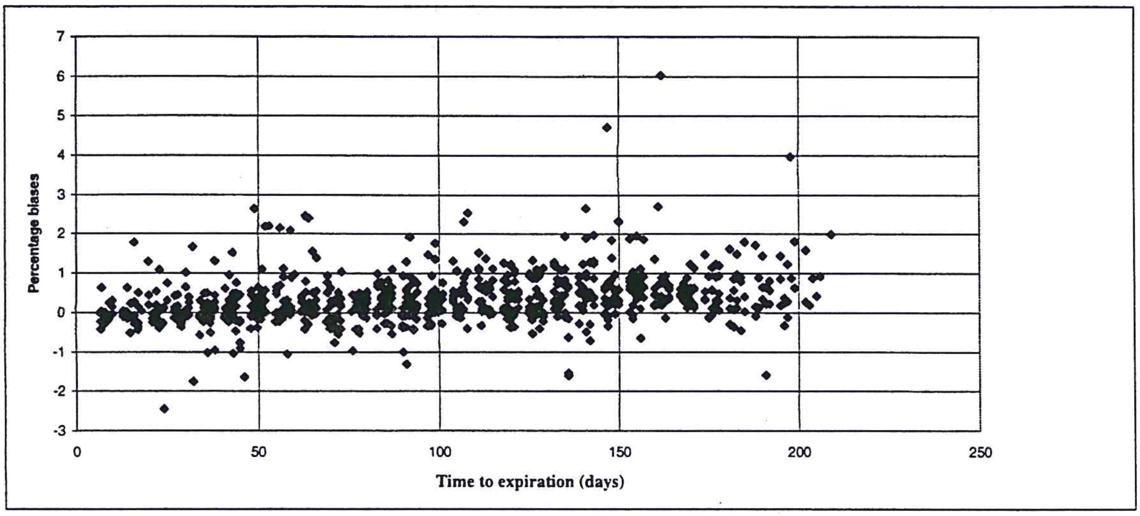

Figure 9:
Biases in the estimated mean distribution respect to the current index
Pre-crash period
Call and Put option prices

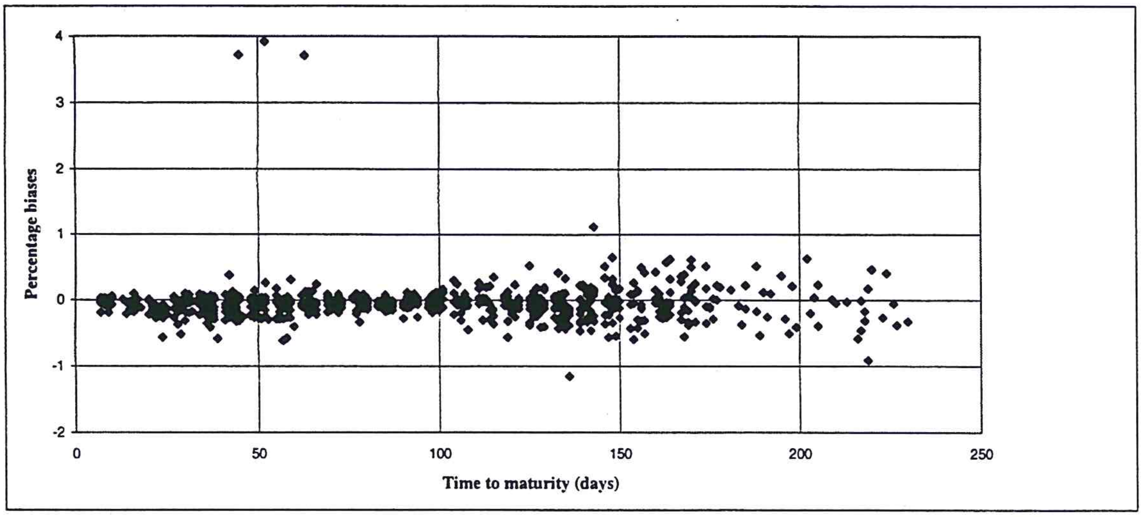

Figure 10:
Biases in the estimated mean distribution respect to the current index
Post-crash period
Call option prices

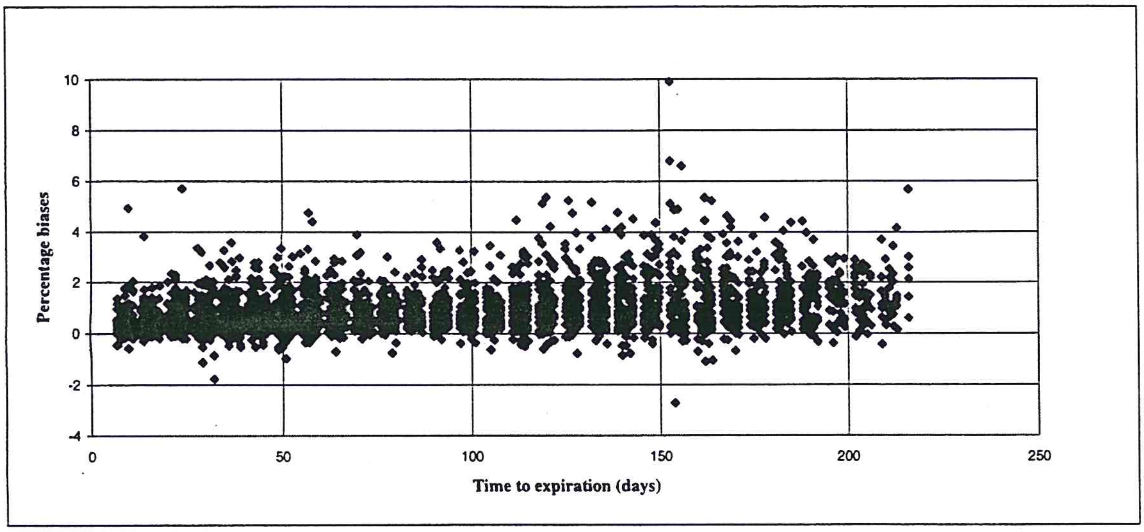

Figure 11:
Biases in the estimated mean distribution respect to the current index
Post-crash period
Put option prices

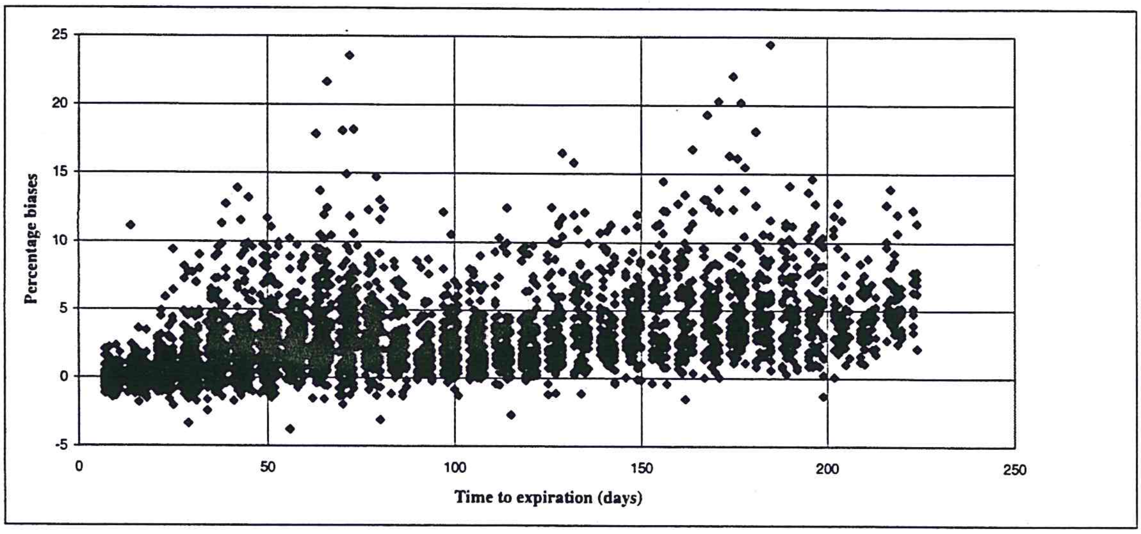

Figure 12:
Biases in the estimated mean distribution respect to the current index
Post-crash period
Call and Put option prices