FINANCIAL OPTIONS RESEARCH CENTRE

University of Warwick

Simple Resettable Cap and Floor Pricing Formulae

Kin Pang

FORC Preprint: 97/72

Simple Resettable Cap and Floor Pricing Formulae

First draft: April 1997

This version: June 1997

Kin PANG*

Financial Options Research Centre,
Warwick Business School,
Coventry, CV4 7AL,
England.

Email: Kin.Pang@warwick.ac.uk
Tel No: +(44)-1203-524118
Fax No: +(44)-1203-524167

* I am very grateful to Stewart Hodges for his constant support and encouragement. I wish to thank FORC for funding and Steve Dalton of Tradition (UK) Limited for introducing me to the topic of this paper. Any errors are my own.

Simple Resettable Cap and Floor Pricing Formulae

Abstract

We examine how the recently developed market-Libor model can be used to price resettable caps and floors to illustrate some of the advantages the market-Libor model offers for pricing Libor derivatives. We derived an exact relationship between the prices of resettable caplets and floorlets. We also derived an exact lower bound, approximate upper bound and two approximations for the prices of resettable caps and floors. Using representative forward rate term structures volatility factors we demonstrated that the first approximation provide prices that are very close to the no-arbitrage prices of the caplets. The second approximation does not perform so accurately but has the advantage that it prices the resettable caps and floors directly of market variables. The second approximation can be implemented on a spreadsheet to provide indicative prices for resettable caps and floors. We finish by discussing how the market-Libor model might be calibrated.

Recently, a number of papers, Brace, Gatarek and Musiela (1995), Jamshidian (1996), Miltersen, Sandmann and Sondermann (1995) and Musiela and Rutkowski (1995), have shown how Libor rates can be modelled consistently with the market convention for the pricing of caps and floors. Libor rates are made lognormal with respect to well chosen probability measures. The model has become known as the market Libor model. Whereas in the more conventional modelling approaches, calibration generally requires a high-dimensional optimisation to ensure that the models price consistently with common derivatives, such as caps and floors, calibration of market Libor model is simple. In the market Libor model, the quoted Black volatilities from the market[1] provide constraints on the Libor volatility factors that are easy to satisfy. Once calibrated, other Libor derivatives can then be priced consistently with the quoted Black volatilities.

We will illustrate the advantages of the market-Libor model by examining how resettable caps and floors can be priced by the model. We derive an exact lower bound and an approximate upper bound for the prices of resettable caps and floors. We show that in the market-Libor model there is an exact functional relationship between resettable caplet and floorlet prices. We also provide approximate formulae for the prices of resettable caplets and floorlets. The approximation formulae are very simple and can be implemented easily on a spreadsheet. One of the approximations can price resettable caps and floors directly of quotes for standard caps and floors.

We begin by outlining the main results from Musiela and Rutkowski (1996) needed for us to price the resettable caps and floors. Readers are referred to the paper for full details. Section 2 provides formulae for the upper and lower bounds for the value of a resettable caplet. Section 3 derives the corresponding results for a resettable floorlet. Section 4 provides numerical examples and Section 5 provides the approximations. Section 6 discusses the advantages and disadvantages of the market-Libor model. Section 7 summarises.

1. RESULTS FROM MUSIELA AND RUTKOWSKI (1996)

We reproduce some results in this section from Musiela and Rutkowski (1996) that we will use to derive the bounds and approximation formulae for the resettable caps and floors. We use the same notation as Musiela and Rutkowski (1996).

Musiela and Rutkowski (1996) assume we are given a *d*-dimensional Wiener process $\underline{W}^*$ defined on a filtered probability space (Ω, $(\Im_t)_{t\in[0,T^*]}$, $\mathbf{P}^*$) satisfying the usual assumptions. They assume the forward bond return defined by

$$F_B(t,T,T^*) \overset{def}{=} \frac{B(t,T)}{B(t,T^*)},\ t \le T \le T^*$$

follows a strictly-positive continuous martingale under $\mathbf{P}^*$. See Musiela and Rutkowski (1996), BP.1 - BP.3. $F_B(t, T, T^*)$ is the return from time T to T^* available at time t by shorting one $1/B(t, T^*)$ units of the T^* maturity PDB and investing the unit cash in the T maturity PDB. It is also the value of the T maturity PDB using the T^* maturity PDB as the numéraire. Since $F_B(t, T, T^*)$ is a martingale with respect to $\mathbf{P}^*$, it follows from Harrison and Kreps (1979) that there is no-arbitrage in the PDB market.

Using standard results on strictly-positive continuous martingale, it is possible to express the dynamics of $F_B(t, T, T^*)$ by the Itô differential equation

$$dF_B(t,T,T^*) = F_B(t,T,T^*)\underline{\gamma}(t,T,T^*)\cdot d\underline{W}_t^*$$

where $\underline{\gamma}(t, T, T^*)$, $t \in [0,T]$, is a R^d-valued predictable process, integrable with respect to the Wiener process $\underline{W}^*$ and the $\cdot$ denotes the inner product.

Using some foresight, note that forward Libor rates define forward bond returns over different time intervals. So to consider the forward bond return between any two time points closer than T^*, let us define

$$F_B(t,T,U) \stackrel{def}{=} \frac{F_B(t,T,T^*)}{F_B(t,U,T^*)} = \frac{B(t,T)}{B(t,U)}, \ \forall\ t \in [0,\ T \wedge U].$$

It follows from an application of Itô's Lemma that

$$dF_B(t,T,U) = F_B(t,T,U)\underline{\gamma}(t,T,U)\cdot(d\underline{W}_t^* - \underline{\gamma}(t,U,T^*)dt)$$

$$= F_B(t,T,U)\underline{\gamma}(t,T,U)\cdot d\underline{W}_t^U \tag{1.1}$$

where

$$\underline{\gamma}(t,T,U) = \underline{\gamma}(t,T,T^*) - \underline{\gamma}(t,U,T^*),\ \forall\ t \in [0,\ T \wedge U]$$

and

$$\underline{W}_t^U = \underline{W}_t^* - \int_0^t \underline{\gamma}(s,U,T^*)ds,\ \forall\ t \in [0,\ U], \tag{1.2}$$

by Girsanov's Theorem, is a Wiener process with respect to the probability measure $\mathbf{P}^U \sim \mathbf{P}^*$ defined by the Radon-Nikodym derivative

$$\frac{d\mathbf{P}^U}{d\mathbf{P}^*} = \exp\left[-\tfrac{1}{2}\int_0^U \left|\underline{\gamma}(s,U,T^*)\right|^2 ds + \int_0^U \underline{\gamma}(s,U,T^*)\cdot d\underline{W}_s^*\right]$$

where $|.|$ denotes the norm in R^d. This change of measure corresponds to a change of numéraire from the T^* maturity PDB to the U maturity PDB. Where convenient, we shall use T^*-measure for $\mathbf{P}^*$ and U-measure for $\mathbf{P}^U$. We also use $E^*[\cdot]$ for expectations taken with respect to the T^*-measure and similarly $E^U[\cdot]$ for expectations taken with respect to the U-measure.

There are various Libor rates with different duration in the market. In the market-Libor model, it is necessary to focus on a particular Libor rate. Let the Libor rate length be δ. Then the forward δ-Libor rate $L(t,\ T)$ at time t, with maturity T, is given implicitly by

$$1+\delta L(t,T) = F_B(t,T,T+\delta).$$

It follows from equation 1.1, substituting $T+\delta$ for U, and using Itô's Lemma, that under the probability $(T+\delta)$-measure, the forward δ-Libor rate follows the process given by

$$dL(t,T) = L(t,T)\frac{1+\delta L(t,T)}{\delta L(t,T)}\underline{\gamma}(t,T,T+\delta)\cdot d\underline{W}_t^{T+\delta}. \quad (1.3)$$

Examining equation 1.3 reveals that the forward δ-Libor rate $L(t, T)$ will be lognormally distributed when

$$\underline{\lambda}(t,T) = \frac{1+\delta L(t,T)}{\delta L(t,T)}\underline{\gamma}(t,T,T+\delta)$$

is deterministic. The class of model obtained where $\underline{\lambda}(t, T)$ is deterministic is known as the market Libor model[1]. We assume henceforth that $\underline{\lambda}(t, T)$ is deterministic and re-write equation 1.3 as

$$\frac{dL(t,T)}{L(t,T)} = \underline{\lambda}(t,T,)\cdot d\underline{W}_t^{T+\delta} \quad (1.4)$$

which can be solved to give the time T spot δ-Libor rate as

$$L(T,T) = L(t,T)\exp\left\{-\tfrac{1}{2}\int_t^T \left|\underline{\lambda}(u,T)\right|^2 du + \int_t^T \underline{\lambda}(u,T)\cdot d\underline{W}_u^*\right\}. \quad (1.5)$$

It is appropriate to provide a few comments here on the market-Libor model before we proceed to see how the simple Libor derivative, a Libor cap, can be priced. We have seen that when $\underline{\lambda}(t, T)$ is deterministic, the forward δ-Libor rate, $L(t, T)$, is lognormally distributed with respect to $(T+\delta)$-measure. $\underline{\lambda}(t, T)$ is the volatility of $L(t, T)$. Note that other maturity forward Libor rates are not lognormally distributed. That is, $L(t, U)$ is not lognormally distributed with respect to the $(T+\delta)$-measure if U is different from T. Nor are other Libor rates with different lengths lognormally distributed. To see this, let $Z(t, T)$ be the $k\delta$ period forward Libor rate from time T to $T+k\delta$ defined implicitly by

$$1 + k\,\delta\, Z(t,\ T) = B(t,\ T)/B(t,\ T + k\delta).$$

Then since

$$\prod_{i=0}^{i=k-1}\left[1+\delta L(t,T+i\delta)\right]=\frac{B(t,T)}{B(t,T+k\delta)}$$

it follows that

$$Z(t,T)=\left[\prod_{i=0}^{i=k-1}\left[1+\delta L(t,T+i\delta)\right]-1\right]\Big/k\delta\,.$$

$Z(t,\ T)$ is not lognormally distributed and is difficult to analyse. These two points we have highlighted are the weaknesses of the model in applications where they require either Libor rates that do not have the same length as δ or where they need to examine different maturing forward Libor rates simultaneously.

The attraction offered by the market-Libor is that the lognormality of the forward Libor rates is consistent with the market quoting convention for cap and floors. It is therefore not surprising that the market Libor model Black gives cap and floor pricing formulae that are consistent with the market convention.

First define

$$T^*_{m\delta} = T^* - m\delta\,.$$

This notation is convenient because $L(t,\ T)$ is lognormal in the $(T+\delta)$-measure and so $L(t, T^*_\delta)$ is lognormal in the T^*-measure.

Consider the caplet which pays $\delta[L(T^*_\delta, T^*_\delta) - K]^+$ at time T^*. This caplet can be used to limit the interest rate that applies to a floating loan over the period [T^*_δ, T^*] to K, the cap rate. It follows from the caplet's time T^* payoff that the time t price of the caplet is given by

$$cpt(t) = \delta\, P(t,T^*)E^*[L(T^*_\delta, T^*_\delta) - K|\Im_t]^+\,. \qquad \textbf{(1.6)}$$

It is a standard result that if equation 1.5, with $T \equiv T^*$, is substituted into equation 1.6 and the expectation taken then we obtain

$$cpt(t) = \delta P(t,T^*)\left\{L(t,T_\delta^*)N[h(t,T_\delta^*)] - KN[h(t,T_\delta^*) - \varsigma(t,T_\delta^*)]\right\} \quad (1.7)$$

where

$$h(t,T_\delta^*) = \left[\log\frac{L(t,T_\delta^*)}{K} + \tfrac{1}{2}\varsigma^2(t,T_\delta^*)\right]\Big/\varsigma(t,T_\delta^*) \quad (1.8)$$

$$\varsigma^2(t,T_\delta^*) = \int_t^{T_\delta^*}\left|\underline{\lambda}(u,T_\delta^*)\right|^2 du \quad (1.9)$$

and $N[\cdot]$ is the normal distribution function. This is the same as the market convention when equation 1.9 is replaced by

$$\varsigma^2(t,T_\delta^*) = \sigma(T_\delta^*)^2(T_\delta^* - t) \quad (1.10)$$

where $\sigma(T_\delta^*)$ is the forward-forward volatility[2] of the caplet that covers the period $[T_\delta^*, T^*]$.

The caplet pricing formula shows that if we can extract the forward-forward volatility of the caplet from quoted market Black volatilities, then equations 1.9 and 1.10 provide a constraint on the forward Libor volatility factors of the market-Libor model. We discuss this observation in more detail in Section 6. We will derive in Section 5 an approximation that will use only these forward-forward volatilities and the current forward Libor rates to price resettable caps and floors.

We now have sufficient material to price resettable caps and floors using the market-Libor model.

2. PRICING OF RESETTABLE CAPLET

A resettable caplet is a standard caplet with the modification that the cap rate is given by the spot δ-Libor rate at a time δ before the caplet maturity. For example, consider the resettable caplet which has at time T_δ^* the payoff given by $\delta[L(T_\delta^*,T_\delta^*) - L(T_{2\delta}^*,T_{2\delta}^*)]^+$. The payoff at time T_δ^* depends on the difference between the spot δ-Libor rates at times T_δ^* and $T_{2\delta}^*$. This resettable caplet has time t value given by

$$rcpt(t) = P(t,T_\delta^*)E^{T_\delta^*}\delta[L(T_\delta^*,T_\delta^*) - L(T_{2\delta}^*,T_{2\delta}^*)|\Im_t]^+ \tag{2.1}$$

We need to solve for $L(T_\delta^*,T_\delta^*)$ and $L(T_{2\delta}^*,T_{2\delta}^*)$ under the T_δ^* - measure to evaluate equation 2.1. We follow Musiela and Rutkowski (1996) and use the notation

$$\varepsilon_T\left(\int_t \underline{\lambda}(u,v)\cdot d\underline{W}_u\right) = \exp\left\{-\tfrac{1}{2}\int_t^T \left|\underline{\lambda}(u,v)\right|^2 du + \int_t^T \underline{\lambda}(u,v)\cdot d\underline{W}_u\right\}$$

for the exponential martingale. For example, equation 1.5 can be re-expressed as

$$L(T_\delta^*,T_\delta^*) = L(t,T_\delta^*)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^*\right). \tag{2.2}$$

$L(T_{2\delta}^*,T_{2\delta}^*)$ is lognormally under the T_δ^* - measure and so analogous to equation 2.2 we can write

$$L(T_{2\delta}^*,T_{2\delta}^*) = L(t,T_{2\delta}^*)\varepsilon_{T_{2\delta}^*}\left(\int_t \underline{\lambda}(u,T_{2\delta}^*)\cdot d\underline{W}_u^{T_\delta^*}\right). \tag{2.3}$$

Equation 2.2 needs to be re-written for the T_δ^* - measure. From equation 1.2, with $U \equiv T_\delta^*$, we have

$$\underline{W}_t^{T_\delta^*} = \underline{W}_t^* - \int_0 \underline{\gamma}(s,T_\delta^*,T^*)ds, \ \forall\ t \in [0,\ T_\delta^*] \tag{2.4}$$

or

$$d\underline{W}_t^{T_\delta^*} = d\underline{W}_t^* - \underline{\gamma}(t,T_\delta^*,T^*)dt, \ \forall\ t \in [0,\ T_\delta^*] \tag{2.5}$$

which when substituted in equation 2.2 and simplified gives

$$\begin{aligned} L(T_\delta^*,T_\delta^*) &= L(t,T_\delta^*)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\exp\left[\int_t^{T_\delta^*} \underline{\lambda}(u,T_\delta^*)\cdot\underline{\gamma}(u,T_\delta^*,T^*)du\right] \\ &= L(t,T_\delta^*)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\exp\left[\int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 \frac{\delta\, L(u,T_\delta^*)}{1+\delta\, L(u,T_\delta^*)}du\right]. \end{aligned} \tag{2.6}$$

We now have the spot Libor rates for equation 2.1 so substituting equations 2.3 and 2.6 into the equation gives

$$rcpt(t) = \delta P(t,T_\delta^*)E^{T_\delta^*}\left[\left. \begin{array}{l} L(t,T_\delta^*)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\exp\{X\} \\ -L(t,T_{2\delta}^*)\varepsilon_{T_{2\delta}^*}\left(\int_t \underline{\lambda}(u,T_{2\delta}^*)\cdot d\underline{W}_u^{T_\delta^*}\right) \end{array} \right| \Im_t \right]^+ \quad (2.7)$$

where

$$X = \int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 \frac{\delta L(u,T_\delta^*)}{1+\delta L(u,T_\delta^*)} du. \quad (2.8)$$

Unfortunately, the expectation in equation 2.7 cannot be evaluated analytically. We will proceed to derive two simple approximations to 2.7 later. First we will derive upper and lower bounds for equation 2.7.

We deal first with the lower bound. Equation 2.7 is an increasing function of X. Since $X > 0$, we can obtain a lower bound for the value of the resettable caplet by evaluating the expectation analytically at $X = 0$. Using the Lemma of the Appendix we obtain the first proposition.

Proposition 1: *A lower bound value for the time t value of a resettable caplet with payoff* $\delta[L(T_\delta^*,T_\delta^*) - L(T_{2\delta}^*,T_{2\delta}^*)]^+$ *at* T_δ^* *is given by*

$$rcpt(t) > \delta P(t,T_\delta^*)\left[L(t,T_\delta^*)N(d_1) - L(t,T_{2\delta}^*)N(d_2)\right] \quad (2.9)$$

where

$$d_1 = \frac{\log\left(\dfrac{L(t,T_\delta^*)}{L(t,T_{2\delta}^*)}\right)}{\sqrt{v(t,T_{2\delta}^*,T_\delta^*)}} + \frac{1}{2}\sqrt{v(t,T_{2\delta}^*,T_\delta^*)} \quad (2.10)$$

$$d_2 = d_1 - \sqrt{v(t,T_{2\delta}^*,T_\delta^*)} \quad (2.11)$$

$$v(t,T_{2\delta}^*,T_\delta^*) = \int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du - 2\int_t^{T_{2\delta}^*} \underline{\lambda}(u,T_\delta^*)\cdot\underline{\lambda}(u,T_{2\delta}^*)du + \int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_{2\delta}^*)\right|^2 du. \quad (2.12)$$

Now the upper bound. If follows from equation 2.8 that

$$0 \le X \le \int_t^{T_\delta^*} \left|\underline{\lambda}(u, T_\delta^*)\right|^2 du .$$

Thus we can obtain an upper bound by evaluating equation 2.7 at $X = \int_t^{T_\delta^*} \left|\underline{\lambda}(u, T_\delta^*)\right|^2 du$. However, this upper bound will be much too large by far. We provide an alternative upper bound; it holds only approximately but in practice, as we shall see in Section 8.5, it too is much larger than a numerical integration of equation 2.7.

Proposition 2: *An approximate upper bound for the time t value of a resettable caplet with payoff* $\delta[L(T_\delta^*, T_\delta^*) - L(T_{2\delta}^*, T_{2\delta}^*)]^+$ *at* T_δ^* *is given by*

$$rcpt_U(t) \lessapprox \frac{Rcpt_M(t) + Rcpt_E(t)}{2} \tag{2.13}$$

where

$$Rcpt_E(t) = \delta P(t, T_\delta^*) f\left[E^{T_\delta^*}\left[X \middle| \Im_t\right]\right] \tag{2.14}$$

$$Rcpt_M(t) = \delta P(t, T_\delta^*) f\left[\int_t^{T_\delta^*} \left|\underline{\lambda}(u, T_\delta^*)\right|^2 du\right] \tag{2.15}$$

$$f(X) = L(t, T_\delta^*)\exp[X] N(d_1) - L(t, T_{2\delta}^*) N(d_2) \tag{2.16}$$

$$d_1 = \frac{\log\left(\dfrac{L(t, T_\delta^*)}{L(t, T_{2\delta}^*)}\right) + X}{\sqrt{v(t, T_{2\delta}^*, T_\delta^*)}} + \frac{1}{2}\sqrt{v(t, T_{2\delta}^*, T_\delta^*)} \tag{2.17}$$

$$d_2 = d_1 - \sqrt{v(t, T_{2\delta}^*, T_\delta^*)} \tag{2.18}$$

$$v(t,T_{2\delta}^*,T_\delta^*) = \int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du - 2\int_t^{T_{2\delta}^*} \underline{\lambda}(u,T_\delta^*)\cdot\underline{\lambda}(u,T_{2\delta}^*)du + \int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_{2\delta}^*)\right|^2 du. \qquad (2.19)$$

Proof: We have

$$rcpt(t) = \delta P(t,T_\delta^*)E^{T_\delta^*}\left[\begin{array}{l} L(t,T_\delta^*)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\exp[X] \\ - L(t,T_{2\delta}^*)\varepsilon_{T_{2\delta}^*}\left(\int_t \underline{\lambda}(u,T_{2\delta}^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\end{array}\right|\Im_t\Bigg]^+$$

$$= \delta P(t,T_\delta^*)E^{T_\delta^*}\left[E^{T_\delta^*}\left[\begin{array}{l} L(t,T_\delta^*)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\exp[X] \\ - L(t,T_{2\delta}^*)\varepsilon_{T_{2\delta}^*}\left(\int_t \underline{\lambda}(u,T_{2\delta}^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\end{array}\right|\Im_t, X\Bigg]^+\Bigg|\Im_t\right]$$

$$= \delta P(t,T_\delta^*)E^{T_\delta^*}\left[f(X)\middle|\Im_t\right] \qquad (2.20)$$

where $f(X)$, $d_1(X)$ and $d_2(X)$ are as defined in equations 2.16 to 2.18 and X is defined by equation 2.8. Note that equation $f(X)$ is an increasing function of X in the same way that that the Black-Scholes European Call option formula is increasing with the current share price. Now for any increasing function $f(X)$,

$$E[f(X)] \leq [f(\text{med}[X]) + f(\max[X])]/2$$

where med[X] is the median of X. We approximate med(X) by E(X), that is, we assume

$$E[f(X)] \lessapprox [f(\text{E}(X)) + f(\max[X]]/2. \qquad (2.21)$$

Now from the definition of X in equation 2.8, we have

$$E^{T_\delta^*}[X|\Im_t] = \int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 E^{T_\delta^*}\left[\frac{\delta L(u,T_\delta^*)}{1+\delta L(u,T_\delta^*)}\middle|\Im_t\right]du$$

$$= \frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)}\int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du \qquad (2.22)$$

since

$$\frac{\delta L(u,T_\delta^*)}{1+\delta L(u,T_\delta^*)} = 1 - \frac{1}{1+\delta L(u,T_\delta^*)} = 1 - \frac{P(u,T^*)}{P(u,T_\delta^*)}$$

is a $P^{T_\delta^*}$ martingale. Furthermore,

$$\max[X] = \int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du . \tag{2.23}$$

The approximation follows by substituting equations 2.22 and 2.23 into equation 2.21.

QED.

We have derived an exact lower bound and an approximate upper bound for the price of resettable caplets. We will proceed to examine how well these bounds perform in Section 4. Before that, we show how the bounds for resettable floorlets can be obtained from the bounds on the resettable caplets.

3. PRICING OF RESETTABLE FLOOR

Resettable floorlet have a similar payoff structure to resettable caplets. The resettable floorlet that covers the same period as the resettable caplet considered in Section 2 has payoff at time T_δ^* the quantity given by $\delta[L(T_{2\delta}^*,T_{2\delta}^*) - L(T_\delta^*,T_\delta^*)]^+$. So resettable floorlets are the counterpart to resettable caplets. Resettable floorlets pay out when the spot δ-Libor rate $L(T_{2\delta}^*,T_{2\delta}^*)$ is greater than $L(T_\delta^*,T_\delta^*)$ whereas for resettable caplets it is the other way round.

We can derive bounds for the price of the resettable floorlet by considering a portfolio that consists a long position in the resettable caplet considered in Section 2 and one short of the resettable floorlet that covers the same period. This portfolio has payoff at time T_δ^* the quantity given by

$$\delta[L(T_\delta^*,T_\delta^*) - L(T_{2\delta}^*,T_{2\delta}^*)]$$

which has the same value as

$$[1+\delta L(T_\delta^*,T_\delta^*)]\delta L(T_\delta^*,T_\delta^*) \text{ at time } T^*$$

by rolling the amount $\delta L(T_\delta^*,T_\delta^*)$ from time T_δ^* to T^* and

$$-\delta L(T_{2\delta}^*,T_{2\delta}^*) \text{ at time } T_\delta^*.$$

We write the payoff in this way because $L(T_\delta^*,T_\delta^*)$ is lognormally distributed with respect to the T^*-measure and $L(T_{2\delta}^*,T_{2\delta}^*)$ is lognormally distributed with respect to the T_δ^*-measure.

The value of the portfolio must be the value of the risk-adjusted discounted expected values of the two cashflows at times T^* and T_δ^*. Therefore

$$rcpt(t) - rflt(t) = P(t,T^*)E^*\left[[1+\delta L(T_\delta^*,T_\delta^*)]\delta L(T_\delta^*,T_\delta^*)\middle|\Im_t\right]$$

$$- P(t,T_\delta^*)E^{T_\delta^*}\left[\delta L(T_{2\delta}^*,T_{2\delta}^*)\middle|\Im_t\right]. \qquad (3.1)$$

We can evaluate equation 3.1 analytically using standard results for the moments of the lognormal distribution equation to give

$$rflt(t) = rcpt(t) + \delta P(t,T_\delta^*)L(t,T_{2\delta}^*)$$

$$-\delta P(t,T^*)\left[L(t,T_\delta^*) + \delta L^2(t,T_\delta^*)\exp\left(\sigma^2(T_\delta^*)(T_\delta^*-t)\right)\right]. \qquad (3.2)$$

The upper and lower bounds for the resettable floorlet are given by substituting the upper and lower bounds for the resettable caplet into equation 3.2.

4. NUMERICAL RESULTS

In this section, we investigate where the no-arbitrage prices of resettable caplets are in relation to be bounds derived in Section 2. We assume that δ is ¼.

Empirical data suggest interest rate dynamics can be explained by two or three factors. In our example we take two. The first and second factors are shown in Figure 1. The first factor accounts for 90% of variations in interest rates and the second factor accounts for the remainder. The resulting volatility term structure and our assumed initial term structure of forward Libor rates are plotted in Figure 2. Our volatility factors are proportional and are assumed to be time stationary, that is, they depend only on the maturities of the forward Libor rates. We have chosen the volatility factors to give a humped volatility term structure that occurs generally in practice.

Using these inputs, we can evaluate the integrals of Propositions 1 and 2 numerically to give the lower and upper bounds for the price of resettable caplets. The results are plotted in Figure 3. We see that the spread between the approximate upper and the exact lower bounds is disappointingly large. This, however, would not be important in practice if the exact price is close to the lower bound. We examine this possibility using Monte Carlo simulations.

4.1 MONTE CARLO SIMULATIONS

We repeat equation 2.1 here for the value of the resettable caplet covering the period $[T_{2\delta}^*, T_\delta^*]$

$$rcpt(t) = P(t, T_\delta^*) E^{T_\delta^*} \delta[L(T_\delta^*, T_\delta^*) - L(T_{2\delta}^*, T_{2\delta}^*) | \Im_t]^+ .$$

We need to be able to simulate simultaneously the values of $L(T_{2\delta}^*, T_{2\delta}^*)$ and $L(T_\delta^*, T_\delta^*)$ under the T_δ^*-measure. Substituting $T_{2\delta}^*$ and T_δ^* for T in equation 1.4 gives the following processes for $L(t, T_{2\delta}^*)$ and $L(t, T_\delta^*)$ respectively:

$$\frac{dL(t, T_{2\delta}^*)}{L(t, T_{2\delta}^*)} = \underline{\lambda}(t, T_{2\delta}^*) \cdot d\underline{W}_t^{T_\delta^*} \tag{4.1}$$

$$\frac{dL(t, T_\delta^*)}{L(t, T_\delta^*)} = \underline{\lambda}(t, T_\delta^*) \cdot d\underline{W}_t^{T^*} . \tag{4.2}$$

We can transform equation 4.2 to the T_δ^*-measure using equation 2.5 to give

$$\frac{dL(t,T_\delta^*)}{L(t,T_\delta^*)} = \left|\underline{\lambda}(t,T_\delta^*)\right|^2 \left[\frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)}\right] dt + \underline{\lambda}(t,T_\delta^*)\cdot d\underline{W}_t^{T_\delta^*}. \qquad (4.3)$$

Equations 4.1 and 4.3 can be re-expressed as

$$d\ln L(t,T_{2\delta}^*) = -\tfrac{1}{2}\left|\underline{\lambda}(t,T_{2\delta}^*)\right|^2 dt + \underline{\lambda}(t,T_{2\delta}^*)\cdot d\underline{W}_t^{T_\delta^*} \qquad (4.4)$$

$$d\ln L(t,T_\delta^*) = \left|\underline{\lambda}(t,T_\delta^*)\right|^2 \left[\frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)} - \frac{1}{2}\right] dt + \underline{\lambda}(t,T_\delta^*)\cdot d\underline{W}_t^{T_\delta^*}. \qquad (4.5)$$

Equations 4.4 and 4.5 allow us to simulate for $L(T_{2\delta}^*, T_{2\delta}^*)$ and $L(T_\delta^*, T_\delta^*)$ jointly. It is better to simulate the logarithms because simulating the logarithms prevents the Libor rates going negative that can occur if we simulate the levels using equations 4.1 and 4.3 instead.

Figure 4 plots the lower bound provided by Proposition 1 and the Monte Carlo estimates together with the two standard error band. For caplets with maturities 0.5 to 5.25 we used step sizes of 0.00125 and for the caplets with maturities 5.5 to 9.5 we used 0.025. For both sets we used 10000 simulations with antithetic variance reduction.

We see, for the Libor curve and volatility factors we have chosen, the Monte Carlo prices are not very close to the exact lower bound. The Monte Carlo prices exceed the exact lower bounds by between 5% and 17%. It is obvious that the upper bound plotted in figure 3 is much too high to be practically useful. The lower bound is also too low so it may be more useful to obtain an approximate formula for the prices of resettable caplets. We provide two approximations in the next section.

5. APPROXIMATIONS

We saw in the previous section that the bounds are disappointingly far away from the Monte Carlo Simulation prices for them to be practically useful. In this section to derive approximations that may be more useful. Here is the first approximation.

Approximation 1: *An approximate value for the time t value of a resettable caplet with payoff* $\delta[L(T_\delta^*,T_\delta^*)-L(T_{2\delta}^*,T_{2\delta}^*)]^+$ *at* T_δ^* *is given by*

$$rcpt(t) \approx \delta P(t,T_\delta^*)\left[L(t,T_\delta^*)\exp\left(\frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)}\int_t^{T_\delta^*}\left|\underline{\lambda}(u,T_\delta^*)\right|^2 du\right)N(d_1)-L(t,T_{2\delta}^*)N(d_2)\right] \tag{5.1}$$

where

$$d_1=\frac{\log\left(\frac{L(t,T_\delta^*)}{L(t,T_{2\delta}^*)}\right)+\frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)}\int_t^{T_\delta^*}\left|\underline{\lambda}(u,T_\delta^*)\right|^2 du}{\sqrt{v(t,T_{2\delta}^*,T_\delta^*)}}+\frac{1}{2}\sqrt{v(t,T_{2\delta}^*,T_\delta^*)} \tag{5.2}$$

$$d_2=d_1-\sqrt{v(t,T_{2\delta}^*,T_\delta^*)} \tag{5.3}$$

$$v(t,T_{2\delta}^*,T_\delta^*)=\int_t^{T_\delta^*}\left|\underline{\lambda}(u,T_\delta^*)\right|^2 du-2\int_t^{T_{2\delta}^*}\underline{\lambda}(u,T_\delta^*)\cdot\underline{\lambda}(u,T_{2\delta}^*)du+\int_t^{T_{2\delta}^*}\left|\underline{\lambda}(u,T_{2\delta}^*)\right|^2 du. \tag{5.4}$$

Proof: We use the approximation

$$E^{T_\delta^*}\left[L(t,T_\delta^*)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\exp[X]-L(t,T_{2\delta}^*)\varepsilon_{T_{2\delta}^*}\left(\int_t \underline{\lambda}(u,T_{2\delta}^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\middle|\Im_t\right]^+$$

$$\approx E^{T_\delta^*}\left[\begin{array}{l}L(t,T_\delta^*)\exp\left\{E^{T_\delta^*}\left(X\middle|\Im_t\right)\right\}\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\\ -L(t,T_{2\delta}^*)\varepsilon_{T_{2\delta}^*}\left(\int_t \underline{\lambda}(u,T_{2\delta}^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\end{array}\middle|\Im_t\right]^+. \tag{5.5}$$

Thus substituting equation 5.5 and equation 2.22 into equation 2.7 gives

$$rcpt(t)\approx\delta P(t,T_\delta^*)E^{T_\delta^*}\left[\begin{array}{l}L(t,T_\delta^*)\exp\left(\frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)}\int_t^{T_\delta^*}\left|\underline{\lambda}(u,T_\delta^*)\right|^2 du\right)\varepsilon_{T_\delta^*}\left(\int_t \underline{\lambda}(u,T_\delta^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\\ -L(t,T_{2\delta}^*)\varepsilon_{T_{2\delta}^*}\left(\int_t \underline{\lambda}(u,T_{2\delta}^*)\cdot d\underline{W}_u^{T_\delta^*}\right)\end{array}\middle|\Im_t\right]^+.$$

Finally, applying the Lemma of the Appendix gives the required formula. **QED**.

Like the upper and lower bounds, Approximation 1 can be evaluated easily once the volatility structure of the forward Libor rates are specified. Before we proceed to examine how well Approximation 1 performs, let us first examine its connection to quoted cap volatilities. It would be very convenient for practitioners, if it were possible to price, even approximately, resettable caps and floors[3] directly from quoted standard cap and floor volatilities.

Actually, nearly all the inputs required for the bounds and Approximation 1 can be obtained directly from market quotes at time t, assuming that the forward-forward volatilities for the standard caplets have been extracted quoted cap volatilities.

To see this, suppose $\sigma(T^*_{2\delta})$ represents the forward-forward volatility for a standard caplet that covers the period $[T^*_{2\delta}, T^*_{\delta}]$ and similarly $\sigma(T^*_{\delta})$ the forward-forward volatility for $[T^*_{\delta}, T^*]$. The caplet pricing formula given by equations 1.7 to 1.9 gives the following constraints on the forward Libor volatility factors:

$$\int_t^{T^*_{2\delta}} \left|\underline{\lambda}(u, T^*_{2\delta})\right|^2 du = \sigma^2(T^*_{2\delta})(T^*_{2\delta} - t) \tag{5.6}$$

$$\int_t^{T^*_{\delta}} \left|\underline{\lambda}(u, T^*_{\delta})\right|^2 du = \sigma^2(T^*_{\delta})(T^*_{\delta} - t)\,. \tag{5.7}$$

These constraints are important in the calibration of the market-Libor model and we will discuss calibration in Section 6. So given the forward-forward caplets and the initial forward Libor rate term structure, we only need

$$\int_t^{T^*_{2\delta}} \underline{\lambda}(u, T^*_{\delta}) \cdot \underline{\lambda}(u, T^*_{2\delta})du = Y, \text{ say,}$$

to be able to evaluate the bounds and Approximation 1.

However, Y cannot be determined without calibrating the model properly. A calibration will typically make assumptions for the forward Libor volatility factors, $\underline{\lambda}(u, T)$, that are consistent with the constraints provided by equations 5.6 and 5.7.

Sometimes practitioners may just want an indication of what the arbitrage free price is without going through the full calibration. To this end, we provide an approximation for Y that gives Approximation 2 for the prices of resettable caplets.

We are looking for an approximation to Y. Note that

$$\left[\int_t^{T_{2\delta}^*} \underline{\lambda}(u,T_\delta^*)\cdot\underline{\lambda}(u,T_{2\delta}^*)du\right]^2 \le \left[\int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_\delta^*)\right|\cdot\left|\underline{\lambda}(u,T_{2\delta}^*)\right|du\right]^2 \le \int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du \int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_{2\delta}^*)\right|^2 du .$$

We have from the caplet forward-forward volatilities

$$\int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_{2\delta}^*)\right|^2 du = \sigma^2(T_{2\delta}^*)(T_{2\delta}^* - t)$$

but not

$$\int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du .$$

We can approximate the latter by

$$\int_t^{T_{2\delta}^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du \approx \frac{T_{2\delta}^* - t}{T_\delta^* - t}\int_t^{T_\delta^*} \left|\underline{\lambda}(u,T_\delta^*)\right|^2 du = \sigma^2(T_\delta^*)(T_{2\delta}^* - t) . \qquad (5.8)$$

and approximate Y by

$$Y = \int_t^{T_{2\delta}^*} \lambda(u,T_\delta^*)\cdot\lambda(u,T_{2\delta}^*)du \approx \sigma(T_{2\delta}^*)\sigma(T_\delta^*)(T_{2\delta}^* - t) . \qquad (5.9)$$

Thus we have

Approximation 2: *An alternative approximation for the time t value of a resettable caplet with payoff* $\delta[L(T_\delta^*, T_\delta^*) - L(T_{2\delta}^*, T_{2\delta}^*)]^+$ *at* T_δ^* *is given by*

$$rcpt_L(t) \approx \delta P(t,T_\delta^*)\left[L(t,T_\delta^*)\exp\left(\frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)}\sigma^2(T_\delta^*)(T_\delta^*-t)\right)N(d_1) - L(t,T_{2\delta}^*)N(d_2)\right] \quad (5.10)$$

$$d_1 = \frac{\log\left(\frac{L(t,T_\delta^*)}{L(t,T_{2\delta}^*)}\right)+\frac{\delta L(t,T_\delta^*)}{1+\delta L(t,T_\delta^*)}\sigma^2(T_\delta^*)(T_\delta^*-t)}{\sqrt{v(t,T_{2\delta}^*,T_\delta^*)}}+\frac{1}{2}\sqrt{v(t,T_{2\delta}^*,T_\delta^*)} \quad (5.11)$$

$$d_2 = d_1 - \sqrt{v(t,T_{2\delta}^*,T_\delta^*)} \quad (5.12)$$

$$v(t,T_{2\delta}^*,T_\delta^*) = \sigma^2(T_\delta^*)(T_\delta^*-t) - 2\rho\sigma(T_\delta^*)\sigma(T_{2\delta}^*)\sqrt{(T_\delta^*-t)(T_{2\delta}^*-t)} + \sigma^2(T_{2\delta}^*)(T_{2\delta}^*-t) \quad (5.13)$$

$$\rho = \sqrt{\frac{T_{2\delta}^*-t}{T_\delta^*-t}}. \quad (5.14)$$

We now examine how well the approximations perform. Figure 5 plots the Monte Carlo estimates and the two approximations. Figure 5 shows that Approximation 1 performs very well. Compared to the Monte Carlo estimates, Approximation 1 underprices the resettable caplets by between 1% and 6%. Approximation 2 does not perform so well and it overprices the resettable caplets by up to 5% and underprices by up to 24%.

Approximation 2 is a little disappointing but it is easy to implement. However, resettable caps are quoted by the average price of the constituent resettable caplets. Figure 6 plots the average resettable caplet prices as given by Approximation 2 and the Monte Carlo Simulations. Comparing the average prices, Approximation 2 underprices by the Monte Carlo prices up to 4% and overprices by up to 5%. Approximation 2 may perhaps be acceptable for providing indicative prices for resettable caps. Approximation 1 performs better but requires a proper calibration of the market-Libor model. We proceed to examine the calibration.

6. CALIBRATION ISSUES

The upper and lower bounds of Section 2 and the Approximation 1 of Section 5 all depend on the terms

$$\int_{t}^{t+k\delta} \underline{\lambda}(u,t+(k+1)\delta)\cdot\underline{\lambda}(u,t+k\delta)du\,,\ k = 1, \ldots, n, \qquad (6.1)$$

where n is the number of resettable caplets making up the longest resettable cap we need to price. The numerical results of Section 4 were produced assuming we knew the forward Libor rate volatility factors, $\underline{\lambda}(u, T)$. In practice, we would have to calibrate the market model and extract the volatility factors, $\underline{\lambda}(u, T)$, from market data.

Forward-forward caplet volatilities only give the following constraints

$$\int_{0}^{k\delta} \left|\underline{\lambda}(u,k\delta)\right|^2 du = \sigma^2(k\delta)k\delta\,,\ k = 1, \ldots, n+1 \qquad (6.2)$$

on the forward Libor volatility factors: $\sigma(k\delta)$ is the forward-forward volatility of a standard caplet covering the period $[k\delta, (k+1)\delta]$ and n is the number of resettable caplets making up the longest resettable cap we need to price. The constraints given by equation 6.2 permit considerable remaining flexibility for the choice of $\underline{\lambda}(u, T)$. Cap prices are not sensitive to the correlation structure of forward Libor rates and so they do not provide enough information to complete the terms in equation 6.1.

The calibration needs more market data. We may attempt to extract $\underline{\lambda}(u, T)$ by supplementing cap prices with historically estimated correlation of changes to forward Libor rates. For example, it may be desirable to assume the forward Libor volatility factors are time stationary so that $\underline{\lambda}(u, T) \equiv \underline{\lambda}(T-u)$. Then we can estimate an correlation matrix of forward Libor rate changes to provide the following additional constraints on the volatility factors

$$\frac{\underline{\lambda}(i\delta)\cdot\underline{\lambda}(j\delta)}{|\underline{\lambda}(i\delta)||\underline{\lambda}(j\delta)|}=\rho_{ij},\ i,j=1..n, \tag{6.3}$$

where ρ_{ij} is the historically estimated correlation between proportional changes of the forward δ-Libor rates of maturities $i\delta$ and $j\delta$.

Calibrating the market-Libor model to caplet forward-forward volatilities and correlations is relatively easy. It is far easier than the calibration of conventional models where cap prices and correlations are highly non-linear functions of the model parameters and where it is difficult to understand how the model parameters affect the fit. Here, for the market-Libor model, it is very clear what the constraints on the volatility factors are.

Alternatively we may supplement the cap prices with market prices of options that are sensitive to the correlation structure of the forward Libor rates. Swaptions are often used because their market is liquid. However, calibrating to swaptions is more difficult because their values are highly non-linear with respect to the volatility factors, $\underline{\lambda}(u, T)$. The calibration would require a difficult non-linear optimisation. Swaptions cannot be priced analytically, but Brace, Gatarek and Musiela (1995) provide an approximate formula that performs well for their term structures. This allows them to calibrate the market-Libor model simultaneously to caps and swaptions prices and a historically estimated correlation matrix.

The lack of an analytical pricing formula for swaptions point to a problem of the market-Libor model. The market-Libor model losses its analytical tractability when it is necessary to price options that depend on other non-Libor financial variables. The market-Libor models are only convenient for applications involving Libor rates. Other financial variables can be complex functions of the Libor rates that are intractable and difficult to simulate. It is probably worth mentioning here that Jamshidian (1996) has produced a market-swap model where forward swaps rates can be made lognormal for well chosen numéraires. In the market-swap model European swaptions are priced consistently with the market convention. Jamshidian (1996) shows that forward Libor rates and forward swap swaps cannot both be lognormal, that is, the market-Libor and market-swap models are inconsistent.

7. SUMMARY

The market Libor model offers some tractability for pricing Libor derivatives. We have derived an exact lower bound and an approximate upper bound for the prices of resettable caps and floors. We have also derived an exact relationship between resettable caplet and floorlet prices within the market-Libor model that depend only observables and forward-forward caplet volatilities that can be readily extracted from quoted cap volatilities. It would be interesting to test the relationship on quoted prices.

We derived two approximations for the prices of resettable caplets and floorlets. We examined the performance of the approximations using realistic Libor volatility factors. We found that the first approximation gives good approximations to the no-arbitrage prices. The second approximation does not perform so well but it is only a function of the current forward Libor term structure and caplet forward-forward volatilities that are readily available to traders. The second approximation has the advantage that it be implemented easily in a spreadsheet to price approximately resettable caps and floors quickly off quotes for standard caps and floors.

We have provided a preliminary discussion on the calibrating issues related to the market-Libor model. We suggested that the market-Libor model would be easy to calibration to quoted cap volatilities and historically estimated forward Libor rate correlation. We argued that the market-Libor model losses some of its attraction when it is necessary to price or to calibrate it to non-Libor derivatives.

More empirical work needs to be done to examine whether the market-Libor model is more suitable to practitioners than other interest rate term structure models.

8. APPENDIX

Lemma: Let $(X, Y) \sim N\left[\begin{pmatrix}\mu_x \\ \mu_y\end{pmatrix}, \begin{pmatrix}\sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2\end{pmatrix}\right]$, then

$$E\left[e^X - e^Y\right]^+ = \exp\left(\mu_x + \frac{\sigma_x^2}{2}\right)\phi\left[\frac{\mu_x - \mu_y + \sigma_x(\sigma_x - \rho\sigma_y)}{\sqrt{\sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2}}\right] - \exp\left(\mu_y + \frac{\sigma_y^2}{2}\right)\phi\left[\frac{\mu_x - \mu_y + \sigma_y(\sigma_y - \rho\sigma_x)}{\sqrt{\sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2}}\right]$$

where $\phi(.)$ is the normal distribution function.

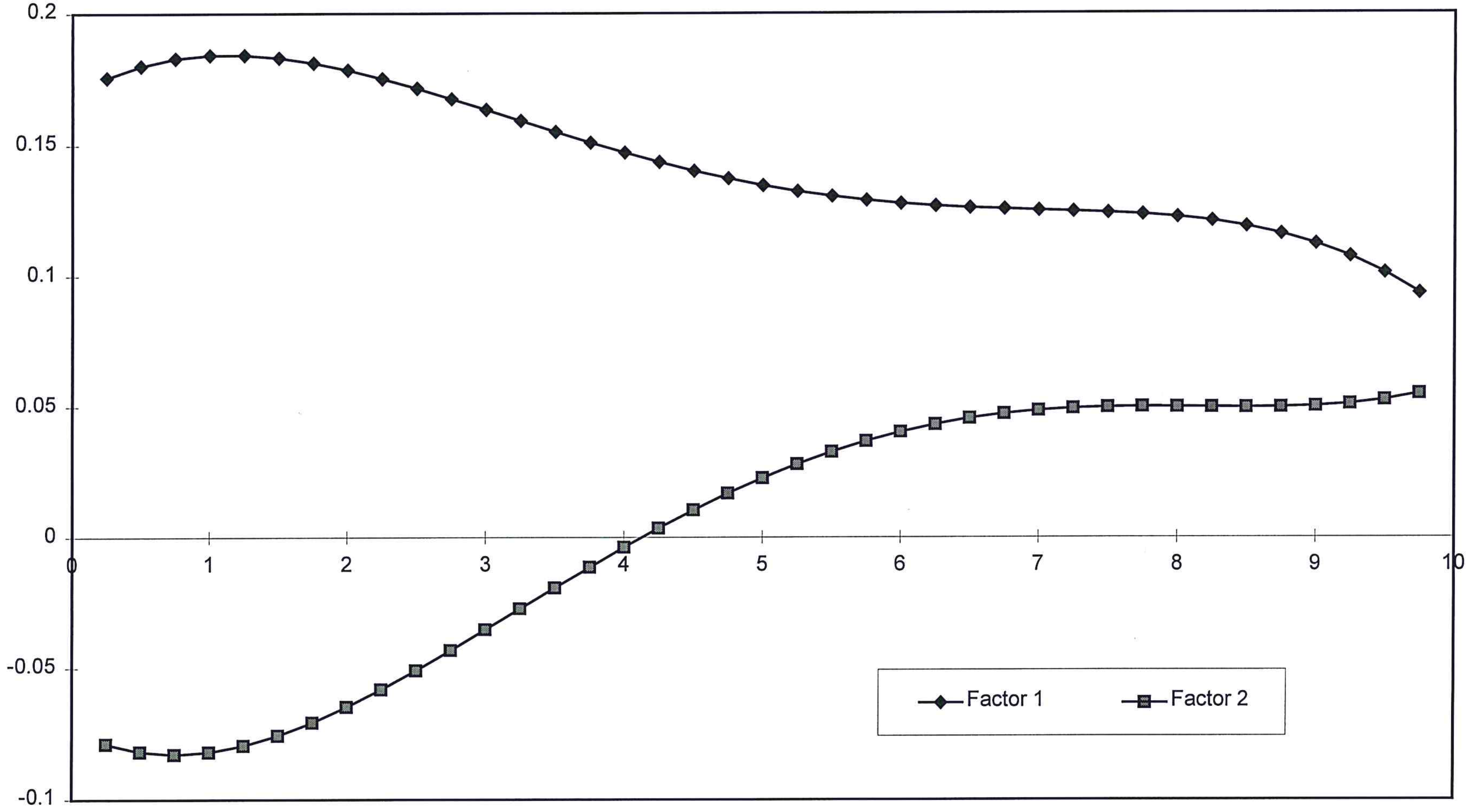
9. ENDNOTES

1. The assumption that $\lambda(t, T)$ is deterministic is arguably unrealistic and therefore inappropriate but it allows analytical tractability.
2. Cap volatilities are quoted on the understanding that the price of a cap is retrieved by using the quoted cap volatility for all the constituent caplets. Therefore, with the market convention, caplets covering the same period in different caps may be priced using different volatilities. In no-arbitrage models, caplets covering the same period should be priced using the same volatility, the forward-forward volatility. We assume the forward-forward volatilities have been extracted from cap quotes.
3. Each resettable cap (floor) is made up of a strip of end-to-end resettable caplets (floorlets) that cover the entire duration of the resettable cap (floor). The price of resettable caps (floors) are given by the sum of the constituent resettable caplets (floorlets).

10. REFERENCES

1. Brace A., Gatarek D. and Musiela M., "*The Market Model of Interest Rate Dynamics*", 1995, Working Paper, *USNW.*
2. Geman H., El Karoui N. and Rochet J-C, "*Changes of Numéraire, Changes of Probability Measure and Option Pricing*", 1995, Journal of Applied Probability.
3. Harrison J.M. and Kreps D.M., 1979, "Martingales and Arbitrage in Multiperiod Securities Markets", *J. Econ. Theory* 20, 381-408.
4. Jamshidian F, "*Libor and Swap Market Models and Measures*", January 1996, Working Paper, Sakura Global Capital.
5. Musiela M and Rutkowski M, "*Continuous-time Term Structure Models*", September 1995, Working Paper, University of New South Wales.

Figure1: Proportional Forward Libor Volatility Factors



Forward Libor Maturity

Figure 2: Forward Libor and Volatility Term Structures

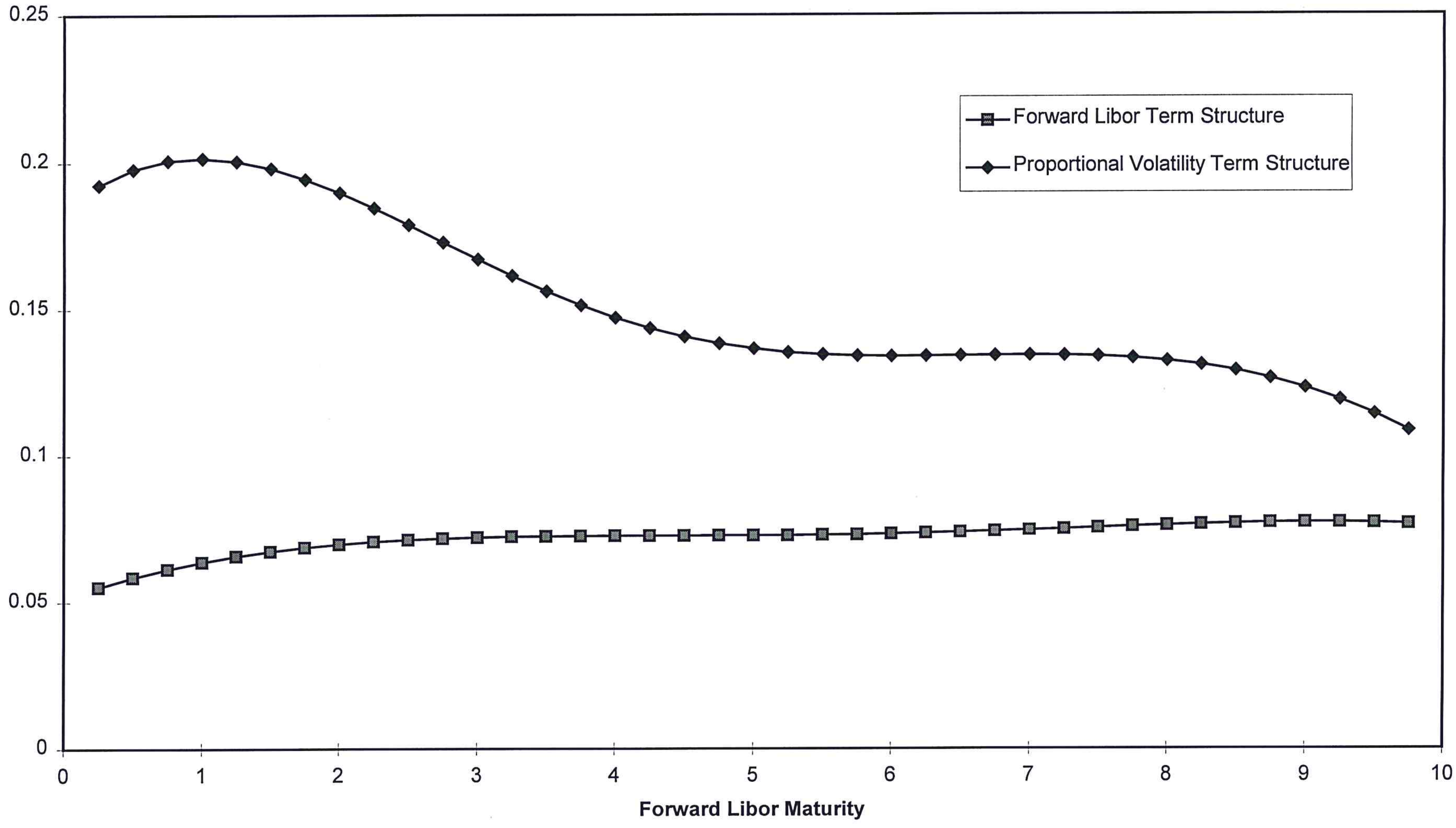


Figure 3: Upper and Lower Bounds to Resettable Caplet Value

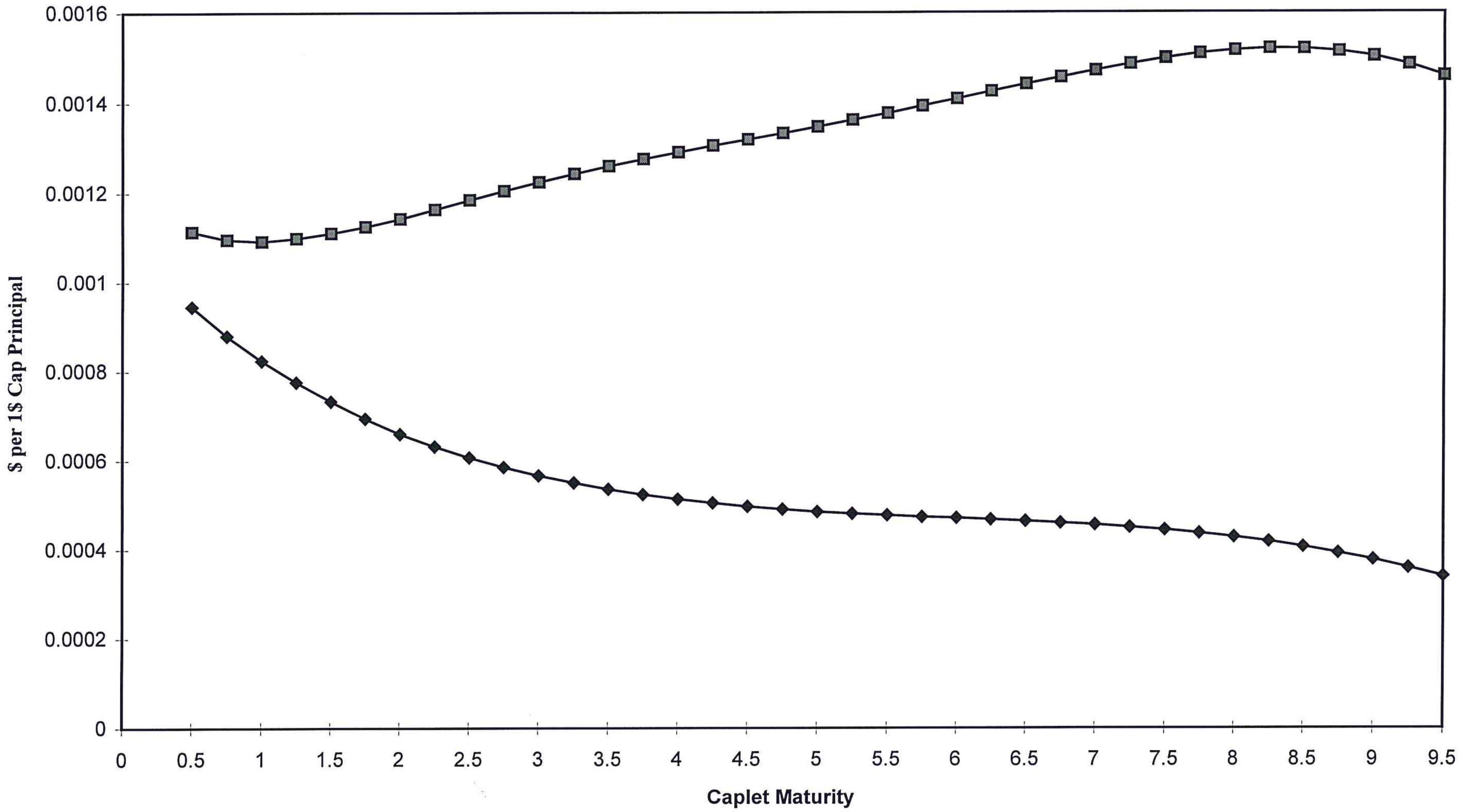


Figure 4: The Lower Bound and The No-Arbitrage Prices

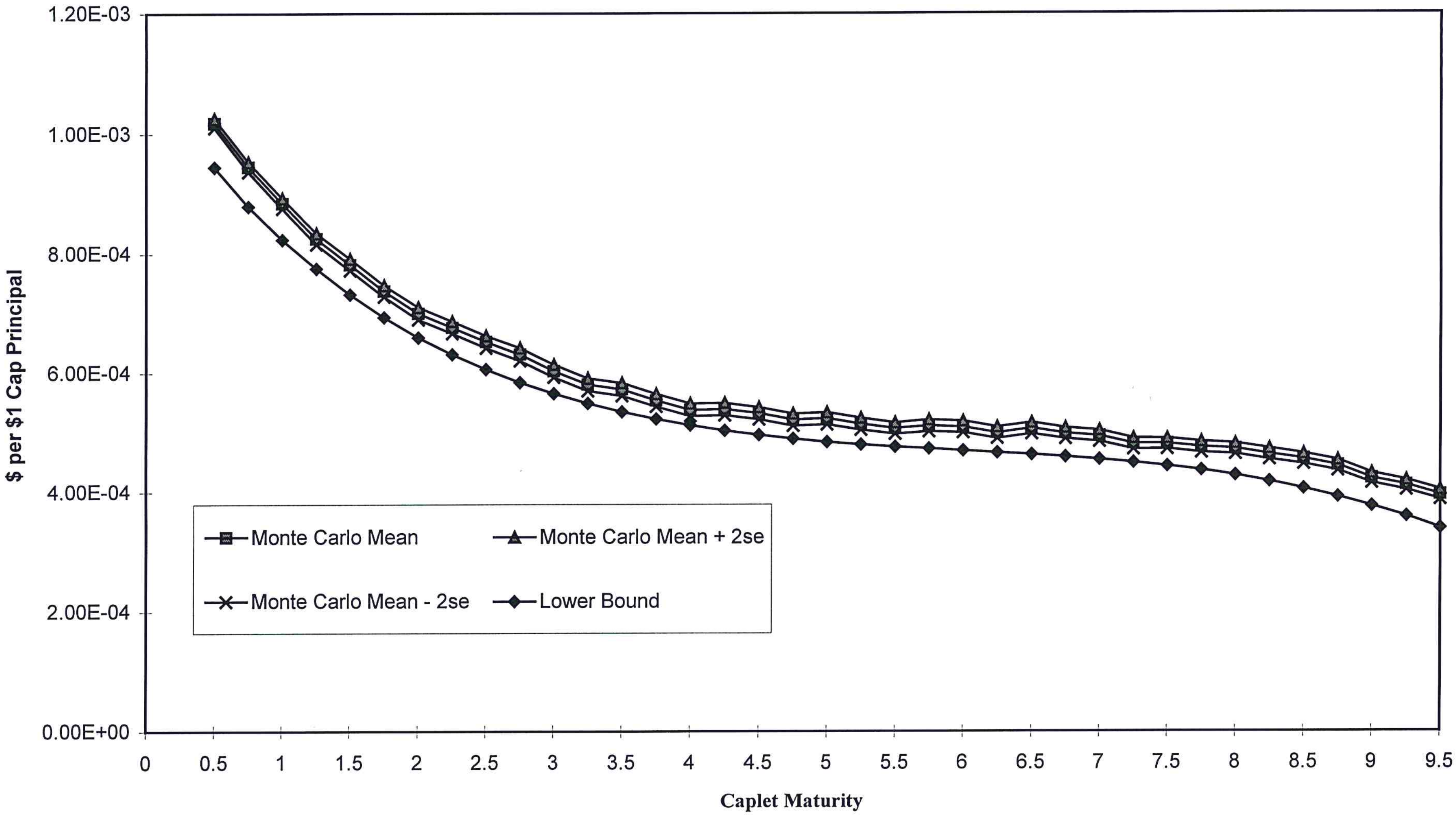


Figure 5: Approximations and No-Arbitrage Resettable Caplet Prices

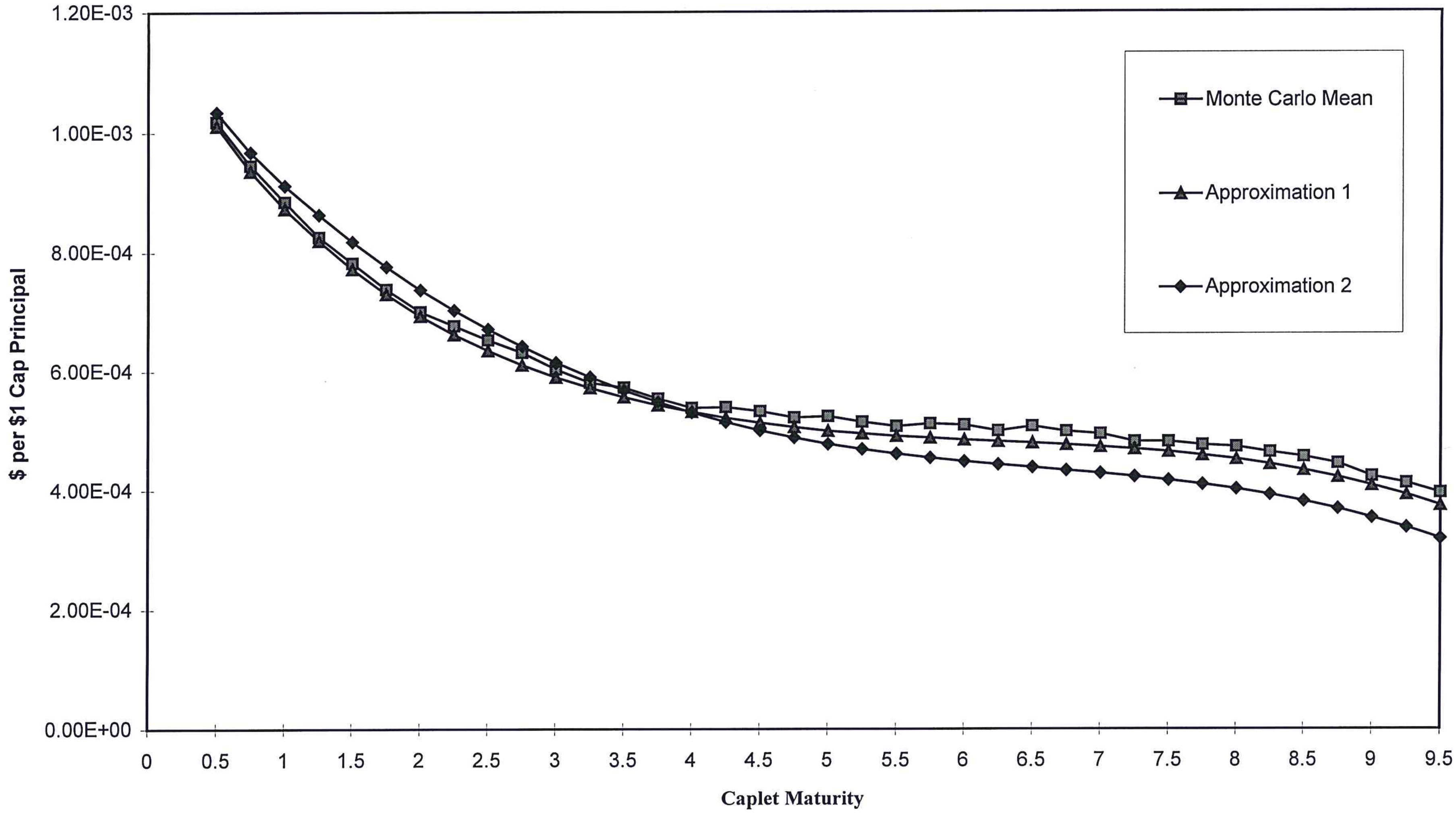


Figure 6: Average Resettable Caplet Prices