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The Potential For Profitable Stock Market Manipulation in the Presence of Positive Feedback Trading

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The potential for profitable stock market manipulation in the presence of positive feedback trading

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Abstract: The aim of this paper is to investigate the implications of positive feedback trading. In particular, the paper investigates the potential for manipulators to profit from the existence of such behaviour. It is shown that prices in the presence of feedback trading may or may not oscillate, and may be stable or unstable. The potential for manipulation depends on the strength of the feedback trading, the delay with which it responds to price changes, and the strength of value-based investor demand.

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1 Introduction

The aim of this paper is to investigate the implications of the presence in a securities market of positive feedback traders, defined as traders whose demand for stock bears a positive relationship to current and/or past prices. In particular, the paper investigates the potential for manipulators to profit from such a situation.

Positive feedback trading can have a number of different causes: it can result from trading strategies such as portfolio insurance and chartism; it can result from 'naive' investors trading on 'noise', as Black (1986) termed information already incorporated into the price; and it can result from the extrapolation of past price trends. The latter is consistent with the findings of Tversky and Kahneman (1982) that excessive emphasis is placed on the recent past when expectations about the future are formed, as well as the experimental evidence of Andreassen and Kraus (1990) which reports that when faced with authentic stock price data subjects tended to chase trends when these could be identified, and evidence found by Case and Shiller (1988) that expectations of future house price appreciation in different U.S. cities was positively related to past price changes.

There are a number of problems associated with incorporating positive feedback trading into models, since it is likely to vary in importance through time and with phases in market behaviour. It is also likely to follow a relatively complicated form that would be difficult to model. Since, however, there is evidence that it forms an important feature of investor behaviour, it should not be ignored, and we should attempt to learn as much about its implications as we can. This simple model should therefore be seen as a preliminary attempt to shed light onto relatively uncharted territory. The use of such a simple model ensures that the results obtained must be interpreted with care, and the more extreme implications discounted: but significant benefits from the exercise will remain.

In the model, the positive feedback traders are assumed to co-exist with value-investors, whose demand is determined by the discrepancy between price and value. The behaviour of these two agent types bears strong similarities to the agent behaviour in De Long, Shleifer, Summers and Waldmann (1989, 1990) and Cutler, Poterba and Summers (1990). We begin by analysing the nature of the price behaviour of this system in the absence of manipulation. We then use results from Hart (1977) to discover the potential for profitable manipulation in such a situation, and finish by using a numerical example of manipulators following the strategy proposed by Baumol (1957) to show how manipulation could occur in an appropriate setting.

2 Feedback traders and passive investors

2.1 The nature of the investors

As in De Long et. al. (1989), we assume that the positive feedback trader demand $\left(D^f\right)$ is positively affected by the price changes in the preceding two periods, and can also include a demand shock. In particular, we assume that it takes the following form: [1]

$$\begin{aligned} D_t^f &= \beta\left(p_{t-1} - p_{t-2}\right) + \delta\left(p_t - p_{t-1}\right) + f_t(t) \\ &= \delta p_t + (\beta - \delta)p_{t-1} - \beta p_{t-2} + f_t(t) \end{aligned} \tag{1}$$

Beta and delta are assumed to be non-negative. The demand shock is a function of time, and is unrelated to market prices. We could think of this as deriving from a separate group of noise traders, but is attributed to the positive feedback traders for convenience. If this defined the behaviour of the only group of agents present in the market, the market price would be extremely unstable: it can be shown that although the price would not oscillate it would be dynamically unstable.

The value-investor, termed passive investors by De Long et. al. (1989, 1990), are not capable of formulating strategies based on the behaviour of the other agents, but simply act on their estimates of underlying stock value, which is unaffected by the market price. In particular, their stock demand in period T $\left(D_T^p\right)$ is given by the following:

$$D_T^p = \alpha\left(V_T - p_T\right) \tag{2}$$

where V represents the passive investors' estimate of the fundamental stock value. The passive investor demand therefore increases at a constant rate as the price moves away from the value estimate. Since the system will be unstable otherwise, and therefore relatively uninteresting, we will make the assumption that alpha exceeds both beta and delta.

2.2 Equilibrium with passive investors and feedback traders

Combining the demand functions of the feedback traders and passive investors gives us a non-speculative excess demand function which is as follows:

[1] Note that the use of beta and delta here differs from that in De Long et. al. (1989). The beta here is equivalent to beta plus delta in that model.

$$\begin{aligned} E(t) &= D_t^f + D_t^p \\ &= \alpha(V_t - p_t) + \beta(p_{t-1} - p_{t-2}) + \delta(p_t - p_{t-1}) + f_t(t) \\ &= -(\alpha - \delta)p_t + (\beta - \delta)p_{t-1} - \beta p_{t-2} + \alpha V_t + f_t(t) \end{aligned} \tag{3}$$

Setting this equal to zero and rearranging gives:

$$p_t = \frac{(\beta - \delta)}{(\alpha - \delta)} p_{t-1} - \frac{\beta}{(\alpha - \delta)} p_{t-2} + \frac{\alpha V_t + f_t(t)}{(\alpha - \delta)} \tag{4}$$

If we assume that the value estimate of the passive investors is constant, and that there are no demand shocks, this can be simplified to:

$$p_t = \frac{(\beta - \delta)}{(\alpha - \delta)} p_{t-1} - \frac{\beta}{(\alpha - \delta)} p_{t-2} + \frac{\alpha V}{(\alpha - \delta)} \tag{5}$$

The time path represented by this second-order difference equation will have different properties depending on the strengths of the two coefficients of feedback trading.

2.3 Oscillations

Following the brief introduction to second-order difference equations given in the appendix, it can easily be shown that under the above example the price path will oscillate when the following condition holds:

$$\begin{aligned} &4\alpha\beta > (\beta + \delta)^2 \\ &\Rightarrow 4\rho_\beta > (\rho_\beta + \rho_\delta)^2 \end{aligned} \tag{6}$$

where:

$$\rho_\beta \equiv \frac{\beta}{\alpha}, \quad \rho_\delta \equiv \frac{\delta}{\alpha}$$

This is equivalent to:

$$\rho_\delta < -\rho_\beta + 2\sqrt{\rho_\beta} \tag{7}$$

A sufficient condition for this to hold is that:

$$\rho_\delta < \rho_\beta < 1. \tag{8}$$

This clearly cannot hold for a beta of zero, and so the price will not oscillate when there is no delayed feedback.

2.4 Stability in an oscillatory system

When the above condition holds, and thus the path oscillates, the oscillations will either be damped, regular, or explosive, depending on whether the magnitude of the coefficient of the twice-lagged term is less than, equal to, or greater than one. The system will therefore be asymptotically stable when the following condition holds:

$$\left|\frac{\beta}{\alpha-\delta}\right| \leq 1 \qquad \Rightarrow \rho_\beta + \rho_\delta \leq 1 \tag{9}$$

The system will exhibit regular cyclical behaviour when this is an equality.

2.5 Stability in a non-oscillatory system

A non-oscillatory system will be stable when the characteristic roots lie within the unit circle, which in this case occurs when the following conditions hold:

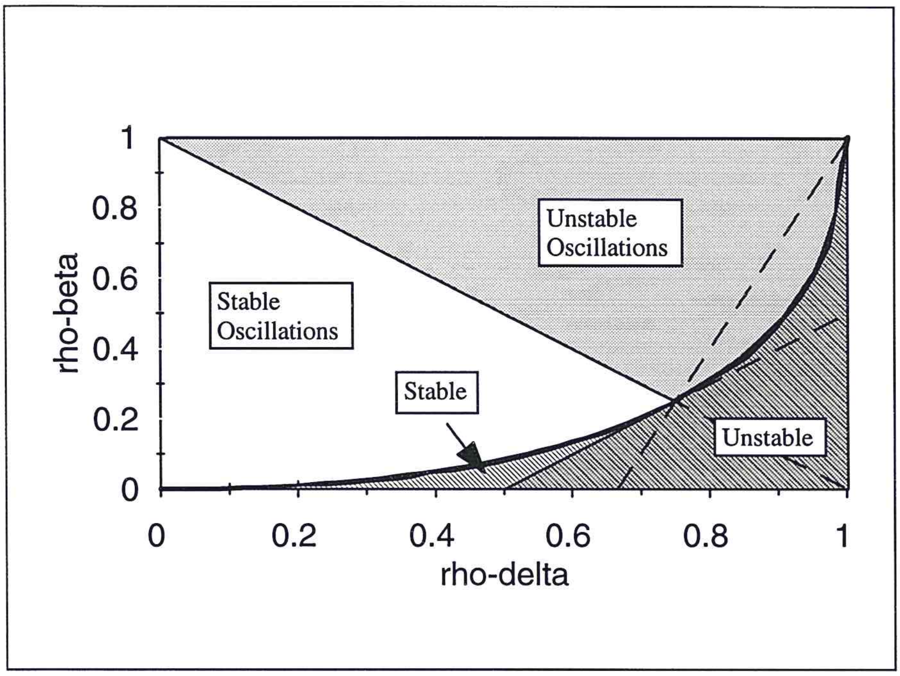
$$\begin{aligned} &\textit{EITHER} \quad \rho_\beta \geq \rho_\delta \\ &\textit{OR} \quad \rho_\beta < \rho_\delta \quad \textit{AND} \quad 2+\rho_\beta - 3\rho_\delta \geq 0 \quad \textit{AND} \quad \rho_\beta \geq \rho_\delta - \frac{1}{2} \end{aligned} \tag{10}$$

2.6 Regions of oscillations and stability

Figure 1 shows the combinations of the coefficients that produce systems that oscillate; and also those that are asymptotically stable, oscillatory or otherwise, as given by the above conditions.

For the case of delayed feedback only ($\delta = 0$) as in De Long et. al. (1990), figure 1 shows that the system will exhibit asymptotically stable oscillations. When there is no delayed feedback trader response to price changes ($\beta = 0$) the system exhibits asymptotic instability when delta exceeds half the value of alpha.

Fig. 1: Regions of stability.



3 System dynamics

In this section we will look at how the system behaves once it has been disturbed from its steady state. In particular we will assume that this occurs as a result of an exogenous demand shock. It would also be possible to look at a value-shock, under which the price would be disturbed following the realisation by passive investors that the underlying stock value had altered. This would produce similar price patterns, but centred around the new underlying value.

3.1 Demand shock

A demand shock can occurs as the result of liquidity trades. Assume that the system is in its steady state, with past prices equal to V_o, when a one-off demand shock N hits the market in period T. The price in this period is determined as follows:

$$\begin{aligned} -\mathrm{N} &= -(\alpha-\delta)p_T + (\beta-\delta)p_{T-1} - \beta p_{T-2} + \alpha V_o \\ &= -(\alpha-\delta)p_T + (\beta-\delta)V_o - \beta V_o + \alpha V_o \qquad (11) \\ \Rightarrow p_T &= V_o + \frac{\mathrm{N}}{(\alpha-\delta)} \end{aligned}$$

For simplicity, we will now assume that delta is zero. The system reduces to the following:

$$p_t = \frac{\beta}{\alpha} p_{t-1} - \frac{\beta}{\alpha} p_{t-2} + V_o \qquad (12)$$

The path of prices in this system will take the following form, for discrete values of t:[2]

$$p_t = Ar^t \cos(t\theta + B) + V_o \qquad (13)$$

where: $r = \sqrt{\frac{\beta}{\alpha}}$ and θ is such that the following holds:

$$\cos\theta = \frac{1}{2}\sqrt{\frac{\beta}{\alpha}}$$

This shows that the periodicity of the cycle is proportional to the square root of beta.

3.2 Numerical example of a demand shock

Assume now that beta is a three-quarters, alpha is one, and the value estimate is initially ten. This gives the following:

$$r = \frac{\sqrt{3}}{2}$$
$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{4}\right) \qquad (14)$$

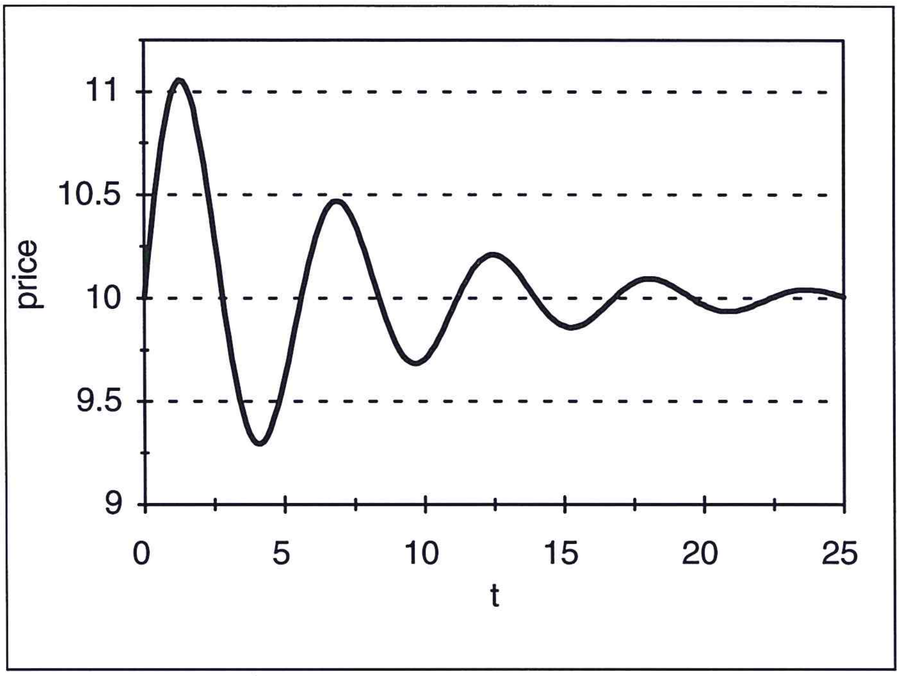
Assume also that a demand shock of one hits the market in period one. The subsequent path of prices is given by the following:

$$p_t = -\frac{8}{\sqrt{39}} r^t \cos\left(t\theta + \frac{\pi}{2}\right) + 10 \qquad (15)$$

This generates the price path shown in figure 2.

[2] See, for example, Goldberg (1958).

Fig. 2: Price with demand shock.



4 Profitability of disturbing the steady state

4.1 Lessons from Hart (1977)

Hart (1977), starting with a system in an initial steady state, attempts to identify conditions for profitable trade-based manipulation. In such a situation, any manipulative activity must be destabilising, and hence the requirement here for a refutation of the Friedman (1953) position that speculation or manipulation that is destabilising must be unprofitable is to find any scope for profitable manipulation. Manipulation is profitable in a system which is explosively unstable; but avoiding disaster in such a (highly implausible) system would be impossible even in its absence. For the case where non-speculator demand, which in this model encompasses the demand of all the traders bar the manipulator, is a linear function of current and past prices, Hart completely characterises the requirements.

Non-speculative demand is independent of wealth, and the non-speculators themselves are assumed never to learn about manipulative behaviour. Hart considers a non-speculator demand function, which for the linear case is of the following form:

$$F(P_t, P_{t-1}, \ldots\ldots, P_{t-n}) = \sum_{i=0}^{n} a_i P_{t-i} + b \qquad (16)$$

where $a_0,\ldots,a_n$, b are constants, and $a_0<0$. The stationary-state price is therefore given by:

$$P^* = -\frac{b}{\sum_{i=0}^{n} a_i} \tag{17}$$

A manipulator is assumed to enter the market in period one, following a period in which the market was in a stationary state. The manipulator is assumed to formulate a strategy that involves leaving the market in or before period T, where $T \leq t$. The sales of the manipulator must equal the demand from the other traders in each period. It is assumed that the manipulator realises his profits, which means that the cumulative sum of his sales over the period must be zero.

Hart shows that the manipulator will be unable to make money when a particular quadratic form is negative semi-definite for all values of T. Hart shows that this quadratic form can be represented by a matrix in which each element is determined by the distance from the diagonal, and in particular takes the following form:

$$\begin{bmatrix} a_0 & a_1/2 & a_2/2 & \ldots. & a_n/2 & 0 & \ldots. & 0 \\ a_1/2 & a_0 & a_1/2 & a_2/2 & \ldots. & a_n/2 & 0 & \ldots. \\ a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 & \ldots. & a_n/2 & 0 \\ \ldots. & a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 & \ldots. & a_n/2 \\ a_n/2 & \ldots. & a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 & \ldots. \\ 0 & a_n/2 & \ldots. & a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 \\ \ldots. & 0 & a_n/2 & \ldots. & a_2/2 & a_1/2 & a_0 & a_1/2 \\ 0 & \ldots. & 0 & a_n/2 & \ldots. & a_2/2 & a_1/2 & a_0 \end{bmatrix} \tag{18}$$

Quadratic forms with this property are known as a Toeplitz forms. Making use of the mathematics literature on Toeplitz forms, Hart characterises the solutions. The main result is found in his Theorem 3.4, in which he states that (for $a_0 < 0$) the manipulator can profit from disturbing the steady state if and only if the equation, $\operatorname{Re} f(z) = 0$, has a solution z satisfying $|z| < 1$,[3] where:

$$f(z) = \sum_{t=0}^{n} a_t z^t. \tag{19}$$

Hart's Lemma 3.5 shows, in addition, (for $a_0 \neq 0$) that the system is explosive in the absence of manipulators if and only if the equation $f(z)=0$ has a solution z satisfying $|z| < 1$. As Hart points out (in Theorem 3.6), if the condition for explosiveness is satisfied, the condition for profitable manipulation must also be satisfied, and so manipulation will always be profitable when the system is explosive.

[3] Where Re $f(z)$ is used to denote the real part of $f(z)$.

Explosiveness is both a necessary and sufficient condition for profitable manipulation in the following situations: when the coefficients of all the lagged terms in the non-speculator demand function are non-negative; when the coefficients of the terms with even lags are non-negative and the coefficients of the terms with odd terms are non-positive; and when the maximum lag in the non-speculator demand function is one.

For the case where the maximum lag is of two periods, Hart's Theorem 3.9 shows that for profitability one or both of the following conditions must hold:

1. The difference equation is explosive.
2. The coefficients a_0, a_1, a_2 satisfy: $a_2 < 0$, $|a_1| \leq 4|a_2|$, and $a_1^2 - 8a_0a_2 + 8a_2^2 > 0$

4.2 Necessary and sufficient conditions

Necessary and sufficient conditions for manipulation to be unprofitable can easily be derived from the above matrix, although these are not given by Hart.

For lags of up to n periods, a sufficient condition for manipulation to be unprofitable is that:

$$\begin{aligned}
&a_0 < 0 \\
&and\ |a_0| \geq |a_1| \\
&and\ |a_0| \geq |a_1| + |a_2| \\
&and\ |a_0| \geq |a_1| + |a_2| + |a_3| \\
&and.... \\
&and\ |a_0| \geq |a_1| + |a_2| + |a_3| + ... + |a_n|
\end{aligned} \tag{20}$$

A necessary condition for manipulation to be unprofitable is that the following conditions hold:

$$\begin{aligned}
&|a_0| \geq \left|\frac{a_1}{2}\right| \\
&and\ |a_0| \geq \left|\frac{a_2}{2}\right| \\
&and\ |a_0| \geq \left|\frac{a_3}{2}\right|
\end{aligned} \tag{21}$$

5 Applying Hart's results

5.1 Converting to the Hart construction

Since Hart deals with incremental demand in each period, while we have been considering the demand for total stock holdings, we now need to express the non-speculator excess demand function in a new form. The incremental demand of the non-speculators is as follows:

$$\begin{aligned} F_t &= E_t - E_{t-1} \\ &= -(\alpha-\delta)(p_t - p_{t-1}) + (\beta-\delta)(p_{t-1} - p_{t-2}) - \beta(p_{t-2} - p_{t-3}) + \alpha(V_t - V_{t-1}) \\ &= -(\alpha-\delta)p_t + (\alpha+\beta-2\delta)p_{t-1} - (2\beta-\delta)p_{t-2} + \beta p_{t-3} + \alpha(V_t - V_{t-1}) \end{aligned} \tag{22}$$

When *V*, the value estimate of the passive investors, is constant, this reduces to:

$$F_t = -(\alpha-\delta)p_t + (\alpha+\beta-2\delta)p_{t-1} - (2\beta-\delta)p_{t-2} + \beta p_{t-3} \tag{23}$$

Equating these with the Hart notation gives:

$$\begin{aligned} a_0 &= -(\alpha-\delta) \\ a_1 &= \alpha+\beta-2\delta \\ a_2 &= -(2\beta-\delta) \\ a_3 &= \beta \end{aligned} \tag{24}$$

5.2 Assessing the profitability of manipulation

We can define a new function, as in Hart:

$$f(z) = -(\alpha-\delta) + (\alpha+\beta-2\delta)z - (2\beta-\delta)z^2 + \beta z^3 \tag{25}$$

As we have seen, Hart shows that manipulation that disturbs the steady state can be profitable if and only if the equation, $\mathrm{Re}\, f(z) = 0$, has a solution z satisfying $|z| < 1$. The real part of the above can be found by first splitting z into its real and imaginary components:

$$z \equiv b + ci \tag{26}$$

The real part of the function can be written as follows:

$$\mathrm{Re}\, f(z) \;= (2\beta-\delta)x - (\alpha-\delta) + (\alpha+\beta-3\beta x-2\delta)b - 2(2\beta-\delta)b^2 + 4\beta b^3 \tag{27}$$

where $x \equiv b^2 + c^2, \quad 0 \le x < 1 \quad and \quad b^2 < x.$

For manipulation that disturbs the steady state to be profitable, there must exist a root b that lies within the unit circle for some feasible value of x. A sufficient condition for this is that the original function $f(z)$ has a real root that satisfies the condition. It can be shown that this is the same condition as for the system to be non-oscillatory and asymptotically unstable.

Hart does not provide the conditions for profitability for the thrice-lagged case, since the general form of this proves too intractable, and so we do not have a ready solution to the problem handy. We can, however, make use of Hart's *Theorem 3.6*, which states that asymptotic instability is a sufficient condition for profitable manipulation.

5.3 Sufficient conditions for non-profitability

For our example, expression 20 above reveals that sufficient conditions for manipulation that disturbs the steady state to be unprofitable are as follows:

$$\begin{aligned} &\alpha - \delta \ge |\alpha + \beta - 2\delta| \\ &and \;\; \alpha - \delta \ge |\alpha + \beta - 2\delta| + |2\beta - \delta| \\ &and \;\; \alpha - \delta \ge |\alpha + \beta - 2\delta| + |2\beta - \delta| + \beta \end{aligned} \tag{28}$$

These are clearly increasing in severity, implying that we need only consider the final condition, which can be expressed as:

$$\rho_\delta \ge 2\rho_\beta \qquad AND \qquad \rho_\delta \le \frac{1}{2} + \frac{\rho_\beta}{2} \tag{29}$$

where $\rho_\beta = \beta/\alpha \;\; and \;\; \rho_\delta = \delta/\alpha$.

This condition is illustrated in figure 3 below.

5.4 Necessary conditions for non-profitability

Following expression 21, a necessary condition for manipulation to be unprofitable is that the following hold:

$$\alpha - \delta \ge \left| \frac{\alpha + \beta - 2\delta}{2} \right|$$
$$\text{and } \alpha - \delta \ge \left| \frac{2\beta - \delta}{2} \right| \qquad (30)$$
$$\text{and } \alpha - \delta \ge \frac{\beta}{2}$$

These can be expressed alternatively as:

$$4\rho_\delta \le \rho_\beta + 3$$

$$\text{AND} \quad \begin{cases} \text{EITHER} & \rho_\beta \ge \frac{\rho_\delta}{2} \quad \text{and} \quad \rho_\beta + \frac{\rho_\delta}{2} \le 1 \\ \text{OR} & \rho_\beta \le \frac{\rho_\delta}{2} \quad \text{and} \quad \frac{3}{2}\rho_\delta - \rho_\beta \le 1 \end{cases} \qquad (31)$$

$$\text{AND} \quad \rho_\delta + \frac{\rho_\beta}{2} \le 1$$

This condition is illustrated in figure 3. In the area outside that specified by the condition, manipulation can profitable.

The dotted line in figure 3 represents the dividing line between stable and unstable systems. This shows that the necessary condition is entirely superseded by the combination of the conditions for asymptotic instability and Hart's *Theorem 3.6*, which, as we have seen, states that asymptotic instability implies profitability.

5.5 Isolating the boundary of profitability

It can be shown that the over-all conditions for manipulation that disturbs the steady state to be profitable are the following:

$$\text{EITHER} \quad \rho_\beta \ge \frac{1}{2}$$

$$\text{OR} \quad \rho_\beta < \frac{1}{2} \quad \text{AND} \quad \rho_\delta \ge 2\sqrt{\rho_\beta(1 - 2\rho_\beta)} \quad \text{AND} \quad \text{either} \quad \rho_\delta < 4\rho_\beta \qquad (32)$$

$$\text{or} \quad \rho_\delta > \rho_\beta + \frac{1}{2}$$

Manipulation that disturbs the steady state can be profitable on the curved border, but not on the straight line.

Fig. 3: Necessary and sufficient conditions for manipulation to be unprofitable.

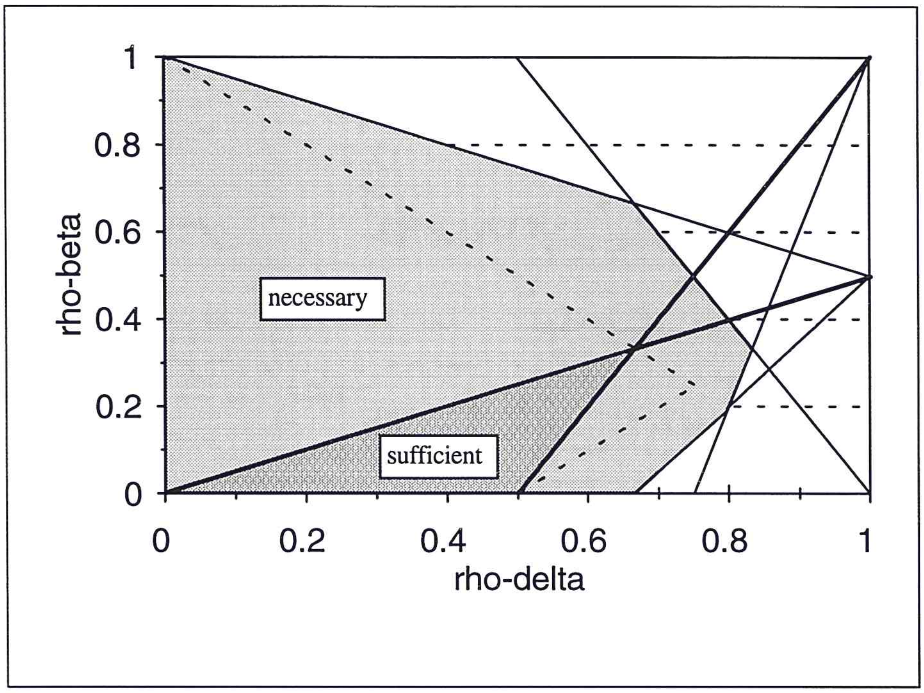


Fig. 4: Region of profitability.

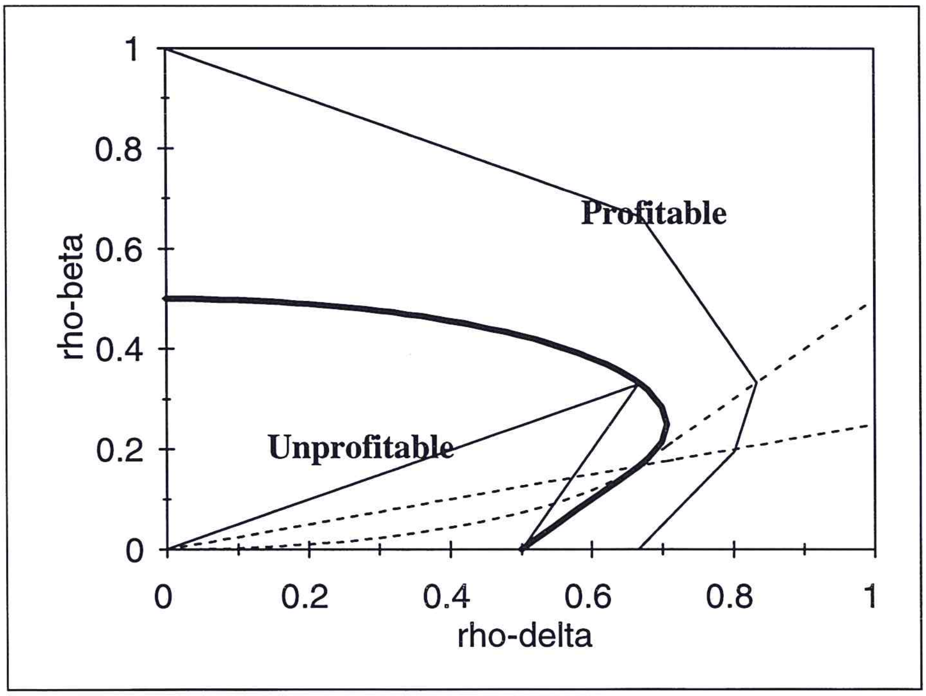
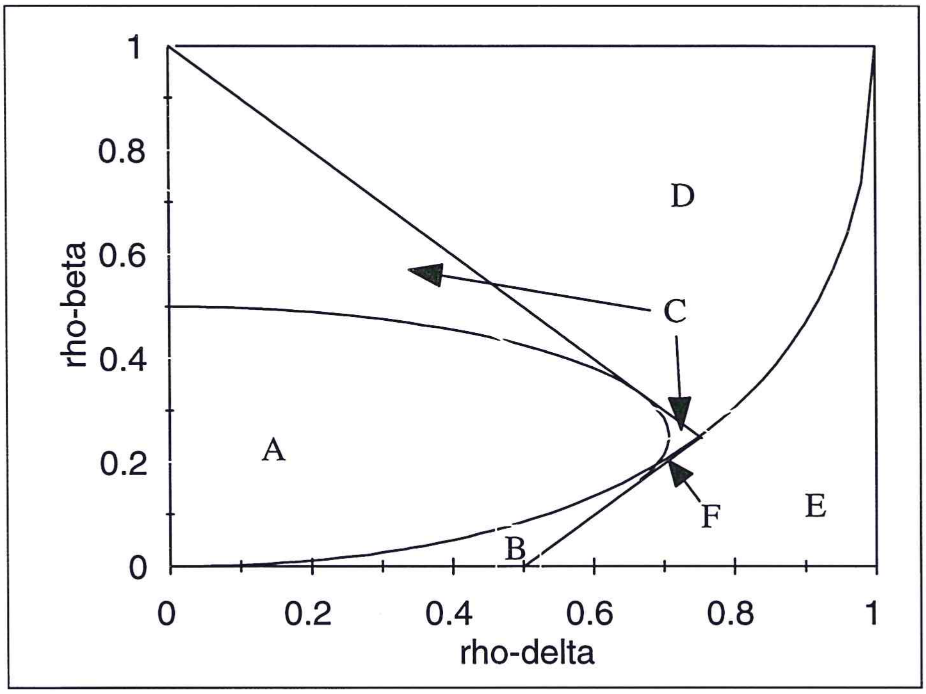


Figure 4 reveals that the sufficient condition identifies only about a third of the unprofitable region, but gives two of the boundary points, (1/2, 0) and (2/3, 1/3). Figure 4 also reveals that an increase in feedback trading can, in some circumstances, move the system from a situation in which manipulation is profitable to one in which it is not. For example, with an alpha of one, and a delta of 0.6, manipulation can be profitable when beta is zero; but if beta rises to, say, 0.2, manipulation can no longer be profitable.

5.6 Combining the conditions

Combining the conditions determining oscillatoriness, stability and profitability reveals the wide variety of system characteristics that are possible. In fact, as figure 5 illustrates, there are six distinct sets of characteristics possible depending on the levels of feedback.

Fig. 5: Combinations of oscillatoriness, stability and profitability.



The characteristics of each of the regions are given in table 1.

Table 1: Characteristics of the regions.

	Oscillations.	Stability.	Profitability.
A	√	√	X .
B	X .	√	X .
C	√	√	√
D	√	X .	√
E	X .	X .	√
F	X .	√	√

6 Profitable manipulator strategies

Previously we found the values for beta and delta under which manipulators can make money from disturbing the steady state. In this section we look at an example of a profitable manipulative strategy that can be employed in such a situation. The particular strategy we use is taken from Baumol (1957), and represents a rule-of-thumb approach rather than one based on optimisation.

6.1 Manipulator strategy from Baumol (1957)

Baumol (1957) looks at a situation in which prices fluctuate cyclically in the absence of manipulators, and shows how manipulators can profit from entering the market in a way the increases price fluctuations. The manipulator behaviour, although not explicitly derived from optimising behaviour, is designed to mimic manipulator behaviour that concentrates purchases just after a price trough, and sales just after a peak, and produces an excess demand function of the following form:

$$\begin{aligned} E_{s,t+1} &= C[(P_{t+1} - P_t) - (P_t - P_{t-1})] \\ &= C[(P_{t+1} - 2P_t + P_{t-1})] \end{aligned} \tag{33}$$

The manipulator demand will therefore follow a cyclical pattern of the same frequency as the price.

6.2 Applying Baumol's manipulator strategy to our model.

Suppose that the manipulator in our system behaves in the same way that she does in Baumol, with the exception that she has initially to disturb the steady state. We can therefore express the manipulator's excess demand function in the following way:

$$\begin{aligned} E_{s,t} &= C[(p_t - p_{t-1}) - (p_{t-1} - p_{t-2})] + X_t - X_{t-1} \\ &= C[(p_t - 2p_{t-1} + p_{t-2})] + X_t - X_{t-1} \end{aligned} \tag{34}$$

where X is the demand shock required to jerk the system out of its steady state, and is assumed to be non-zero only in period one, the first period of manipulative activity. The total demand from the manipulator in period t is therefore given by the following:

$$F_{s,t} = C(p_t - p_{t-1}) + X_t \tag{35}$$

Adding this to the non-speculator total demand function and setting it equal to the stock supply, which we set equal to zero for convenience, allows us to determine the new price process:

$$-(\alpha-\delta-C)p_t+(\beta-\delta-C)p_{t-1}-\beta p_{t-2}+\alpha V_t+X_t=0$$

$$\Rightarrow p_t=\frac{(\beta-\delta-C)}{(\alpha-\delta-C)}p_{t-1}-\frac{\beta}{(\alpha-\delta-C)}p_{t-2}+\frac{\alpha V_t+X_t}{(\alpha-\delta-C)} \quad (36)$$

The price will oscillate when the following condition holds:

$$(\beta-\delta-C)^2<4\beta(\alpha-\delta-C) \quad (37)$$

A sufficient for this to hold is that:

$$\beta>\delta+C \quad (38)$$

When the price oscillates, the cycles are of the following form:

$$p_t=A\left(\frac{\beta}{\alpha-\delta-C}\right)^{\frac{t}{2}}\cos(t\theta+B) \quad (39)$$

where θ is given by $\cos\theta=\dfrac{\beta-\delta-C}{2\sqrt{\beta(\alpha-\delta-C)}}$ and *A* and *B* are given by the initial conditions.

6.3 Numerical example

Let us now look at the effect of manipulative involvement of this sort, within the system used to provide the previous examples. We assume that the manipulator purchases one unit of the stock, in period one, to disturb the system from its steady state. The manipulator demand in subsequent periods is determined, via prices, by the value of *C*. For simplicity we will set the fundamental value to zero, which does not alter the nature of the cycles illustrated. *C* is set at 0.2. The parameters are therefore as follows:

Fig. 2: Parameter values.

α	β	δ	V	C
1	0.75	0	0	0.2

The plot shows the deviation of the price from its mean of the fundamental value *V*, rather that the price itself, in order to aid comparison of this cycle with the others given. The contribution of each period to the total profit is calculated in the following way:

$$\textit{Single period profit} = -E_{s,t}\,p_t \quad (40)$$

The cumulative profit of the manipulator is therefore given by:

$$\text{Cumulative profit to period } T = \sum_{t=1}^{T} -E_{s,t} p_t \tag{41}$$

Figure 6 shows the path of manipulator demand, prices and incremental profit.

Fig. 6: Price, demand and incremental profit with C of 0.2.

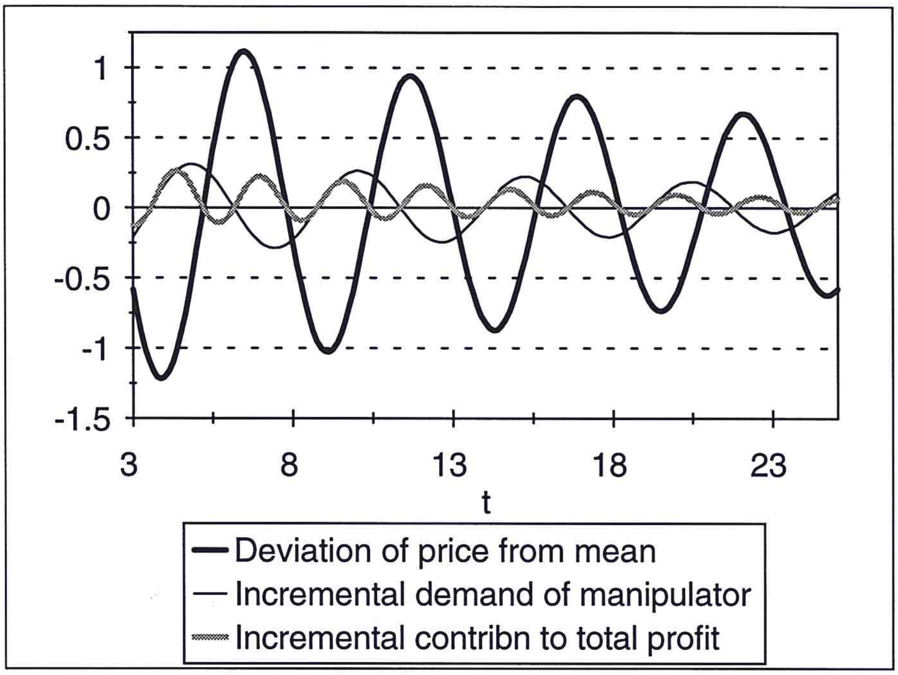
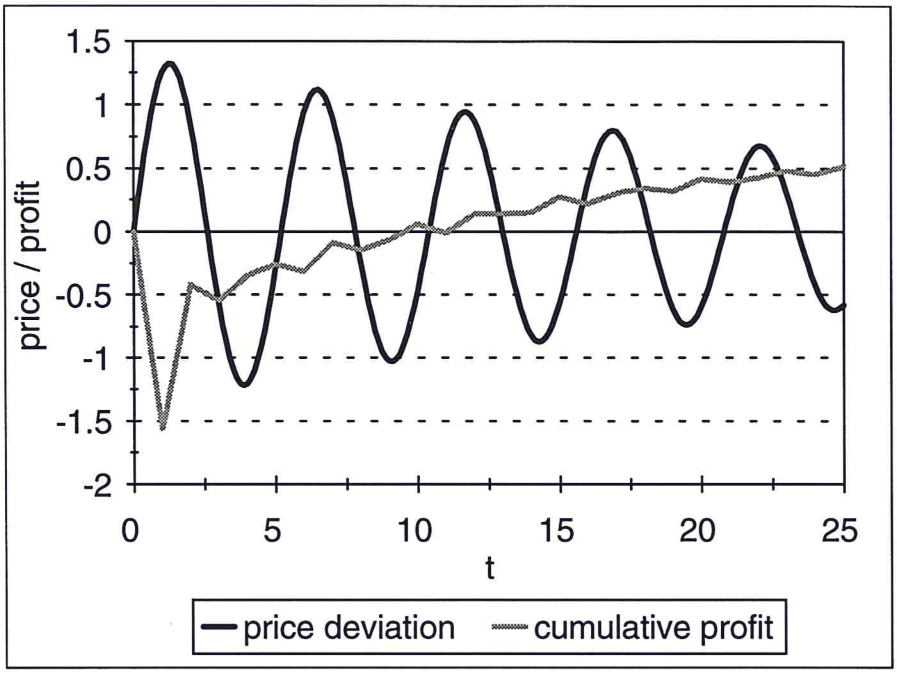


Figure 6 shows that in this example the manipulative activity reduces the rate at which the price cycle is damped. Since the grey line - representing the incremental change in profits - has a mean greater than zero, the manipulator earns positive profits once the system has been destabilised.

In order to assess the profitability of the manipulative strategy as a whole, the cumulative profit must be analysed. This is shown in figure 7, and reveals that the manipulator recoups the early loss and moves into profit by the twelfth period.

In this framework the manipulator can make greater profits than in the example given by trading sufficiently strongly to cause an increase in the amplitude of price movements over time. This highlights a weakness of this model framework. When the other agents are capable of learning about the manipulator's behaviour, we would not expect such a situation to persist indefinitely.

Fig. 7: Cumulative profit with C of 0.2.



7 Conclusion

This paper has revealed the richness of potential price behaviour in the presence of positive feedback trading. It has also shown that the presence of positive feedback trading does not necessarily provide an opening for manipulators, since the potential for this to be profitable will depend on the delay with which prices feed back into demand and the strength of value-trading by passive investors.

Unfortunately, the example given in section 6 reveals that when manipulation is possible its most profitable form leads to price fluctuations that increase over time. However, if other factors were to be taken into consideration, such as the riskiness of the strategy or the likelihood of the strategy being discovered, this conclusion might change. A non-linear passive investor demand function might even be sufficient. In any case, this feature highlights the need for more sophisticated modelling of the effects of positive feedback trading.

Appendix

Second-order difference equations

Take a general second-order difference equation such as the following:

$$p_t = a_1 p_{t-1} + a_2 p_{t-2} \tag{A1}$$

The characteristic roots are the roots of the following expression, which is derived from the above:

$$m^2 - a_1 m - a_2 = 0 \tag{A2}$$

The characteristic roots are therefore:[4]

$$\mu_{1,2} = \frac{1}{2}\left(a_1 \pm \sqrt{a_1^{\ 2} + 4a_2}\right) \tag{A3}$$

The system will oscillate around the stationary level of zero when the characteristic roots are complex, which occurs when the discriminant is negative, which is when the following condition holds:

$$a_1^{\ 2} + 4a_2 < 0 \tag{A4}$$

When the system does not oscillate, it is asymptotically stable whenever both the characteristic roots lie within the unit circle.

[4] See, for example, Levy (1992).

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