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Rationality testing under asymmetric loss

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Abstract

This paper shows that if agents have an asymmetric loss function, standard empirical tests for the rationality of their expectations are in general invalid. We further show that under a popular class of asymmetric loss functions, the linex family, a valid test can be carried out by estimating an appropriate ARCH-in-Mean model. The contrast between the standard test and our ARCH-M test is illustrated using the Goldsmith–Nagan forecasts of US Treasury bill yields. © 1998 Published by Elsevier Science S.A. All rights reserved.

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1. Introduction

This paper shows that if agents have an asymmetric loss function, standard empirical tests for the 'rationality' of their expectations are in general invalid. We further show that under a popular class of asymmetric loss functions, a valid test can be carried out by estimating an appropriate ARCH-in-Mean model. The contrasts between the standard test and our ARCH-M test are illustrated using the Goldsmith–Nagan forecasts of US Treasury bill yields.

Much theory in economics and finance assumes that agents act as if they had symmetric loss functions. Sometimes this is plausible, and it has the advantage of allowing 'least-squares' principles to be applied in estimation and hypothesis testing. However, there are also many situations where the assumption of symmetric loss is implausible. This is especially true in financial markets, where risk may be one-sided, or monetary profits may be nonlinear functions of underlying prices or exchange rates (see, for example, Stockman, 1987).

Granger (1969) demonstrates that for *unconditional* Gaussian processes, any optimal forecast under asymmetric loss will exhibit a *constant bias*, the size of which will depend on the parameters of the loss function, and the constant prediction error variance. Thus if underpredictions of interest rate changes are more costly than overpredictions, optimal forecasts would show a bias to overprediction, as agents seek to avoid the more expensive type of error. A recent paper by Christofferson and

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orthogonal to all information known at t, and in particular to the expected change in y, $(f_{t,t+n} - y_t)$. We should therefore find in Eq. (3) that $a_0 = a_1 = 0$. An equivalent procedure is the so-called 'unbiasedness test', which involves running the regression

$$y_{t+n} - y_t = b_0 + b_1(f_{t,t+n} - y_t) + v_{t+n} \tag{4}$$

and jointly testing $b_0 = a_0 = 0$, and the slope coefficient $b_1 = (1 - a_1) = 1$.

However, if the loss function is of the form Eq. (1), so that the optimal forecast has the form Eq. (2), the slope coefficient in Eq. (3) has probability limit:

$$\text{plim}(a_1) = \frac{-\,Var\left\{\frac{\alpha}{2}E_t\sigma^2_{t+n}\right\}}{Var\left\{E_t(\mu_{t+n} - \mu_t) + \frac{\alpha}{2}E_t\sigma^2_{t+n}\right\}} < 0 \tag{5}$$

Consequently, the slope coefficient in the unbiasedness test (4) has

$$\text{plim}(b_1) = \frac{Var\{E_t(\mu_{t+n} - \mu_t)\}}{Var\left\{E_t(\mu_{t+n} - \mu_t) + \frac{\alpha}{2}E_t\sigma^2_{t+n}\right\}} < 1 \tag{6}$$

In general, then, the slope coefficients in Eq. (3) and Eq. (4) will be lower under the asymmetric linex loss function than under a symmetric loss function. Indeed in the limiting but plausible case where y_t follows a random walk without drift, $E_t(\mu_{t+n} - \mu_t) = 0$, and the plims of the coefficients under the linex loss function are dramatically different, at -1 and 0 respectively.

The implication of this is clear. Many tests on published forecasts find that in regressions like Eq. (3) and Eq. (4), $a_1 < 0$ or $b_1 < 1$. The foreign exchange forecasters studied by Frankel and Froot (1987) and Froot and Frankel (1989) are a good example. Our theory shows that we cannot infer that this indicates irrationality. It may instead indicate rational forecasting in an environment where losses are asymmetric, and forecast variances change over time.

We might conjecture that in the case of foreign exchange forecasts, disinterested economic commentators might have a symmetric loss function, while individuals trading on their forecasts might have an asymmetric loss function. But in general without knowledge of the agents loss function, and when a target series exhibits time-varying volatility, there is observational equivalence in conventional test equations between suboptimal forecasts under a symmetric loss function, and optimal forecasts under an asymmetric loss function.

3. An ARCH-M rationality test

It is possible to design tests equivalent to Eq. (3) and Eq. (4) which are appropriate to a situation where there is time-varying bias in predictions, provided we maintain our assumption of a linex loss function, and make an additional (testable) assumption about the process driving the variance of the target variable.

First note that the rationally expected forecast error from Eq. (2) is $E_t\ (y_{t+n} - f^*_{t,t+n}) = (\alpha/2)E_t(\sigma^2_{t+n})$. Adding this time-varying bias to the right-hand-side of Eq. (3) yields

$$\sigma_{t+1}^2 = \underset{(0.2028)}{0.2526} + \underset{(0.1813)}{0.5250}u_t^2 + \underset{(0.2304)}{0.3399}\sigma_t^2 \tag{10'}$$

The coefficient on the variance term in the test equation is positive, but not statistically significant. Its presence, however, is sufficient to make the slope coefficient a_1 much less negative. The Lagrange multiplier test statistic for $a_0 = a_1 = 0$ is 1.8573, well below the 5% critical level of 5.99 for the $\chi^2(2)$ distribution, and the hypothesis of rationality cannot now be rejected.

The precise system we have used is not entirely satisfactory. Quite apart from the low significance of the variance term in the mean equation, the GARCH coefficient b_2 is barely significant. Other specifications are possible, and we have experimented with several. For example, we added a shift dummy to Eq. (10′) to account for the increased variance of bill yields during the monetary base targeting regime of 1979–82. We also used in the quarterly model Eq. (8) estimates of the variance of bill yields from a GARCH model parameterised on *monthly* data, which is much more informative about volatility. Our main result, that the hypothesis of rationality cannot be rejected, is robust to all these specification changes. It therefore seems likely that the highly negative slope coefficient in the conventional test Eq. (3′) is indeed an artefact of the kind of bias which we have shown may be induced by an asymmetric loss function.

4. Conclusions

The primary purpose of this paper is to draw attention to an important implication of recent work on asymmetric loss functions. If forecasters have asymmetric loss functions, conventional rationality tests may be seriously misleading. A second purpose is to establish that for a tractable class of asymmetric loss functions, the linex family, rationality testing is possible if allowance is made for possible ARCH-in-Mean effects. An empirical exercise on the Goldsmith–Nagan data has conveniently illustrated the point of our theoretical argument, that failure to allow for asymmetric losses under time-varying volatility may lead to an incorrect rejection of the hypothesis that forecasts are rational.

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References

Bollerslev, T., Chou, R.Y., Kroner, K.F., 1992. ARCH modelling in finance: a review of the theory and empirical evidence. Journal of Econometrics 52, 5–59.

Christofferson, P.F., Diebold, F., 1994. Optimal prediction under asymmetric loss, National Bureau of Economic Research Working Paper 167.

Dua, P., 1988. Multiperiod forecasts of interest rates. Journal of Business and Economic Statistics 6, 381–384.

Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 987–1007.