Calculating the Misspeci…cation in Beta from Using a Proxy for the Market Portfolio

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Abstract

This study investigates the e¤ects of the market portfolio being unknown on the estimation of beta in the CAPM. We provide an analysis of the impact of using a proxy for the market portfolio when the market portfolio is known. This allows us to ask and answer "if what" questions, such as if portfolio A is the true market portfolio, what happens to beta if we use portfolio B as a proxy for A. We show that for a given universe of investible assets, frequently used equally weighted and value weighted portfolios are far from the Markowitz market portfolio and thus the betas calculated with the equally weighted and value weighted portfolios are quite di¤erent from those obtained with the Markowitz portfolio. These calculations are based on sequential assumptions that one portfolio is a proxy whilst another is the actual market.

Keyword CAPM, Beta, Proxy Market, Ine¢ciency

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1 Introduction

The purpose of this paper is to undertake a misspecification analysis of the impact on asset beta of using a proxy for the market portfolio. It has been widely accepted since Roll's (1977) work that the market is not observable. However, the impact that this will have on the estimated asset betas has not been explored.

We carry out an analysis of the following form. Suppose, for example, that the market is some value weighted portfolio such as the S&P500. Then, what impact will there be on betas if we (mistakenly) use an equally weighted portfolio? Whilst this fails to solve the harder problems associated with the unknown market portfolio, our results do give us new insights.

In such an analysis, bias in beta will be determined by such considerations as the mean-variance (MV) efficiency of the proxy and the MV efficiency of the equity whose beta we are calculating. This separates our results from the conventional CAPM which requires the MV efficiency of the market but puts no restraints on the MV efficiency of the asset being priced.

In the next section, we consider two cases; the case where the means of the proxy and the market are equal (section 2.1), and the more general case where the means may differ (section 2.2). Section 3 is devoted to empirical calculation of the biases whilst conclusions follow in section 4.

2 The Effects of Inefficient Market Proxy on the Estimation of Beta

2.1 For Portfolios with the Same Expected Returns

Suppose that there are N risky assets whose returns are represented as a vector r where expected return and variance are represented as $E(r) = \mu$ and $Var(r) = \Sigma$. An investor constructs a portfolio by choosing a vector $w = (w_1; w_2; \ldots; w_N)'$, where w_i is the proportion (weight) that the investor invests his wealth in asset i, so that $e'w = 1$ where $e = (1; 1; ; ; 1)'$. In what follows, p will be a proxy portfolio, p^* a mean-variance efficient portfolio, and q an arbitrary portfolio.

Consider a portfolio, p^*, which is on the efficient frontier. The rate of return on the portfolio is $r_{p^*} = w_{p^*}' r$, where w_{p^*} is the appropriate weight vector. Then, means and variances of r_{p^*} are given as

$$r_{p^*} \sim (w_{p^*}'\mu; w_{p^*}'\Sigma w_{p^*}) \equiv (\mu_{p^*}; \sigma^2_{p^*}). \qquad (1)$$

Then in the mean-variance world, the investor will choose the following weighting vector to minimise his risk (variance) for given expected return μ_{p^*},[1]

$$w_{p^*} = \lambda^* \Sigma^{-1} e + v^* \Sigma^{-1} \mu \tag{2}$$

where

$$\begin{aligned} v^* &= \frac{A\mu_{p^*} - B}{AC - B^2}, \\ \lambda^* &= \frac{C - B\mu_{p^*}}{AC - B^2}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} A &= e' \Sigma^{-1} e, \\ B &= e' \Sigma^{-1} \mu, \\ C &= \mu' \Sigma^{-1} \mu. \end{aligned} \tag{4}$$

We next consider an arbitrary portfolio (i.e., the proxy portfolio), p, whose return is represented as $r_p = w_p' r$. Suppose that the expected return of the portfolio is the same as that of p^*, where p^* is described in (1), that is, $\mu_p = w_p' \mu = \mu_{p^*}$. Let

$$\begin{aligned} \Delta &= w_p - w_{p^*}, \\ r' \Delta &= r' w_p - r' w_{p^*} = r_p - r_p^*, \\ \mu' \Delta &= \mu' w_p - \mu' w_{p^*} = \mu_p - \mu_p^* = 0, \\ e' \Delta &= e' w_p - e' w_{p^*} = 1 - 1 = 0. \end{aligned} \tag{5}$$

Also note that if Δ is orthogonal to w_{p^*}, we have

$$\mathrm{Cov}(r' \Delta, r' w_{p^*}) = \mathrm{Cov}(r' w_p, r' w_{p^*}) - \mathrm{Cov}(r' w_{p^*}, r' w_{p^*}) = 0,$$

and thus

$$w_p' \Sigma w_{p^*} = w_{p^*}' \Sigma w_{p^*}. \tag{6}$$

In addition,

$$\mathrm{Cov}(r' \Delta, r' \Delta) = \mathrm{Cov}(r' w_p, r' \Delta) - \mathrm{Cov}(r' w_{p^*}, r' \Delta)$$

is equivalent to

$$\Delta' \Sigma \Delta = w_p' \Sigma \Delta - w_{p^*}' \Sigma \Delta \tag{7}$$

[1] See any finance textbook for a detailed explanation.

Consider another arbitrary portfolio corresponding to the priced asset, q, which is obtained with weight vector w_q; $r_q = w_q' r$. Then the covariances between portfolio q and p and the variance of p are

$$\begin{aligned} Cov(r_q; r_p) &= Cov(w_q' r; w_p' r) \\ &= w_q' \Sigma w_p \\ &= w_q' \Sigma (w_{p^*} + \Delta) \\ &= w_q' \Sigma (\lambda^* \Sigma^{-1} e + v^* \Sigma^{-1} \mu) + w_q' \Sigma \Delta \\ &= \lambda^* w_q' e + v^* w_q' \mu + w_q' \Sigma \Delta \\ &= \lambda^* + v^* \mu_q + w_q' \Sigma \Delta; \end{aligned} \tag{8}$$

and

$$\begin{aligned} Cov(r_p; r_p) &= Cov(w_p' r; w_p' r) \\ &= (w_{p^*}' + \Delta') \Sigma (w_{p^*} + \Delta) \\ &= w_{p^*}' \Sigma w_{p^*} + \Delta' \Sigma \Delta + 2 \Delta' \Sigma w_{p^*} \\ &= Cov(w_{p^*}' r; w_{p^*}' r) + \Delta' \Sigma \Delta: \end{aligned} \tag{9}$$

since $\Delta' \Sigma w_{p^*} = Cov(\Delta' r; w_{p^*}' r) = 0$ for the efficient portfolio p^*. To see this we can show that the following is true

$$\begin{aligned} w_{p^*}' \Sigma w_p &= w_{p^*}' \Sigma (w_{p^*} + \Delta) \\ &= w_{p^*}' \Sigma w_{p^*} + w_{p^*}' \Sigma \Delta \\ &= w_{p^*}' \Sigma w_{p^*}; \end{aligned}$$

where the last equation comes from equation (6).

The beta obtained with two arbitrary portfolios which have the same expected return is misspecified unless p is an efficient portfolio.

Theorem 1 The beta of portfolio q on the proxy portfolio p with the same expected return as the mean-variance efficient portfolio p^* is

$$\beta_{qp} = \beta_{qp^*} \frac{1}{1+\theta} + \beta_{q\Delta} \frac{\theta}{1+\theta} \tag{10}$$

where

$$\begin{aligned} \beta_{qp^*} &= \frac{w_q' \Sigma w_{p^*}}{w_{p^*}' \Sigma w_{p^*}}; \\ \beta_{q\Delta} &= \frac{w_q' \Sigma \Delta}{\Delta' \Sigma \Delta}; \\ \theta &= \frac{\Delta' \Sigma \Delta}{w_{p^*}' \Sigma w_{p^*}}: \end{aligned} \tag{11}$$

Proof Since

$$Cov(w_q'r; w_p'r) = Cov(w_q'r; w_{p^*}'r) + Cov(w_q'r; \delta'r);$$

we have

$$\begin{aligned}
\beta_{qp} &= \frac{Cov(w_q'r; w_p'r)}{Cov(w_p'r; w_p'r)} \\
&= \frac{w_q'\Sigma w_{p^*} + w_q'\Sigma\delta}{w_{p^*}'\Sigma w_{p^*} + \delta'\Sigma\delta} \\
&= \frac{w_q'\Sigma w_{p^*}}{w_{p^*}'\Sigma w_{p^*}}\frac{w_{p^*}'\Sigma w_{p^*}}{w_{p^*}'\Sigma w_{p^*} + \delta'\Sigma\delta} + \frac{w_q'\Sigma\delta}{\delta'\Sigma\delta}\frac{\delta'\Sigma\delta}{w_{p^*}'\Sigma w_{p^*} + \delta'\Sigma\delta} \\
&= \beta_{qp^*}\frac{1}{1+\theta} + \beta_{q\delta}\frac{\theta}{1+\theta}. \text{ QED.}
\end{aligned}$$

Remark 1 If q is mean-variance efficient, then $\beta_{q\delta} = 0$ since $Cov(w_q'r; \delta'r) = \delta'\Sigma w_q = 0$, and we have

$$\beta_{qp} = \beta_{qp^*}\frac{1}{1+\theta}. \qquad (12)$$

Note that θ is always positive if portfolio p is not efficient. We can think of two cases. The first case is $\beta_{qp^*} > \beta_{q\delta}$. In this case, $\beta_{qp^*} > \beta_{qp} > \beta_{q\delta}$ and β_{qp} is biased downwards. This result implies that if we use an inefficient market proxy whose expected return is the same as that of the market in testing the CAPM, the beta obtained using the market proxy is always less than it should be. Therefore, in the risk-return relationship in the tests of the CAPM, for the calculated risk (i.e., β_{qp}), returns appear to be rewarded more than they should be. The second case is $\beta_{qp^*} < \beta_{q\delta}$. Here, β_{qp} is biased upwards since we have $\beta_{qp^*} < \beta_{qp} < \beta_{q\delta}$. In this case, the beta obtained using the inefficient market proxy is larger than it should be and returns appear to be rewarded less than they should be. The question on the misspecification of the estimated beta with an inefficient market proxy can be answered empirically if certain assumptions are made.

We now turn to specific examples. Suppose that p^* is the Markowitz portfolio with the weighting vector w_{p^*},

$$\begin{aligned}
w_{p^*} &= \frac{\Sigma^{-1}(\mu - r_f e)}{e'\Sigma^{-1}(\mu - r_f e)} \qquad (13)\\
&= \frac{\Sigma^{-1}(\mu - r_f e)}{B - r_f A}
\end{aligned}$$

and p is (a) an equally-weighted portfolio ($w_i = \frac{1}{N}$ for all i), or (b) a value-weighted portfolio.

Example 1 When p is the equally-weighted portfolio, p^e, and p^* is the market portfolio, we have

$$
\begin{aligned}
\mu_{p^e} &= w_{p^e}' \mu \\
&= \frac{1}{N} e' \mu.
\end{aligned}
$$

Then

$$
\begin{aligned}
\Delta &= w_{p^e} - w_{p^*} \\
&= \frac{1}{N} e - \frac{\Sigma^{-1}(\mu - r_f e)}{e' \Sigma^{-1}(\mu - r_f e)}
\end{aligned}
$$

Note that

$$
\begin{aligned}
Var(r_{p^*}) &= w_{p^*}' \Sigma w_{p^*} \\
&= \frac{(\mu - r_f e)' \Sigma^{-1}(\mu - r_f e)}{(B - r_f A)^2} \\
&= \frac{C - 2 r_f B + r_f^2 A}{(B - r_f A)^2}, \\
Var(r_{p^e}) &= \frac{1}{N^2} e' \Sigma e,
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta' \Sigma \Delta &= (w_{p^e}' - w_{p^*}') \Sigma (w_{p^e} - w_{p^*}) \\
&= \left(\frac{1}{N} e - \frac{\Sigma^{-1}(\mu - r_f e)}{e' \Sigma^{-1}(\mu - r_f e)} \right)' \Sigma \left(\frac{1}{N} e - \frac{\Sigma^{-1}(\mu - r_f e)}{e' \Sigma^{-1}(\mu - r_f e)} \right) \\
&= \frac{1}{N^2} e' \Sigma e - 2 \frac{\mu_{p^e} - r_f}{B - r_f A} + \frac{C - 2 r_f B + r_f^2 A}{(B - r_f A)^2} \\
&= \frac{1}{N^2} e' \Sigma e - 2 \frac{\mu_{p^e} - r_f}{B - r_f A} + Var(r_{p^*}).
\end{aligned}
$$

Thus

$$
\begin{aligned}
\theta^e &= \frac{\Delta' \Sigma \Delta}{w_{p^*}' \Sigma w_{p^*}} \qquad (14) \\
&= \frac{\frac{1}{N^2} e' \Sigma e - 2 \frac{\mu_{p^e} - r_f}{B - r_f A} + Var(r_{p^*})}{Var(r_{p^*})} \\
&= \frac{Var(r_{p^e})}{Var(r_{p^*})} - 2(\mu_{p^e} - r_f) \frac{(B - r_f A)}{C - 2 r_f B + r_f^2 A} + 1
\end{aligned}
$$

From the above, it is clear when we use an equally weighted portfolio instead of the Markowitz portfolio, our beta will be misspecified unless θ^e is very close to zero. However, we do not know the magnitude of this bias.

Example 2 On the other hand, if p is a value weighted portfolio, p^v, and p^* is the Markowitz portfolio, we have

$$\mu_{p^v} = w'_{p^v}\mu.$$

Then

$$\Delta = w_{p^v} - w_{p^*}$$

Note that

$$Var(r_{p^v}) = w'_{p^v}\Sigma w_{p^v},$$

and

$$\begin{aligned} \Delta'\Sigma\Delta &= (w_{p^v} - w_{p^*})'\Sigma(w_{p^v} - w_{p^*}) \\ &= w'_{p^v}\Sigma w_{p^v} - 2w'_{p^v}\Sigma w_{p^*} + w'_{p^*}\Sigma w_{p^*}. \end{aligned}$$

Thus

$$\begin{aligned} \theta^v &= \frac{\Delta'\Sigma\Delta}{w'_{p^*}\Sigma w_{p^*}} \qquad (15) \\ &= \frac{w'_{p^v}\Sigma w_{p^v}}{w'_{p^*}\Sigma w_{p^*}} - 2\frac{w'_{p^v}\Sigma w_{p^*}}{w'_{p^*}\Sigma w_{p^*}} + 1 \end{aligned}$$

Therefore, unless $w_{p^v} = w_{p^*}$, we have $\theta^v > 0$ and the beta calculated with the value weighted portfolio (market proxy) will be misspecified by an amount involving θ^v and $\beta_{q\Delta}$.

2.2 Analysis for Portfolios with Different Expected Returns

Using portfolios with the same expected returns, we may calculate misspecification from using the inefficient market proxy as in the above. However, in practice this is a cumbersome constraint. In this subsection, we generalised the above analysis for portfolios with different expected returns.

We use the same settings and notations unless they are explained differently; let p^* be an efficient portfolio and p and q are arbitrary portfolios, i.e., market proxy and priced portfolios, respectively. Note that the expected returns of these three portfolios may be different. Again let

$$\begin{aligned} \Delta &= w_p - w_{p^*}; \\ e'\Delta &= 0; \end{aligned}$$

and we have

$$
\begin{aligned}
r'\Delta &= r'w_p - r'w_{p^*} = r_p - r_p^*, \\
\mu'\Delta &= \mu'w_p - \mu'w_{p^*} = \mu_p - \mu_p^* \neq 0.
\end{aligned}
$$

The covariance of portfolios p and q is the same as equation (8). However, the variance of portfolio p is

$$
\begin{aligned}
\mathrm{Cov}(r_p, r_p) &= (w_{p^*}' + \Delta')\Sigma(w_{p^*} + \Delta) \qquad (16)\\
&= w_{p^*}'\Sigma w_{p^*} + \Delta'\Sigma\Delta + 2\Delta'\Sigma w_{p^*} \\
&= w_{p^*}'\Sigma w_{p^*} + \Delta'\Sigma\Delta + 2\Delta'\Sigma(\lambda^*\Sigma^{-1}e + v^*\Sigma^{-1}\mu) \\
&= w_{p^*}'\Sigma w_{p^*} + \Delta'\Sigma\Delta + 2\lambda^*\Delta'e + 2v^*\Delta'\mu \\
&= w_{p^*}'\Sigma w_{p^*} + \Delta'\Sigma\Delta + 2v^*\Delta'\mu.
\end{aligned}
$$

Using the above covariance, we can derive the beta of an arbitrary portfolio q against an arbitrary portfolio p. The following theorem shows that the beta is always misspecified unless the portfolio p is efficient, i.e., $\Delta = 0$ and thus $p = p^*$.

Theorem 2 The beta of portfolio q on the portfolio p with different expected returns is

$$\beta_{qp} = \beta_{qp^*}\frac{1}{1+\theta+\delta} + \beta_{q\Delta}\frac{\theta}{1+\theta+\delta} \qquad (17)$$

where

$$
\begin{aligned}
\beta_{qp^*} &= \frac{w_q'\Sigma w_{p^*}}{w_{p^*}'\Sigma w_{p^*}}, \\
\beta_{q\Delta} &= \frac{w_q'\Sigma\Delta}{\Delta'\Sigma\Delta}, \\
\theta &= \frac{\Delta'\Sigma\Delta}{w_{p^*}'\Sigma w_{p^*}}, \\
\delta &= 2\frac{v^*\Delta'\mu}{w_{p^*}'\Sigma w_{p^*}},
\end{aligned}
$$

where v^* is in (3).

Proof The beta of portfolio q on portfolio p is

$$
\begin{aligned}
\beta_{qp} &= \frac{\mathrm{Cov}(w_q'r, w_p'r)}{\mathrm{Cov}(w_p'r, w_p'r)} \\
&= \frac{w_q'\Sigma w_{p^*} + w_q'\Sigma\Delta}{w_{p^*}'\Sigma w_{p^*} + \Delta'\Sigma\Delta + 2v^*\Delta'\mu}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{w_q'\Sigma w_{p^*}}{w_{p^*}'\Sigma w_{p^*}} \frac{w_{p^*}'\Sigma w_{p^*}}{w_{p^*}'\Sigma w_{p^*} + \Delta'\Sigma\Delta + 2v^*\Delta'\mu} \\
&\quad + \frac{w_q'\Sigma\Delta}{\Delta'\Sigma\Delta} \frac{\Delta'\Sigma\Delta}{w_{p^*}'\Sigma w_{p^*} + \Delta'\Sigma\Delta + 2v^*\Delta'\mu} \\
&= \beta_{qp^*} \frac{1}{1+\theta+\delta} + \beta_{q\Delta} \frac{\theta}{1+\theta+\delta}. \text{QED.}
\end{aligned}
$$

Remark 2 If q is mean-variance efficient, then $\beta_{q\Delta} = 0$. In this case we have

$$
\beta_{qp} = \beta_{qp^*} \frac{1}{1+\theta+\delta}
$$

which represents the beta of efficient portfolio q on an arbitrary portfolio p. Note that if p and q are efficient portfolios and $p = p^*$ (i.e., $\Delta = 0$), then $\theta = \delta = 0$ and $\beta_{qp} = \beta_{qp^*}$.

Again if p is inefficient, then θ is always positive, but δ may be positive or negative. Equations (10) and (17) suggest that δ is an adjustment factor for different expected means between portfolios. Since the sum of $\frac{1}{1+\theta+\delta}$ and $\frac{\theta}{1+\theta+\delta}$ is not equal to one, β_{qp} is not a weighted average of β_{qp^*} and $\beta_{q\Delta}$. In addition, since δ may be any number, we cannot say β_{qp} is misspecified upwards or downwards.

As in the previous subsection, suppose that p^* is the Markowitz portfolio with the weighting vector w_{p^*} and p is an equally-weighted portfolio ($w_i = \frac{1}{N}$ for all i), or a value-weighted portfolio. Let us first investigate the case of an equally weighted portfolio.

Example 3 When p is an equally-weighted portfolio, p^e, θ^e is equivalent to (14). Note that

$$
\begin{aligned}
\Delta'\mu &= \mu'\left(\frac{1}{N}e - \frac{\Sigma^{-1}(\mu - r_f e)}{e'\Sigma^{-1}(\mu - r_f e)}\right) \\
&= \mu_{p^e} - \frac{C - r_f B}{B - r_f A}; \\
v^*\Delta'\mu &= \left(\frac{1}{B - r_f A}\right)\left(\mu_{p^e} - \frac{C - r_f B}{B - r_f A}\right) \\
&= \frac{\mu_{p^e} B - \mu_{p^e} r_f A - C + r_f B}{(B - r_f A)^2}; \\
w_{p^*}'\Sigma w_{p^*} &= \frac{C - 2r_f B + r_f^2 A}{(B - r_f A)^2}.
\end{aligned}
$$

Thus the adjustment factor δ^e is

$$
\begin{aligned}
\delta^e &= 2\frac{v^{*} \cdot' \mu}{w'_{p^*} \Sigma w_{p^*}} \\
&= 2\frac{\frac{\mu_{p^e} B - \mu_{p^e} r_f A - C + r_f B}{(B - r_f A)^2}}{\frac{C - 2 r_f B + r_f^2 A}{(B - r_f A)^2}} \\
&= 2\frac{\mu_{p^e} B - \mu_{p^e} r_f A - C + r_f B}{C - 2 r_f B + r_f^2 A}.
\end{aligned}
$$

Example 4 On the other hand, if p is a value weighted portfolio, p^v, we have

$$\delta^v = 2\frac{(w'_{p^v} - w'_{p^*})\mu}{C - 2 r_f B + r_f^2 A}$$

Therefore, unless $w_{p^v} = w_{p^*}$, we have $\delta^v \neq 0$ and the beta calculated with the value weighted portfolio (market proxy) will be misspecified.

3 Empirical Tests

In this study we use both equally weighted and value weighted market portfolios to investigate the effects of using proxy market portfolios on the calculation of beta. The empirical tests in this study assume that there are only a finite number of known assets so that we can make equally weighted, value weighted, and Markowitz market portfolios from the same universe of assets.

For the universe of assets, we consider two examples. We use 420 equities included in the S&P500 index and 220 equities in the FTSE350 at 23 May 2000, which are available to us for our sample period. Monthly log-returns for these equities are calculated from May 1990 to April 2000. For the riskless returns, we use three month US and UK Treasury Bills, respectively.

The rates of return on the riskless asset are not constant over time and deflated excess returns are calculated as follows; the observed excess rates of return on the individual equities deflated by unity plus the riskless interest rates, $R_{it} = (r_{it} - r_{ft})/(1 + r_{ft})$. Excess rates of return on the market portfolio are also deflated in the same way. This is a method to make moments of the rate of returns intertemporal constants under a changing riskless interest rate (see Fama, 1970). Under certain circumstances, it allows us to use one period models for time-series data.

To reduce measurement errors in beta, risk asset portfolios are formed for each country with a grouping procedure similar to the procedures used by Black, Jensen, and Scholes (1972). First, we calculate beta using 5 years monthly de‡ated excess returns of the individual assets against monthly de‡ated excess returns of the equally weighted market portfolio.[2] Then stocks are ranked into deciles (10 groups) on the basis of beta estimates, and thus, the number of risk portfolios is 10: Using this method, we have 42 equities in a S&P500 risk portfolio and 22 equities in a FTSE350 risk portfolio, respectively. The subsequent 12 months de‡ated excess returns for the risk portfolios are calculated for each group. This procedure is repeated for the entire sample period beginning each May. This provides 60 monthly de‡ated excess returns from May 1995 to April 2000 for each of the risk portfolios.

All our subsequent calculations on beta are carried out with the risk portfolios above. That is, betas, expected returns, value weights, etc., in the following are calculated for the risk portfolios obtained with a proxy market portfolio (i.e., the equally weighted portfolio), not for individual equities. Thus for di¤erent proxy market portfolios, the components of the risk portfolios can change. Using the risk portfolios is not only to reduce measurement error but also to avoid complicated calculation involving the inversion of large variance-covariance matrices of equity returns.

For the ten risk portfolios, we …rst investigate the e¤ects of various proxy market portfolios on the estimates of beta. We use S&P100, S&P500, RUSSEL1000, RUSSEL2000, RUSSEL3000, Dow-Jones America as the proxy market portfolio for the S&P500 risk portfolios, and FT30, FT100, FT350, FT-All Share for the proxy market portfolio for the FTSE350 risk portfolios.[3] We also use FTSE-World, Dow-Jones World, and MSCI World index total return for both the S&P500 and FTSE350 risk portfolios.

Table 1 reports expected returns and betas for the ten risk portfolios with di¤erent proxy market portfolios. As expected, for the domestic market portfolios such as RUSSEL1000 (US) and S&P100 (UK), the means of the individual equity betas are close to 1, see the next to the last column of table 1. On the other hand, when the world market portfolios are used, the means of the individual equity betas are far less than one and lie between 0.72 and 0.8. The di¤erence in beta between these market portfolios comes from the lower correlation of the domestic market with the world market.

We next investigate the risk and return relationship for the ten risk portfolios for the given proxy market portfolios. For the sample period, the US market shows a linear relationship between risk and return; when the beta

[2] The value weighted portfolio can be used to calculate beta and group equities according to the obtained beta. However, we believe that the results are similar to those reported in tables 2 to 4. The expected returns and betas calculated with the value weighted porto‡io can be referred to table 1, since all indices in table 1 are value weighted indices.

[3] All these proxy market portfolio returns are total index returns except RUSSEL1000, RUSSEL2000, and RUSSEL3000 which are price index returns.

of an risk portfolio is low (high), expected returns of the risk portfolio are also low (high). However, this does not seem to be true for the other cases such as US risk portfolios with the world market portfolios, UK risk portfolios with the UK market portfolios, and UK risk portfolios with the world market portfolios. In particular, the risk-return relation does not seem to hold in the UK market; many correlation coefficients are negative or close to zero.

Our second empirical tests investigate how far the betas calculated with equally weighted and value weighted market portfolios are different from those with the Markowitz portfolio, which we exampified in the previous section. Again we use the same ten risk portfolios as described above.

From now on, we assume that there are only 10 risk portfolios in the market. Such a procedure of analysing a large universe of stocks is standard in the finance literature, see Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and Kraus and Litzenberger (1976), for example. The Markowitz portfolio in this market can be composed by applying the weight vector in (13) to the ten risk portfolios, and equally weighted and value weighted portfolio can also be obtained with appropriate weight vectors.

Table 2 reports some properties of the 10 risk portfolios with the weights on the portfolios. Panel A of table 2 shows that Markowitz weights (w_{p^M}) are quite different from value weights (w_{p^v}) and also equal weights (w_{p^e}, i.e., $w_{p^e} = (0.1, \ldots, 0.1)'$). Note that in many cases, the weights are negative, representing short sale which may not be allowed in practice. Panel B shows expected return and standard error of returns and correlation matrix of the three market portfolios. Interestingly, for the S&P500 risk portfolios, risk (standard error) increases as expected return increases, whilst for the FTSE350 risk portfolios, this is not true. Mean-variance analysis suggests that the Markowitz portoflio is always preferred to the other portfolios in the UK market. The difference between the equally weighted market portfolio and value weighted market portfolio is marginal in both the US and the UK. This small difference can also be detected in the correlation matrix. The correlation coefficient between these two portfolios is very close to 1. On the other hand, these portfolios are not highly correlated with the Markowitz portfolio; the correlation coefficients are between 0.3 and 0.5.

We now investigate if there is any bias in estimated beta from using the equally weighted or value weighted portfolio. For each risk portfolio (q) in panel A of table 2, we consider the following three cases; 1) the equally weighted portfolio is used for the proxy market portfolio (p), but the true market portfolio is the Markowitz portfolio (p^*) (table 3), 2) the value weighted portfolio is used for the proxy market portfolio (p), but the true market portfolio is the Markowitz portfolio (p^*) (table 4), and finally

3) the equally weighted portfolio is used for the proxy market portfolio (p), but the true market portfolio is the value weighted portfolio (p^*) (table 5). We do not include the three other possible cases to spare the reader from excessive tables and results.

Tables 3 to 5 report the results. Our estimates in tables 3 to 5 can be summarised with equation (17) as follows: for the S&P500 risk portfolios,

$$\begin{aligned} \beta_{qp} &= 2.119\beta_{qp^*} + 2.217\beta_{q\Delta} \text{ (case 1, panel A of table 3)}, \\ \beta_{qp} &= 2.155\beta_{qp^*} + 2.192\beta_{q\Delta} \text{ (case 2, panel A of table 4)}, \\ \beta_{qp} &= 0.981\beta_{qp^*} + 0.003\beta_{q\Delta} \text{ (case 3, panel A of table 5)}, \end{aligned}$$

and for the FTSE350 risk portfolios,

$$\begin{aligned} \beta_{qp} &= 0.718\beta_{qp^*} + 0.986\beta_{q\Delta} \text{ (case 1, panel B of table 3)}, \\ \beta_{qp} &= 0.879\beta_{qp^*} + 0.948\beta_{q\Delta} \text{ (case 2, panel B of table 4)}, \\ \beta_{qp} &= 0.817\beta_{qp^*} + 0.019\beta_{q\Delta} \text{ (case 3, panel B of table 5)}. \end{aligned}$$

These equations show that the values of $\frac{\theta}{1+\theta+\delta}$ in case 3 are very small and thus the effects of $\beta_{q\Delta}$ on the estimation of β_{qp} may be trivial despite the large values of $\beta_{q\Delta}$ in case 3. Therefore, when the equally weighted portfolio is used for the proxy market portfolio (p), but the true market portfolio is the value weighted portfolio (p^*), we may expect $\beta_{qp} \approx \frac{1}{1+\theta+\delta}\beta_{qp^*}$. However, when the true market portfolio is the Markowitz portfolio (p^*) but the equally weighted or value weighted portfolio is used for the proxy market portfolio, $\frac{\theta}{1+\theta+\delta}$ is not trivial and $\beta_{q\Delta}$ affects the estimates of β_{qp} as in equation (17). In addition, the values of θ and δ in cases 1 and 2 are similar such that the equally weighted and value weighted proxy portfolios result in similar misspecification in betas against the true Markowitz portfolio.

Tables 3 to 5 also suggest that in many cases, most of the estimates of $\beta_{q\Delta}$ are significantly large and positive. The difference portfolio Δ is not trivial. This means that portfolio q is not mean-variance efficient, see Remark 2.

As explained above, for the S&P500 risk portfolios, there seems to be a risk-return relationship when the equally weighted and value weighted market portfolios are used as the (assumed true) market portfolio, whilst for the FTSE350 risk portfolios, we do not find a relationship between risk and return; see the last column of tables 3 to 5.

When the Markowitz portfolio is used as the (assumed true) market portfolio for the risk portfolios (see tables 3 and 4), the correlation coefficients are one, since in the mean-variance world, risk and return are linear. Intestingly, in this case, estimated betas are all less than those calculated with the equally weighted and value weighted portfolios; i.e., $\beta_{qp} > \beta_{qp^*}$. In addition, the correlation coefficients between expected return and $\beta_{q\cdot}$ are all large and negative for all four cases, suggesting systematic bias in the estimation of the beta when we use the equally weighted and value weighted market portfolios.

In most cases, for a risk portfolio, betas on the value weighted portfolio are larger than those on the equally weighted portfolio. When we represented p^e for equally weighted portfolio and p^v for value weighted portfolio, our empirical results suggest $\beta_{qp^v} > \beta_{qp^e} > \beta_{qp^*}$ with few exceptions. Note that the proxy market portfolios in table 1 are value weighted portfolios and thus may result in larger estimates of beta when the true market portfolio is the Markowitz portfolio. However, the difference between β_{qp^v} and β_{qp^e} is marginal and in most cases indistinguishable. In addition, the values of δ are always negative in our empirical results. In particular, δ is not large enough to make $1 + \theta + \delta$ negative. That is, $1 + \theta > -\delta$: The adjustment factors do not change significantly for the four cases in tables 3 and 4.

Table 5 shows some possible deviation of beta when the equally weighted portfolio is used as the market portfolio instead of the value weighted portfolio. The difference between them seems to be marginal; betas on the risk portfolio are similar. This is because the two adjustment factors are close to zero.

Tables 1 to 4 indicate that the equally weighted portfolio or value weighted portfolios may provide larger systematic risk. The true beta calculated with the true market portfolio may be much smaller than we generally obtain with the market portfolio. The exaggerated beta may require higher return, or may make investors seem to be less risk averse. In addition, table 5 suggests that there is actually no difference in the estimated beta between the equally weighted portfolio and the value weighted portfolio as a proxy market portfolio.

4 Conclusions

In this study we showed that a proxy market portfolio, which here is the equally weighted or value weighted portfolio and is not on the efficient frontier, can provide a relatively larger beta than the Markowitz portfolio. However, we do not find any significant difference between equally weighted portfolio and value weighted portfolio when one is the proxy market portfolio and the other is the market, whichever way round we proceed.

Our study has allowed us to answer certain questions. Theoretical considerations imply that the aggregate demand portfolio in a CAPM should be value weighted whilst two fund money separation implies that the market portfolio is the Markowitz portfolio. We can analyse the "what-if" situation whereby we assume the market is one of those and let the other be the proxy.

The unsuprising conclusion is that value and equal weighted portfolios are proxies for each, neither are proxies for the Markowitz portfolio, at least not for the universe of assets and time periods considered in this paper.

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Table 1 The Estimates of Beta on Various Market Portfolios

	Market Portfolio	No of Obs	Expected Excess Returns										Beta										Mean of Individual Equity Betas	Correlation between Expected Return and Beta
			1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10		
S&P 500	S&P100	60	0.52	0.51	0.64	0.65	0.53	0.71	0.79	0.46	1.04	1.94	0.50	0.67	0.80	0.77	0.81	0.94	0.90	1.04	1.09	1.23	0.96	0.71
	S&P500	60	0.41	0.41	0.82	0.66	0.58	0.76	0.56	0.83	0.97	1.81	0.59	0.79	0.82	0.89	0.88	0.97	1.04	1.22	1.06	1.26	1.03	0.73
	RUSSEL1*	60	0.40	0.38	0.81	0.51	0.63	0.91	0.46	0.94	0.92	1.84	0.58	0.78	0.79	0.88	0.89	0.95	1.00	1.20	1.15	1.32	1.03	0.78
	RUSSEL2*	60	0.38	0.42	0.80	0.61	0.79	0.76	0.77	0.63	1.10	1.54	0.12	0.27	0.34	0.39	0.45	0.44	0.42	0.52	0.82	0.96	0.61	0.93
	RUSSEL3*	60	0.47	0.18	0.83	0.70	0.61	0.82	0.73	0.70	0.88	1.88	0.52	0.77	0.79	0.86	0.92	0.97	1.01	1.15	1.20	1.35	1.03	0.73
	DJ-America	40	0.14	0.62	0.28	0.15	0.54	0.40	0.32	0.72	1.24	1.74	0.49	0.58	0.67	0.69	0.75	0.81	0.87	0.85	0.95	1.23	0.95	0.84
	DJ-WORLD	40	0.51	0.73	0.88	0.38	0.72	0.51	0.06	0.58	0.59	1.17	0.50	0.58	0.68	0.77	0.77	0.82	0.88	0.88	0.96	1.24	0.80	0.31
	MSCI-World	60	0.90	0.53	0.71	0.73	0.82	0.61	0.52	0.63	0.61	1.73	0.52	0.73	0.84	0.92	0.97	1.01	1.01	1.08	1.11	1.28	0.80	0.37
	FTSE-World	60	0.86	0.53	0.89	0.50	1.02	0.51	0.48	0.77	0.49	1.73	0.50	0.66	0.85	0.97	0.89	1.09	1.00	1.05	1.12	1.25	0.77	0.24
FTSE 350	FT30	60	1.74	0.95	0.74	0.96	1.00	0.70	0.78	0.86	0.64	0.84	0.77	0.72	0.69	0.75	0.92	0.96	1.07	1.06	1.09	1.22	0.93	-0.39
	FT100	39	1.69	0.68	0.82	0.72	0.35	0.26	0.27	0.76	0.88	0.68	0.59	0.81	0.78	0.88	0.94	0.76	0.80	0.80	0.89	1.06	0.93	-0.51
	FT350	27	1.73	0.65	0.61	1.16	-0.37	0.52	0.34	0.43	0.61	1.12	0.92	0.88	1.09	1.08	0.96	0.97	0.95	0.92	1.15	1.54	0.97	0.26
	FT-ALL	60	1.71	1.03	0.97	0.87	0.44	0.91	0.76	0.67	0.97	0.89	0.80	1.04	0.82	0.95	0.96	0.96	1.12	1.18	1.03	1.33	1.04	-0.46
	FTSE-World	60	1.08	1.01	1.36	0.59	1.12	0.37	0.62	0.95	1.12	0.99	0.53	0.59	0.58	0.52	0.58	0.61	0.63	0.69	0.75	0.93	0.74	0.13
	DJ-World	40	0.67	1.45	1.12	0.37	1.27	0.39	0.75	1.12	0.60	1.37	0.55	0.50	0.60	0.51	0.52	0.57	0.66	0.61	0.64	0.93	0.72	0.29
	MSCI-World	60	1.12	1.00	0.90	1.09	0.74	0.51	0.79	0.93	1.05	1.07	0.58	0.54	0.56	0.59	0.59	0.59	0.63	0.67	0.75	0.97	0.75	0.30

Notes: Samples are 120 monthly log-returns from May 1990 to April 2000. For some market portfolios, sample periods are shorter than this. However, all sample periods end at April 2000. Note that we need 60 observations to compose portfolios as in Black, Jensen, and Scholes (1972) and the number of observations used to calculate expected excess returns and betas is 60 observations less than the total observations, which is represented in 'No of Obs'. For the calculation of beta, all returns are deflated as in Fama (1970). RUSSEL1*, RUSSEL2* and RUSSEL3* are RUSSEL1000, RUSSEL2000 and RUSSEL3000 price index returns, respectively, whilst for the other markets, total returns are used. 'Mean of Individual Equity Betas' represents the averaged value of individual equity betas on the market portfolio.

Table 2 Properties of Risk Portfolios

A. Expected Returns and Weights on 10 Risk Portfolios

S&P500 Equities

	Portfolios									
	1	2	3	4	5	6	7	8	9	10
Expected Return (m)	0.431	0.261	0.686	0.599	0.718	0.731	0.703	0.537	0.587	1.715
Averaged Value-Weights (w_{pv})	0.085	0.111	0.126	0.124	0.091	0.133	0.102	0.095	0.066	0.067
Markowitz Weights (w_{pM})	0.706	-1.661	2.338	-0.014	0.181	1.475	-0.132	-0.703	-2.497	1.306

FTSE350 Equities

	Portfolios									
	1	2	3	4	5	6	7	8	9	10
Expected Return (m)	1.159	1.194	0.577	0.787	0.781	0.894	0.752	1.124	0.981	0.836
Averaged Value-Weights (w_{pv})	0.200	0.173	0.142	0.087	0.081	0.092	0.069	0.068	0.055	0.034
Markowitz Weights (w_{pM})	0.675	1.338	-0.292	-0.529	-0.005	0.603	-0.564	0.165	0.117	-0.510

B. Properties of Equally Weighted, Value Weighted, and Markowitz Portfolios

S&P500 Equities

			Correlation Matrix		
	Expected Return	Standard Error	Equally Weighted Market Portfolio	Value Weighted Market Portfolio	Markowitz Portfolio
Equally Weighted Market Portfolio	1.216	4.539	1.000		
Value Weighted market Portfolio	1.203	4.513	0.999	1.000	
Markowitz Portfolio	3.506	6.605	0.341	0.329	1.000

FTSE350 Equities

			Correlation Matrix		
	Expected Return	Standard Error	Equally Weighted Market Portfolio	Value Weighted Market Portfolio	Markowitz Portfolio
Equally Weighted Market Portfolio	1.428	4.105	1.000		
Value Weighted market Portfolio	1.451	3.688	0.994	1.000	
Markowitz Portfolio	2.307	3.478	0.432	0.491	1.000

Notes: We use 420 equities included in the S&P500 index and 220 equities included in the FTSE350 index. For each equity, monthly log-returns are calculated from May 1990 to April 2000. The ten risk portfolios are obtained with the same grouping method as in Black, Jensen, and Scholes (1972). Note that the first 60 monthly observations are used to estimate the CAPM, and thus each of the the risk portfolio returns above consists of 60 observations from May 1995 to April 2000. See section 3 for a detailed explanation on the grouping method.

Table 3 The Effects of Equally Weighted Market Portfolio on the Estimation of Beta When the True Market Portfolio is the Markowitz Portfolio

A. S&P500 Equities

	Portfolios										Correlation between Expected Return and Beta
	1	2	3	4	5	6	7	8	9	10	
Expected Return (m_q)	0.431	0.261	0.686	0.599	0.718	0.731	0.703	0.537	0.587	1.715	-
Beta on the Equally Weighted Market Portfolio (b_{qp})	0.622	0.823	0.852	0.983	1.014	1.086	1.122	1.136	1.178	1.184	0.518
Beta on the Markowitz Market Portfolio (b_{qp*})	0.145	0.088	0.230	0.201	0.241	0.246	0.236	0.181	0.197	0.576	1.000
Beta on the Difference betweeen the Markowitz and Equally Weighted Portfolios (b_{qD})	0.148	0.300	0.172	0.262	0.237	0.267	0.292	0.355	0.358	-0.017	-0.754
Adjustment Factor 1 (θ)	1.004										-
Adjustment Factor 2 (δ)	-1.532										-

B. FTSE350 Equities

	Portfolios										Correlation between Expected Return and Beta
	1	2	3	4	5	6	7	8	9	10	
Expected Return (m_q)	1.159	1.194	0.577	0.787	0.781	0.894	0.752	1.124	0.981	0.836	-
Beta on the Equally Weighted Market Portfolio (b_{qp})	0.728	0.712	0.755	0.928	0.994	0.959	1.091	1.273	1.225	1.334	-0.055
Beta on the Markowitz Market Portfolio (b_{qp*})	0.650	0.670	0.324	0.441	0.438	0.502	0.422	0.630	0.550	0.469	1.000
Beta on the Difference betweeen the Markowitz and Equally Weighted Portfolios (b_{qD})	0.265	0.235	0.530	0.620	0.689	0.607	0.800	0.832	0.842	1.012	-0.381
Adjustment Factor 1 (θ)	1.374										-
Adjustment Factor 2 (δ)	-0.981										-

Notes: We use 420 equities included in the S&P500 index and 220 equities included in the FTSE350 index. For each equity, monthly log-returns are calculated from May 1990 to April 2000. The ten risk portfolios are obtained with the same grouping method as in Black, Jensen, and Scholes (1972). Note that the first 60 monthly observations are used to estimate the CAPM, and thus each of the the risk portfolio returns above consists of 60 observations from May 1995 to April 2000. See section 3 for a detailed explanation on the grouping method. p* represents the "true" market portfolio, and p represents the proxy market portfolio.

Table 4 The Effects of Value Weighted Market Portfolio on the Estimation of Beta When the True Market Portfolio is the Markowitz Portfolio

A. S&P500 Equities

	Portfolios										Correlation between
	1	2	3	4	5	6	7	8	9	10	Expected Return and Beta
Expected Return (m_q)	0.431	0.261	0.686	0.599	0.718	0.731	0.703	0.537	0.587	1.715	-
Beta on the Value Weighted Market Portfolio (b_{qp})	0.640	0.845	0.867	0.998	1.031	1.100	1.136	1.145	1.173	1.144	0.453
Beta on the Markowitz Market Portfolio (b_{qp*})	0.145	0.088	0.230	0.201	0.241	0.246	0.236	0.181	0.197	0.576	1.000
Beta on the Difference betweeen the Markowitz and Value Weighted Portfolios (b_{qD})	0.149	0.299	0.169	0.257	0.233	0.260	0.286	0.345	0.341	-0.045	-0.792
Adjustment Factor 1 (θ)	1.017										-
Adjustment Factor 2 (δ)	-1.553										-

B. FTSE350 Equities

	Portfolios										Correlation between
	1	2	3	4	5	6	7	8	9	10	Expected Return and Beta
Expected Return (m_q)	1.159	1.194	0.577	0.787	0.781	0.894	0.752	1.124	0.981	0.836	-
Beta on the Value Weighted Market Portfolio (b_{qp})	0.859	0.799	0.839	1.023	1.093	1.048	1.194	1.384	1.324	1.442	-0.027
Beta on the Markowitz Market Portfolio (b_{qp*})	0.650	0.670	0.324	0.441	0.438	0.502	0.422	0.630	0.550	0.469	1.000
Beta on the Difference betweeen the Markowitz and Value Weighted Portfolios (b_{qD})	0.304	0.221	0.585	0.670	0.747	0.640	0.869	0.876	0.887	1.086	-0.416
Adjustment Factor 1 (θ)	1.079										-
Adjustment Factor 2 (δ)	-0.941										-

Notes: We use 420 equities included in the S&P500 index and 220 equities included in the FTSE350 index. For each equity, monthly log-returns are calculated from May 1990 to April 2000. The ten risk portfolios are obtained with the same grouping method as in Black, Jensen, and Scholes (1972). Note that the first 60 monthly observations are used to estimate the CAPM, and thus each of the the risk portfolio returns above consists of 60 observations from May 1995 to April 2000. See section 3 for a detailed explanation on the grouping method. p* represents the "true" market portfolio, and p represents the proxy market portfolio.

Table 5 The Effects of Equally Weighted Market Portfolio on the Estimation of Beta When the True Market Portfolio is the Value Weighted Market Portfolio

A. S&P500 Equities

	Portfolios										Correlation between Expected Return and Beta
	1	2	3	4	5	6	7	8	9	10	
Expected Return (m_q)	0.431	0.261	0.686	0.599	0.718	0.731	0.703	0.537	0.587	1.715	-
Beta on the Equally Weighted Market Portfolio (b_{qp})	0.622	0.823	0.852	0.983	1.014	1.086	1.122	1.136	1.178	1.184	0.518
Beta on the Value Weighted Market Portfolio (b_{qp^*})	0.640	0.845	0.867	0.998	1.031	1.100	1.136	1.145	1.173	1.144	0.453
Beta on the Difference betweeen the Markowitz and Equally Weighted Portfolios (b_{qD})	-2.286	-2.592	0.388	1.045	0.840	2.321	2.645	4.189	9.961	22.592	0.885
Adjustment Factor 1 (θ)	0.003										-
Adjustment Factor 2 (δ)	0.016										-

B. FTSE350 Equities

	Portfolios										Correlation between Expected Return and Beta
	1	2	3	4	5	6	7	8	9	10	
Expected Return (m_q)	1.159	1.194	0.577	0.787	0.781	0.894	0.752	1.124	0.981	0.836	-
Beta on the Equally Weighted Market Portfolio (b_{qp})	0.728	0.712	0.755	0.928	0.994	0.959	1.091	1.273	1.225	1.334	-0.055
Beta on the Value Weighted Market Portfolio (b_{qp^*})	0.859	0.799	0.839	1.023	1.093	1.048	1.194	1.384	1.324	1.442	-0.027
Beta on the Difference betweeen the Markowitz and Equally Weighted Portfolios (b_{qD})	1.374	3.152	3.672	4.849	5.287	5.399	6.073	7.450	7.525	8.202	-0.182
Adjustment Factor 1 (θ)	0.023										-
Adjustment Factor 2 (δ)	0.201										-

Notes: We use 420 equities included in the S&P500 index and 220 equities included in the FTSE350 index. For each equity, monthly log-returns are calculated from May 1990 to April 2000. The ten risk portfolios are obtained with the same grouping method as in Black, Jensen, and Scholes (1972). Note that the first 60 monthly observations are used to estimate the CAPM, and thus each of the the risk portfolio returns above consists of 60 observations from May 1995 to April 2000. See section 3 for a detailed explanation on the grouping method. p* represents the "true" market portfolio, and p represents the proxy market portfolio.