Implied Volatility Trees and Pricing Performance: Evidence from the S&P 100 Options*

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Abstract

This paper examines the pricing performance of various discrete-time option models that accept the variation of implied volatilities with respect to the strike price and the time-to-maturity of the option (implied volatility tree models). To this end, data from the S&P 100 options are employed for the first time. The complex implied volatility trees are compared to the standard Cox-Ross-Rubinstein model and the ad-hoc traders model. Various criteria and interpolation methods are used to evaluate the performance of the models. The results have important implications for the pricing accuracy of the models under scrutiny and their implementation.

Keywords: Implied volatility, Implied volatility trees, Option Pricing.

JEL Classification: G13

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1. Introduction

There are two general approaches to developing an option-pricing model: the traditional and the smile-consistent approach (also termed implied models approach; see Bates, 2003, for a survey of the development of the approaches taken in option pricing). In the traditional approach, first a stochastic process for the underlying traded/non-traded asset is assumed. Then arbitrage/equilibrium arguments are used to develop the option-pricing model. In the smile-consistent approach, the market prices of European options are taken as given, and are used to infer the process that the underlying asset follows. The derived process is used to price and hedge American and exotic options consistently with the market European option prices.

Models that fall within the first approach include the geometric Brownian motion Black-Scholes (1973) model, jump diffusion models (e.g., Merton, 1976, Bates, 1996b), stochastic volatility models (e.g., Hull and White, 1987, Heston, 1993), and combinations of stochastic volatility and jump processes models (e.g., Bates, 1996a, Scott, 1997). The models that are more complex to the Black Scholes (1973) have been developed so as to explain the variation of implied volatilities with respect to the strike price and the expiry date (implied volatility smiles/skews, term structure of implied volatilities, see e.g., Rubinstein, 1985, 1994, and Jackwerth, 1999, for a review of the literature), as well as their dynamics (see e.g., Skiadopoulos et al., 1999). However, a number of studies (see e.g., Bakshi et al., 1997, Das and Sundaram, 1999, Buraschi and Jackwerth, 2001) have confirmed the anecdotal evidence that models falling within the traditional approach cannot account for the empirically observed implied volatility smiles, or for their evolution over time. As a response, the "smile-consistent" models have emerged. Models falling within this category may be further classified as deterministic and stochastic volatility "smile-consistent" models. The former category (see e.g., Derman and Kani, 1994, Rubinstein, 1994, Barle and Cakici, 1995, Jackwerth, 1997, among others) assumes that volatility is a deterministic function of the asset price and time. The latter assumes that volatility evolves stochastically (see e.g., Dupire, 1992, Derman and Kani, 1998, Schönbucher, 1999, Skiadopoulos and Hodges, 2001, Britten-Jones and Neuberger, 2002, Rossi, 2002, Panigirtzoglou and Skiadopoulos, 2004). The models are developed in either discrete (deterministic/stochastic implied volatility trees) or continuous time (see also Jackwerth, 1999, and Skiadopoulos, 2001, for surveys on the smile-consistent models). The implementation of the models requires the choice of an interpolation method so as to obtain a continuum of implied volatilities across strikes and maturities (see e.g., Rubinstein, 1994, Avellaneda et al., 1997).

Despite the fact that the literature on smile-consistent models is growing fast, the empirical performance of these models has not been tested extensively. To the best of our knowledge, there are only four papers on the empirical performance of deterministic volatility smile-consistent models; the stochastic volatility smile-consistent models have not been tested since their implementation is subject to various computational limitations.

Dumas et al. (1998) used S&P 500 European options data to compare a *class* of "smile-consistent" deterministic volatility models with the Black-Scholes model (1973), and an ad-hoc procedure that smooths the Black-Scholes (1973) implied volatilities across strikes and times-to-maturity (ad-hoc model). The models are calibrated to the data on some date and the accuracy in terms of pricing European options is assessed for some future date. They found that the complex "smile-consistent" deterministic volatility models perform no better than the ad-hoc procedure in pricing and hedging terms. Hull and Suo's (2002) study is close to the one by Dumas et al. (1998) in that they compare a class of implied deterministic volatility models with the Black-Scholes model as well. However, they test the ability of implied models to price *exotic* options accurately in the presence of model error where the data are assumed to be generated by a stochastic volatility process. The process is fitted to S&P 500 and foreign currency options, separately, on some date and the performance is evaluated for the same date. They find that the implied models are superior to the Black-Scholes model; the out-performance depends on the type of the exotic option to be priced.

Lim and Zhi (2001) and Brandt and Wu (2002) have assessed the pricing performance of various *specific* deterministic implied volatility trees using data on the FTSE-100 options (see Chriss, 1997, for an excellent description of deterministic implied volatility trees). The methodology that is followed in both papers consists of calibrating first the models under scrutiny to the market prices of European style options. Then, the calibrated models are used to price the American style options. The two studies differ in the sample period, in the models employed, in the metrics that they use to assess the models, and in the interpolation methods that are applied to obtain the required implied volatilities. In particular, Lim and Zhi (2001) have compared the pricing and (delta) hedging performance of the Derman and Kani (1994) and Jackwerth (1997) generalized binomial model, and the standard binomial Cox-Ross-Rubinstein (CRR, 1979) models. Their data set was comprised of daily prices of the FTSE 100 Index options over the period January–November 1999. A quadratic curve was used to describe the variation of implied volatilities as a function of the strike price; the term structure was assumed to be linear. The results were mixed depending on the type of option to be priced (call or put), the criterion under which the assessment was carried out, and the moneyness and the time-to-maturity of the option. In general, in pricing terms, the Derman and Kani implied volatility tree was found to perform best for American call options with earlier maturity than the maturity span of the implied trees. On the other hand, the generalized binomial tree performed best for American at-the-money put options pricing for any maturity. Brandt and Wu (2002) have examined the pricing performance of the Barle and Cakici (1995), the CRR, and the ad-hoc models by using daily prices of the FTSE 100 index options over the period October 1995–September 1997. To this end, they employed a complex functional form (Legendre polynomials) to interpolate the implied volatilities across the strike prices and the times-to-maturity. In line with Dumas et al. (1998), they found that the implied binomial tree model performs no better than the ad-hoc model.

This paper adopts the approach taken by Lim and Zhi (2001) and Brandt and Wu (2002), and it applies it to a different data set, the liquid Standard & Poor's 100 (S&P 100) option data traded in the Chicago Board of Exchange (CBOE). The S&P 100 Index is a capitalization-weighted index of 100 stocks from a broad range of industries. Many traders regard it as the best gauge for the performance of the US stock market. Given its importance, the S&P 100 options data set has been used in a number of studies to examine the properties of option models that fall within the traditional approach (see Bates, 1996c, for a detailed survey). Surprisingly, this data set has not been used so far for the purposes of evaluating the performance of implied models.

The pricing performance of two deterministic volatility implied trees (Derman and Kani, DK, 1994, and Barle and Cakici, BC, 1995) versus the CRR and the ad-hoc models is investigated. All four models are very popular among practitioners and this is the primary reason that we subject them to investigation. The DK model was one of the first deterministic volatility implied trees to be introduced while the BC model was suggested as an improved algorithm over the DK model. The CRR model is used as a benchmark for the more complex implied models. The ad-hoc model compromises between the simplicity and a theoretically sound structure of an option pricing model so as to be consistent with the empirically documented behavior of implied volatilities.

To perform our study, the models are calibrated to the European market option prices and then the American options are priced. Various metrics are employed to assess the relative pricing performance of the models. Two different interpolation methods (linear and cubic splines) are also used to investigate the effect of the interpolation method on the pricing performance of each model. For instance, Brandt and Wu (2002) found that a more complex interpolation scheme does not necessarily improve the performance of the model. The results have important implications for the pricing accuracy of the models under scrutiny and their implementation.

The remainder of the paper is structured as follows. The implied tree models employed are briefly reviewed in Section 2. Section 3 describes the data set and the filtering constraints that are applied. Section 4 outlines the methodology and the criteria that are used to assess the pricing performance of the various models. The results are presented and discussed in Section 5. Section 6 concludes and highlights the implications of the results.

2. Various Binomial Tree Models: Description

2.1 The Derman and Kani (1994) Implied Tree

Derman and Kani (DK, 1994) build a recombining implied binomial tree that uses as input the market prices of European-style index options across all available strikes and expirations. Their algorithm uses forward and backward induction simultaneously to deduce the deterministic volatility structure of the process that the underlying asset follows[1]. Their model has uniformly spaced levels Δt apart. To construct it, they assume that they have already implied the tree's nodes and the transition probabilities out to time level n. The known price $S_{i,n}$ at node i and level n can evolve into an "up" node with price $S_{i+1,n+1}$, or into a "down" node with price $S_{i,n+1}$ at level $(n+1)$. The (unknown) probability of making a transition into the "up" node is denoted by p_i. The aim is to determine the nodes of the $(n+1)$th level at time t_{n+1} and the corresponding transition probabilities.

The DK model can be expressed formally as follows: The martingale condition delivers the forward price $F_{i,n}$ of the stock expiring at time n+1 as

$$F_{i,n} = p_i S_{i+1,n+1} + (1-p_i) S_{i,n+1} \tag{1}$$

where F_i is given by

$$F_{i,n} = S_{i,n} e^{(r_i - q_i)\,\Delta t} \tag{2}$$

and q_i, r_i are the stock's dividend yield and the risk-free rate at the ith time step, respectively. The notation indicates that we extend the original DK algorithm to take into account a term structure of interest rates and dividends (see also Brandt and Wu, 2002, and the discussion therein).

Let $C(S_{i,n}, t_{n+1})$ and $P(S_{i,n}, t_{n+1})$ be the known market prices of a European call and a European put option, respectively, struck today at $K=S_{i,n}$ and expiring at t_{n+1}. The values of those options can be calculated from interpolating the smile curve implied from options expiring at time t_{n+1} across strikes. The theoretical binomial value of a call option struck at K and expiring at t_{n+1} is given by:

[1] Backward and forward induction are the discrete analogues of the Kolmogorov backward and forward equations, respectively. The binomial backward equation states that the price at any period n is the discounted value of the average of the prices at the two up and down nodes in the next period n+1. The binomial forward is the "dual" or the "adjoint" of the binomial backward equation. It states that the price of an Arrow-Debreu security of any maturity (n+1) is the average of the discounted Arrow-Debreu prices of maturity n that correspond to the previous two up and down nodes.

$$C(K,t_{n+1}) = e^{-r\Delta t}\sum_{j=1}^{n}\{Q_{j,n}p_j + Q_{j+1,n}(1-p_{j+1})\}\max(S_{j+1,n+1}-K,0) \tag{3}$$

where the sum is taken over all nodes j at the $(n+1)$ level and $Q_{j,n}$ is the price at node j of the Arrow-Debreu security expiring at t_{n+1}.

Setting the strike price $K = S_{i,n}$, only the up node and all nodes above it contribute to the call value. Therefore, by simultaneously solving equations (1) and (3) for $S_{i+1,n+1}$ and the transition probability p_i, we obtain formulae that can be written in a general form as follows (see Derman and Kani, 1994, for the exact formulae)

$$S_{i+1,n+1} = f(r,\Delta t, S_{i,n+1}, C(S_{i,n},t_{n+1}), \Sigma_c, Q_{i,n}, S_{i,n}, F_{i,n}) \tag{4}$$

$$p_i = f(F_{i,n}, S_{i,n+1}, S_{i+1,n+1}) \tag{5}$$

where $\Sigma_c = \sum_{j=i+1}^{n} Q_{j,n}(F_{j,n} - S_{i,n})$. The Arrow-Debreu prices $Q_{j,n}$ are calculated by applying forward induction. Equations (4) and (5) can be used to find iteratively $S_{i+1,n+1}$ and p_i for all the nodes above the center of the tree. This iterative procedure requires knowledge of $S_{i,n+1}$ at one initial node. Centering conditions are used towards this end; the conditions depend on whether the number of nodes is even or odd[2].

Similarly, for all the nodes below the central node at level n the stock prices are calculated by using the known market put prices. The analogous formula that determines a lower stock price from a known upper one is given by

$$S_{i,n+1} = f(r,\Delta t, S_{i+1,n+1}, P(S_{i,n},t_{n+1}), \Sigma_p, Q_{i,n}, F_{i,n}) \tag{6}$$

where $\Sigma_p = \sum_{j=1}^{i-1} Q_{j,n}(S_{i,n} - F_{j,n})$

[2] Space limitations do not allow us to report the centering conditions. See Derman and Kani (1994) for more details.

The transition probability calculated by equation (5) should fall between zero and one. However, it may often be the case that the calculated probabilities do not fall within this interval (commonly termed as "bad probabilities"); this indicates arbitrage opportunities. In this case, Derman and Kani suggest overriding the stock price that yields a bad probability and choosing a stock price that keeps the logarithmic spacing between this node and its adjacent node the same as that between corresponding nodes at the previous level, i.e. $\ln S_{i+1,n+1} - \ln S_{i,n+1} = \ln S_{i+1,n} - \ln S_{i,n}$.

2.2 The Barle and Cakici (1995) Implied Tree

Barle and Cakici (BC, 1995) note that the DK algorithm fails to reproduce the smile accurately in the case where the interest rate is high. The reason is that with higher interest rates, negative probabilities are encountered more frequently, thus overriding the corresponding stock/option prices. Hence, the constructed tree does not fully incorporate the information from the smile. In order to correct for this problem they propose three modifications to the DK method[3].

First, they choose the option to be struck at $K=F_{i,n}$. Second, rather than fixing the center of the tree at the current stock price as DK do, they allow it to follow the evolution of the mean of the risk-neutral distribution by setting it to $Se^{rt_{n+1}}$. Third, in the case where there is a missing stock price due to the violation of the arbitrage condition, they set $S_{i+1,n+1} = \dfrac{F_{i,n} + F_{i+1,n}}{2}$. BC's modifications are equivalent to working with the futures rather

[3] The transition probabilities calculated from equation (5) can be either negative or greater than one causing small errors, which accumulate, and lead to serious discrepancies (i.e. option prices that are negative). In the case where $S_i \le F_i \le S_{i+1}$, the corresponding transition probability p_i, given by equation (5), is between zero and one. In the cases where $S_i < F_i$ or $F_i > S_{i+1}$, p_i is negative or greater than one, respectively.

than the spot price. In a standard binomial tree this trick guarantees non-negative transition probabilities (see Hull, 2003). Despite the proposed modifications, "bad probabilities" may still be encountered in the BC tree and hence they should be overridden. This results in losing information about the smile at the corresponding nodes. Fortunately, on most occasions the bad probabilities occur for far away-from-the-money options that do not trade heavily.

2.3 The Cox-Ross-Rubinstein (1979) Model

Cox, Ross, Rubinstein (CRR, 1979) developed the binomial tree that is the discrete-time analogue of the Black Scholes (1973) model. The asset price may move either up or down. The size of the movement is determined by the time step Δt that is used to divide the time to maturity T into $n = \frac{T}{\Delta t}$ time steps, and by a constant volatility σ. In the limit, as $\Delta t \to 0$, the CRR model converges to the Black Scholes model. For the purposes of our study, the CRR model is used as a benchmark for the more complex models that incorporate the variation of the implied volatility with respect to the strike. The CRR is built by considering σ to be the implied volatility of the at-the-money option with time-to-maturity that corresponds to the one of the option to be priced.

2.4. The Ad-Hoc (AH) Binomial Tree

The Ad-hoc model is a modification of the CRR tree (see Hull, 2003, page 336). It has been adopted by traders to price American/exotic options by taking into account the variation of the implied volatility with respect to the strike and time-to-maturity. The implied volatilities are extracted from European options by using the Black-Scholes model. Then, for the option contract under scrutiny (i.e. for a specific strike and time-to-maturity) the corresponding implied volatility is obtained by interpolating the already constructed implied volatility surface. Finally, the interpolated implied volatility is inserted in the CRR model and the option is priced. This procedure is ad-hoc because it is internally inconsistent. It uses the Cox-Ross-Rubinstein binomial tree that assumes constant volatility, but it takes as input the implied volatility that depends on the strike and the maturity of the option under scrutiny.

3. The Data Set

3.1 Source Data

We use daily data on European and American style options on the Standard & Poor's 100 (S&P 100) index from the Chicago Board of Exchange Trade (CBOE) for the period August 15th, 2001 – July 21st, 2003. This data set contains the following daily information for each option traded: the expiration date, the strike price, the last bid and ask prices, the trading volume, and the S&P 100 closing price. The average of the bid-ask option price is used as the option's market price. This is a standard approach taken to reduce the impact of measurement errors on the implied volatilities calculated subsequently, which are necessary to implement the models employed (see Figlewski, 1997, for a detailed discussion).

The American style options on the S&P 100 (ticker symbol *OEX*) were the first index options introduced in CBOE, on March 11th, 1983. The European style options on the S&P 100 (ticker symbol *XEO*) were introduced on August 15th, 2001. The last trading day of the OEX and XEO options is the business day (usually a Friday) preceding the expiration date. The expiration date is the Saturday following the third Friday of the expiration month. Up to four near-term months plus up to one month on the March quarterly cycle are traded every day. The strike prices are spaced at intervals of five index points and ten/twenty points for the far-term months OEX/XEO options. Both OEX and XEO options are traded from 8:30 a.m-3.15 p.m. central time (Chicago time).

In addition, we use London Euro-currency interest rates (middle rates) on the US dollar obtained from Datastream to proxy for the risk-free rate. Daily interest rates for 7-days, one-month, three months, six months and one year were used, while those for other maturities were obtained by linear interpolation.

3.2 Screening the Data

The raw data is screened for data errors for the purposes of the subsequent analysis. Options with zero trading volume, with less than five trading days to maturity, and less than $0.5 premium were discarded. For each day, the OEX options with time to maturity that does not exceed the maximum time-to-maturity of the XEO options are retained. This is necessary since the implied models constructed from European options cannot price American options with longer time-to-maturity.

The underlying S&P 100 of the OEX and XEO options is a dividend-paying asset. Therefore, the dividend yield to be realized over the life of the option is required as an input so as to implement the implied volatility tree models. Towards this end, historical data on dividends may be used to proxy for the expected future dividend yield. However, this type of data is backward-looking and there is no reason to assume that they forecast accurately the expected future dividends at the time the option is priced. Hence, following Ait-Sahalia and Lo (1998), the implied dividend yield calculated from put-call parity is used as a proxy.

For each day t, put-call parity states that

$$C_t + Ke^{-r_{t,\tau}\tau} = P_t + S_t e^{-q_{t,\tau}\tau} \tag{7}$$

where τ is the time-to-maturity, C_t and P_t are the European call and put options prices at time t with time-to-expiration τ, respectively, S_t is the spot price of the S&P 100 index on day t, $r_{t,\tau}$ is the discount rate on trading day t corresponding to maturity τ, K is the strike price of the option, and $q_{t,\tau}$ is the dividend yield on day t corresponding to time-to-maturity τ. The implied dividend yield is obtained from the at-the-money (ATM) market option prices. This is because the ATM options have the greatest liquidity and therefore their prices are expected not to contain significant measurement errors due to non-synchronous trading. For each day t and time-to-maturity τ, the price of the ATM call and put was calculated from the prices of the nearest-the-money call and put options by linear interpolation. To perform the linear interpolation, at least two call option and two put option should be trading for each maturity date. Therefore, maturities not complying with this condition were deleted from the dataset. Maturities where negative implied dividend yields were encountered (suggesting the existence of arbitrage opportunities) were also deleted. Finally, the standard upper and lower arbitrage bounds for the European option prices (Merton, 1973) were checked using the dividend yield obtained.

To construct the implied trees, the implied volatilities need to be backed out using Merton's (1973) model. Towards this end, only out-of-the-money and at-the-money European option prices are used. This is done so as to minimize the effect of measurement errors on the implied volatilities calculated; in-the-money options are notoriously illiquid and their prices are therefore prone to greater measurement errors (see e.g., Skiadopoulos et al., 1999, Brandt and Wu, 2002, and Panigirtzoglou and Skiadopoulos, 2004, for the same practice). Furthermore, in-the-money options are redundant due to put-call parity. Figure 1 shows the implied volatility surface of the S&P 100 for a representative date, that of April 22nd, 2003;

cubic spline interpolation was used to obtain a continuous volatility surface. We can see that the Black-Scholes assumption of a flat implied volatility surface is violated; the implied volatilities depend on both the strike and the maturity. For any given time-to-maturity, the implied volatility decreases as the strike price increases (implied volatility skew). Also, for any given strike, the implied volatility increases as the time to maturity increases. Moreover, the volatility skew attenuates as the time-to-maturity increases. These results are in accordance with those reported in the literature.

In summary, the raw data contained 123,608 XEO and 167,250 OEX contracts. Applying the above discussed constraints left us with 3,015 XEO and 11,739 OEX contracts. Table 1 shows the summary statistics for the European (XEO) and American (OEX) style options data. The average quoted bid/ask mid-point price, the average bid/ask, and the average percentage bid/ask (spread divided by the bid/ask mid-point) are reported. The results are classified according to the type of option (call or put), and the maturity class. Short-, medium- and long-term refer to options with less than 40 days, with between 40 and 70 days, and with more than 70 days to expiration, respectively.

4. Methodology

4.1 Models and Implementation

The pricing performance of the DK, BC, CRR, and AH models is compared. At any given day, each one of the models is calibrated to the European XEO options for any traded maturity; the implied volatility trees were checked that they price the set of input European options well within the bid/ask spread. Then, each one of the calibrated models is used to price the available American OEX options in our dataset. The pricing errors are assessed using various criteria.

Each one of the above four tree models is built using 50 time steps. Since options are traded across a discrete set of strike prices and times-to-maturity, an interpolation method needs to be chosen. The implied volatilities are interpolated either linearly (denoted with *L*) or using cubic splines (denoted with *CS*) with respect to the strike price[4]; for instance, DK-L denotes the Derman-Kani model implemented by using linear interpolation. Therefore, for any given date and maturity, eight binomial trees are implemented and compared: DK-L, BC-L, AH-L, CRR-L, DK-CS, AH-CS, and CRR-CS. Following Lim and Zhi (2001), the implied volatilities are interpolated linearly with respect to the maturity dimension. This is also consistent with the construction of various implied volatility indices (e.g., the method to construct the "old" VIX, currently termed VXO, see Whaley, 1993).

4.2 Assessing the Pricing Performance: Criteria

Following Brandt and Wu (2002), six measures are used to assess the pricing performance of the models employed. These are the following:

1. The mean valuation error *MVE* is the average difference between the market and model prices. The *MVE* is positive if the model overprices, and negative if it underprices a set of options on average.
2. The root mean squared valuation error *RMSVE* is the square-root of the average squared difference between the market and model prices.
3. The frequency in bid/ask *FIBA* is the frequency the model price falls within the observed bid/ask spread. The greater the FIBA, the better the model performs.

[4] Cubic splines assume that each segment between the data points is represented as a cubic polynomial. They have been commonly used to obtain smooth implied volatility surfaces (see e.g., Bates, 1991).

4. The mean outside error *MOE* is the average error outside of the bid/ask spread. This is defined as follows: if the model price is below the bid or above the ask quote, the error is defined as the model price minus the bid or ask quote, respectively. If the model price is within the bid/ask spread, the error is set to zero. *MOE* reveals whether the bid/ask violations measured by FIBA are symmetric (i.e. the model overprices as much as it underprices the options) or whether there are systematic biases in the mispricing relative to the bid/ask spread.
5. The root mean squared outside error *RMSOE* measures the variability of the errors outside the bid/ask spread.
6. The mean relative outside error *MROE* is the average outside error divided by the market price. It measures the same form of mispricing as *MOE* but in relative terms.

5. Pricing Performance: Results and Discussion

5.1 Aggregate Results

Table 2 shows the results on the comparative pricing performance of the various models in the case where the options data are considered across the whole spectrum of maturities and strike prices. The Table is structured in three panels: Panel A reports the results across the whole data set (calls and puts, aggregate results) and Panels B and C report the results for the call and put data, respectively. The results for calls and puts are reported separately because the pricing performance of the models may depend on the type of option to be priced. For instance, Lim and Zhi (2002) found that the pricing of the American put option is less accurate than that of the American call in the case where deterministic implied trees are used.

We can see that in the case where linear interpolation is applied, all models perform quite similarly in terms of the MVE, RMSVE, MOE, RMSOE and MROE metrics; the AH model presents marginally the best and the CRR model the worst performance. On the other hand, under the FIBA metric the AH model outperforms significantly the other models. Interestingly, in the case where cubic spline interpolation is applied, the CRR model outperforms the other models in all but the FIBA metric where the AH model performs best.

However, we can also see that it is preferable to use linear rather than cubic spline interpolation; Brandt and Wu (2002) had also found that it is preferable to use simpler interpolation schemes. The former yields smaller pricing errors for all models and under almost all metrics; this holds especially under the RMSVE and the RMSOE metrics. The poor performance of cubic interpolation may be explained by the fact that it yields more unacceptable transition probabilities. Therefore, more information has to be excluded compared to the case where linear interpolation is used. Finally, the comparison of the DK versus the BC model shows that the latter performs marginally better in most of the cases; some exceptions appear in the case where call options are considered. This is to be expected since the BC model has adopted modifications so as to reduce the occurrence of bad probabilities. The results discussed here hold, irrespective of whether one examines the whole data set, or the calls and puts separately.

5.2 Pricing Errors of Individual Models

In this Section, we focus on the pricing performance of the individual pricing models for given moneyness and maturity levels. The results are discussed first with respect to the moneyness level, and then with respect to the time-to-maturity. These two dimensions are examined separately since they are expected to affect the pricing performance of any pricing model; for instance, in the presence of an implied volatility smile, it is well known that the Black-Scholes model underprices away from-the-money options and overprices close-to-the-money options. Moreover, the Black-Scholes pricing biases are more severe for long maturity options (see e.g., Rubinstein, 1985).

Moneyness Dimension

Tables 3, 4, 5, and 6 show the behavior of the DK, BC, CRR and AH models with respect to the moneyness level, respectively. Call options are regarded in-the-money (ITM) when $S/K > 1.02$, at-the-money (ATM) when $0.98 \leq S/K \leq 1.02$ and out-of-the-money (OTM) when $S/K < 1.02$. Put options are characterized ITM when $S/K < 0.98$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K > 1.02$. The results for the American calls and puts are reported separately under each criterion and for each interpolation method.

We can see that in general, the DK, BC, and AH models (Tables 3, 4, and 6, respectively) perform better in the case where linear rather than cubic interpolation is applied. Regarding the CRR model (Table 5), we can see that the results on the performance of linear versus cubic spline interpolation are mixed. Hence, the discussion of the (absolute values of the) results focuses on the linear interpolation case. Moreover, the results depend on the type of option (put or call), the moneyness level, and the criterion under which they are assessed. This is in line with the implications drawn in Lim and Zhi (2002).

In particular, regarding the MVE and RMSVE criteria, in the case of the DK, BC, and AH models, the most severe mis-pricing occurs for the ITM options while the results improve for OTM options. The CRR model performs best for ATM options. These results hold for both calls and puts. Regarding the FIBA criterion, in the case of the call options the AH-FIBA increases as they get ITM (it exceeds 70%); this is also the case for the implied tree models.

In the case of put options, the DK, BC, and AH-FIBA are maximized for ATM options. The CRR-FIBA criterion is maximized for ATM call and ITM put options; OTM call and put options present a poor CRR-FIBA performance.

Regarding the MOE metric, the systematic biases relative to the bid-ask spread are greatest for ITM options and they decrease as the option gets OTM in the cases where the DK and BC models are used. On the other hand, in the case of the CRR model, the MOE reaches its maximum value for OTM options. In the case of the AH model, the MOE is maximized for OTM calls and ITM puts. The RMSOE is maximized for ITM options and it is minimized for OTM options in the case where the DK, CRR, and AH models are used. In the case of the BC model, the RMSOE is maximized for ATM calls and ITM puts. Finally, the MROE criterion is maximized for OTM and it is minimized for ITM options.

Maturity Dimension

Figure 2 shows the pricing error (model price minus market price) of ITM call and put options separately using the DK-CS model with respect to the time-to-maturity. The first row corresponds to call options and the second row to put options. The horizontal axis measures the ratio *S/K* and the vertical axis measures the pricing error. We can see that the range of pricing errors decreases significantly as the time-to-maturity increases. This is in line with the results that have been previously reported in the literature on the behavior of the pricing errors incurred by the Black-Scholes model. The results are quite similar for OTM, ATM, and ITM options for both interpolation methods and for all pricing models[5]. Hence, given the space limitations no additional figures are reported.

[5] This is not the case for the AH-ATM call options; it only holds for AH-ITM calls. Two more exceptions occur. The first appears in the BC model: the medium-term options have the narrowest range of valuation error in the case where cubic spline interpolation is applied. The second exception occurs for the CRR model where the

6. Conclusions

This paper has been motivated by the extensive literature on smile consistent models and has compared the pricing performance of four widely used pricing models. The Derman and Kani (DK, 1994) and Barle and Cakici (BC, 1995) deterministic volatility implied binomial trees were compared to the standard Cox-Ross-Rubinstein (CRR, 1979) and the traders' ad-hoc (AH) implied model. To this end, the S&P 100 options data set was used for the first time. A number of criteria were employed to compare the models under investigation. Linear and cubic splines were used separately, to obtain a continuum of implied volatilities across strikes. The comparative performance of the models was first investigated by using the whole data set (aggregate level analysis). Then, the performance of each model was analyzed for given maturity and moneyness levels.

We found that for any given model and metric, linear interpolation should be preferred to cubic spline interpolation since the former yields smaller pricing errors. Focusing on the linear interpolation case, on the aggregate level the four models compete closely, with the AH model performing better and the CRR model worse. The BC model performs marginally better than the DK model despite the additional modifications to the DK that it has adopted. Moving on to the pricing performance of each individual model with respect to moneyness, we found that the results are mixed depending on the type of option (put or call), and on the criterion under which the model is assessed. The evidence is far clearer in the case where the performance of each model is evaluated with respect to the time-to-maturity; the pricing errors decrease as the time-to-maturity increases.

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range of pricing error is smaller for the short-term OTM calls (linear interpolation case) and for the ATM medium-term calls (cubic interpolation case).

The results of this study have at least three implications. First, the interpolation method does affect the pricing performance of the models under investigation. In fact, a more complex interpolation method may not necessarily improve the pricing performance of the model. This is in line with the results found in Brandt and Wu (2002). Second, the presence of implied volatility smiles does not dictate that implied volatility tree models should necessarily be used on an *aggregate* level. Simpler models that lack of a sound theoretical foundation such as the ad-hoc model may be used instead without loss of accuracy in pricing terms. This is in accordance with the results found in previous studies that compared competing pricing models, which found that simpler is better (see e.g., Backshi et al., 1997, and Dumas et al., 1998, Brandt and Wu, 2002). Finally, the pricing performance of *each* model depends on the metric under which it is assessed, on the type of option to be priced (call or put), its moneyness level, and the time-to-maturity. This is in line with the implication drawn from Lim and Zhi (2002) who also performed a horse race of implied volatility models and stated "the results…indicate that different methods should be used for different applications, and some cautions should be exercised". Future research should investigate the hedging performance of implied models for various hedging schemes (e.g., delta, gamma, vega). The performance of deterministic volatility implied models versus the more complex stochastic volatility implied models should also be studied. In the interests of brevity, these extensions are best left for future research.

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Table 1: Summary Statistics for the European and American style S&P 100 options

Maturity	Short		Medium		Long	
	Call	*Put*	*Call*	*Put*	*Call*	*Put*
Retained Options (% of Same Maturity Options)	1,300 (89.97%)	1,439 (91.66%)	122 (8.44%)	118 (7.52%)	23 (1.59 %)	13 (0.83%)
Average Mean Price	5.405	5.771	8.868	10.582	17.058	30.2
Average Bid/Ask Spread	0.631	0.641	0.937	1.016	1.524	2.662
Average % Bid/Ask Spread	0.172	0.157	0.144	0.123	0.137	0.094
Subtotal (% of Options)	2,739 (90.85%)		240 (7.96%)		36 (1.19%)	
Retained Options (% of Same Maturity Options)	4,588 (44.44%)	5,735 (55.56%)	590 (44.46%)	737 (55.54%)	37 (41.57%)	52 (58.43%)
Average Mean Price	15.491	15.719	13.908	22.079	17.957	29.198
Average Bid/Ask Spread	0.941	0.902	1.079	1.267	1.307	1.552
Average % Bid/Ask Spread	0.112	0.110	0.143	0.116	0.095	0.082
Subtotal (% of Options)	10,323 (87.94%)		1,327 (11.30%)		89 (0.76%)	

This Table shows the summary statistics for daily data on European (XOE) and American style (OEX) S&P 100 index options from August 15, 2001 to July 21, 2003. For each type of option and maturity class, we report (i) the average quoted bid/ask mid-point price, (ii) the average bid/ask spread, and (iii) the average percentage bid/ask spread (spread divided by the bid/ask mid-point). The percentage of retained call and put options is also reported within the parentheses. Short, Medium and Long-Term refers to options with less than 44 days, with between 44 and 81 days, and with more than 81 days to expiration, respectively.

Table 2: Pricing Performance of Derman-Kani (DK), Barle-Cakici, Cox-Ross-Rubinstein (CRR) & Ad-Hoc (AH) Models

Model	MVE	RMSVE	FIBA (%)	MOE	RMSOE	MROE
			Panel A: Aggregate Results			
DK-L	-0.142	1.215	49.511	0.022	0.943	-0.024
BC-L	-0.140	1.204	49.254	0.023	0.928	-0.025
AH-L	-0.156	1.132	51.287	-0.004	0.854	-0.036
CRR-L	-0.177	1.594	24.430	-0.101	1.254	-0.040
DK-CS	0.741	8.223	50.480	0.882	8.136	0.160
BC-CS	1.088	7.639	49.794	1.218	7.519	0.281
AH-CS	0.960	15.888	59.144	1.134	15.839	0.467
CRR-CS	-0.386	1.610	23.941	-0.254	1.289	-0.070
			Panel B: Call Options			
DK-L	-0.376	0.795	51.370	-0.135	0.569	-0.028
BC-L	-0.372	0.809	51.409	-0.130	0.586	-0.026
AH-L	-0.395	0.687	53.628	-0.162	0.452	-0.044
CRR-L	0.019	1.334	20.996	0.167	0.924	0.166
DK-CS	0.552	8.394	52.740	0.763	8.312	0.167
BC-CS	0.833	7.381	52.547	1.032	7.270	0.335
AH-CS	0.058	7.947	61.386	0.315	7.905	0.233
CRR-CS	-0.205	1.386	21.459	-0.009	0.988	0.124
			Panel C: Put Options			
DK-L	0.046	1.440	48.023	0.149	1.142	-0.020
BC-L	0.045	1.418	47.529	0.144	1.115	-0.023
AH-L	0.035	1.358	49.413	0.122	1.056	-0.030
CRR-L	-0.033	1.759	27.177	-0.315	1.430	-0.205
DK-CS	0.892	8.081	48.672	0.978	7.993	0.155
BC-CS	1.291	7.835	47.591	1.366	7.711	0.238
AH-CS	1.681	20.065	57.350	1.789	20.016	0.654
CRR-CS	-0.531	1.756	25.927	-0.451	1.457	-0.226

The models are implemented using linear (L) interpolation (DK-L, BC-L, AH-L, CRR-L) and Cubic Spline (CS) interpolation (DK-CS, BC-CS, AH-CS, CRR-CS). We measure the pricing errors by (i) the mean valuation error (MVE), (ii) the root mean squared valuation error (RMSVE), (iii) the frequency in bid/ask spread (FIBA), (iv) the mean outside error (MOE), (v) the root mean squared outside error (RMSOE), and (vi) the mean relative outside error (MROE). The pricing performance is examined for the whole data set (call and put options, aggregate results), and for the call and put options, separately.

Table 3: Pricing Performance of the Derman & Kani (1994) Model

Moneyness	MVE		RMSVE		FIBA (%)		MOE		RMSOE		MROE	
	L	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*
Panel A: Call Options												
OTM	-0.222	0.701	0.668	6.659	36.326	44.444	-0.109	0.743	0.550	6.636	-0.048	0.312
ATM	-0.299	-0.108	0.834	6.161	64.896	53.285	-0.138	0.150	0.587	6.096	-0.009	0.003
ITM	-0.734	0.793	0.870	12.093	69.385	68.169	-0.184	1.286	0.589	11.912	-0.005	0.019
Panel B: Put Options												
OTM	-0.117	0.650	0.813	6.978	41.603	47.274	-0.029	0.693	0.684	6.951	-0.047	0.247
ATM	0.393	0.595	1.428	6.204	62.985	48.921	0.333	0.678	1.188	6.123	0.020	0.035
ITM	0.163	1.636	2.253	10.967	51.789	51.607	0.414	1.820	1.711	10.780	0.010	0.036

We measure the pricing errors by (i) the mean valuation error (MVE), (ii) the root mean squared valuation error (RMSVE), (iii) the frequency in bid/ask spread (FIBA), (iv) the mean outside error (MOE), (v) the root mean squared outside error (RMSOE), and (vi) the mean relative outside error (MROE). OTM, ATM, ITM stand for out-of-the-money, at-the-money, and in-the-money, respectively. Call options are regarded ITM when $S/K > 1.02$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K < 1.02$. Put options are characterized ITM when $S/K < 0.98$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K > 1.02$. The pricing errors are classified according to their moneyness (OTM, ATM, ITM), their type (Call or Put), and the employed interpolation method (linear, L, or cubic spline, CS).

Table 4: Pricing Performance of the Barle & Cakici (1995) Model

Moneyness	MVE		RMSVE		FIBA (%)		MOE		RMSOE		MROE	
	L	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*
Panel A: Call Options												
OTM	-0.213	1.389	0.703	7.619	36.326	44.968	-0.098	1.421	0.585	7.568	-0.045	0.629
ATM	-0.293	0.595	0.855	7.999	64.806	52.115	-0.132	0.844	0.599	7.881	-0.009	0.036
ITM	-0.740	-0.045	0.844	6.220	69.599	67.382	-0.188	0.437	0.574	6.036	-0.005	0.010
Panel B: Put Options												
OTM	-0.124	1.262	0.763	7.838	41.249	46.510	-0.038	1.298	0.626	7.784	-0.052	0.382
ATM	0.398	1.245	1.389	7.960	62.726	47.627	0.333	1.311	1.149	7.844	0.020	0.066
ITM	0.171	1.390	2.252	7.742	50.819	49.970	0.419	1.558	1.706	7.450	0.010	0.037

We measure the pricing errors by (i) the mean valuation error (MVE), (ii) the root mean squared valuation error (RMSVE), (iii) the frequency in bid/ask spread (FIBA), (iv) the mean outside error (MOE), (v) the root mean squared outside error (RMSOE), and (vi) the mean relative outside error (MROE). OTM, ATM, ITM stand for out-of-the-money, at-the-money, and in-the-money, respectively. Call options are regarded ITM when $S/K > 1.02$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K < 1.02$. Put options are characterized ITM when $S/K < 0.98$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K > 1.02$. The pricing errors are classified according to their moneyness (OTM, ATM, ITM), their type (Call or Put), and the employed interpolation method (linear, L, or cubic spline, CS).

Table 5: Pricing Performance of the Cox-Ross-Rubinstein (1979) Model

Moneyness	MVE		RMSVE		FIBA (%)		MOE		RMSOE		MROE	
	L	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*
Panel A: Call Options												
OTM	0.947	0.719	0.687	0.783	6.809	10.587	0.745	0.551	0.614	0.661	0.333	0.261
ATM	-0.244	-0.561	0.843	1.054	49.865	48.515	-0.118	-0.308	0.547	0.813	-0.003	-0.019
ITM	-1.552	-1.697	0.985	1.057	25.179	20.744	-0.711	-0.840	0.857	0.947	-0.020	-0.025
Panel B: Put Options												
OTM	-1.185	-1.340	0.966	1.046	3.790	1.772	-0.978	-1.118	0.863	0.945	-0.376	-0.407
ATM	0.435	0.105	1.356	1.388	55.824	48.835	0.321	0.157	1.115	1.149	0.018	0.004
ITM	1.015	0.816	2.217	2.180	59.066	63.554	0.713	0.607	1.786	1.748	0.018	0.015

We measure the pricing errors by (i) the mean valuation error (MVE), (ii) the root mean squared valuation error (RMSVE), (iii) the frequency in bid/ask spread (FIBA), (iv) the mean outside error (MOE), (v) the root mean squared outside error (RMSOE), and (vi) the mean relative outside error (MROE). OTM, ATM, ITM stand for out-of-the-money, at-the-money, and in-the-money, respectively. Call options are regarded ITM when $S/K > 1.02$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K < 1.02$. Put options are characterized ITM when $S/K < 0.98$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K > 1.02$. The pricing errors are classified according to their moneyness (OTM, ATM, ITM), their type (Call or Put), and the employed interpolation method (linear, L, or cubic spline, CS).

Table 6: Pricing Performance of the Ad-hoc Model

Moneyness	MVE		RMSVE		FIBA (%)		MOE		RMSOE		MROE	
	L	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*	*L*	*CS*
Panel A: Call Options												
OTM	-0.262	0.417	0.474	6.041	39.469	59.110	-0.151	0.510	0.349	6.025	-0.078	0.462
ATM	-0.271	-0.564	0.762	0.977	66.157	59.586	-0.140	-0.284	0.503	0.777	-0.009	-0.020
ITM	-0.751	-0.134	0.830	12.790	70.744	67.167	-0.201	0.419	0.567	12.708	-0.006	-0.003
Panel B: Put Options												
OTM	-0.146	0.975	0.665	14.419	43.457	60.196	-0.070	1.063	0.527	14.438	-0.064	1.133
ATM	0.426	0.118	1.329	1.371	63.848	55.910	0.327	0.201	1.098	1.128	0.020	0.010
ITM	0.160	4.344	2.194	33.258	52.517	52.032	0.405	4.520	1.649	33.151	0.010	0.042

We measure the pricing errors by (i) the mean valuation error (MVE), (ii) the root mean squared valuation error (RMSVE), (iii) the frequency in bid/ask spread (FIBA), (iv) the mean outside error (MOE), (v) the root mean squared outside error (RMSOE), and (vi) the mean relative outside error (MROE). OTM, ATM, ITM stand for out-of-the-money, at-the-money, and in-the-money, respectively. Call options are regarded ITM when $S/K > 1.02$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K < 1.02$. Put options are characterized ITM when $S/K < 0.98$, ATM when $0.98 \leq S/K \leq 1.02$ and OTM when $S/K > 1.02$. The pricing errors are classified according to their moneyness (OTM, ATM, ITM), their type (Call or Put), and the employed interpolation method (linear, L, or cubic spline, CS).

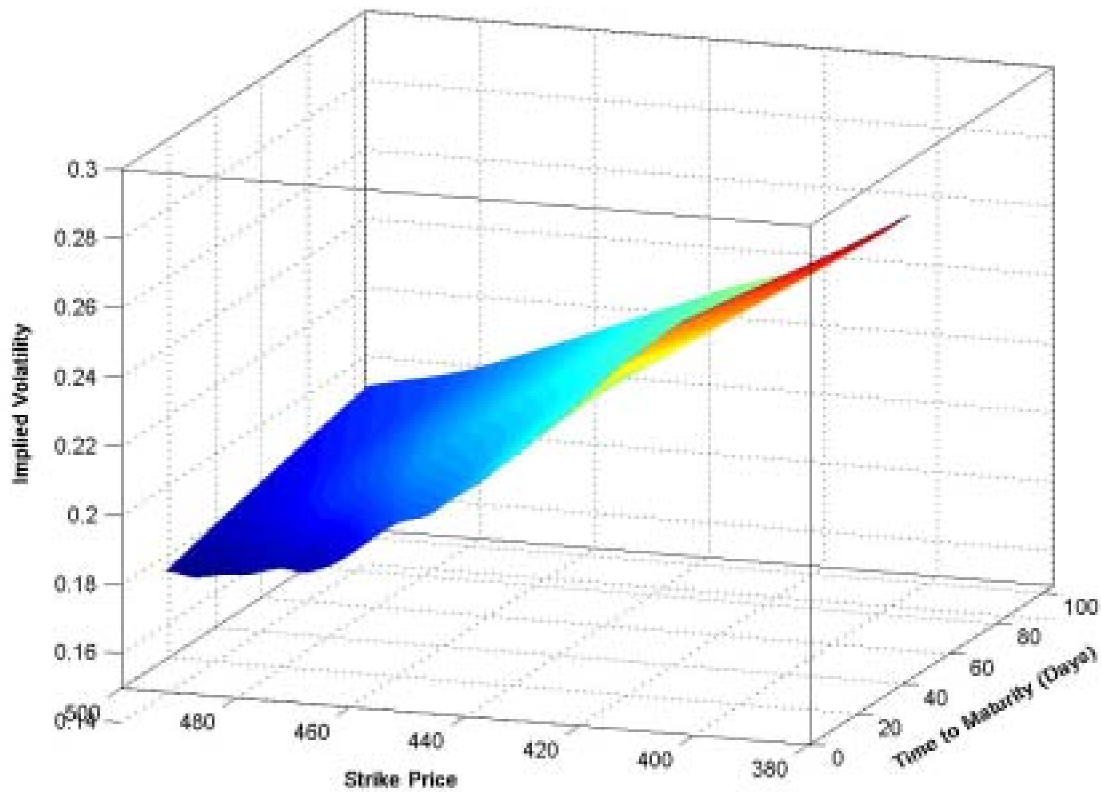

Figure 1: Implied Volatility Surface obtained from the European Style S&P 100 Options (XEO) on April 22nd, 2003.

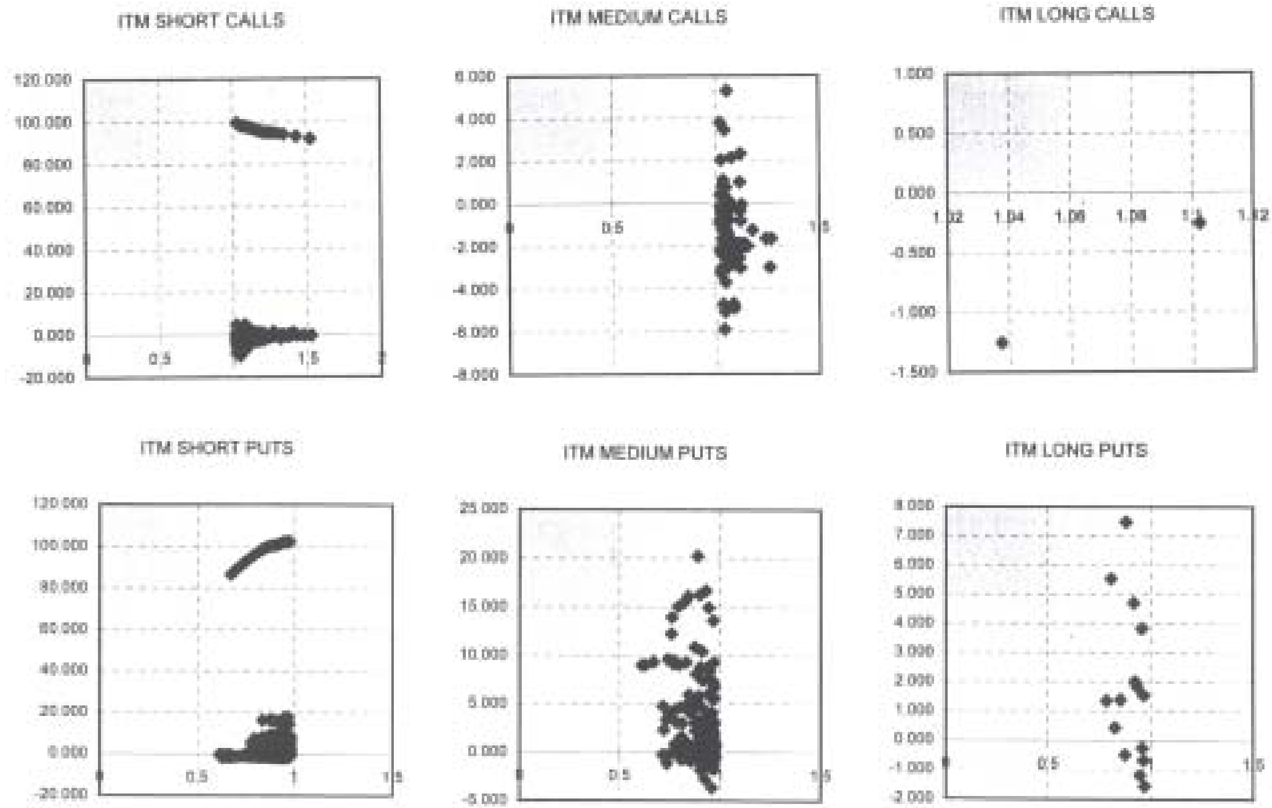

Figure 2: Pricing Errors of in-the-money (ITM) call and put options as a function of the moneyness level for various times-to-maturity. Short, Medium and Long-Term refers to options with less than 44 days, with between 44 and 81 days, and with more than 81 days to expiration, respectively. The options have been priced by the Derman and Kani model using cubic splines interpolation (DK-CS).