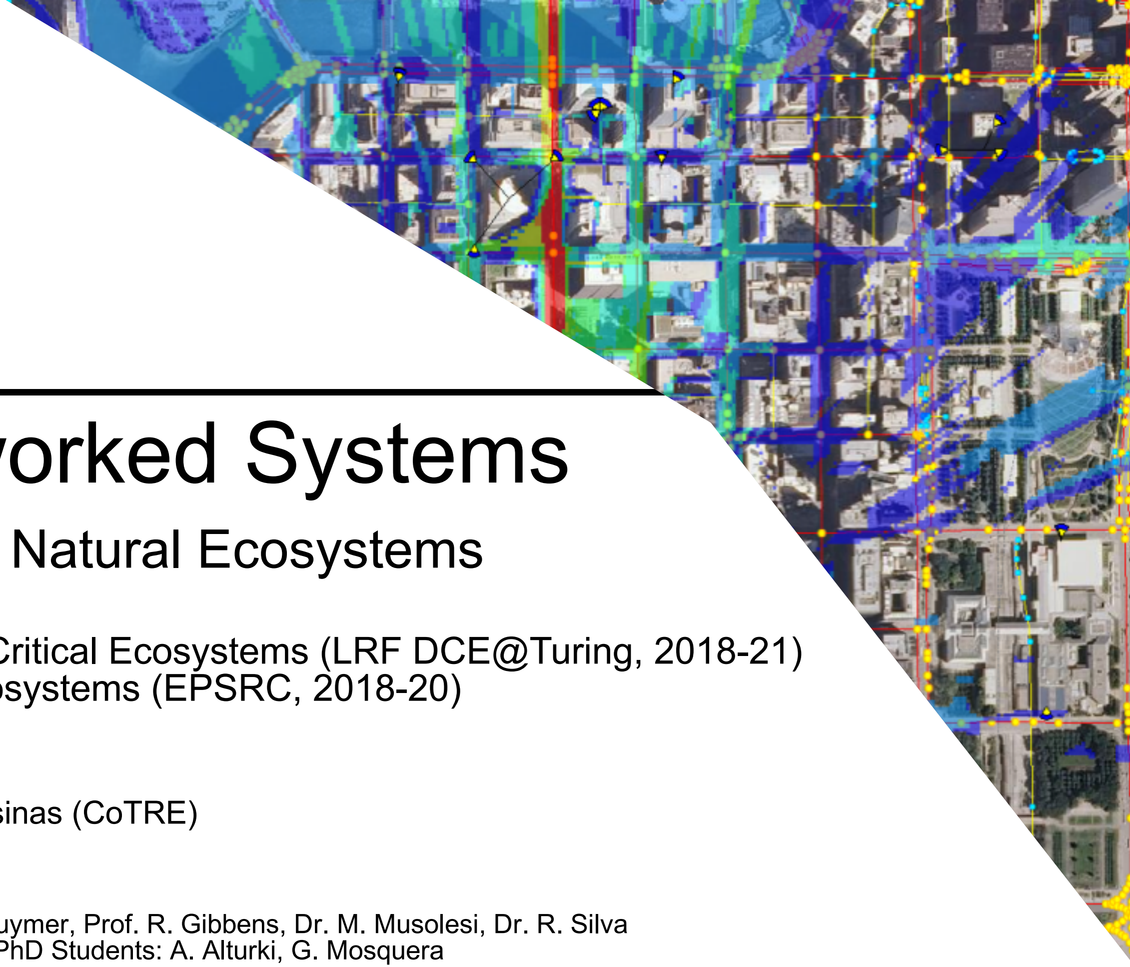


Resilience of Networked Systems

Critical Built Infrastructures & Natural Ecosystems

CHANCE - Coupled Human And Natural Critical Ecosystems (LRF DCE@Turing, 2018-21)
CoTRE - Complexity Twin to Resilient Ecosystems (EPSRC, 2018-20)

Dr. Weisi Guo
Dr. Alessio Pagani (CHANCE), Dr. Giannis Moutsinas (CoTRE)
Alan Turing Institute and University of Warwick

Co-Investigators: Sir A. Wilson, Prof. J. McCann, Prof. I. Guymer, Prof. R. Gibbens, Dr. M. Musolesi, Dr. R. Silva
Other Collaborators: Dr. S. Johnson and Prof. L. Varga, & PhD Students: A. Alturki, G. Mosquera

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Both Individual Function & Global Topology Contribute to Resilience

1. **Review**
2. **Deterministic Analysis: Telecoms, Food Webs, Gene Regulation**
3. **Data-Driven Analysis: Rail Transport, Water Distribution**

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Critical Infrastructure & Ecosystems

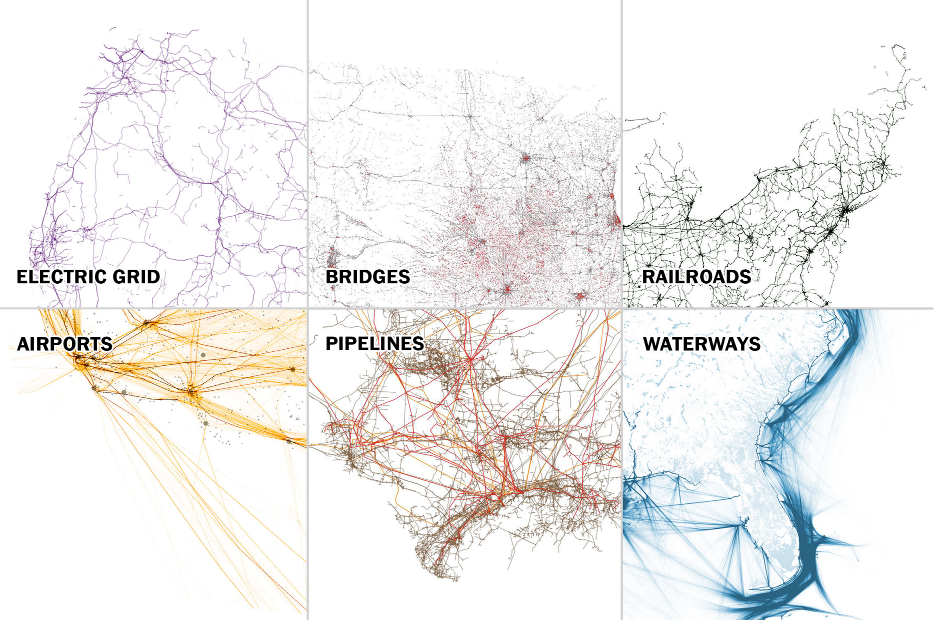
Many of our built infrastructures are networked together:

- Water Supply
- Transportation
- Electricity Supply
- Telecommunications

They combine local **functional elements** with interdependent **coupling elements**. Together, they form the backbone of our modern civilization, providing services to billions.

These often sit alongside natural ecosystems with network dimensions:

- Food Webs
- Organizational Structure
- Gene Regulation



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Challenge of Cascades on Networks

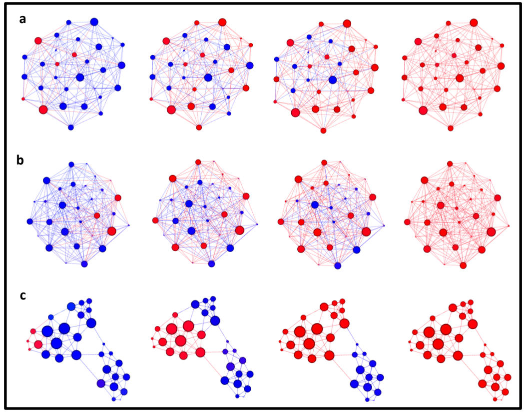
Background: Cascade effects (undesirable, not designed) are a common problem on networked systems.

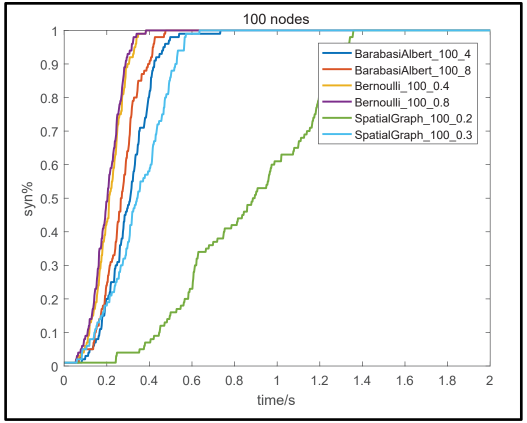
Examples: virus (cyber and epidemic), pollution, false information…etc.

Literature: we know how effects spread, but we don't know that well how it affects and is affected by the performance and behaviour of the individual components.

Goal: What we want to understand is their individual resilience behaviour and their coupled resilience behaviour.

Challenge: The dimensionality of the problem is huge (networks with millions of nodes), and the behaviour can be complex models. How do we gain meaningful insight?





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What is Networked Resilience & Robustness

1. Simple Graphs
2. Dynamical Systems

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

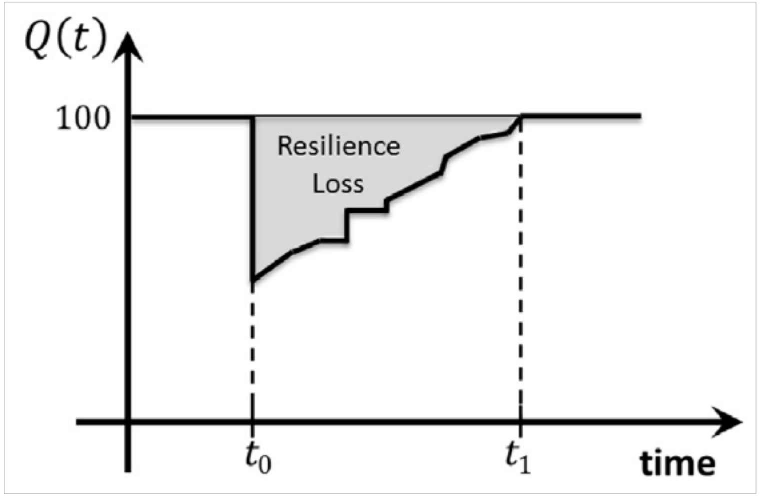
Brief Review

A review of definitions and measures of system resilience

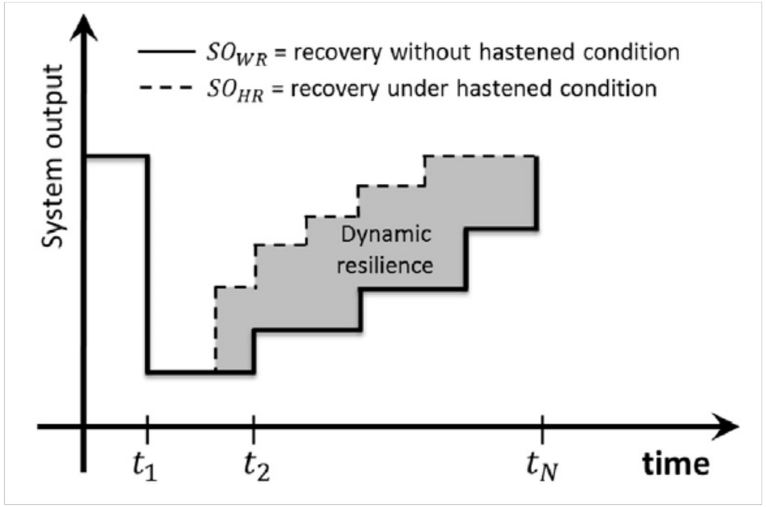
Seyedmohsen Hosseini [a], Kash Barker [a,*], Jose E. Ramirez-Marquez [b,c]

[a] School of Industrial and Systems Engineering, University of Oklahoma, United States
[b] School of Systems and Enterprises, Stevens Institute of Technology, United States
[c] Tec de Monterrey, School of Science and Engineering, Zapopan Guadalajara, Mexico

Bouncing Back: systems that return to their operating state after a negative shock is the commonality in resilient behaviour. Rate of return, asymptotic convergence, are all important metrics.



Hasten & Bounce Forward using Networks: couplings on the network can help individuals bounce back faster, as well as move forward by connecting them (rewiring) to new elements that can improve their resilience in the future.



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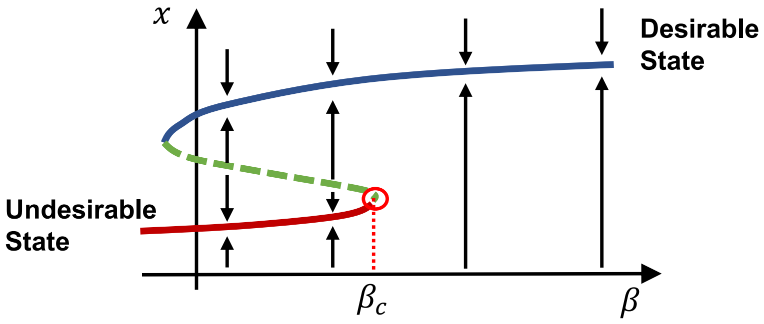
Resilience

Resilience is important as systems constantly face stressors (demand spikes) and perturbations.

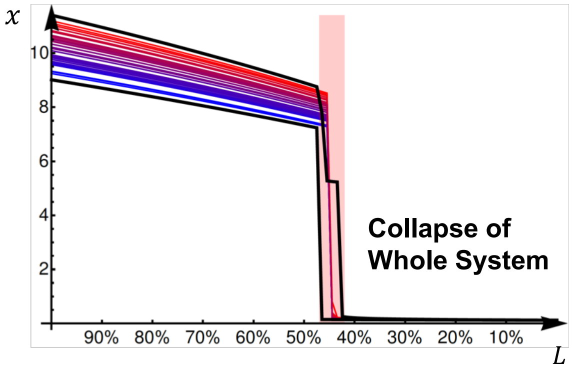
Each system has a desirable equilibrium state and want to avoid undesirable states. Cascades can cause system wide poor performance.



Robustness

Robustness is important as systems constantly face complete failures at the sub-system or connector level.

Each sub-system requires connectivity to function properly. Failures can lead to cascade failures.



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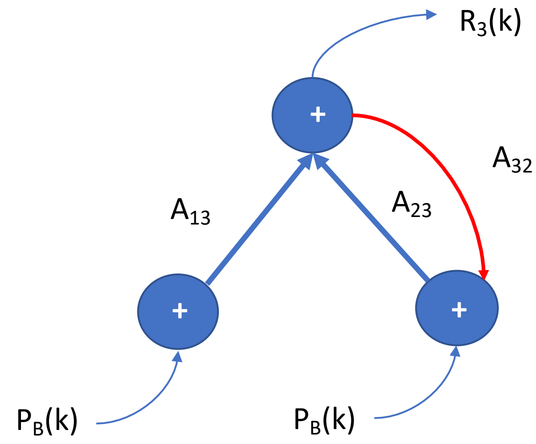
Brief Review of Resilience



Stability: defined as return to a desirable equilibrium after some perturbation or shock.

Random Graphs: In random graphs, it was shown (May 1972) that instability (largest eigenvalue) scales with size of the network (*N*) and average connectivity (*C*):

$$\propto \sqrt{NC}$$

Later expanded to random graphs with delays.

Linear Dynamics on Small Structured Graphs with Defined I-O: we know that linear stability is defined by the largest root of the transfer function.

Large Structured Graphs with No I-O: we don't know. So we currently check no. loops / trophic coherence (2nd part of talk), but we can also develop some new theories (1st part).


Trophic coherence determines food-web stability

Samuel Johnson, Virginia Domínguez-García, Luca Donetti, and Miguel A. Muñoz

PNAS December 16, 2014 111 (50) 17923-17928; published ahead of print December 2, 2014
https://doi.org/10.1073/pnas.1409077111


Looplessness in networks is linked to trophic coherence

Samuel Johnson and Nick S. Jones

PNAS published ahead of print May 16, 2017 https://doi.org/10.1073/pnas.1613786114

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

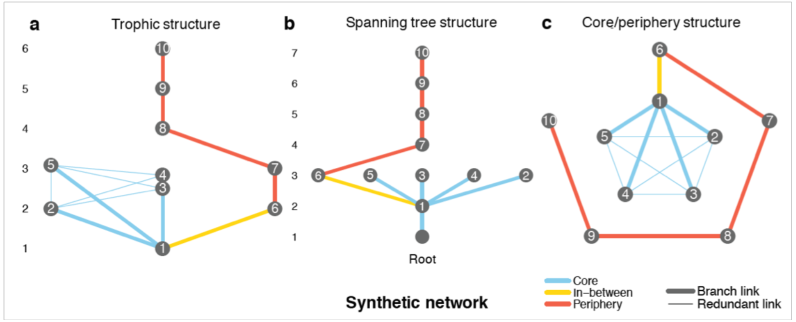
Brief Review of Robustness

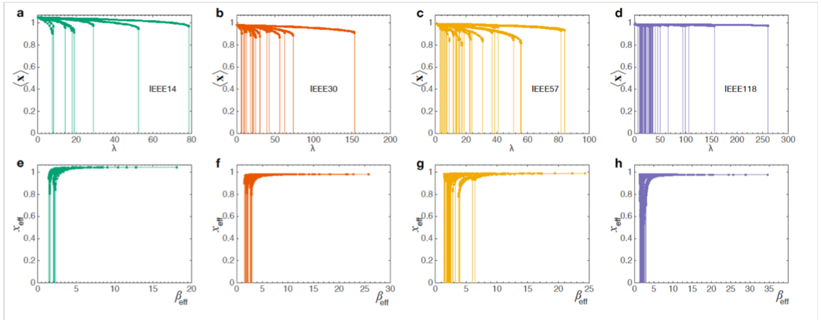
Attack/Removal Process: random removal/targeted removal leads to slow destruction of network.

Failure Process: loss of complete connectivity = loss of functionality (assumes coupling determines function).

Relation to Network Structure: easy to see the role of network topology on the overall performance (e.g. small world network robust to random attacks, vulnerable to targeted attacks).

Cascades on Electricity Networks: cascade failure on electricity grid networks (Gao et al. Nature: SI 2016).





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Gap in Knowledge

1. Just Networks: Current knowledge is mostly limited to weighted or unweighted graphs (they may change over time).
2. The nodes and edges do not have dynamical behaviours.
3. Dynamics Matter: many (most?!) dynamical systems have non-linear behaviours and multiple equilibrium states.
4. In stable equilibriums, they cannot be easily disturbed, whereas in unstable equilibriums, they can be easily disturbed.
5. This already allows you to understand how modeling dynamics is important.

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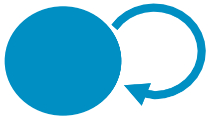
What do we know already?



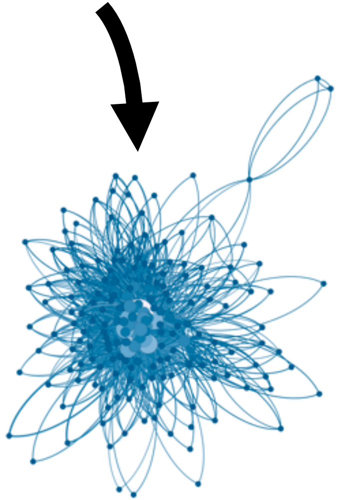
$$\frac{dx}{dt} = f(x,\beta)$$

Engineers and ecologists have a good understanding of:

- Local Dynamics & Governing Equations
- Stability and Control
- Data and Experience
- Stressors & Perturbations

But, as we connect systems in increasingly larger networks, we **don't know**:

- Gives Insight over Pure Predictive Approaches (ARIMAs, HMMs, DGPs, CNNs).
- Relationship between: Topology and Dynamics
- How Local Effects affect Network Wide Cascades



$$\frac{dx_i}{dt} = f(x_i) + \sum_{j=1}^{N} a_{ij}\, g(x_i, x_j)$$

Open Questions:

Is system resilience more sensitive to <u>network topology</u> or <u>component dynamics</u>?
How can this knowledge inform the design of new critical infrastructure systems?
What are the wider applications of this framework (ecology, biology, society)?

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Part 1: Deterministic Analysis

1. Telecommunications (Engineered)
2. Bee Pollination (Natural)
3. Gene Regulation (Natural)

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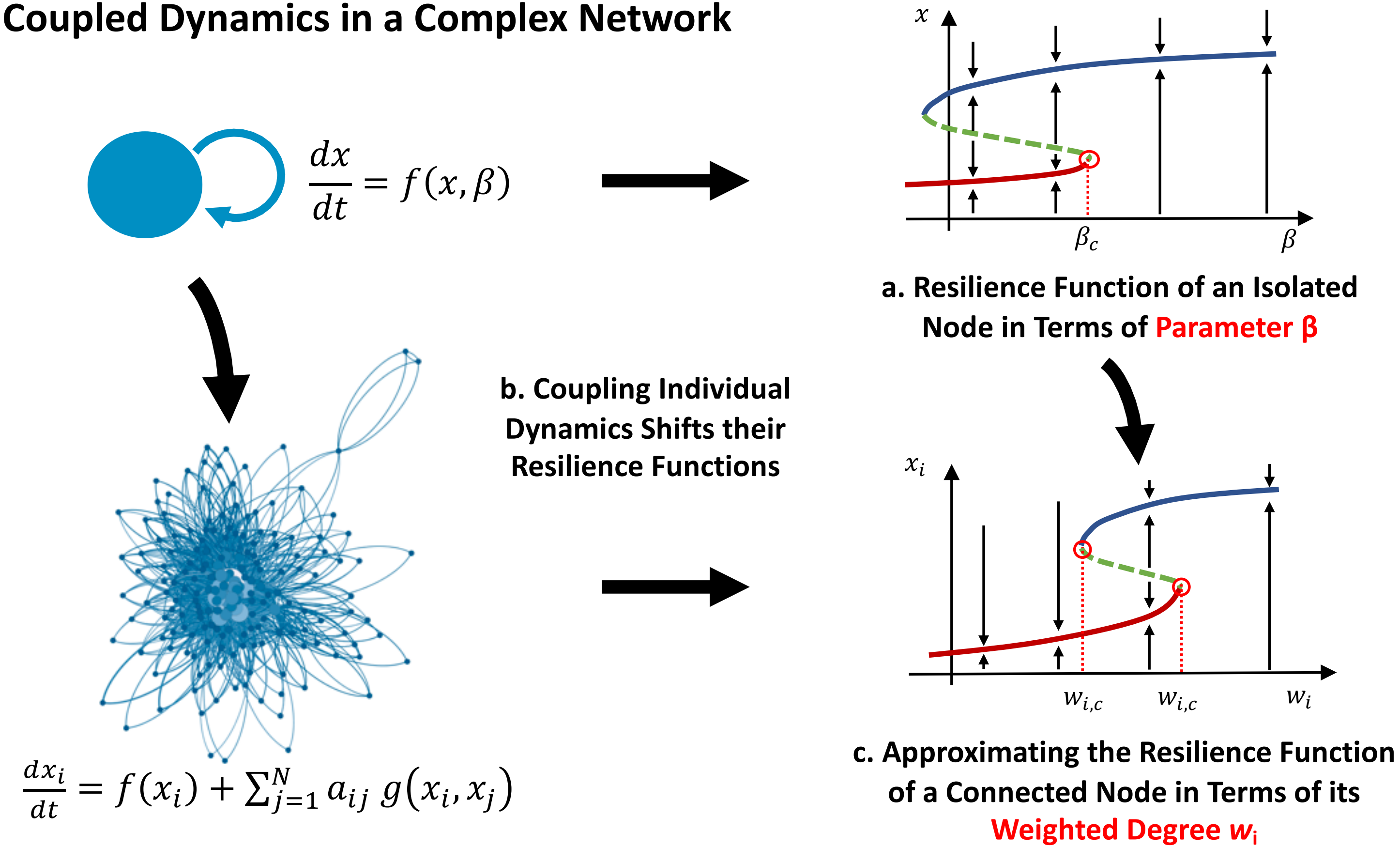
(i) Coupled Dynamics in a Complex Network



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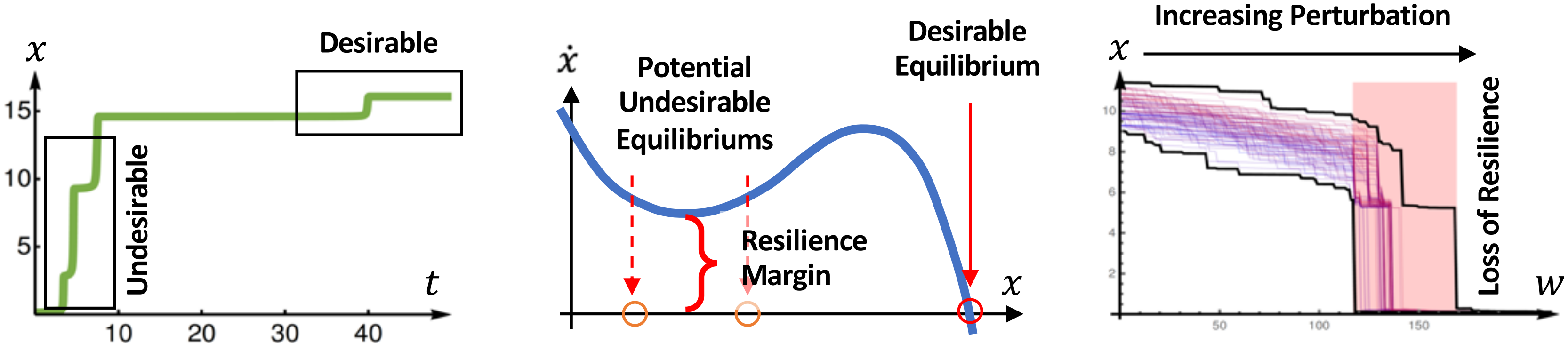
Mapping Resilience to Robustness

Familiar notions of dynamic time response (bounce back) is mapped to changing equilibrium states and a resilience margin. Networked systems can have cascade dynamics (resilience), but when it causes cascade unrecoverable failures (robustness).

(ii) Characteristic Functions



a. Dynamic Response: Recovery of Resilience

b. Rate Dynamics: Equilibriums Shift with Perturbations

c. Resilience Function: Under Perturbations (w)

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Current Literature

Discovering the explicit relationship between:

- **Average Network Dynamics** as a function:
- Local Node Dynamics
- Network Topology

This was applied to a variety of ecological and biological dynamics in 2 key papers (Nature Physics 2013) and (Nature 2016).

Basic idea is to develop a mean field approximation.

nature physics ARTICLES
PUBLISHED ONLINE: 8 SEPTEMBER 2013 | DOI:10.1038/NPHYS2741

Universality in network dynamics

Baruch Barzel[1,2] and Albert-László Barabási[1,2,3]★

Despite significant advances in characterizing the structural properties of complex networks, a mathematical framework that uncovers the universal properties of the interplay between the topology and the dynamics of complex systems continues to elude us. Here we develop a self-consistent theory of dynamical perturbations in complex systems, allowing us to systematically separate the contribution of the network topology and dynamics. The formalism covers a broad range of steady-state dynamical processes and offers testable predictions regarding the system's response to perturbations and the development of correlations. It predicts several distinct universality classes whose characteristics can be derived directly from the continuum equation governing the system's dynamics and which are validated on several canonical network-based dynamical systems, from biochemical dynamics to epidemic spreading. Finally, we collect experimental data pertaining to social and biological systems, demonstrating that we can accurately uncover their universality class even in the absence of an appropriate continuum theory that governs the system's dynamics.

LETTER
doi:10.1038/nature16948

Universal resilience patterns in complex networks

Jianxi Gao[1]*, Baruch Barzel[2]* & Albert-László Barabási[1,3,4,5]

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

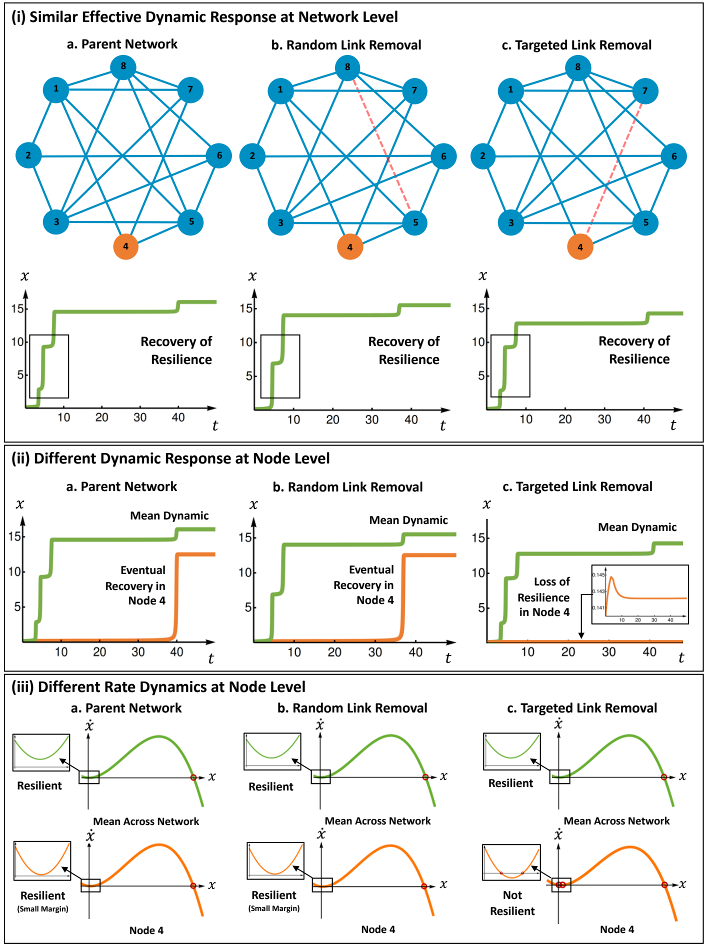
Mean Network Dynamics Hides Node Level Behaviour

Near identical networks and dynamics can hide different node level dynamics.

Here we show how the resilience of node 4 can vary between being resilient (bouncing back) to collapsing.

The overall network dynamics (Nat. Phy.13 & Nat16 papers) predicts would be the same. Indeed, one expects mean field to give similar expectations.

What we wish to do is to improve on this and give node level accurate predictions, because most interventions are made at the node level.



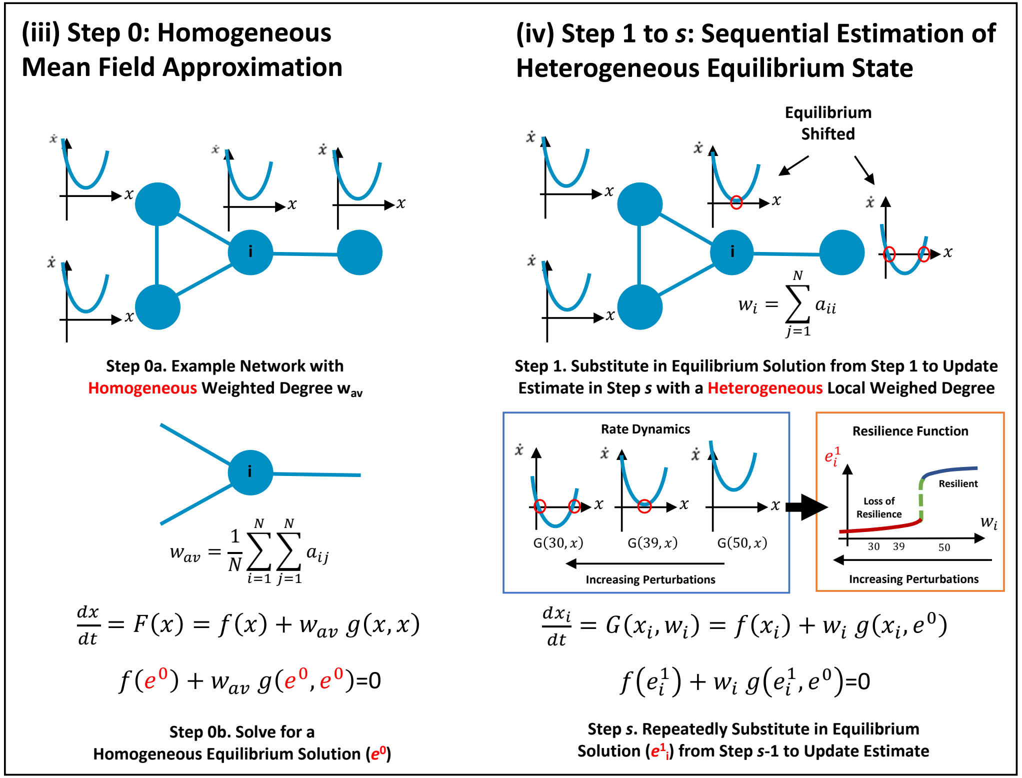
CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Sequential Estimation: Heterogeneous Mean Field

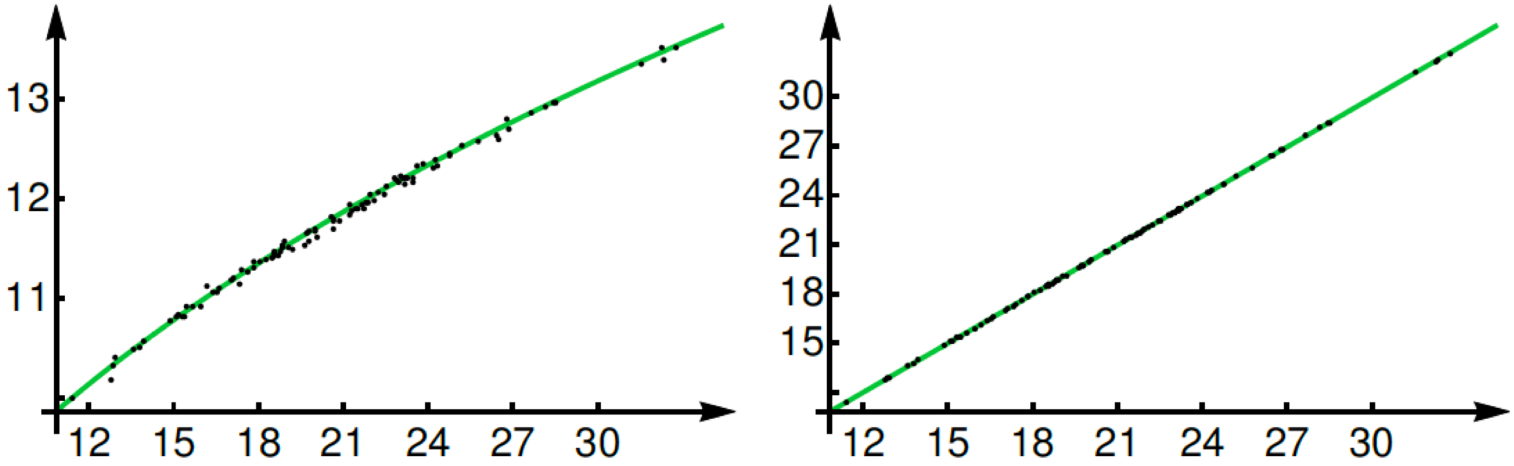
We first take a homogeneous estimate to give a mean field understanding of equilibrium states. The trick here is finding a network wide topological measure.

We then iteratively substitute this back into the network using local network measures to create heterogeneous solutions.

Improves over current methods [1] by giving node level prediction, which helps to inform action [2].



Accuracy of Equilibrium on Two Networks



(a) Mutualistic interactions dynamics. (b) Gene regulatory dynamics.

Fig. 3: The first order approximation of two dynamical systems on the same Erdös-Rényi graph with 100 vertices and $p = 0.2$. The horizontal axis is the weighted in-degree of a vertex, w_i, and the vertical axis is the value of the equilibrium at this vertex. The equilibrium computed numerically is shown in black and the blue line is the graph of the function $\chi^{\{1\}}$.

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Case Study: Telecommunications (Wireless Load Balancing)

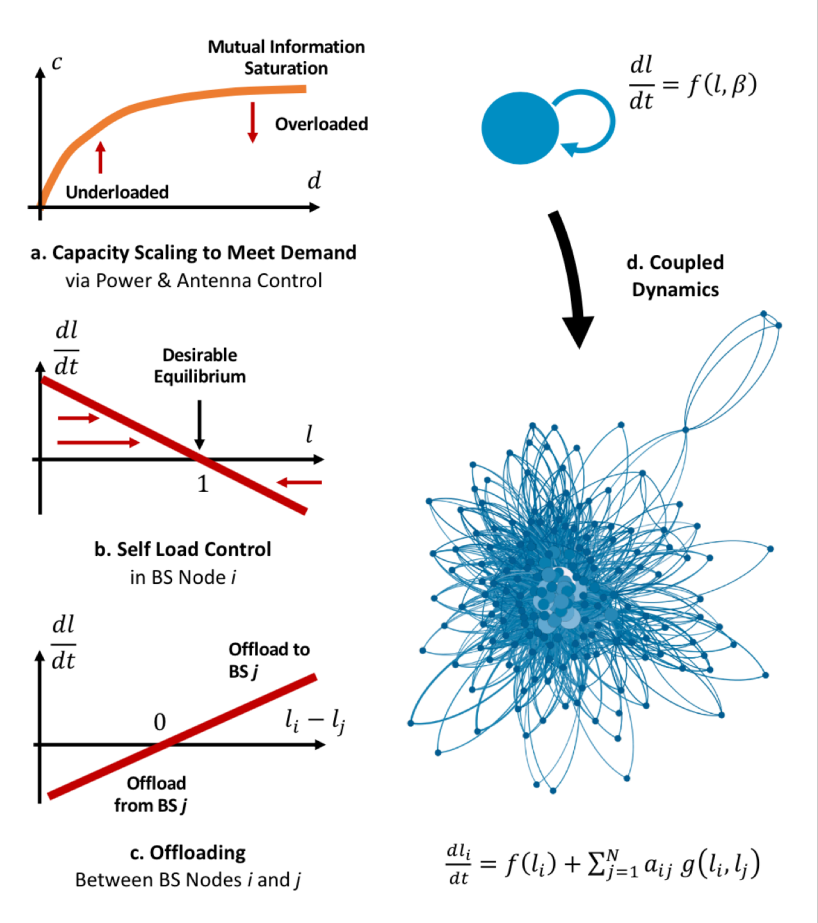
Here we study a mobile network, whereby the load demand (l) dynamics is governed by [7]:

- Load balancing inside a cell (capacity scaling using adaptive modulation coding schemes / power control / antenna switching)
- Load balancing between coupled cells
- Data demand from consumers

The load dynamics in each cell can be described by its own attempt to satisfy demand (RHS 1st term) and the coupling with other cells (RHS 2nd term).

$$\dot{l}_i = f(l_i) + \sum_{j=1}^{N} a_{ji} g(l_j - l_i),$$

$$\dot{l}_i = \beta(1 - l_i) + \sum_{j=1}^{N} a_{ji} \alpha(l_j - l_i),$$



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Stability Criteria for Load and Capacity Dynamics

A. Stability

In order to determine the stability of the equilibrium we compute the eigenvalues of the Jacobian at the equilibrium. Let F_i be the i-th component of the function F of equation (4), then we have

$$\begin{aligned} \frac{\partial}{\partial l_i} F_i(L)\Big|_{L=r\mathbf{1}} &= f'(r) - \sum_{j=1}^{N} a_{ji} g'(0) \\ &= f'(r) - \alpha \sum_{j=1}^{N} a_{ji} \\ &= f'(r) - \alpha w_i. \end{aligned} \tag{6}$$

where we define $w_i = \sum_{j=1}^{N} a_{ji}$.

When $k \neq i$ we have

$$\frac{\partial}{\partial l_k} F_i(L)\Big|_{L=r\mathbf{1}} = \sum_{j=1}^{N} \delta_{jk} a_{ji} g'(0) = \alpha a_{ki},$$

where δ_{ki} is the Kronecker delta. This equation together with equation (6) shows that the Jacobian has the form

$$J(r\mathbf{1}) = f'(r)\mathrm{Id} - \alpha D + \alpha A = f'(r)\mathrm{Id} - \alpha\Lambda,$$

where Id is the identity matrix, D is the weighted in-degree matrix and Λ the weighted in-Laplacian of the graph. Notice that the spectrum of $J(r\mathbf{1})$ is a spectral shift of the spectrum of $\alpha\Lambda$.

We assume that $\phi_i(l_i) = d_i/l_i$. This implies that $\phi_i^{-1}(c_i) = d_i/c_i$ and $\phi_i'(l_i) = -d_i/l_i^2$. Then the system (7) becomes

$$\begin{aligned} \dot{c}_i &= -\frac{c_i^2}{d_i}\left(\beta\left(1 - \frac{d_i}{c_i}\right) + \sum_{j=1}^{N} a_{ji}\alpha\left(\frac{d_j}{c_j} - \frac{d_i}{c_i}\right)\right) \\ &= \beta c_i\left(1 - \frac{c_i}{d_i}\right) + \sum_{j=1}^{N} \alpha\, a_{ji} c_i \left(1 - \frac{c_i d_j}{c_j d_i}\right). \end{aligned} \tag{8}$$

At first glance it seems that the above equation implies that the self-dynamics of a BS is given by $f(c_i) = \beta c_i(1 - c_i/d_i)$ and it has two equilibria, d_i which is stable and 0 which is unstable. The equilibrium $(d_i, \ldots, d_N)$ corresponds to the stable equilibrium of the system (3) and from this we deduce that it is not just asymptotically stable but also a global attractor of the system. The equilibrium 0, however, is not an admittable one because it appears also as a denominator and in this case the right-hand side of (8) cannot be evaluated. In a sense the 0 "equilibrium" of the system (8) corresponds to infinity in the system (3).

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Stochastic Geometry Networks with Eigenvalues Bounded by Gershgorin Circle

Eigenvalues are bounded by disks centred on the diagonal of the matrix:

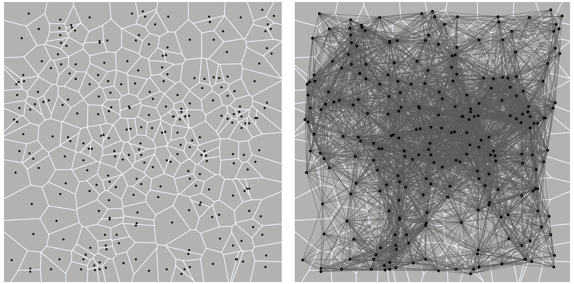
- Sum the absolute value of other row or column members
- Smallest determines radius of circle
- Each circle contains one eigenvalue.

In our case, all the Laplacian eigenvalues are positive. We show with PPP & PCP networks.

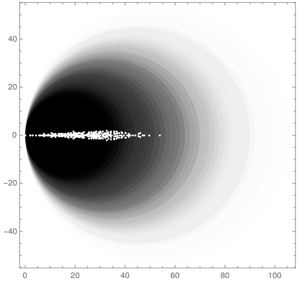
A. Gershgorin Circle Theorem

For the Laplacian it is known that 0 is an eigenvalue and that all eigenvalues have non-negative real part. The first fact can be checked easily, since the vector **1** is in the kernel of the transposed Laplacian. The second fact a consequence of Gershgorin circle theorem, [12].

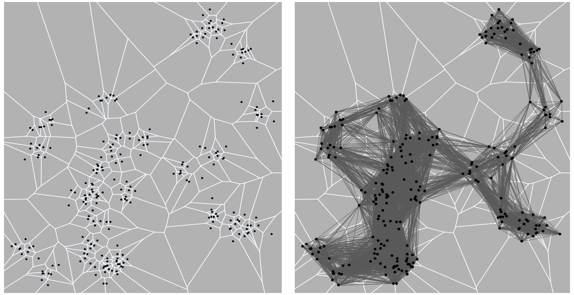
For each row of the matrix we construct the disc that has the diagonal element as centre and the sum of the absolute values of the remaining elements as radius. Gershgorin theorem states that each such disc contains at least one eigenvalue of the matrix. Since the transpose of a matrix has the same eigenvalues, we can do the same with the columns instead of the rows. In the case of the Laplacian matrix, since the sum of a row is zero and the diagonal elements are all non-negative, each disc has centre on the positive real axis and is tangent to the imaginary axis.



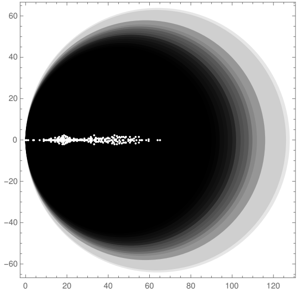
a1. PPP Distributed Cells & Random Neighbor Association



a2. Eigenvalue Distribution



b1. PCP Distributed Cells & Random Neighbor Association



a2. Eigenvalue Distribution

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Case Study: Bee Pollination

Here we study bee pollination network, whereby the population dynamics is governed by:

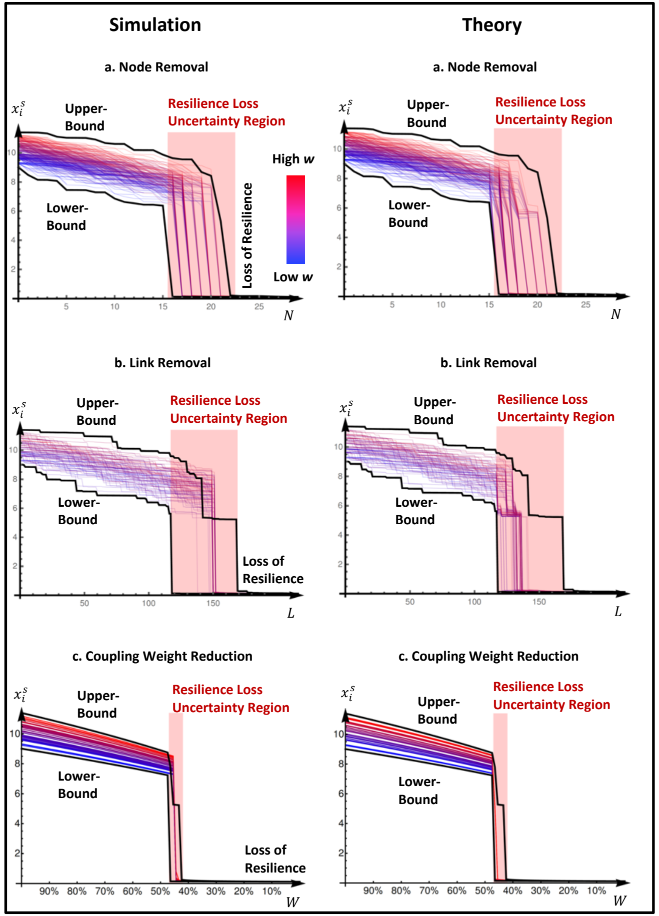
- Carrying Capacity of Bees (*K*)
- Allee Effect (critical hive threshold, *C*)
- Mutualistic Interactions

$$\frac{dx_i}{dt} = x_i(1 - \frac{x_i}{K})(\frac{x_i}{C} - 1) + \sum_j^N a_{ji} \frac{x_i x_j}{D_i + E_i x_i + H_j x_j} + B_i,$$

We show excellent predictability of both the resilience collapse subject to 3 standard perturbation simulations:

- Node Removal: dying of bee colonies
- Link Removal: cut-off from interactions/migration
- Weight Reduction: lowering of interactions

Gives insight into Colony Collapse Disorder.



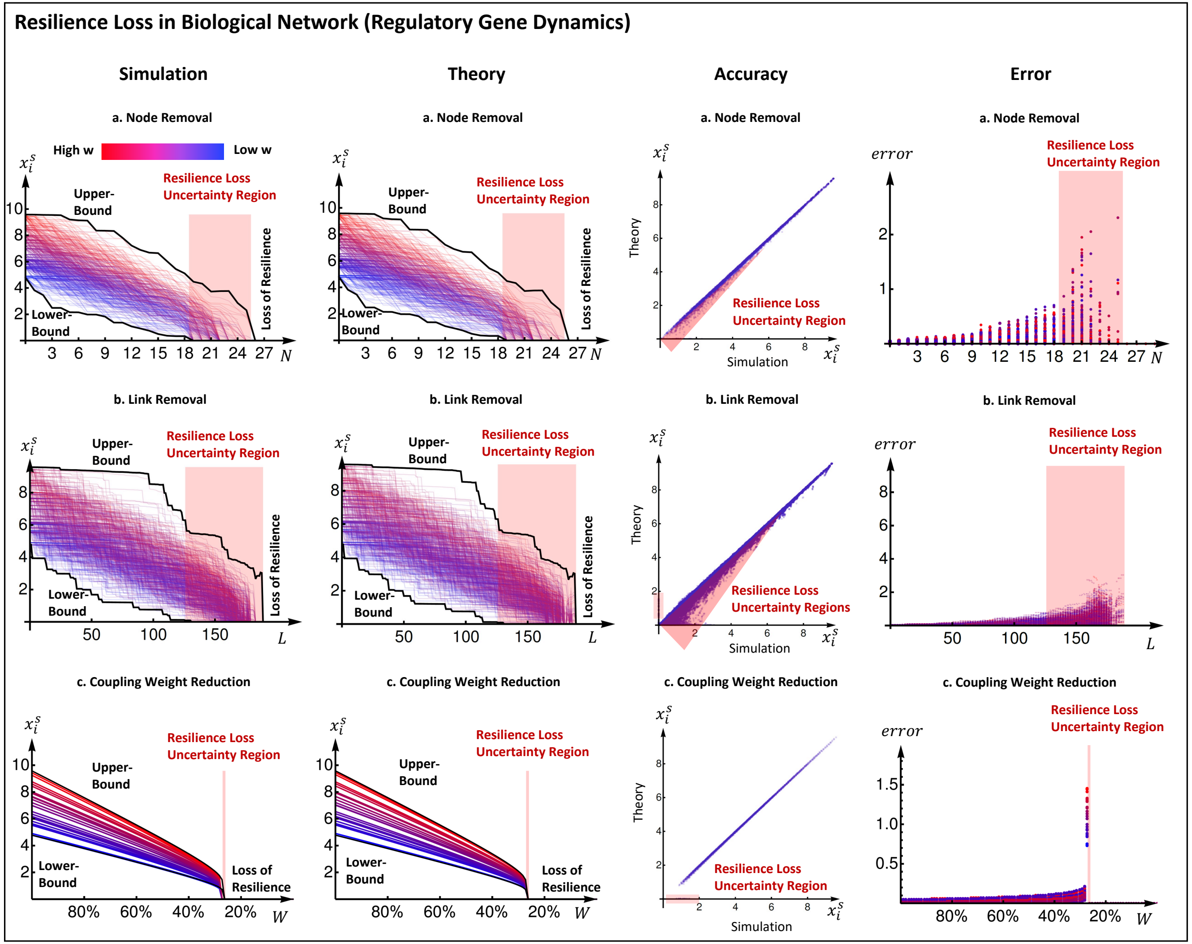
CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Case Study: Gene Regulation

Michaelis-Menten Kinetics infer gene regulatory network.

Smoother collapse profile leads to more accurate predictions.

Error increases towards collapse regime (as network becomes very small and mean field estimation becomes less meaningful).



CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Part 2: Phenomenological Relations*

1. Rail Transportation (Engineered)
2. Water Networks (Engineered)

* High-Dimensional or without Closed Form Solutions

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Resilience

In absence of well defined measures of resilience, we must use data to help us find proxy measures. We use hierarchical coherence as a proxy for measuring the stability of feedback loops on large complex networks. The **hierarchical level*** (trophic level) is defined as:

$$s_i = 1 + \frac{1}{k_i^{in}} \sum_j a_{ij} s_j.$$

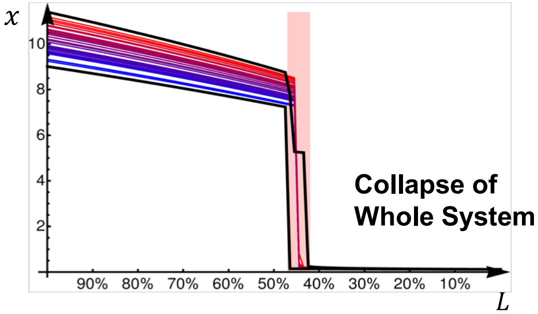
The **incoherence** of the network (instability) is defined as [3]:

$$q = \sqrt{\frac{1}{L} \sum_{ij} a_{ij} x_{ij}^2 - 1}$$

Robustness

Robustness can be well simulated using random and targeted node/link removal.

Identifying the average number of steps until collapse or decay to 50% is quite common. Other mesoscopic proxies such as core-periphery size and rich club coefficient can also be used [4].



CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

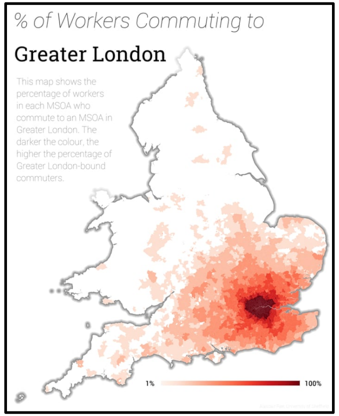
Case Study: Transport (Rail)

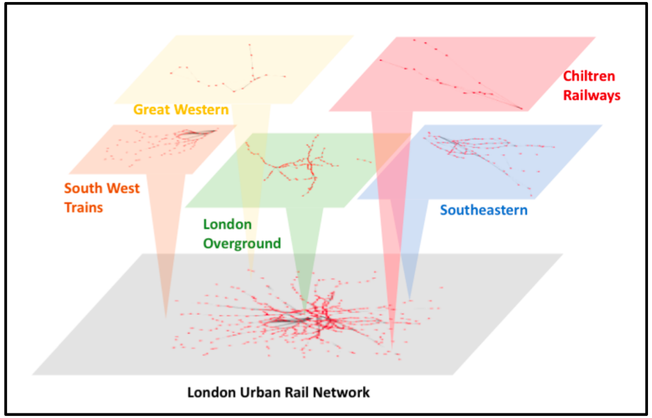
Here we study morning commuter rail travel, using census data and transport planning API to examine [5]:

- Which railway route I(include which train and what service) people will take (if any)
- How long it will take to get there

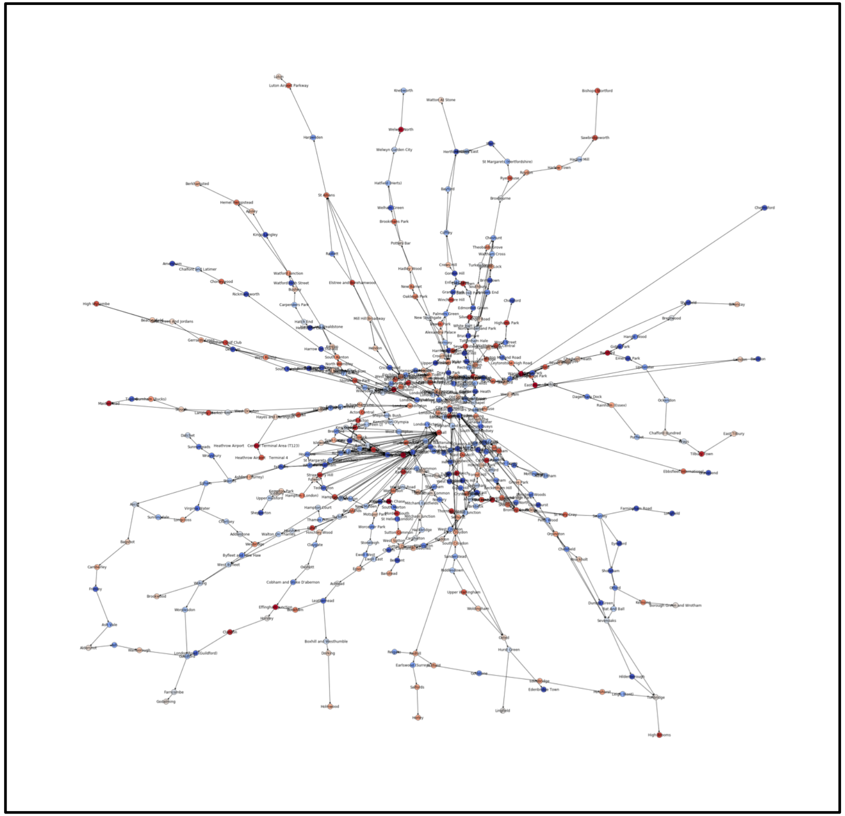
We construct a hierarchical multi-scale graph, where:

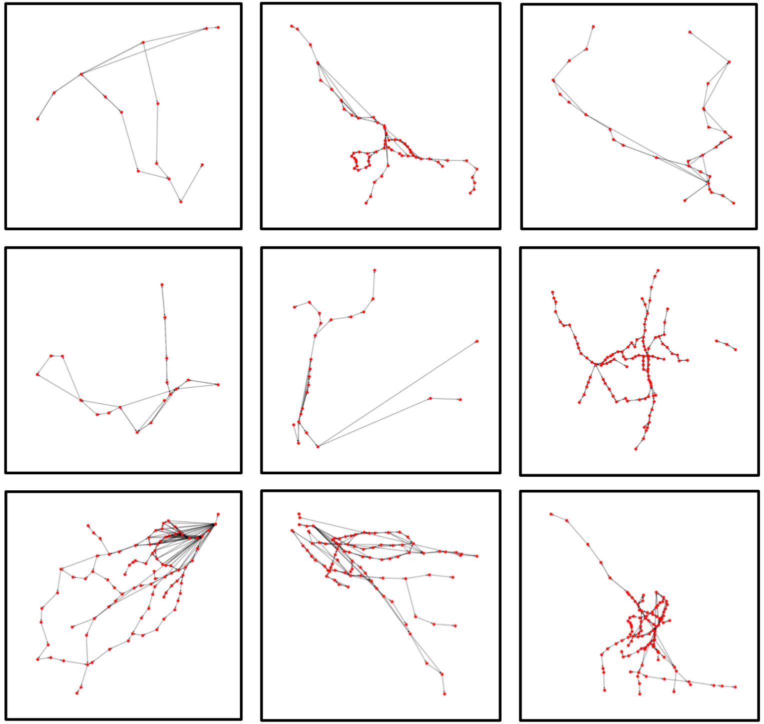
- Multiple transport links overlap on common stations
- Minor flows are removed (counter commuter flow <30 passengers).



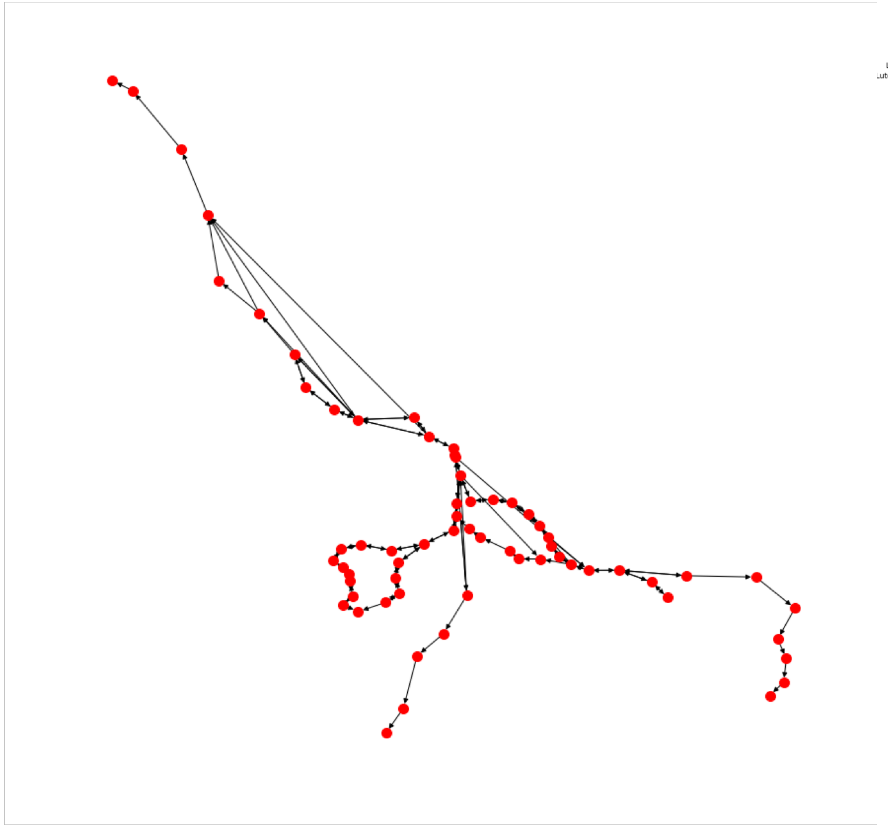


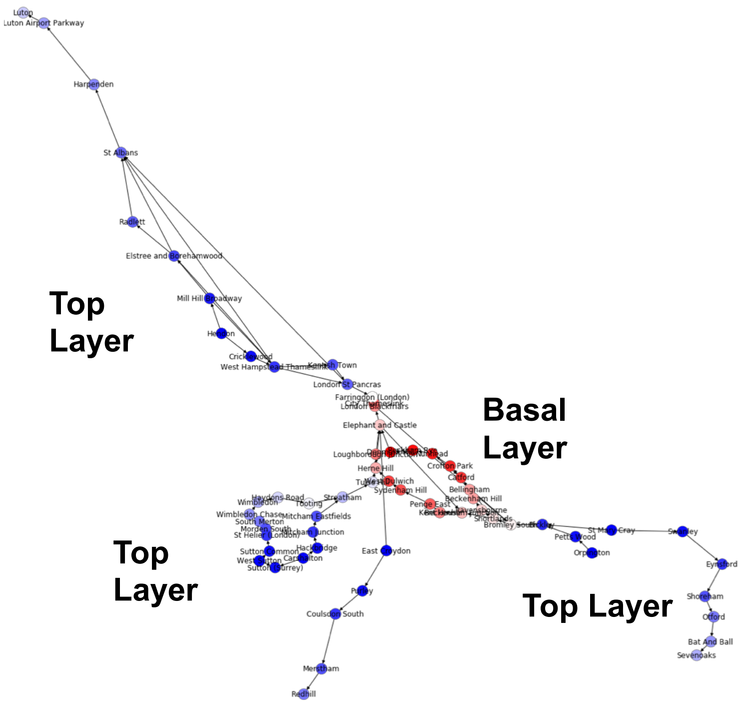
Overall and Individual Rail Network Topologies





Hierarchical Graph (Example: Thameslink)





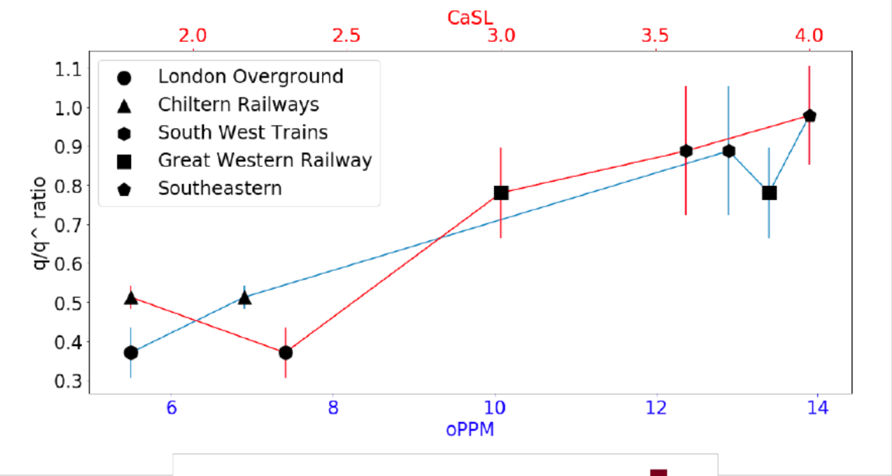
Results

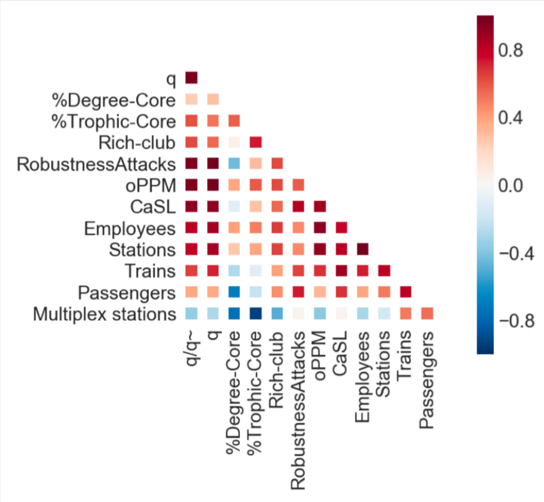
We analyse [5]:

- Resilience vs. Robustness against consumer satisfaction & late train data
- Greater London ~1 Hour Commuter Range

As a pure data-driven study, we show:

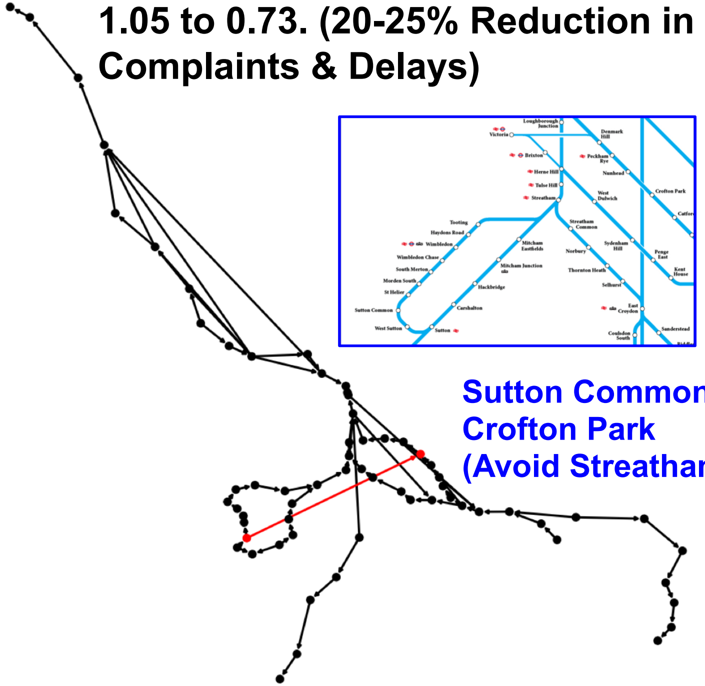
- Rail performance is strongly correlated to resilience, but not robustness;
- Pointing towards a pathway to reduce interdependency between rail services to reduce cascade effects.





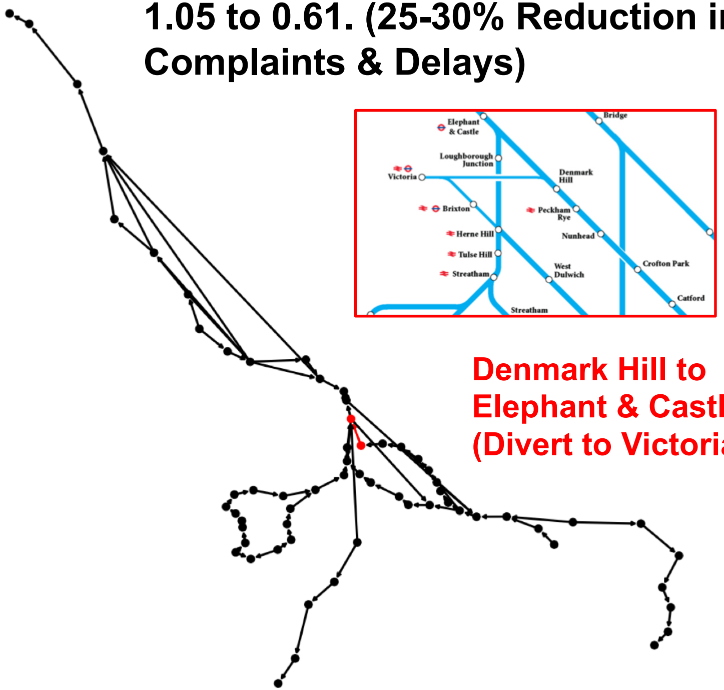
Minimum Change for Improvement (Example: Thameslink)

New Link Reduces Incoherence from 1.05 to 0.73. (20-25% Reduction in Complaints & Delays)



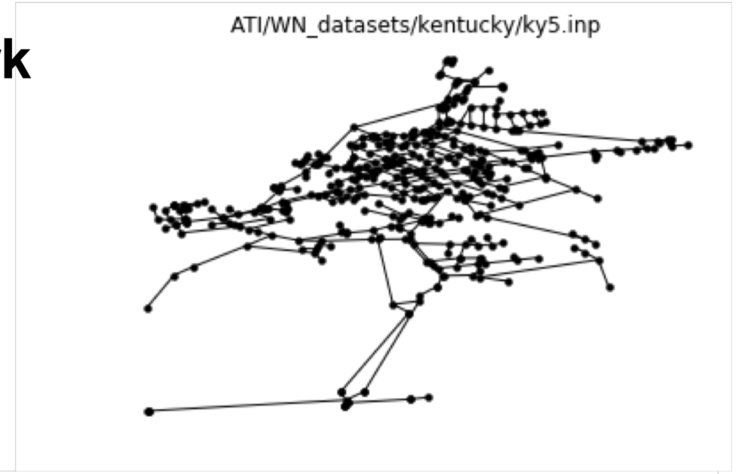
Sutton Common & Crofton Park (Avoid Streatham)

Remove Link Reduces Incoherence from 1.05 to 0.61. (25-30% Reduction in Complaints & Delays)



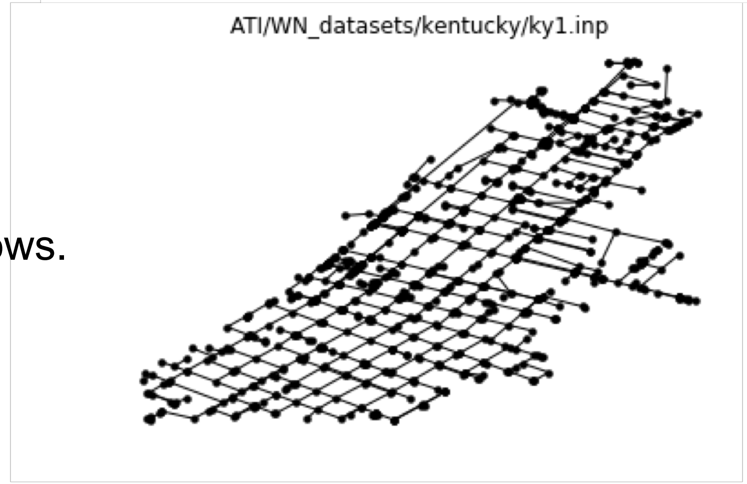
Denmark Hill to Elephant & Castle (Divert to Victoria)

Case Study: Water Distribution Network



Here we study different urban and rural water distribution networks (WDN) from the world with data from [6] and using EPANET to simulate the WDN performance:

- WDN topology and units
- Simulate demand variation across WDN
- Define failures in terms of pressure in pipes.



Nodes:

Junctions have water demand, **Reservoirs** provide water, **Pumps** increase pressure, **Valves** manage flows.

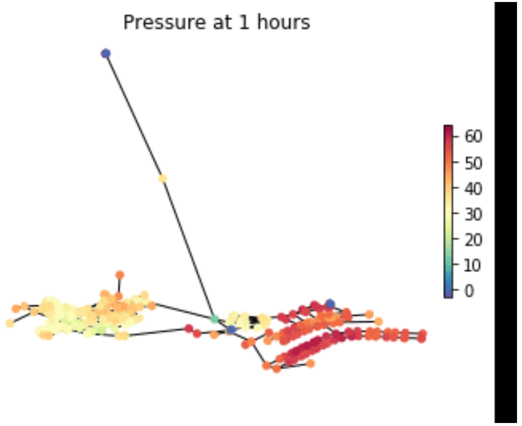
Edges:

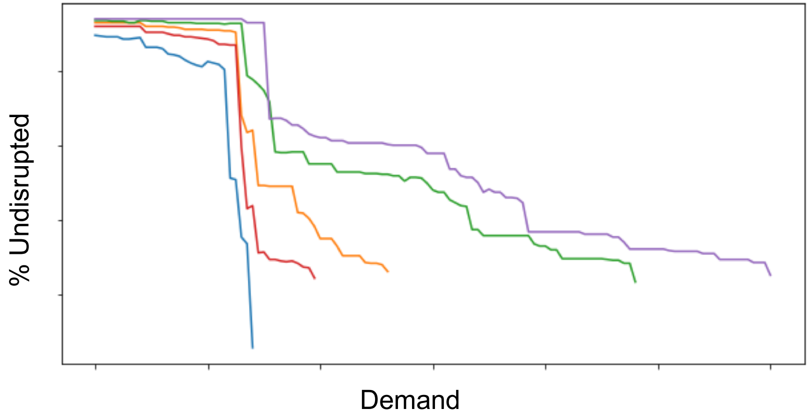
Pipes that connect nodes. Pipe properties include diameter, length, roughless and minor loss.

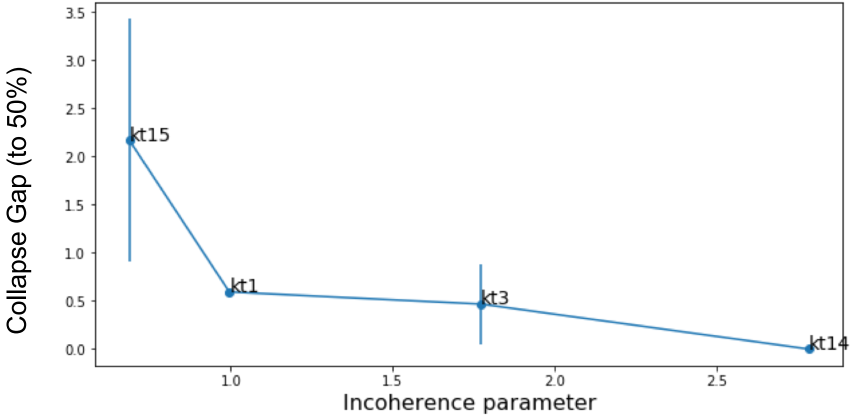
Water Distribution Cascade Failure

As a pure data-driven study, we show:

- Cascade failure performance is strongly correlated to resilience (data driven structural parameter).
- Pointing towards a pathway to improve topological structure by increasing WDN structural coherence.
- This can be achieved using **dynamic topology reconfiguration**.







Summary & Looking at Current & Future Work

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Butterfly Effect on Networks

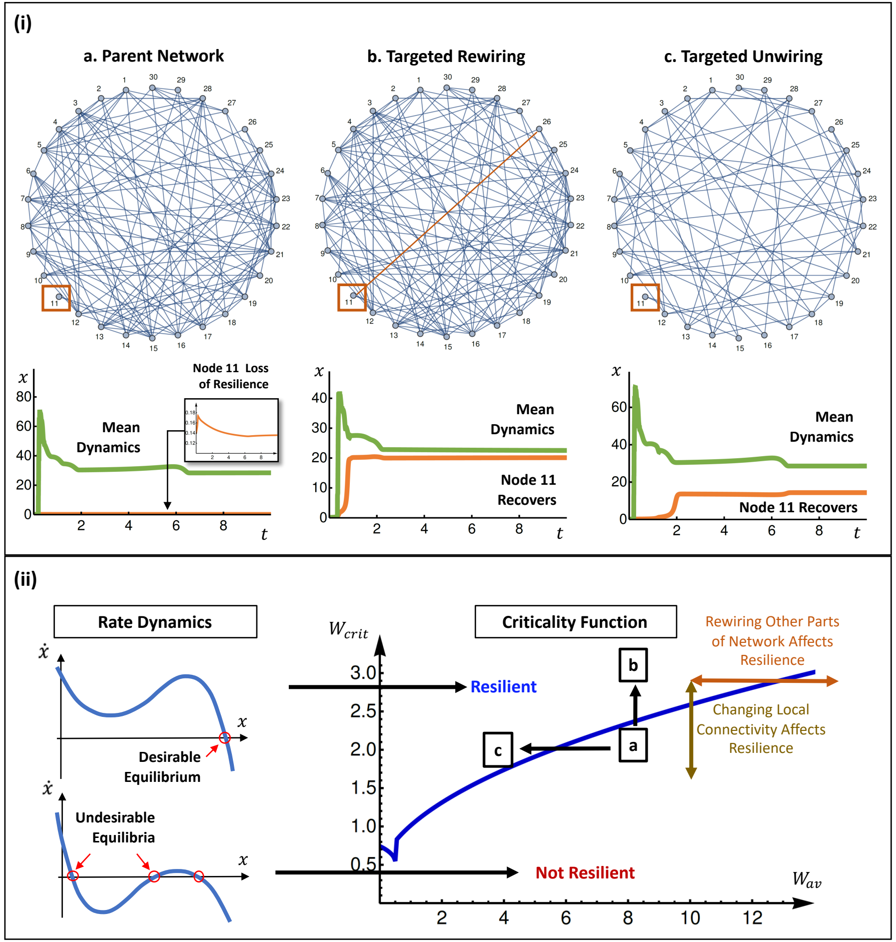
Small changes in other parts of the network can cause improvements or collapse in another part.

Here, we show a positive case, where Node 11 (orange) starts as collapsed.

First, we show **intuitively** how rewiring 11 can restore its functionality.

Then we show **non-intuitively** how collapsing other nodes can also restore 11's functionality due to cascade effects.

This can be understood from a criticality function perspective.



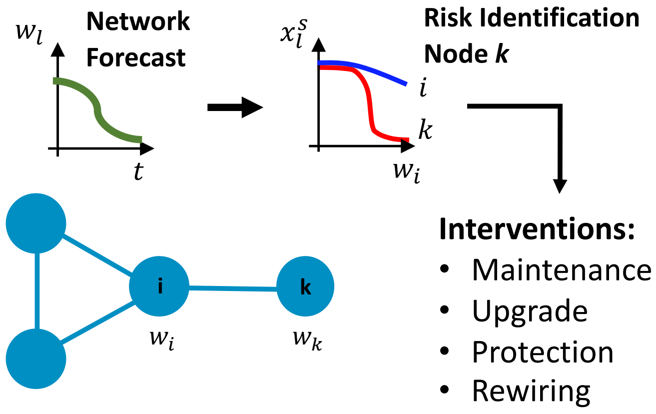
CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

Summary, Next Steps & Impact

We have only been working on these important questions for 7-10 **months and have a long way to go.**

Better understand the relationship between:

- **Resilience & Robustness: Built & Natural Ecosystems**
- **Local Dynamics and Global Topology in Networked Ecosystems**



We can identify vulnerable nodes at the risk of losing resilience and this may go on to inform infrastructure operators.

We still need to couple ecosystems together and model higher dimensional dynamics.

We want the work here to inform real-time sensing and control systems as well as the design of new systems (topology and components).

CHANCE funded by Lloyds Register Foundation. CoTRE funded by EPSRC (EP/R041725/1).

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[2] “Node Level Resilience Loss in Dynamic Complex Networks,” G. Moutsinas et al., NPG Sci. Rep., submitted, 2018

[3] ”Trophic Coherence Determines Food-Web Stability,” S. Johnson et al., PNAS, 2014

[4] “Drought Rewires the Core of Food Webs,” X. Lu et al., Nature Climate Change, 2016

[5] “Resilience or Robustness: Identifying Topological Vulnerabilities in Urban Rail Networks”, A. Pagani, et al., Royal Society Open Science, under revision, 2018

[6] Water Distribution Network Data Source: University of Kentucky - http://www.uky.edu/WDST/

[7] “Stability of Traffic Load Balancing in Complex Wireless Networks,” G. Moutsinas et al., IEEE Wireless Comm. Lett., submitted, 2018

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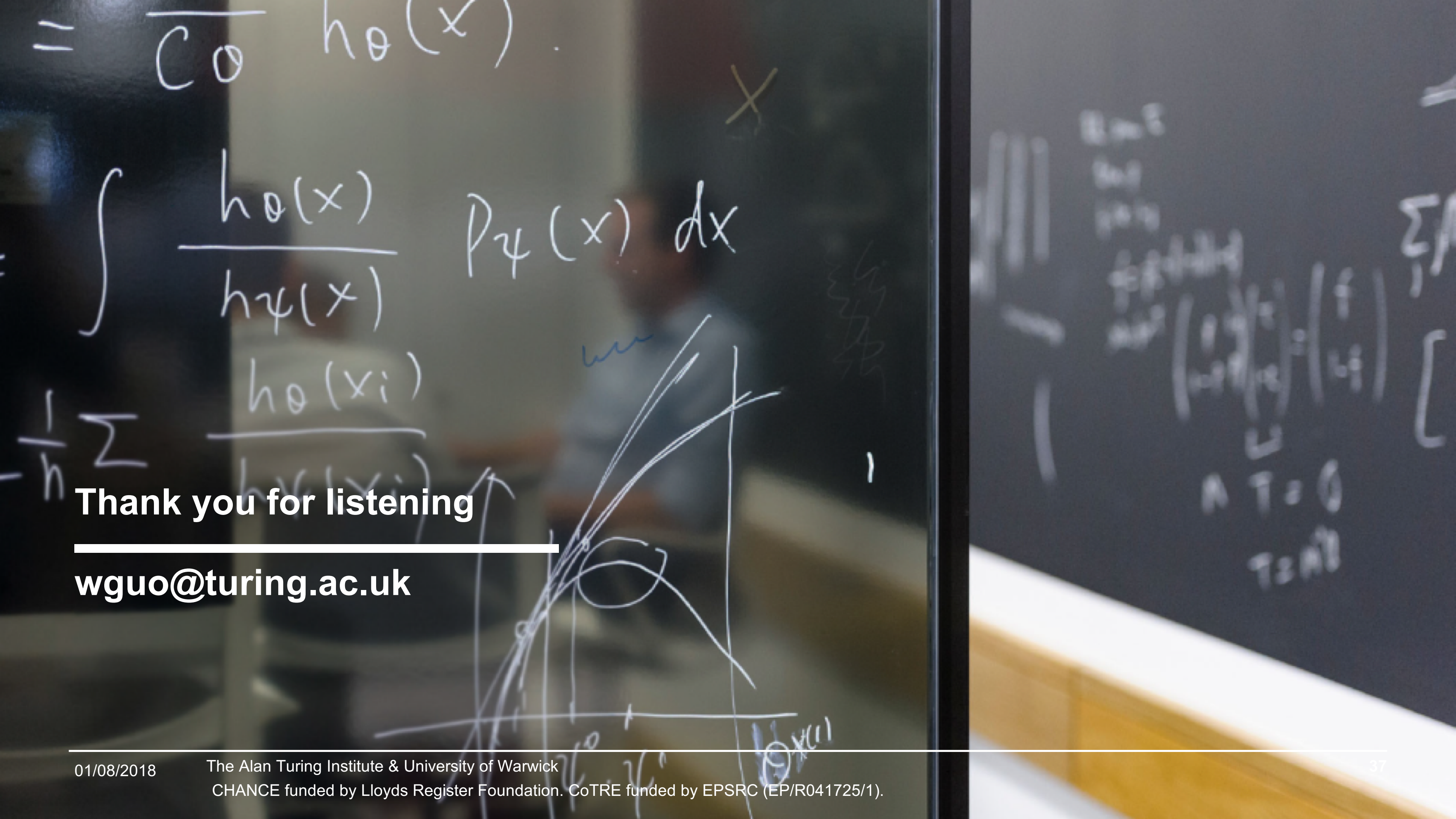

Thank you for listening

wguo@turing.ac.uk

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