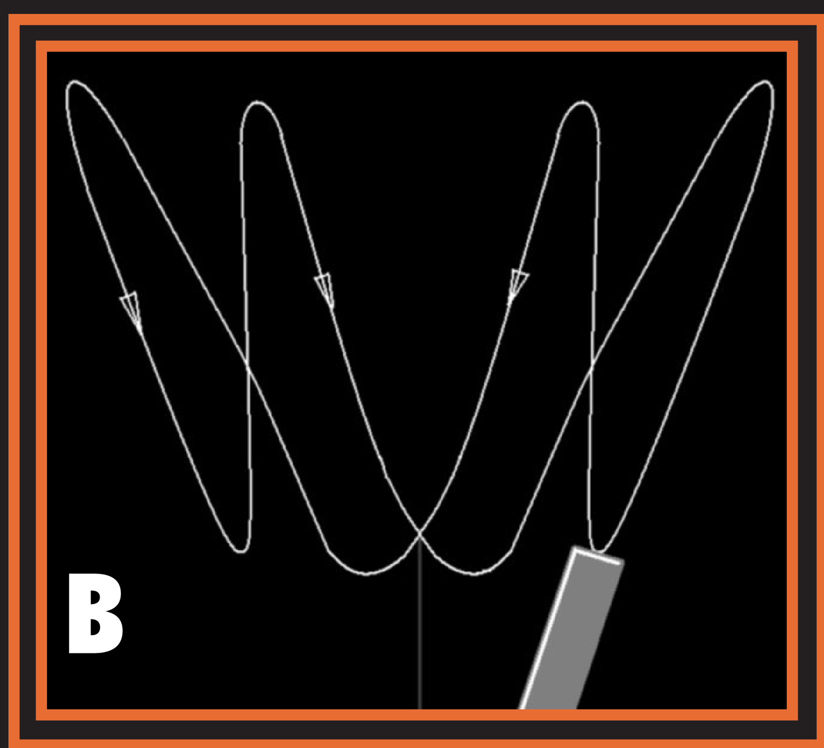
An investigation into the possibility of stabilising pendulums in their upright position using a system without a feedback element. Looking into the idea of a link between these pendulum models and noisy-driven systems displaying Stochastic Resonance.

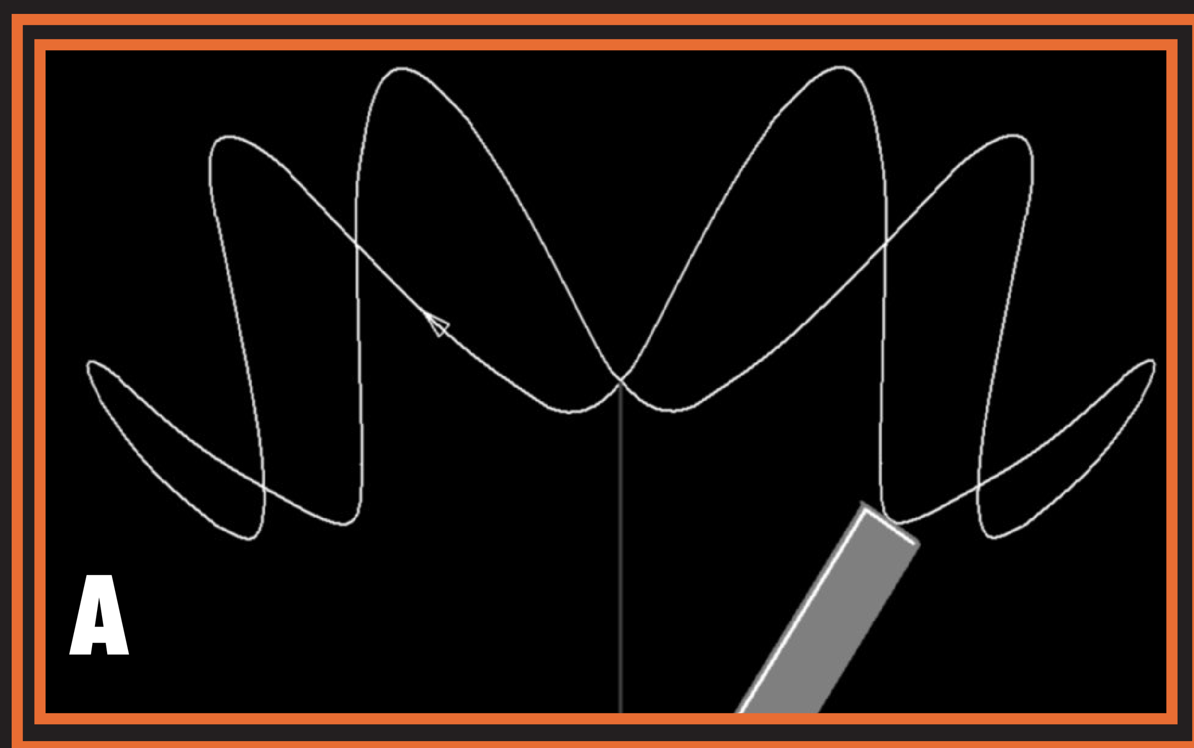
Single and multiple pendulum setups were tested with partial success. Whilst stability was displayed for the single pendulum, the available testing machinery was not capable of sufficiently high frequency/acceleration values to fully stabilise the inverted double pendulum. Despite this lack of full stability it was shown that there are likely multiple modes of stability in multiple setups. The idea of a noisy input producing some form of Stochastic Resonance was not tested, though the ability of the system to exist in multiple stable states is a positive step towards the link being identified.

SENSE

Stability Emergence from Noisy System Excitation

Stabilising an inverted pendulum is a problem that is commonly considered within the realm of the physical sciences. Most often it is dealt with using a horizontally free trolley and a system of feedback which compensates for the unstable nature of the pendulum in this system. The thought that a pendulum could be stabilised without the use of feedback is a curious idea but one which has been shown to be possible.



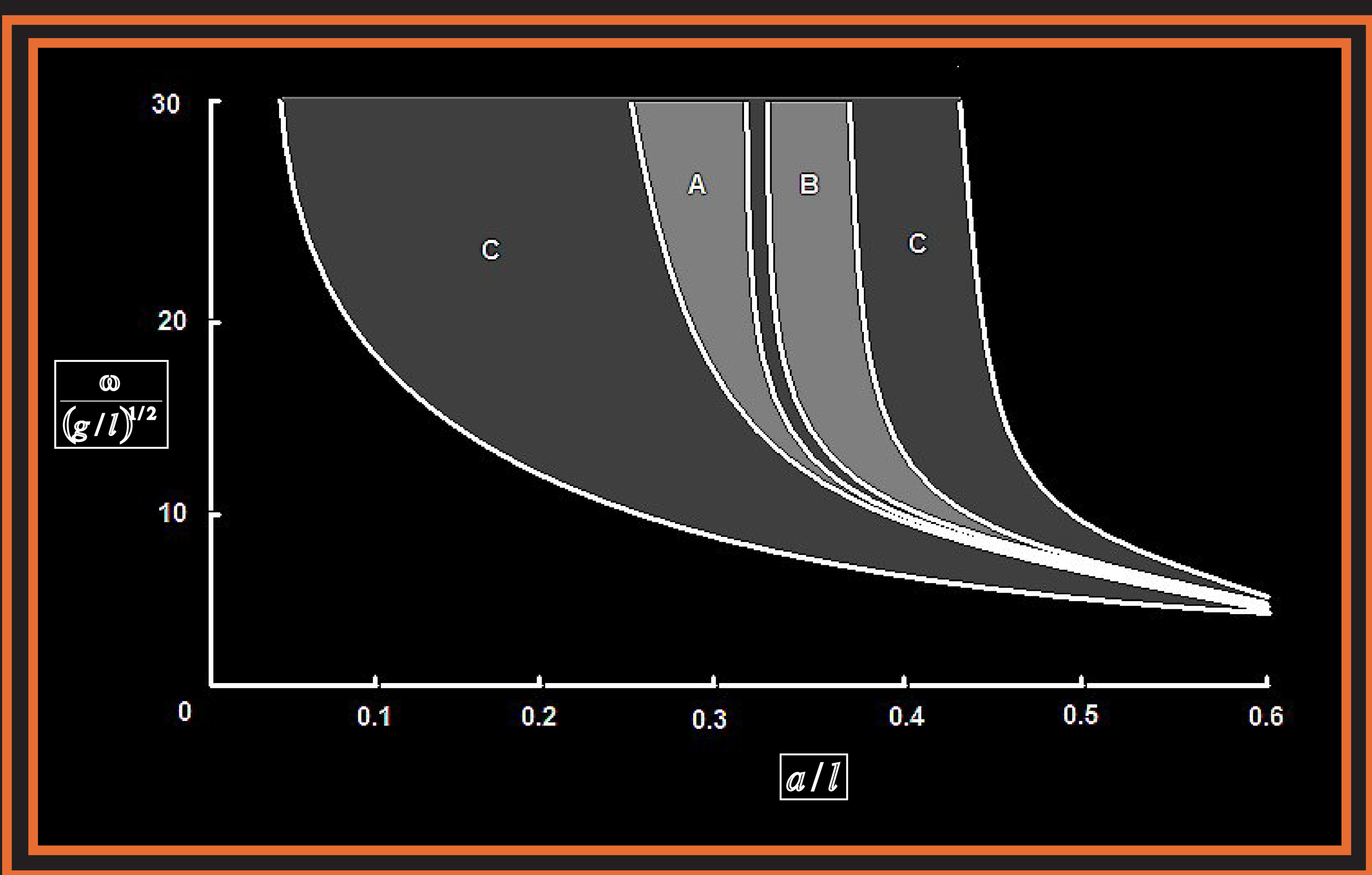


Through vibration of the base at high frequency multiple modes of stable motion can be achieved. The graph below displays three distinct forms of stable motion.

A: represents a triple cycle on either side of the upright vertical

B: represents a double cycle on either side of the upright vertical

C: represents a motion of continually decreasing amplitude about the upright vertical.



By examining the natural frequencies of a system of inverted pendulums connected at their ends, the required frequency and amplitude of vertical base vibration for inversion stability can be found. A differential equation to find these frequencies can be formed in the usual way; for this particular investigation a double system was used, in which;

$$\frac{d^2\theta_1}{dt^2} + m\frac{d^2\theta_2}{dt^2} + \frac{g}{l}\theta_1 = 0$$

And,

$$\frac{d^2\theta_2}{dt^2} + \frac{d^2\theta_1}{dt^2} + \frac{g}{l}\theta_2 = 0$$

The following rearrangement, in which θ_1 and θ_2 are replaced by $A\cos\omega t$ and $B\cos\omega t$ respectively will give the natural frequencies, of which there are two for a double pendulum setup:

$$\omega^2 = \frac{g/l}{1 \pm \sqrt{m}} \qquad \left[m = \frac{m_2}{m_1 + m_2} \right]$$

Describe pendulum motion in the non-inverted state. Here θ_1 and θ_2 represent the angular displacement of each from the downward vertical, g represents the gravitational field strength and l is the length of each pendulum.

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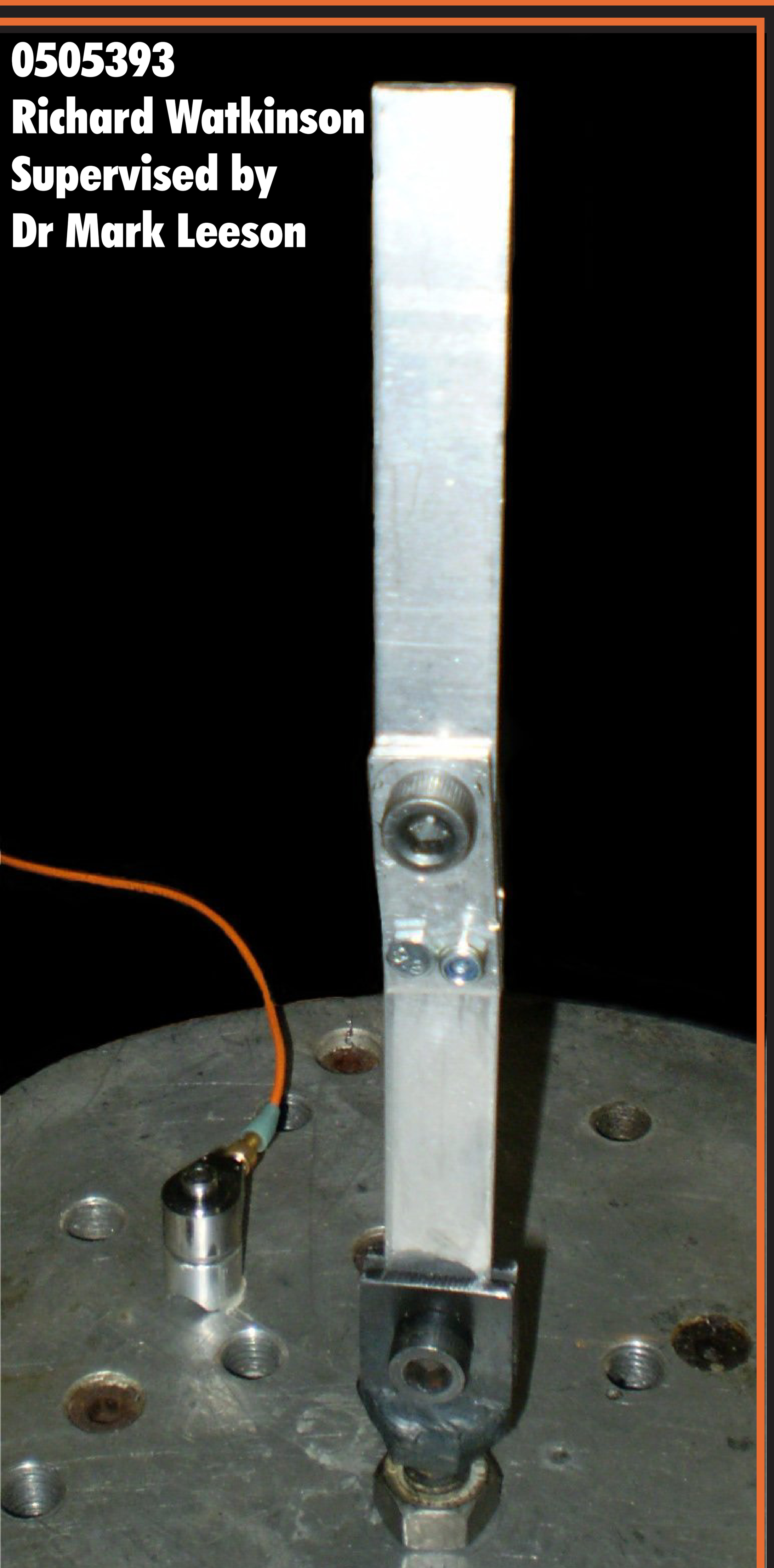
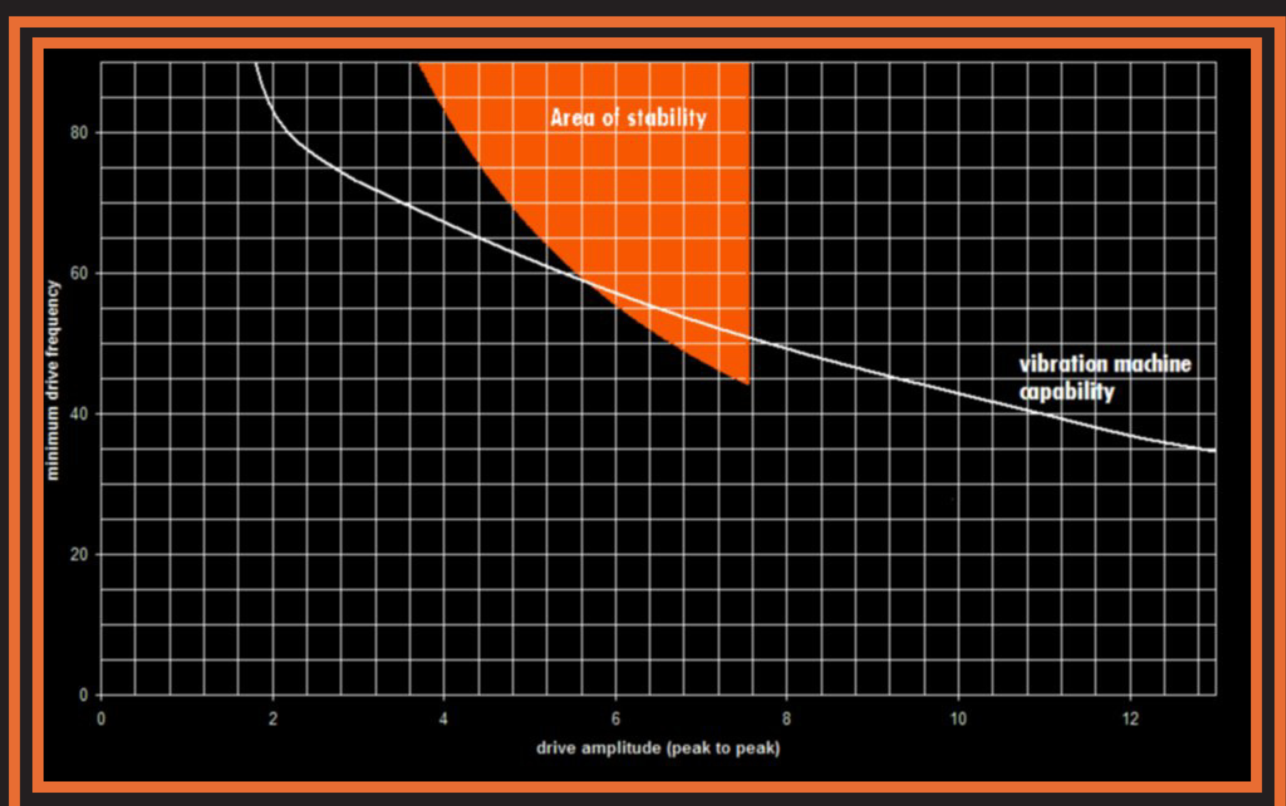


From these known natural frequencies of vibration it is then possible to derive a set of equations to define the area of stability for the system of inverted pendulums, these are given by:

$$a \le \frac{0.450g}{\omega_{max}^2} \quad \text{And,} \quad a\omega \ge \frac{\sqrt{2g}}{\omega_{min}}$$

In which ω_{max} is the larger of the natural frequencies, and ω_{min} is the smaller. These equations hold for systems in which the square of the natural frequencies are spread far apart.

A pendulum setup was designed with the following stability requirements:



The orange highlighted area shows what was required of the input signal to allow stability, whilst the white line represents the capability of the vibration machine used during the investigation. Despite the relatively small area in which the double pendulum could be stabilised in its upright position, it was instantly obvious that there the system was capable of multiple forms of stable motion as demonstrated above for the single pendulum setup. Sweeping-frequency tests showed the system moving between these states without becoming unstable.

Stochastic Resonance involves taking a system with distinct stable states in which there is a probability of the system being in one or the other at any time. An 'energy barrier' preventing movement between the two is overcome using the extra energy that the noise provides. Positions of stability at low energy (A & B) in which the energy barrier (C) must be overcome for periodic 'hopping' to occur.

From the point of observing the inverted modes of stability, the hope was that the theory of Stochastic Resonance could apply to the system when a lower than required signal frequency and amplitude was applied. By allowing the system to 'jump' between the distinct stability modes, an optimised level of noise could produce a steady system of motion which was not achievable with only a signal input. Unfortunately there was not an opportunity to introduce noise to the setup due to restrictions on the test equipment imposed by its owner. The idea that this could be achieved is certainly a question for future research.