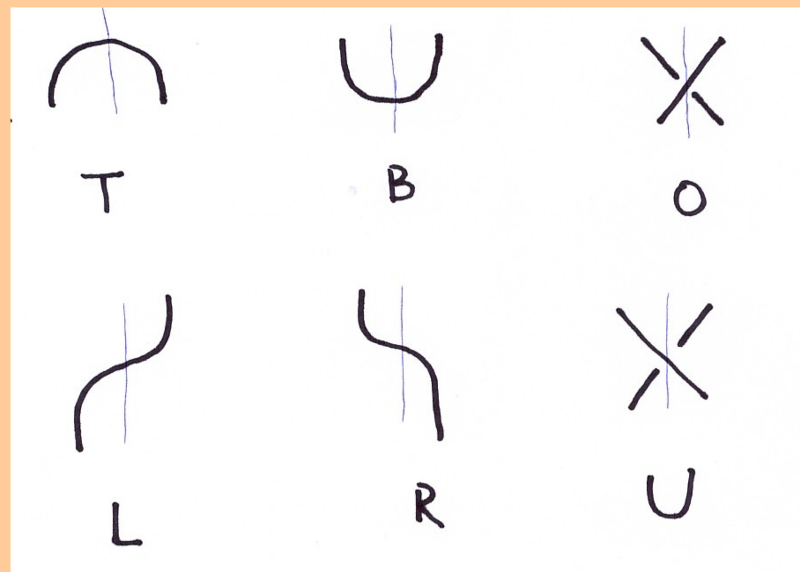
URSS

RANDOM KNOTTING

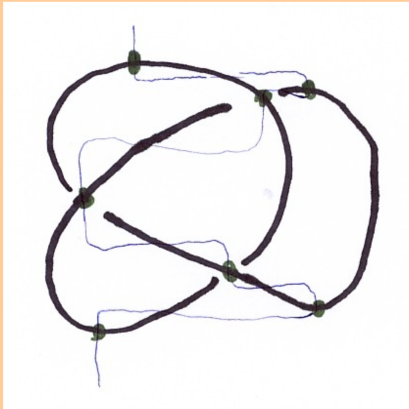
THE UNIVERSITY OF WARWICK

KNOT REPRESENTATION

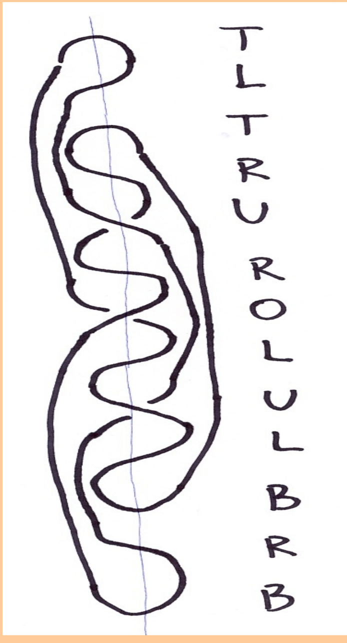
Assign a height function to a given link, mark every crossing and critical point. Then as you go along the link starting from the top (maximum) use the following rules to translate a link into a word over a six letter alphabet:



An example of how one can translate a link into a word:



We get: *T L T R U R O L U L B R B*



Now given a word as above we can draw the link back as presented on the right.

COUNTING LINKS

A form of backtrack algorithm can be used to count and list all the words of a given length (say n) that correspond to a link. The conditions for the algorithm are the following:
1) if at any stage the number of lines on any of the sides is less than 0
2) if after k-th letter is evaluated the number of lines on any of the sides is greater than n-k

We can see the results (length vs number of components) below:

	1	2	3	4	5	6
2	1					
3	2					
4	6	2				
5	16	10				
6	48	48	5			
7	156	196	42			
8	554	796	280	14		
9	2136	3228	1536	168		
10	8590	13560	7860	1440	42	
11	36076	58800	38500	9900	660	
12	154380	263252	187526	61072	6930	132

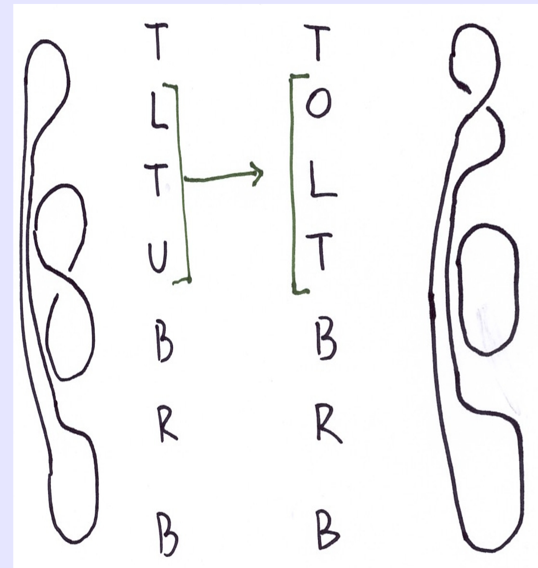
RANDOM KNOTS

Take a knot of a given length and perform a number of changes in the word representing this knot. We need to make sure that:

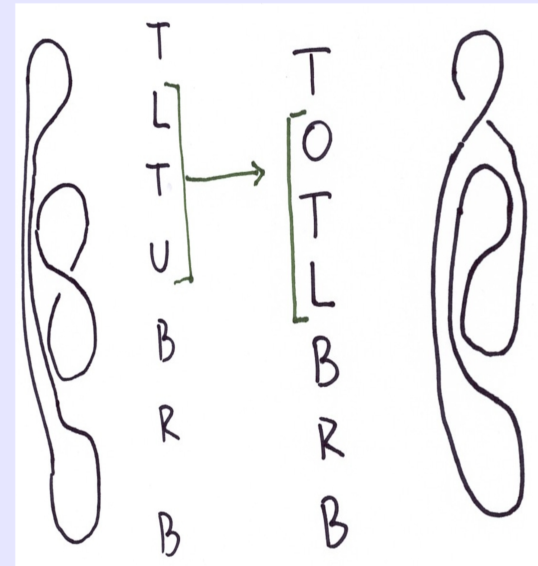
1) After the changes have been made the resulting word still represents a knot.
2) The resulting word (knot) is random enought.

To deal with the first problem we start with links. Given any link of length n we can take a subword of the word representing this link and replace it with a different word (random). The resulting word is still a link provided that the number of lines athe the beginning and at the end of the subword agree with those of the new one. We also require that the number of lines doesn't go below 0.

Consider the following examples:



A slightly different change changes the number of components:



TRICOLORING

A tricoloring is defined to be a coloring using 3 colours such that at each crossing either 1 or 3 colours appear. It is one of the simplest knot invariants. Some possible colorings at a crossing are presented below.

3 colors

1 color at a crossing:

One important property of the tricoloring invariant is that it can in some cases indicate whether a given knot is not an uknot. In particular the tricoloring invariant distinguishes between the unknot TB and the trefoil knot which has 9 colorings. The table below presents the number of tricolorings for words of length from 9 to 12 (all words of length less than 9 have 3 tricolorings).

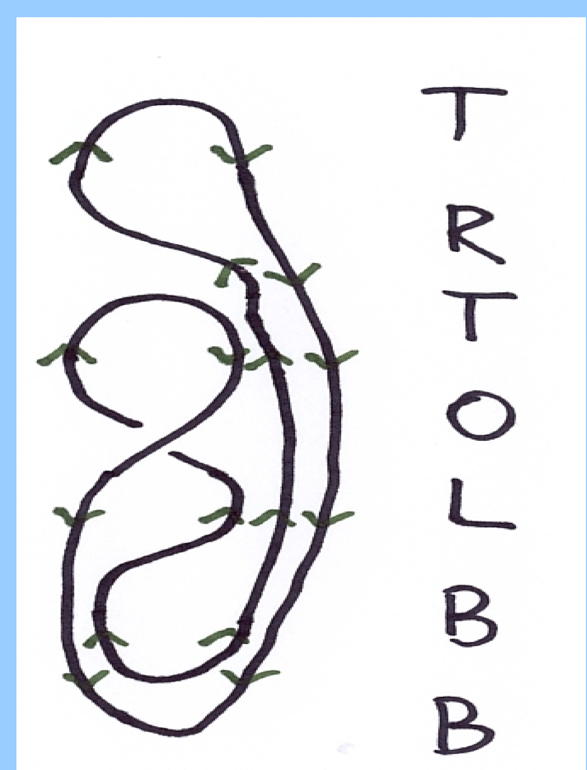
	3	9
9	2132	4
10	8558	32
11	35776	300
12	152492	1888

JONES POLYNOMIAL

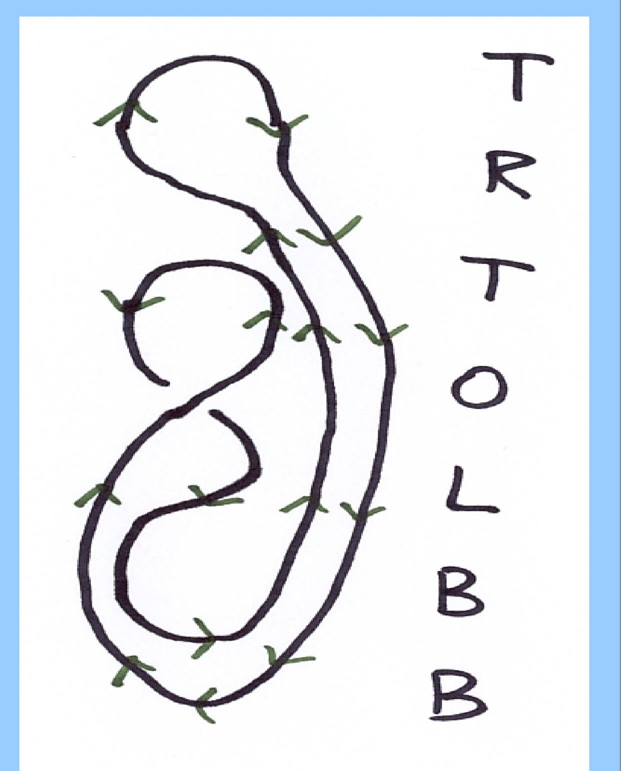
In order to calculate the Jones Polynomial of a given knot one needs to change the representation of a knot from a string of letters into a monomial in an algbera. So tha TUB would correspond to T*U*B where T,B,O,U,L,R are variables in a 6-dimensional algebra.

One also needs to assign an orientation to a given knot. This is done as follows:

Whenever T occurs an additional orientation is added. If all orientationns agree on all the Bs the orientation is accepted.



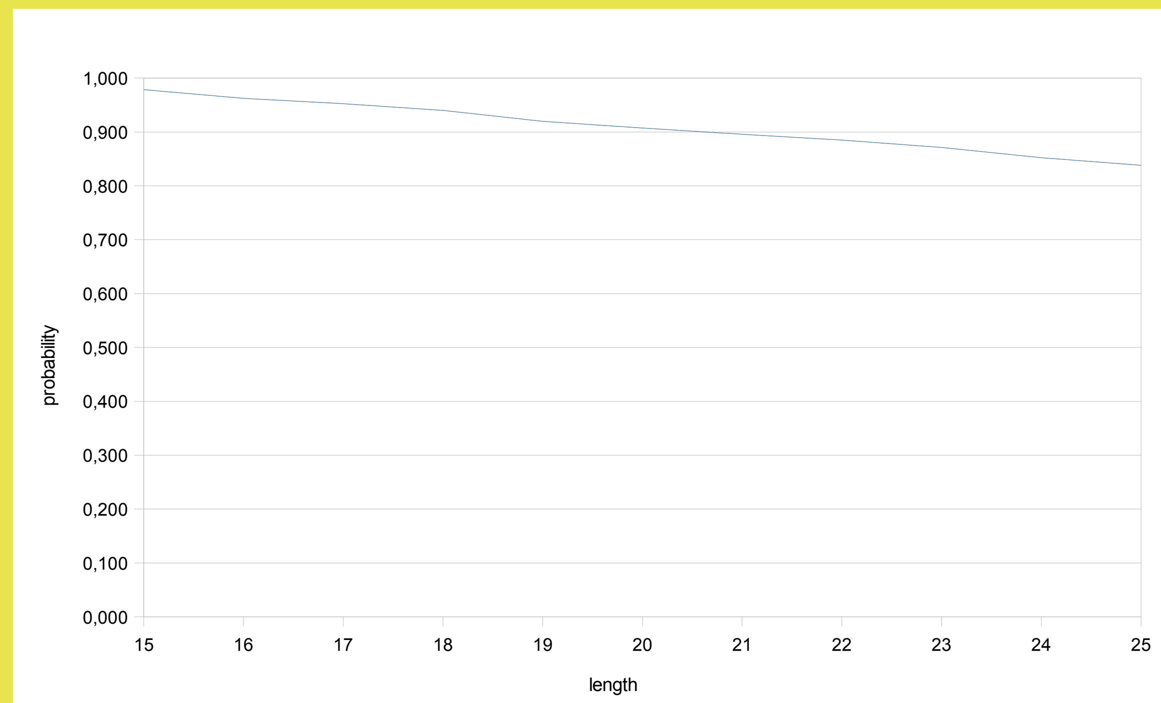
Otherwise it is discarded.

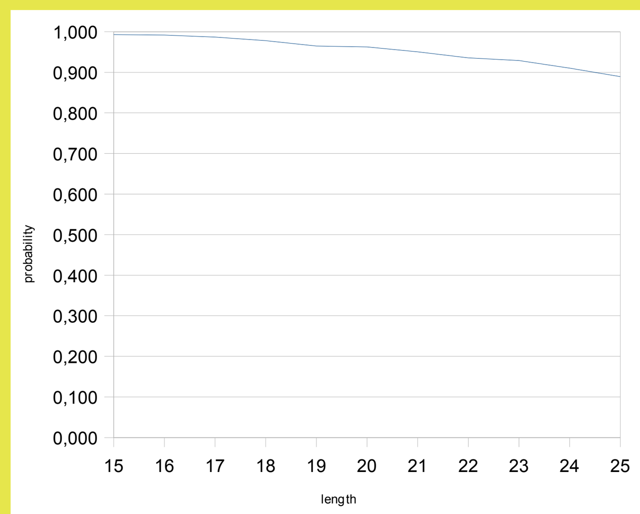


Once we have the orientation assigned we can compute the Jones Polynomial.

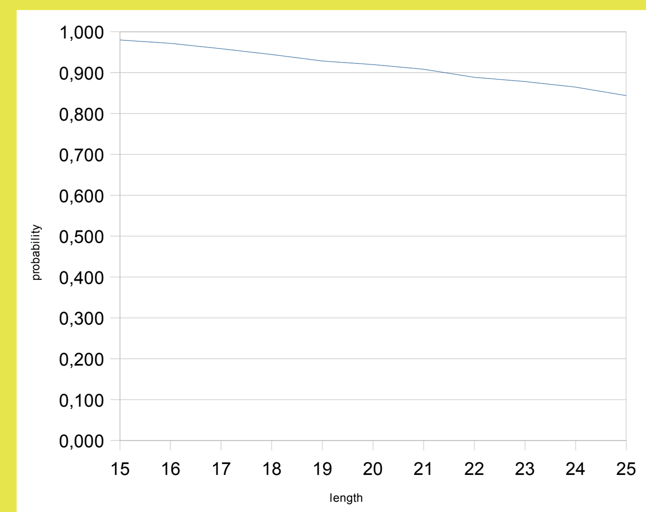
FINAL RESULTS

Using the Jones Polynomial we can determine how many words of a given length represent an unknot and how a probability of getting a knot (not an unkot) changes as the length of the word increases. We can see these results on the left:





We can alter the number of occurances of different letters to see if the behavior of the probability function changes. We can see the graphs for the probability of O and U increased on the left and with L and R on the right. Unfortunately the function still behaves linearly.



Author: Tomasz Lechowski
Supervisor: dr Bruce Westbury