Applying variational principles to fermion masses and mixings

Daniel Busbridge

Email: d.w.busbridge@warwick.ac.uk

Web: go.warwick.ac.uk/ep/pg/phugfl

1. Objective

To find a flavour-symmetric action which, when extremised covariantly, predicts the masses and mixing angles in both the quark and leptonic sectors.

2. Extending the Standard Model

The Standard Model of particle physics has been incredibly successful at predicting phenomena observed in particle accelerators, particle showers, and many other events. The fermion masses and mixing angles, however, lie outside its predictive scope, and have to be 'fixed by hand' from experimental data. This is unsatisfactory, and the theory needs extending.

3. Quark and Lepton mixing

The phenomena of mixing arises from a mismatch in quantum states; particles behave as 'weak interaction eigenstates,' which are linear combinations of flavour eigenstates, e.g. for quarks

interaction eigenstates

$$\boldsymbol{u}' = U^u \boldsymbol{u}, \quad \boldsymbol{d}' = U^d \boldsymbol{d}, \qquad \boldsymbol{u}^T = (u, c, t), \quad \boldsymbol{d}^T = (d, s, b)$$

U^u and U^d contain the contribution of each flavour to the interaction eigenstates.

The simplified Lagrangian for the electroweak interaction is

$$\mathcal{L} = \boldsymbol{u}'^{\dagger} \boldsymbol{d}' W + \boldsymbol{u}'^{\dagger} \Gamma_u \boldsymbol{u}' + \boldsymbol{d}'^{\dagger} \Gamma_d \boldsymbol{d}',$$

Γ_u and Γ_d are the quark mass matrices, whose eigenvalues are masses.

Substituting the interaction eigenstates,

$$\mathcal{L} = \boldsymbol{u}^{\dagger} \underbrace{U^{u\dagger} U^d}_{\text{CKM matrix}} \boldsymbol{d} W + \boldsymbol{u}^{\dagger} \underbrace{U^{u\dagger} \Gamma_u U^u}_{D_u} \boldsymbol{u} + \boldsymbol{d}^{\dagger} \underbrace{U^{d\dagger} \Gamma_d U^d}_{D_d} \boldsymbol{d}.$$

new mass matrices

The CKM matrix describes mixing in the quark sector. The mixing matrix for leptons is the PMNS matrix.

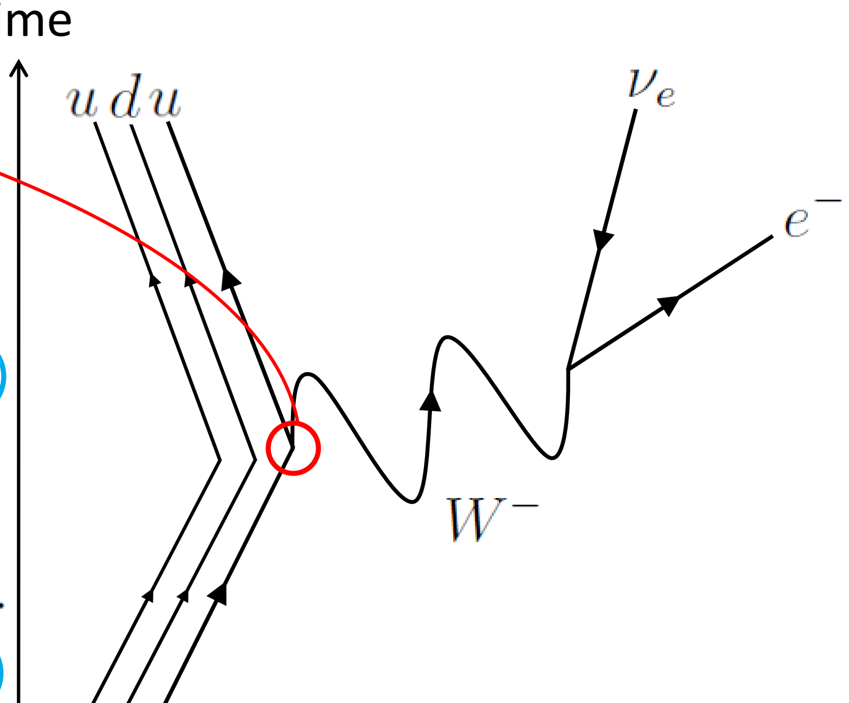

Fig.1 Beta decay via the electroweak interaction

4. Relationships between masses and mixing

In the basis where D_d is diagonal, the CKM matrix diagonalises D_u, such that

$$D_d = \mathrm{diag}(m_d, m_s, m_b), \quad V^{\dagger} D_u V = \mathrm{diag}(m_u, m_c, m_t).$$

There follows an implication that the mixing is 'encoded' in the masses.

Making unitary transformations to the mass matrices leave the mixing unchanged. Consequently, obtaining the mixing matrices in a basis independent way is probably the only physically consistent approach.

5. Evidence of extremisation

The CKM and PMNS matrices are

small

trimaximally mixed

$$|V_{\mathrm{CKM}}| \approx \begin{array}{c|ccc} & d & s & b \\ \hline u & 0.97 & 0.23 & 0.0036 \\ c & 0.23 & 0.97 & 0.042 \\ t & 0.0087 & 0.041 & 0.999 \end{array} \qquad |V_{\mathrm{PMNS}}| \approx \begin{array}{c|ccc} & \nu_1 & \nu_2 & \nu_3 \\ \hline e & \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \mu & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \tau & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{array}$$

where the PMNS matrix is the phenomenologically viable tribimaximal ansatz motivated by solar and atmospheric neutrino data.

Distinctive features of the mixing are the small V_{ub} and V_{e3} elements, implying that CP violation is being minimised, and the trimaximally mixed ν_2 eigenvector, hinting at an underlying maximisation process.

6. Quantities for constructing actions

The Jarlskog invariant, J, is a measure of CP violation, and is proportional to the trace of the cube of the commutator of mass matrices, e.g. for leptons $J \propto \mathrm{Tr}[L, N]^3$, where L and N are the lepton and neutrino mass matrices.

Conveniently, the trace of cubics of commutators of powers of mass matrices correspond to a CP violating quantity, and the trace of the quadratics of these commutators corresponds to a CP conserving quantity, e.g.

CP conserving $\mathrm{Tr}[L^2, N][L, N]$ CP violating $\mathrm{Tr}[L, N^2][L, N]^2$

It is important to note that traces are basis independent quantities.

7. Extremising CP conserving actions

When extremised with respect to L, i.e. $\partial_L \mathcal{A} = 0$,

$$\left.\begin{array}{r} \mathrm{Tr} C_{11} C_{21} + q\, \mathrm{Tr} C_{21}^2 \\ \mathrm{Tr} C_{11}^2 + q\, \mathrm{Tr} C_{11} C_{21} \\ \mathrm{Tr} C_{11}^2 + q\, \mathrm{Tr} C_{21}^2 \end{array}\right\} \approx \text{Tribimaximal mixing} \qquad C_{mn} = -i[L^m, N^n].$$

q is predicted from the extremisation, and is necessary for dimensionality.

It was found, however, the N derivatives are only satisfied by zero mixing, implying that the actions were incomplete, or some assumptions were incorrect.

8. Deviations from an S3 group matrix

One assumption not required by the solution is N is an S3 group matrix (the sum of each row and column is equal). If N takes on the parameterisation

$$N = \begin{pmatrix} a & z + id & y - id \\ z - id & b & x + id \\ y + id & x - id & c \end{pmatrix} \qquad \left.\begin{array}{l} a = x + \sigma \\ b = y + \sigma \\ c = z + \sigma \end{array}\right\} \text{S3 condition}$$

then deviations from this form can be made using the substitutions

$$\begin{array}{rcl} a & = & x + \sigma'_1 \\ b & = & y + \sigma'_2 \\ c & = & z + \sigma'_3 \end{array} \qquad \begin{array}{rcl} \sigma_1 & = & \sigma'_1 - \sigma'_2 \\ \sigma_2 & = & \sigma'_3 - \sigma'_1 \end{array} \qquad \begin{array}{rcl} a - b & = & x - y + \sigma_1 \\ b - c & = & y - z - \sigma_1 - \sigma_2 \\ c - a & = & z - x + \sigma_2 \end{array}$$

$\approx$ equal $\approx$ zero

and Taylor expanding around S3 ($\sigma_1 = \sigma_2 = 0$). This did not solve the triviality in the N derivatives, but may be useful with the correct action.

9. CP conserving and violating combinations

Extending an action to incorporate CP violating quantities produced

$$C_{11}^2 + q\, C_{11} C_{21} + r\, (C_{11}^3 + \epsilon\, C_{11}^2 C_{21}) \approx \text{Tribimaximal mixing.}$$

Although this did not solve the problem, it came closer to a consistent solution.

10. Project Experience

Throughout the project I was able to work on my own ideas, giving me a lot of creative freedom. I thoroughly enjoyed participating in this research, which was beneficial in clarifying whether I wanted to pursue a career in this field.

11. Acknowledgements

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Supervisor: Professor Paul Harrison

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