

Introduction

Double-Beta decay is a nuclear decay process in which two individual Beta events occur at the same time. Two neutrons in the nucleus of a heavy element turn into protons, releasing an electron and a neutrino each. In two-neutrino double-Beta decay, the events occur without interaction, but in neutrinoless double-Beta decay, the neutrino emitted by one decay is absorbed in the other.

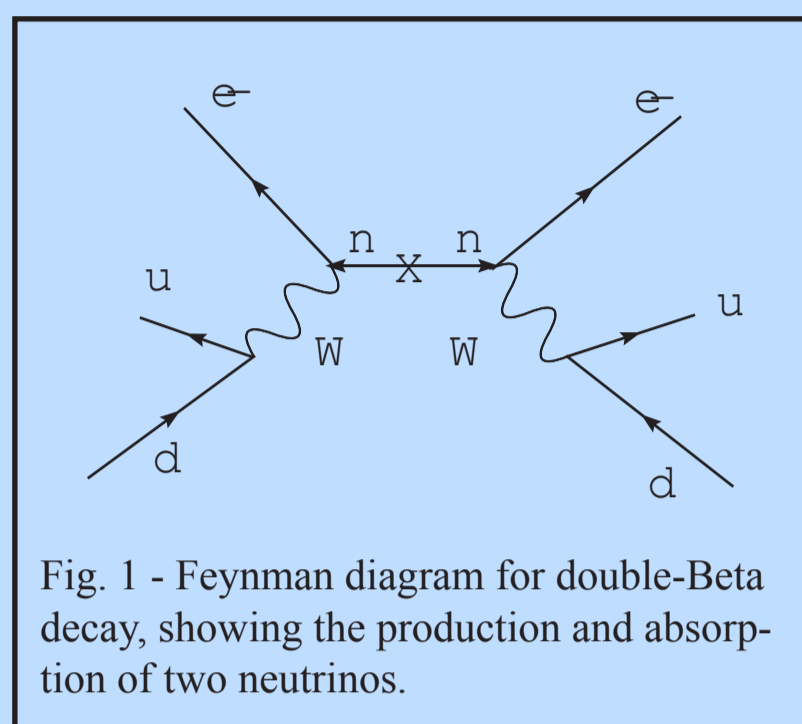


Fig. 1 - Feynman diagram for double-Beta decay, showing the production and absorption of two neutrinos.

Neutrinoless double-Beta decay has not been observed, and is prohibited by the currently accepted "Standard Model" of particle physics. However, if observed experimentally, it would prove that the neutrino, a particle with no charge and very little mass, is a so-called Majorana particle; it is its own antiparticle.

Generating Double-Beta Events

Double-Beta decay has a half-life of around 10^{25} years, which makes it very difficult to observe experimentally. A large amount of material is required in order to decrease the timescale in which a decay is detected. One alternative is to use computer simulations to model the expected physics behind the decay events and produce a high number of artificial events which may be fed into data acquisition and processing software to test and tune the later stages of the detection procedure.

As such, a program capable of creating large numbers of double-beta decay events in a short time, based on the theoretical model of double-Beta decay, is very useful for testing and fine-tuning experimental searches for (particularly) neutrinoless double-Beta. The decay events depend on parameters such as the mode of decay and the energy available for decay. The energies of the two electrons released in the double-Beta process would be calculated as a function of the decay mode and the available energy (the Q-value); an event encapsulates the knowledge of the energies T_1 and T_2 of the two electrons, and their angular distribution in 3-dimensional space.

One application of such a program would be to produce events which may then be tracked through some detector material, using for example the CERN GEANT4 software.

Existing FORTRAN software performed some of these tasks, but was not as flexible (or as fast) as the end result of this project.

Monte Carlo Sampling

The primary aim of the project was to design and implement a program which generates the correlated energies and momenta of double-Beta decay events, using the C++ programming language.

Mathematical expressions for the probability distributions of each decay mode are given in [1]. The majority of the project consisted of developing

a framework to implement these mathematical probability distributions in C++ using Monte Carlo techniques [2]. The method involves picking numbers at random and testing whether they lie within the probability distribution

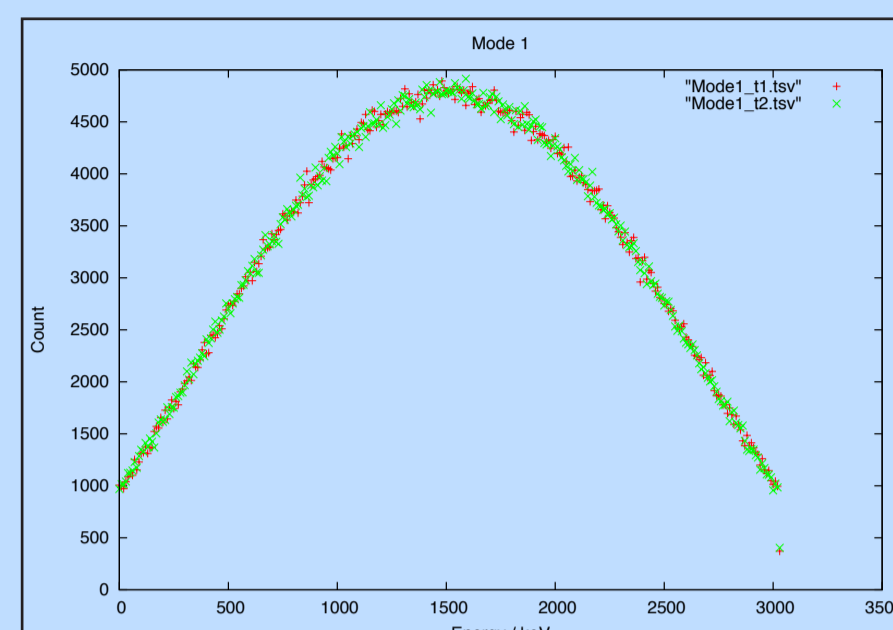


Fig. 2 - Neutrinoless decay with neutrino mass, 0^+-0^+ transition, 2n mechanism.

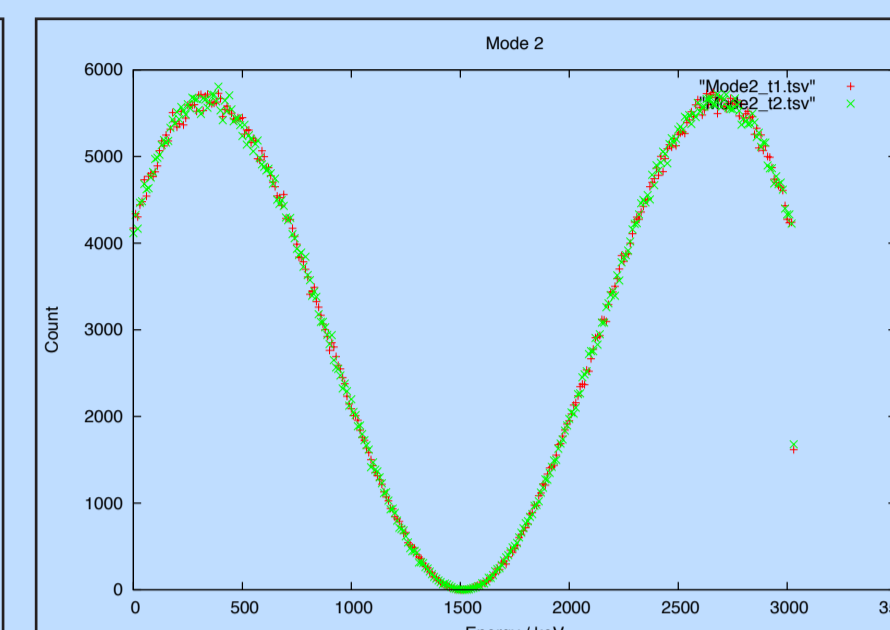


Fig. 3 - Neutrinoless decay with right-handed currents, 0^+-0^+ transition, 2n mechanism.

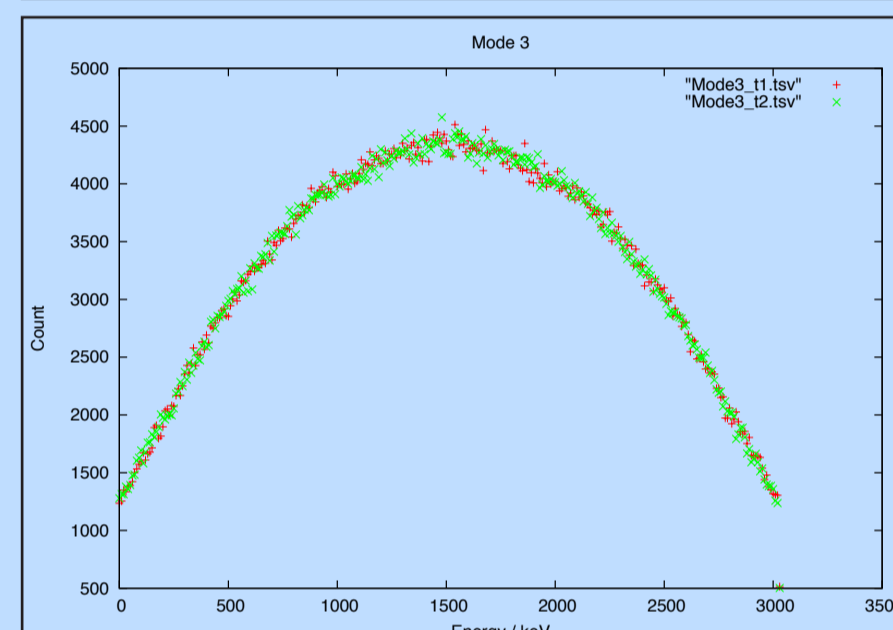


Fig. 4 - Neutrinoless decay with right-handed currents, 0^+-0^+ transition, N^* mechanism.

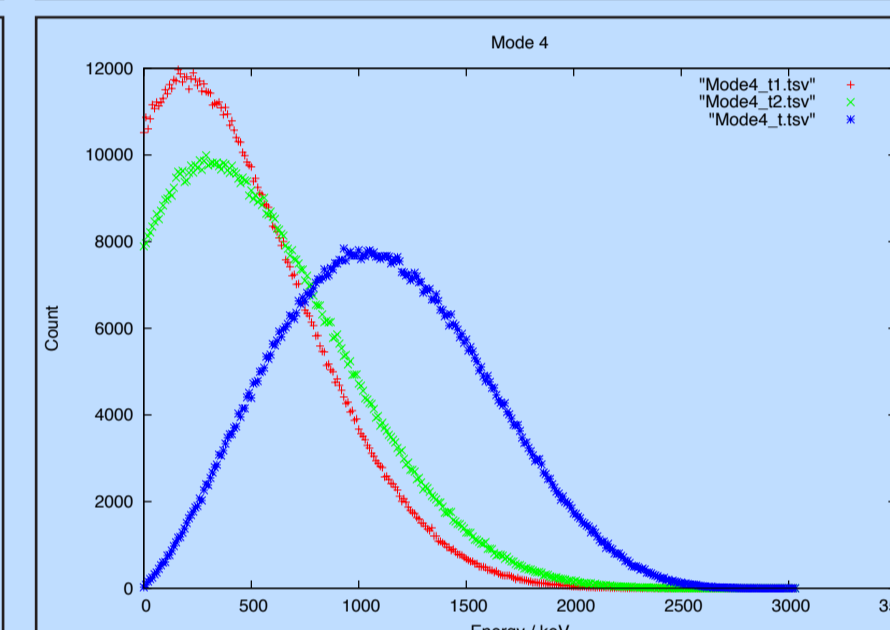


Fig. 5 - Two-neutrino decay, 0^+-0^+ transition, 2n mechanism.

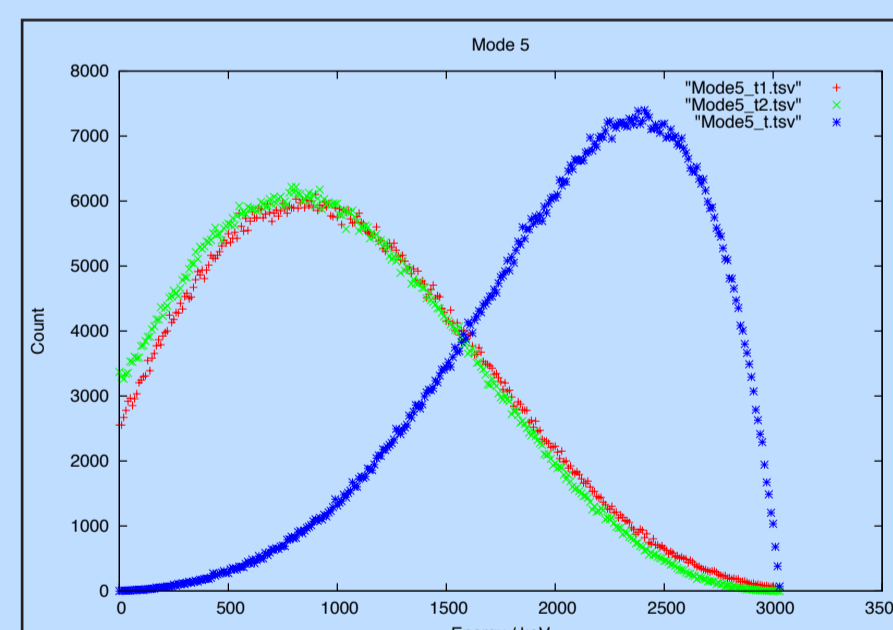


Fig. 6 - Neutrinoless decay with emission of Majoron, 0^+-0^+ transition, 2n mechanism.

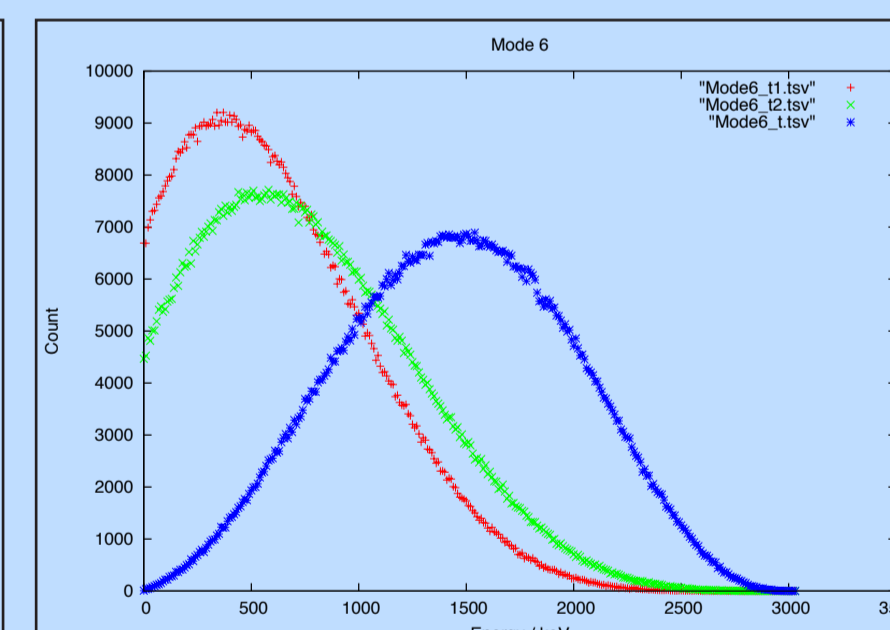


Fig. 7 - Neutrinoless decay with double Majoron emission, 0^+-0^+ transition, 2n mechanism; Charged $L=2$ Majoron and massive vector Majoron

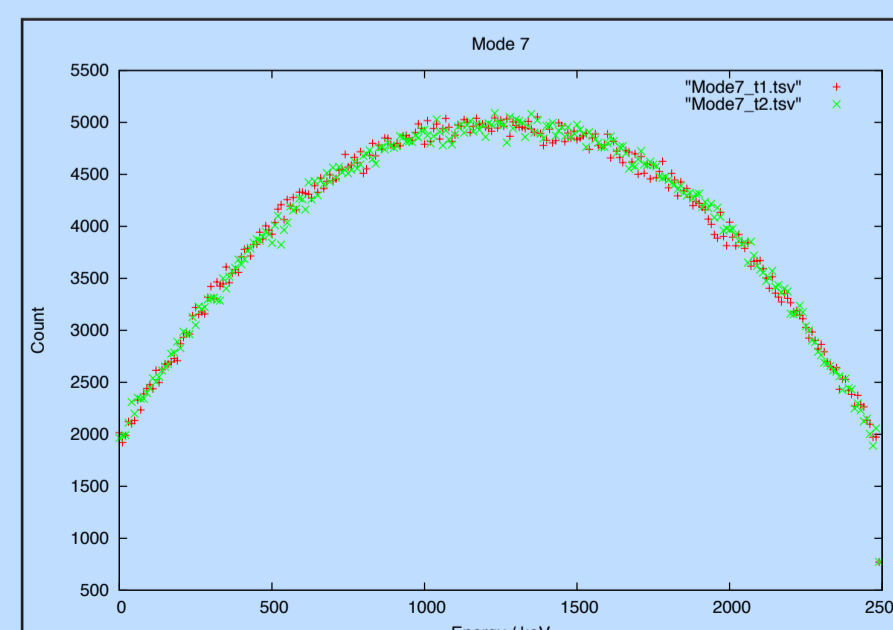


Fig. 8 - Neutrinoless decay with right-handed currents, 0^+-0^+ transition, 2n mechanism.

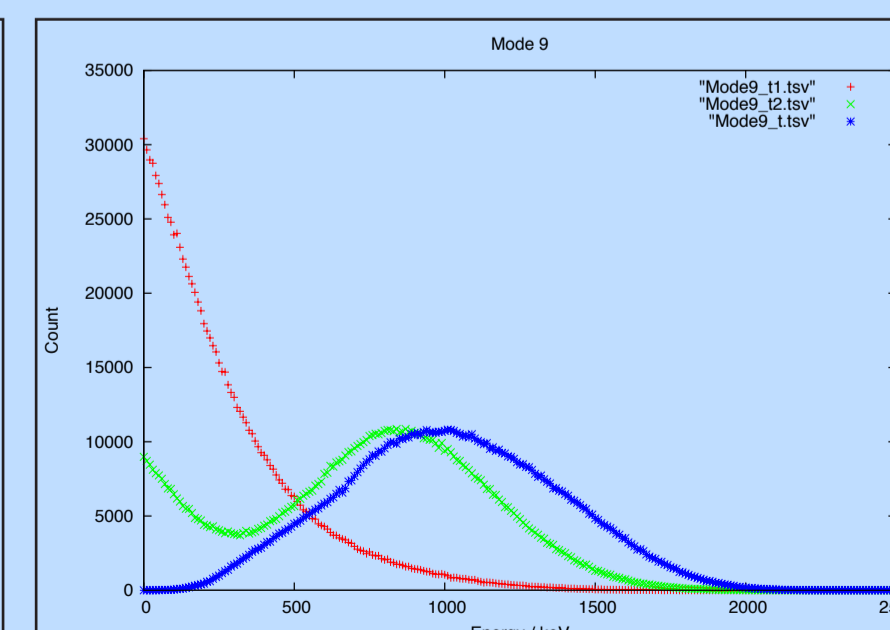


Fig. 9 - Two-neutrino decay, 0^+--2^+ transition, 2n mechanism and N^* mechanism.

curve expressed mathematically. In order to do this, the curves must be normalised to unit area (that is, integrating the equation for the probability distribution and setting it equal to one, thus obtaining a scale factor for normalisation).

This Monte Carlo sampling process is used to pick a point under the single-electron probability distribution F_1 , then the resulting T_1 value is used to fix a curve for the F_{12} distribution. A second number is picked from F_{12} , giving the energy T_2 of the second electron. Angular distributions are picked in a similar manner, with the first angle picked at random in the sphere and the second determined through Monte Carlo sampling of a correlated angular distribution. The events are output as a series of numbers representing the energy and the angles θ , Φ from an arbitrarily orientable axis, and output is also provided as a frequency of events in a given energy window, to allow for plots of the Monte Carlo sampling of large numbers of events to be compared against the theoretical probability distribution curves.

Results

The program, known internally as Pebble (momentum-Energy Beta-Beta Leptonic Events) developed during the time spent on this project was brought to a working state in which nine modes of decay are available at any given Q-value. The number of events to generate is also a parameter, and there exists an interface library to connect other programs into Pebble. Event generation is both fast and accurate, and the graphs in the centre column show the frequency distributions of events generated using Pebble's Monte Carlo sampling. Mode 8 (not shown) is identical to Mode 3, but uses the lower energy of 2494 keV because the excitation of the daughter nucleus required for a 0^+-2^+ transition reduces the Q-value for the decay.

The results from the Pebble event generator match with the theoretical probability distributions, which are used internally in the von Neumann acceptance-rejection testing approach to Monte Carlo sampling; a random T_1 in the range 0 to T_0 (here, 3034 keV) is picked along with a random number u in the range 0 to 1. The theoretical probability curve is normalised to unit area, and a test is performed to determine whether the value u times a normalised box around the distribution represents a point under the curve itself at the given T_1 value. If the point passes the test, the number is accepted and used as a T_1 energy value, otherwise the whole process is repeated. A similar approach is used for T_2 and the angles θ , Φ . The momenta are calculated from these energies and the generated angles, and the whole dataset is output to tab-separated data files.

References

- [1] V.I. Tretyak and Yu. G. Zdesenko, Atomic Data and Nuclear Data Tables **61**, 43 (1995)
- [2] W.-M. Yao et al., Journal of Physics G, **33**, 1 (2006)