Ionisation Cross Sections and the Composition of Dense Plasmas

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Plasma basics

Everyone knows about the three states of matter: solids, liquids and gases. But in actual fact, there are quite a few more! Plasmas are the simplest state of these extra states, and are often thought of as the fourth state of matter. When a solid is heated, bonds in the material are broken, and it turns into a liquid (melting). When a liquid is heated, further bonds are broken, and it turns into a gas (boiling). Similarly, when a gas is heated, the bonds holding the electrons around the nuclei are "broken" and the gas turns into a plasma (ionisation). Plasmas are really just gases consisting of charged particles, and have lots of interesting properties. Most of the visible universe is in fact made of plasmas.

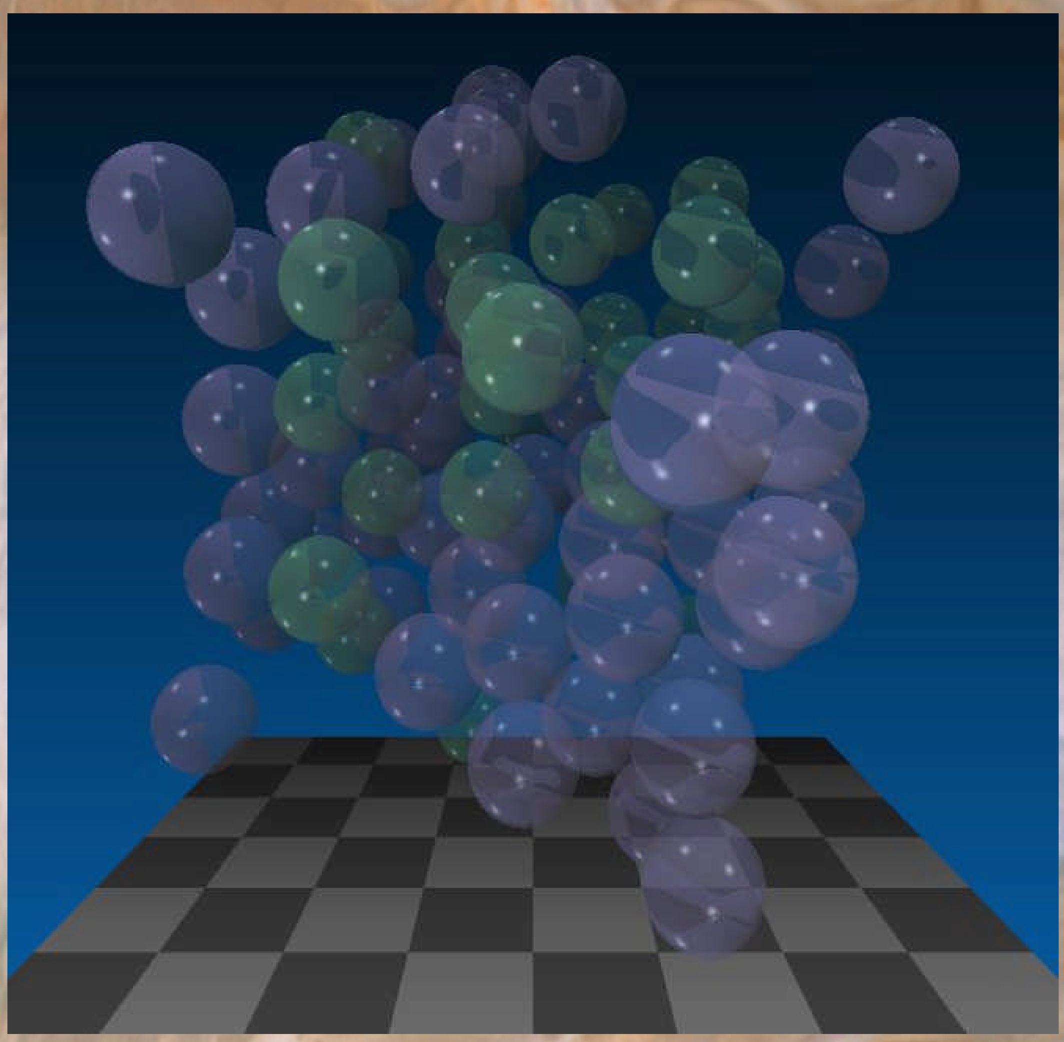
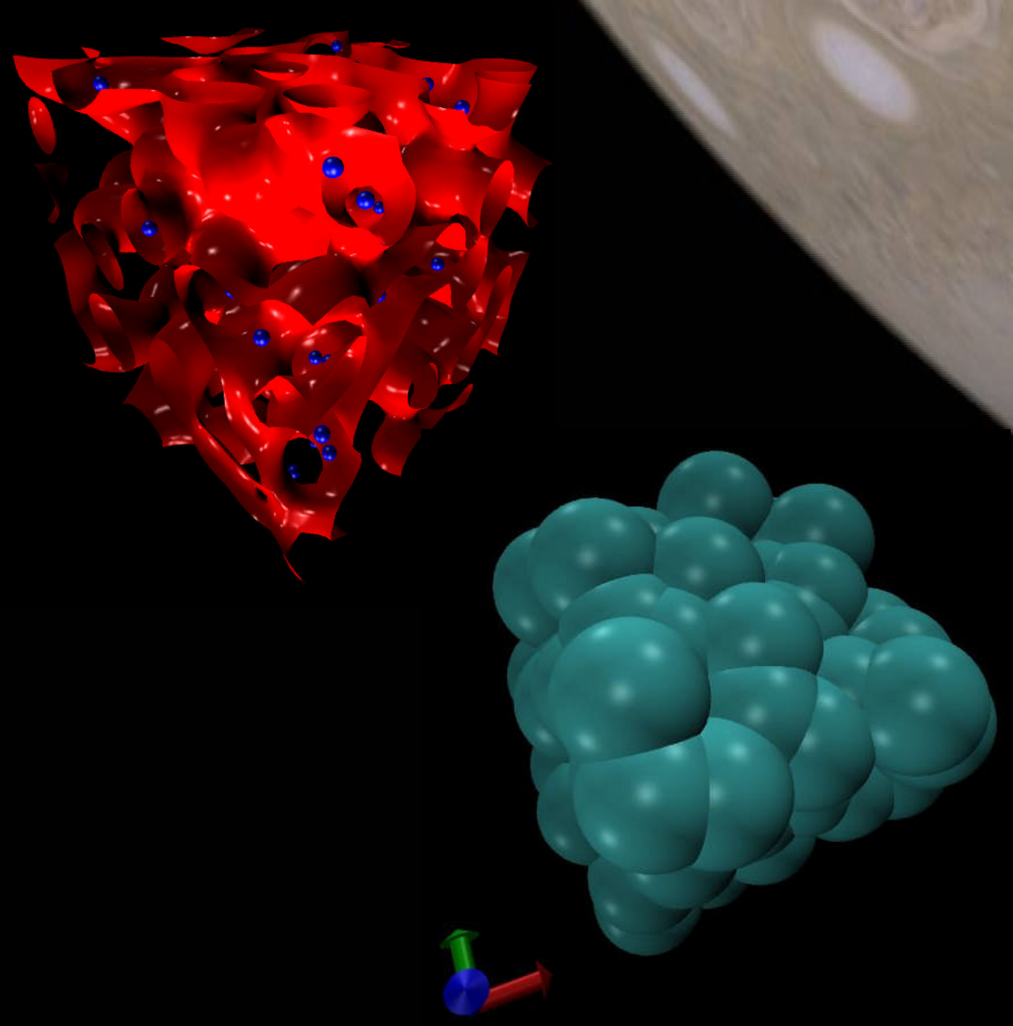


Fig. 1. A snapshot of proton-electron plasma generated using semi-commercial software, CPMD, and rendered using VMD.

Below are two different views of the same plasma. On the left, the wavefunctions of the electrons are shown. This is like the contours of pressure on a weather map, and shows us areas with equal probability of finding electrons. The image on the right depicts the positions of the ions in the plasma.



There are a few parameters that are important when considering plasmas. These define almost everything we need to know about the plasma.

Possibly the most important parameter is the Debye length, λ_D. This is the length over which electrons screen out the electric charge from the ions in the plasma.

The next is the degeneracy parameter, $n\lambda_e^3$, which is a measure of how degenerate the plasma is. Every object has an associated de Broglie wavelength, which is a measure of the "fuzziness" of that object. $n\lambda_e^3$ is simply the number of electrons in the thermal de Broglie wavelength of the electrons in a given plasma. The last parameter we need to consider is the coupling parameter, Γ, the ratio of the potential energy due to the coulomb interaction to the kinetic energy of the plasma.

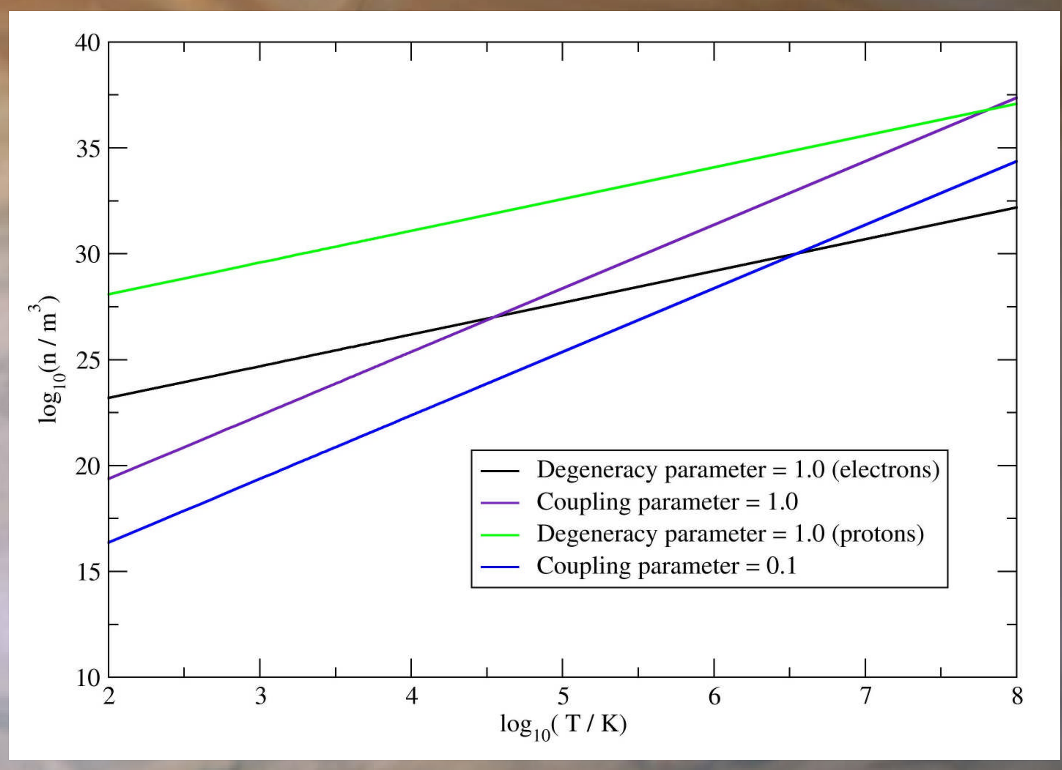


Fig. 2. Density-temperature plane. The lines show regimes where the degeneracy parameter is unity for plasmas consisting of electrons and protons respectively. Also shown are lines of constant coupling parameter.

Ionisation cross-section

The graph below (fig. 3) shows ionisation cross sections for several atomic species as a function of energy. These curves were generated by a C++ program of the empirical formula for the electron-impact ionisation cross-section[1]:

$$\sigma = \sum_{i=1}^{N} a_i q_i \frac{\ln(E/P_i)}{EP_i}\left\{1 - b_i \exp\left[-c_i\left(E/P_i - 1\right)\right]\right\}$$

where $E \geq P_i$.

The plasma is kept in a state of ionisation by the constant collision of high-energy free electrons with the atoms and ions in the plasma.

References

[1] Lotz, W., *Empirical Formula for the Electron-Impact Ionisation Cross-Section*, Zeitscrift Fur Physik, 206, 205-211, 1967

[2] Saha, M. N., *On a Physical Theory of Stellar Spectra*, Proceedings of the Royal-Society, vol. 49, p135-153, 1921

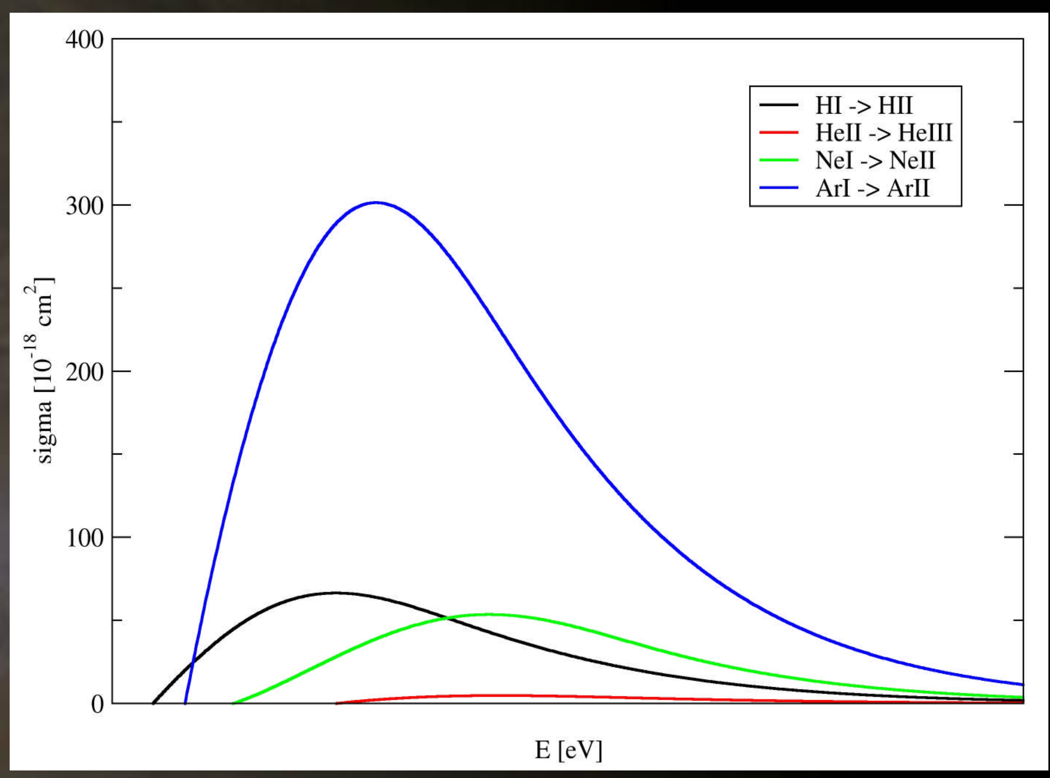


Fig. 3. Ionisation cross-sections for different atomic species.

Saha equation

One of the most important equations in plasma physics is the Saha equation. This equation tells us about the degree of ionisation in a plasma under equilibrium conditions. It was first derived in 1920 by Indian astrophysicist Meghnad Saha[2]

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

For hydrogen the Saha equation can be reduced to

$$\frac{n_p n_e}{n_H} = \frac{2}{\Lambda_e^3} e^{-\chi_i/kT}$$

It is useful to rearrange this equation so that the left hand side is a function of the ionisation degree which is defined as

$$x = \frac{n_p}{n_p + n_H}$$

So the Saha equation becomes

$$\frac{x}{1-x} = \frac{2}{n_e \Lambda_e^3} e^{-\chi_i/kT}$$

This equation cannot be solved analytically and must be solved numerically. This is done for a range of given number densities of nuclei (see fig. 4).

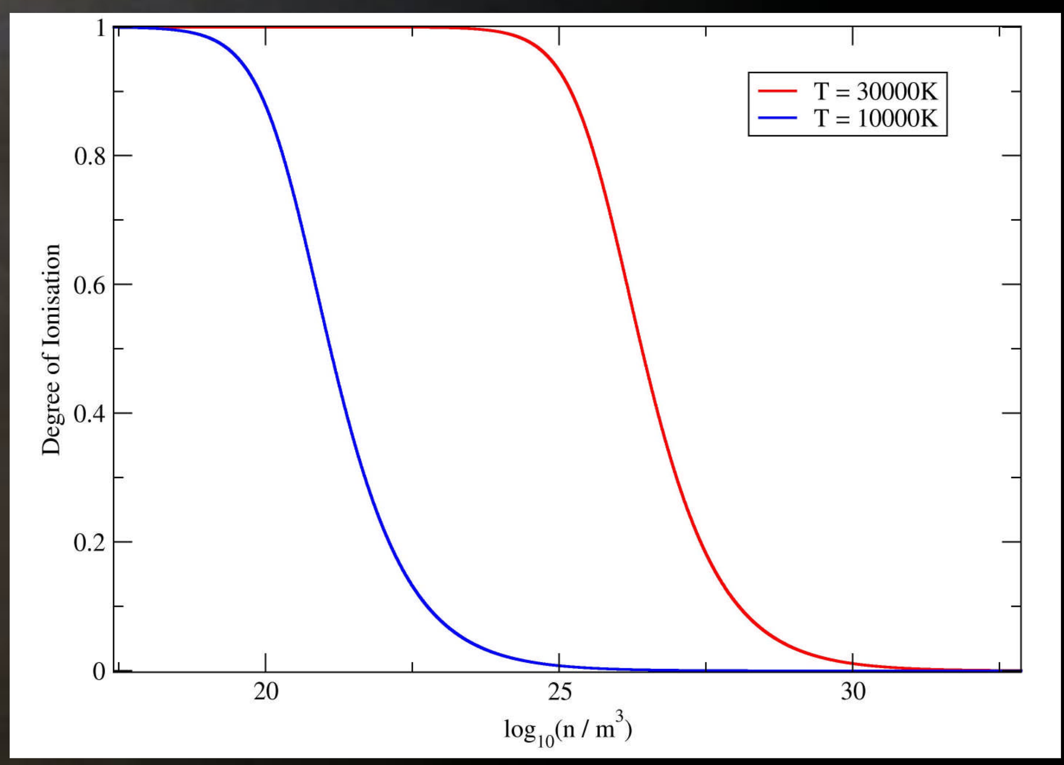


Fig. 4. The degree of ionisation plotted against the total number density of nuclei using the Saha equation for hydrogen.