ES2A7 - Fluid Mechanics Example Classes

Model Answers to Example Questions (Set I)

<u>Question 1:</u> Wind Tunnel

A simple wind-tunnel is depicted schematically in Figure 1. The flow speed in the working section is assumed to be constant with a value of V_B = 60m/s at point B. The cross-sectional area of the working section is 1 m^2.

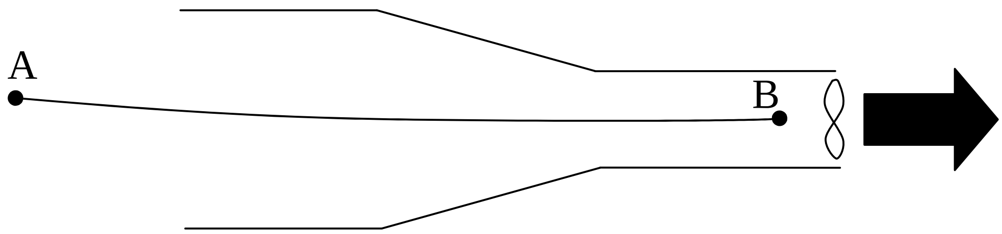

Figure 1: Sketch of simple wind-tunnel

i) Neglecting irreversible losses, such as those due to viscous effects, calculate the gauge pressure in the working section. (Assume a value of $\rho = 1.2$ kg/m^3 for the density of air).

ii) Calculate the mass flow rate across the cross-section in the working section.

i) Consider a typical streamline AB. Point B is in the working section and point A is located in surroundings where the pressure equals atmospheric pressure p_A and the flow speed is zero. According to the Bernoulli equation one then gets

$$p_A + \frac{1}{2}\rho V_A^2 = p_B + \frac{1}{2}\rho V_B^2$$

with $V_A = 0$ and $V_B = 60\,\text{ms}^{-1}$

$\therefore$ The gauge pressure is

$$p_B - p_A = -\frac{1}{2} \times 1.2\,\frac{kg}{m^3} \times \left(60\,\frac{m}{s}\right)^2 = -2160\,\text{Pa} \quad .$$

This means that the pressure in the working section is 2160 Pa below the atmospheric pressure. (N.B.: The modulo of this pressure difference corresponds approximately to the pressure resulting from a 21 cm high water column.)

ii) The mass flow rate Q across the cross-section in the working section is given by:

$$Q = \rho \times V_B \times cross - section \;\; area \;\; = \;\; 1.2 \frac{kg}{m^3} \times 60 \frac{m}{s} \times 1m^2 = 72 \frac{kg}{s}$$

<u>Question 2:</u> Plunger

A plunger is moving through a cylinder as schematically illustrated in the Figure 2. The velocity of the plunger is V_p = 10 ms^{-1}. The oil film separating the plunger from the cylinder has a dynamic viscosity of μ=0.3 N.s.m^{-2}. Assume that the oil-film thickness is uniform over the entire peripheral surface of the plunger. Calculate the force and the power required to maintain this motion.

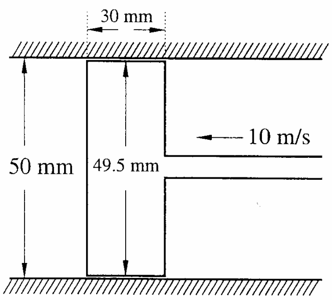

Figure 2: Plunger moving through cylinder.

The oil film is sufficiently thin such that we can assume a **linear velocity profile** for the flow of oil in the film. One can calculate the frictional resistance by computing the shear stress at the plunger surface by means of **Newton's law of viscosity**.

$$\tau = - \mu \frac{\partial V}{\partial r} = 0.3 \frac{Ns}{m^2} \frac{10 \frac{m}{s}}{\frac{1}{2}\left(50 \cdot 10^{-3} \; m - 49.5 \cdot 10^{-3} \; m\right)} = 12000 \frac{N}{m^2}$$

The frictional force can now be calculated by multiplying the shear stress with the surface of the plunger.

$$F = \tau \; A = 12000 \frac{N}{m^2} \;\; 2 \, \pi \, r \, h = 12000 N \;\; 2 \, \pi \frac{1}{2} 49.5 \cdot 10^{-3} \; m \quad 30 \cdot 10^{-3} \; m = 55.98 \; N$$

The power P required to drive the piston is

$$P = F \; V_P = 55.98 \; N \;\; 10 \frac{m}{s} = 559.8 \; W$$

<u>Question 3:</u> Flow over Wing

Assume that a plane is flying with a constant velocity of $v_0 = 55\,\text{ms}^{-1}$ at standard sea level conditions (Density air: ρ_0=1.23 kg m^{-3}, Pressure: $p_0 = 1.01\times10^5\,\text{Nm}^{-2}$). At some point on one of the plane's wings the pressure is measured as $p = 0.95\times10^5\,\text{Nm}^{-2}$. Calculate the flow velocity v at this point.

The question can be solved by relating the flow conditions far upstream of the wing to the flow conditions at the point on the wing considered by the Bernoulli equation. Note that we implicitly assume that wing is stationary and air is blowing at wing with velocity v_0 Bernoulli gives:

$$p_0 + \frac{1}{2}\rho_0 v_0^2 = p + \frac{1}{2}\rho_0 v^2$$

Solving for v gives

$$v = \sqrt{\frac{2(p_0 - p)}{\rho_0} + v_0^2} = \sqrt{\frac{2\left(1.01\times10^5\,\frac{N}{m^2} - 0.95\times10^5\,\frac{N}{m^2}\right)}{1.23\frac{kg}{m^3}} + \left(55\frac{m}{s}\right)^2} = 113.1\frac{m}{s}$$

<u>Question 4:</u> Sphere in Fluid

A sphere moves through oil. The constant velocity of the sphere is $u = 1\,\text{mms}^{-1}$. The dynamic viscosity of the oil is $\mu = 0.05$ Nsm^{-2} and its density is $\rho_o = 900\,\text{kgm}^{-3}$. The radius of the sphere is $r = 10$ mm and its density is $\rho_s = 1200\,\text{kgm}^{-3}$. (i) Use Figure 3 to estimate the drag forces acting on the sphere. (ii) Calculate the buoyancy force acting on the sphere.

(i) We need to calculate the Reynolds number associated with the motion in order to use the graphs in Figure 3.

$$\text{Re} = \frac{u\,d}{\nu} = \frac{\rho_o u d}{\mu} = \frac{900\,kgm^{-3}\,0.001\,ms^{-1}\,0.02\,m}{0.05\,Nsm^{-2}} = \frac{0.018}{0.05} = 0.36$$

For $\text{Re} = 0.36$ the graph gives $60 < C_D < 70$. The definition of the drag coefficient is

$$C_D = \frac{Drag\ Force}{\frac{\rho_0}{2}u^2 \times Cross-sectional\ Area\ exposed\ to\ flow}$$

$$C_D = \frac{Drag\ Force}{\frac{\rho_0}{2} u^2 \times \pi r^2}$$

$$\Rightarrow \quad Drag\ Force = C_D \times \frac{\rho_0}{2} u^2 \times \pi r^2 = C_D \times 1.4137 \times 10^{-7}\,\text{N}$$

Thus, with the limits for C_D obtained from the graph

$$8.4823 \times 10^{-6}\,\text{N} < Drag\ Force < 9.896 \times 10^{-6}\,\text{N}$$

Note: Exact Drag Coefficient is $C_D = \frac{24}{\text{Re}} = \frac{24}{0.36} = 66.67$. Also note, as 1kg corresponds to 10N the result for drag force is 'equivalent' to 'weight' of $8.4823 \times 10^{-7}\,\text{kg} < 'Weight' < 9.896 \times 10^{-7}\,\text{kg}$ or $8.4823 \times 10^{-4}\,\text{g} < 'Weight' < 9.896 \times 10^{-4}\,\text{g}$

(ii) The buoyancy force F_B is given by

$$F_B = g \times \text{Volume of fluid displaced} \times \text{Density of fluid displaced}$$

$$F_B = g \times \frac{4}{3}\pi r^3 \times \rho_o = \left(9.81\,ms^{-2}\right)\left(\frac{4}{3}\pi (0.01m)^3\right)\left(900\,kgm^{-3}\right) = 0.03698\,N$$

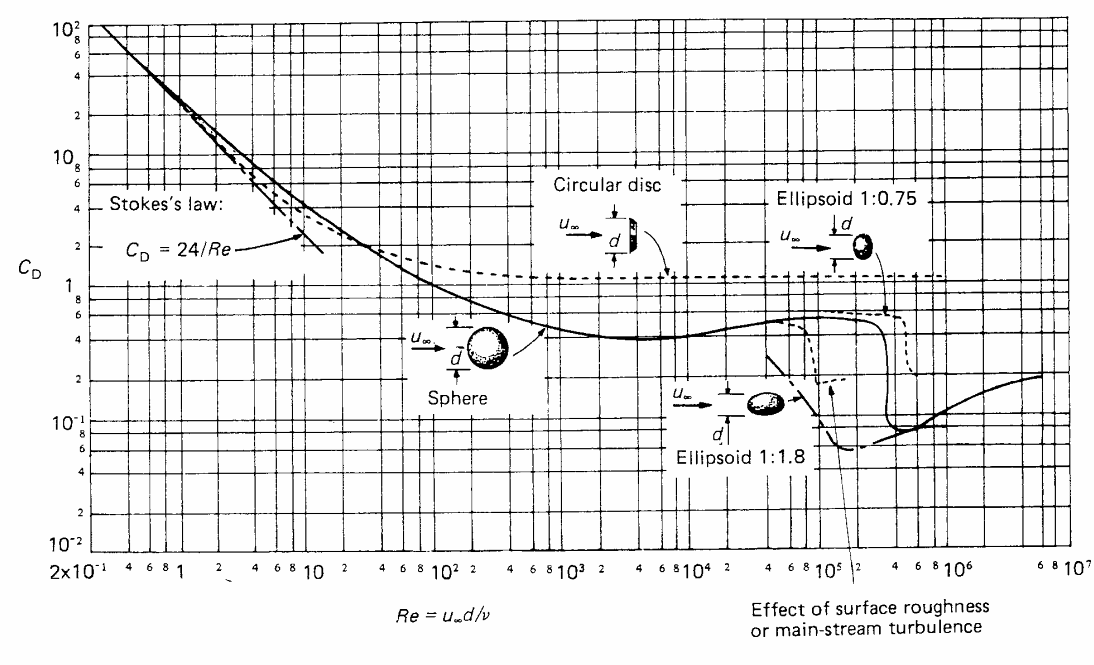

Figure 3: Drag coefficient of smooth, axially symmetric bodies (From: Massey, Mechanics of Fluids, Chapman & Hall, 1989, 6th Edition)

Question 5: **Fire Engine**

A fire engine pump develops a head of 50 m, i.e. it increases the energy per unit weight of the water passing through it by 50 N m N^{-1}. The pump draws water from a sump at A (Fig. 4) through a 150 mm diameter pipe in which there is a loss of energy per unit weight due to friction $h_1 = 5u_1^2/2g$ varying with the mean velocity u_1 in the pipe, and discharges it through a 75 mm nozzle at C, 30 m above the pump, at the end of a 100 mm diameter delivery pipe in which there is a loss of energy per unit weight $h_2 = 12u_2^2/2g$. Calculate (a) the velocity of the jet issuing from the nozzle at C and (b) the pressure in the suction pipe at the inlet to the pump at B.

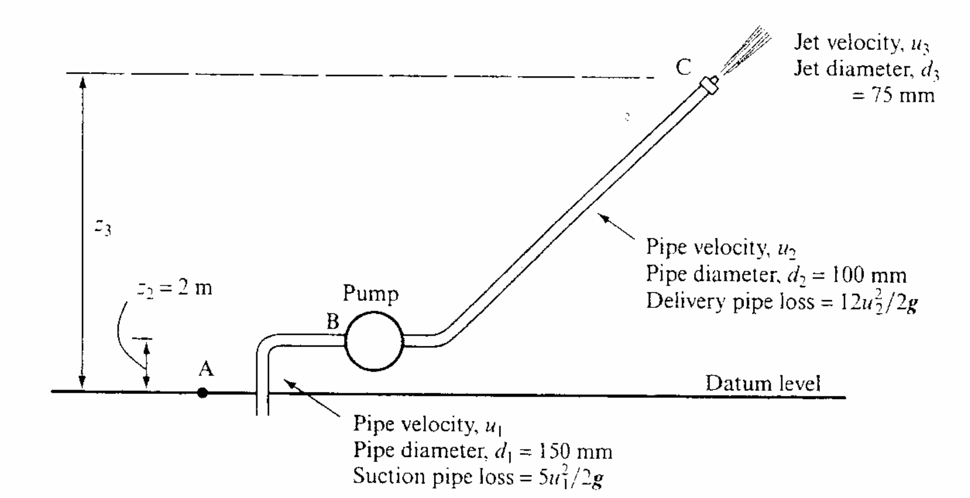

Figure 4: Fire engine Pump

(Note: Question taken from Douglas,Gasiorek & Swaffield, Fluid Mechanics, Prentice Hall, 4' edition, 2001, see pages 170-173)

(a) We can apply Bernoulli's equation between two points, one of which will be C, since we wish to determine the jet velocity u_3 and the other a point at which the conditions are known, such as a point A on the free surface of the sump where the pressure will be atmospheric, so that $p_A = 0$, the velocity u_1 will be zero if the sump is large, and A can be taken as the datum level so that $z_1 = 0$. Then,

$$\begin{bmatrix}\text{Total energy}\\ \text{per unit of weight}\\ \text{at A}\end{bmatrix} = \begin{bmatrix}\text{Total energy}\\ \text{per unit of weight}\\ \text{at C}\end{bmatrix} + \begin{bmatrix}\text{Loss in inlet}\\ \text{pipe}\end{bmatrix} - \begin{bmatrix}\text{Energy per unit}\\ \text{of weight supplied}\\ \text{by pump}\end{bmatrix} + \begin{bmatrix}\text{Loss in}\\ \text{discharge}\\ \text{pipe}\end{bmatrix} \quad (I)$$

$$\begin{bmatrix}\text{Total energy}\\ \text{per unit of weight}\\ \text{at A}\end{bmatrix} = \frac{p_A}{\rho_0 g} + \frac{u_1^2}{2g} + z_1 = 0$$

$$\begin{bmatrix}\text{Total energy}\\ \text{per unit of weight}\\ \text{at C}\end{bmatrix} = \frac{p_c}{\rho_0 g} + \frac{u_3^2}{2g} + z_3 = 0$$

With p_c = Atmospheric pressure = 0 and z_3 =30+2=32m.
Therefore,

$$\begin{bmatrix}\text{Total energy}\\ \text{per unit of weight}\\ \text{at C}\end{bmatrix} = 0 + u_3^2\,/2g + 32 = u_3^2\,/2g + 32 \text{ m.}$$

$$[\text{Loss in inlet pipe}] = h_1 = 5u_1^2/2g$$

$$[\text{Energy per unit of weight supplied by pump}] = 50\text{m}$$

$$[\text{Loss in delivery pipe}] = h_2 = 12u_2^2/2g$$

Substituting in (I),

$$0 = (u_3^2/2g + 32) + 5\,u_1^2/2g - 50 + 12u_2^2/2g,$$

$$u_3^2 + 5u_1^2 + 12u_3^2 = 2g \times 18. \qquad \text{(II)}$$

From the continuity of flow equation,

$$(\pi/4)\,d_1^2 u_1 = (\pi/4)\,d_2^2 u_2 = (\pi/4)\,d_3^2 u_3$$

therefore,

$$u_1 = \left(\frac{d_3}{d_1}\right)^2 u_3 = \left(\frac{75}{150}\right)^2 u_3 = \frac{1}{4} u_3$$

$$u_2 = \left(\frac{d_3}{d_2}\right)^2 u_3 = \left(\frac{75}{100}\right)^2 u_3 = \frac{9}{16} u_3$$

Substituting in equation (II),

$$u_3^2\left(1+5\times\left(\frac{1}{4}\right)^2 + 12\times\left(\frac{9}{16}\right)^2\right) = 2g\times 18$$

$$5.109u_3^2 = 2g\times 18$$

$$u_3 = 8.314 \text{m.s}^{-1}$$

(b) If P_B is the pressure in the suction pipe at the pump inlet, applying Bernoulli's equation to A and B,

$$\begin{bmatrix}\text{Total energy}\\ \text{per unit of weight}\\ \text{at A}\end{bmatrix} = \begin{bmatrix}\text{Total energy}\\ \text{per unit of weight}\\ \text{at B}\end{bmatrix} + \begin{bmatrix}\text{Loss in inlet}\\ \text{pipe}\end{bmatrix}$$

$$0 = (p_B / g\rho + u_1^2 / 2g + z_2) + 5u_1^2 / 2g,$$

$$p_B / g\rho = -z_2 - 6u_1^2 / 2g,$$

$$z_2 = 2\text{m},\ u_1 = \frac{1}{4} u_3 = 8.134/4 = 2.079\text{m.s}^{-1}$$

$$p_B / g\rho = -(2 + 6 \times 2.079^2 / 2g) = -(2 + 1.32) = -3.32\text{m},$$

$$p_B = -1000 \times 9.81 \times 3.32 = 32.569\text{kN.m}^{-2}$$ below atmospheric pressure.

<u>Question 6:</u> Multi-Fluid Manometer

A multi-fluid manometer is set up as shown in Figure 5. The pressure at its right end is $p_2 = 0.9 \times 10^5$ Nm^{-2}. The densities of the fluids are $\rho_A = 1000$ kg m^{-3}, $\rho_B = 900$ kg m^{-3} $\rho_C = 13000$ kg m^{-3}. One measures $h_1 = 0.5$ m $h_2 = 0.3$ m and $h_3 = 0.6$ m. Find the pressure p_1.

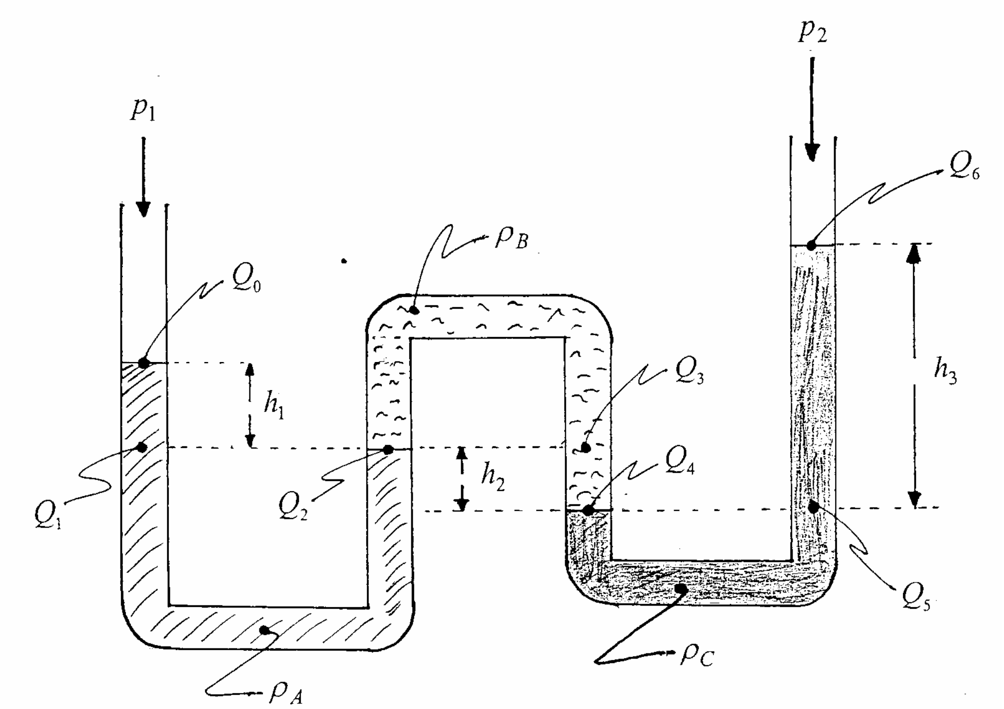

Figure 5: Multi-fluid manometer

Start at Point Q_0 at surface of fluid at left end where pressure is p_1. Then move along the tube and add or subtract appropriate $\rho - g - h$-terms until reaching Point Q_6 and setting the result equal to pressure p_2.

- From $Q_0 \rightarrow Q_1$: Point Q_1 lies by height h_1 lower than Q_0. Hence pressure here is $\Delta p_{0,1} = \rho_A \; g \; h_1$ higher than pressure in Q_0 (where pressure is p_1).
- From $Q_1 \rightarrow Q_2$: Both points at same height hence $\Delta p_{1,2} = 0$.
- From $Q_2 \rightarrow Q_3$: Both points at same height hence $\Delta p_{2,3} = 0$.
- From $Q_3 \rightarrow Q_4$: Point Q_4 lies by height h_2 below Point Q_3. Hence pressure increases by $\Delta p_{3,4} = \rho_B \; g \; h_2$ with respect to pressure in Q_3.
- From $Q_4 \rightarrow Q_5$: Both points at same height hence $\Delta p_{4,5} = 0$.
- From $Q_5 \rightarrow Q_6$: Point Q_6 lies by height h_3 above Point Q_5. Hence pressure decreases by $\Delta P_{5,6} = -\rho_C \; g \; h_3$ with respect to pressure in Q_5.
- Now, at Point Q_6 is pressure is p_2.

Summing up all terms gives and equating to pressure p_2 in Point Q_6 gives:

$$p_1 + \Delta p_{0,1} + \Delta p_{1,2} + \Delta p_{2,3} + \Delta p_{3,4} + \Delta p_{4,5} + \Delta p_{5,6} = p_2$$

and, hence : $$p_1 + \rho_A \; g \; h_1 + 0 + 0 + \rho_B \; g \; h_2 + 0 - \rho_C \; g \; h_3 = p_2$$

Solving for p_1 yields :

$$\begin{aligned}
p_1 &= p_2 - \rho_A \; g \; h_1 - \rho_B \; g \; h_2 + \rho_C \; g \; h_3 \\
&= p_2 + g \left(\rho_C h_3 - \rho_A h_1 - \rho_B h_2 \right) \\
&= 90 \; kPa + 9.81 \frac{m}{s^2} \left(13000 \frac{\text{kg}}{\text{m}^3} 0.6 \text{ m} - 1000 \frac{\text{kg}}{\text{m}^3} 0.5 \text{ m} - 900 \frac{\text{kg}}{\text{m}^3} 0.3 \text{ m} \right) \\
&= 90 \; kPa + \left(76518 \frac{\text{kg}}{\text{m s}^2} - 4905 \frac{\text{kg}}{\text{m s}^2} - 2648.7 \frac{\text{kg}}{\text{m s}^2} \right) \\
&= 90 \; kPa + (68964.3 \; Pa) = 158964.3 \text{ Pa} = 158.9643 \text{ kPa}
\end{aligned}$$