

ES2A7 - Fluid Mechanics Example Classes

Model Answers to Example Questions (Set II)

(If you spot any typing errors etc in this document I would be grateful if you would send me an email concerning this such that I can update my file. Thank you! My email address is pjt@eng.warwick.ac.uk)

Question 1: DIMENSIONAL ANALYSIS

Use dimensional analysis to determine the period t for small oscillations of a simple pendulum (Fig. 1) of length l . Assume that the period depends on the length of the pendulum, the mass of the oscillating body and the gravitational acceleration of the Earth. (N.B.: You will find that the correct answer to the question will imply that the period must in fact be independent of the mass of the oscillating body.)

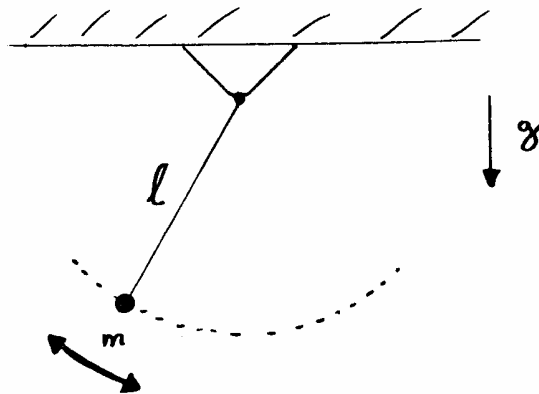


Figure 1: Sketch of Pendulum Geometry

Quantities involved in the problem are:

	QUANTITY	UNITS	DIMENSIONS
t :	period of oscillation	[s]	T
l :	length of pendulum	[m]	L
m :	mass of oscillating body	[kg]	M
g :	gravitational acceleration	$\left[\frac{m}{s^2}\right] = [m s^{-2}]$	$L T^{-2}$

Assume:

$$t \propto l^\alpha \cdot m^\beta \cdot g^\gamma$$

thus

$$(T)^1 = (L)^\alpha \cdot (M)^\beta \cdot (L \cdot T^{-2})^\gamma$$

such that

$$L^0 \cdot M^0 \cdot T^1 = L^\alpha \cdot M^\beta \cdot L^\gamma \cdot T^{-2\gamma}$$

and hence

$$L^0 \cdot M^0 \cdot T^1 = L^{\alpha+\gamma} \cdot M^\beta \cdot T^{-2\gamma}$$

compare exponents to obtain:

$$\text{From 'M' one gets : } \underline{\beta=0}$$

$$\text{From 'T' one gets : } -2\gamma = 1 \text{ and thus } \underline{\gamma = -\frac{1}{2}}$$

$$\text{From 'L' one gets : } 0 = \alpha + \gamma \text{ and thus } \underline{\alpha = -\gamma = \frac{1}{2}}$$

The final result is thus

$$t \propto l^{\frac{1}{2}} \cdot m^0 \cdot g^{-\frac{1}{2}}$$

such that

$$t = \text{const} \cdot \sqrt{\frac{l}{g}}$$

The (non-dimensional !) constant can in principle be determined from one single experiment or one can obtain it from some other theoretical considerations. It turns out that it has a value of $\text{const} = 2\pi \approx 6.28$

N.B.: Dimensional analysis yields the result that the period of oscillation must be independent of the mass!

Question 2: Dynamic Similarity

The flow around an airship with a diameter $d = 3\text{ m}$ and a length $l = 20\text{ m}$ needs to be studied in a wind tunnel. The airspeed range to be investigated is at the docking end of its range, a maximum of $v_p = 2\text{ ms}^{-1}$. Calculate the mean model wind tunnel speed if the model is made to 1/10 scale. Assume the same air pressure and temperature for model and prototype.

Dynamic similarity requires that the Reynolds number of model and prototype need to be the same. This gives

$$\frac{v_m l_m \rho}{\mu} = \frac{v_p l_p \rho}{\mu}$$

Resolving for v_m yields

$$v_m = v_p \frac{l_p}{l_m} \frac{\rho_p}{\rho_m} \frac{\mu_m}{\mu_p}$$

As the air pressure is assumed to be the same for prototype conditions and model conditions one thus gets

$$v_m = 2 \frac{m}{s} \times \frac{3 m}{0.3 m} \times 1 \times 1 = 20 \frac{m}{s}.$$

Question 3: Manometer

In Fig. 2 the liquids at A and B are water and the manometer liquid is oil. Given are the density of water $\rho_w = 1000 \text{ kgm}^{-1}$, the density of the oil $\rho_o = 800 \text{ kgm}^{-1}$, $h_1 = 300 \text{ mm}$, $h_2 = 200 \text{ mm}$, $h_3 = 600 \text{ mm}$, the gravitational acceleration of the Earth $g = 9.81 \text{ ms}^{-2}$. Determine the pressure difference $p_A - p_B$ in Pascals.

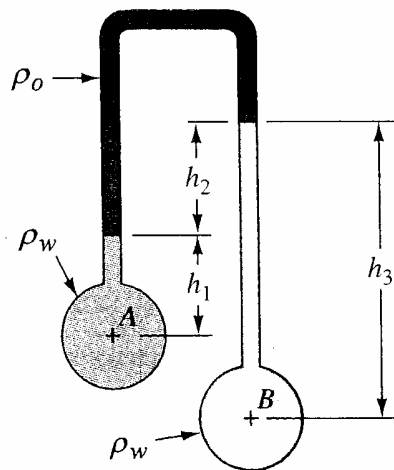


Figure 2: Differential Manometer

The system is in equilibrium. Hence it is required that

$$\begin{aligned} p_A - \rho_w g h_1 - \rho_o g h_2 &= p_B - \rho_w g h_3 \\ \Rightarrow p_A - p_B &= \rho_w g h_1 + \rho_o g h_2 - \rho_w g h_3 = g (\rho_o h_2 + \rho_w (h_1 - h_3)) \\ &= 9.81 \text{ ms}^{-2} (800 \text{ kgm}^{-3} 0.2 \text{ m} + 1000 \text{ kgm}^{-3} (0.3 \text{ m} - 0.6 \text{ m})) \\ &= 9.81 \text{ ms}^{-2} (800 \text{ kgm}^{-3} 0.2 \text{ m} + 1000 \text{ kgm}^{-3} (0.3 \text{ m} - 0.6 \text{ m})) \\ &= 9.81 \text{ ms}^{-2} (160 \text{ kgm}^{-2} - 300 \text{ kgm}^{-2}) = -1373 \frac{\text{kg}}{\text{s}^2 \text{ m}} = -1373.4 \text{ Pa} \end{aligned}$$

Question 4: Sliding Board

A board with an area $A = 1 \text{ m} \times 1 \text{ m}$ slides down an inclined ramp as is schematically illustrated in Figure 3. The ramp is inclined at an angle $\alpha = 14^\circ$. The weight of the board is $W = 40 \text{ N}$ and it slides with a constant velocity of $U = 3.0 \text{ cm s}^{-1}$. The board is separated from the ramp by a thin film of oil with a dynamic viscosity of $\mu = 0.05 \text{ N s m}^{-2}$. Assume that edge effects are negligible. Assume further that the distribution of the fluid velocity across the oil-filled gap between the ramp and the board is linear. Calculate the gap width d between the board and the ramp. (Hint: The board slides at a constant velocity when the component of the weight parallel to the inclined ramp is equal to the resisting shear force.)

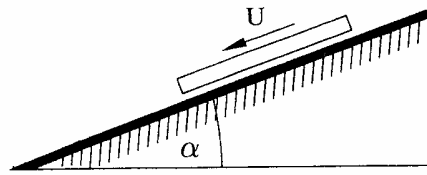


Figure 3: Board Sliding on Ramp

The mathematical interpretation of the hint given is

$$F_{\text{tan gential}} = F_{\text{shear}} \quad (1)$$

The component of the weight tangential to the ramp is given by

$$F_{\text{tan gential}} = W \sin \alpha.$$

The shear stress τ can be calculated from Newton's law of viscosity

$$\tau = \mu \frac{dU}{dy}.$$

As we are assuming a linear velocity distribution in the oil film this gives

$$\tau = \mu \frac{\Delta U}{\Delta y}.$$

The shear force F_{shear} is then

$$F_{\text{shear}} = \tau A = \mu \frac{\Delta U}{\Delta y} A = \mu \frac{U}{d} A.$$

Substituting the expressions for $F_{\text{tan gential}}$ and F_{shear} in the above expression (1) gives

$$W \sin \alpha = \mu \frac{U}{d} A$$

from which one finds

$$d = \frac{\mu U A}{W \sin \alpha} = \frac{0.05 \frac{\text{Ns}}{\text{m}^2} \cdot 0.03 \frac{\text{m}}{\text{s}} \cdot 1 \text{m}^2}{40 \text{N} \sin 14^\circ} = 0.000155 \text{ m} = 0.155 \text{ mm}$$

Question 5: Discharge from Nozzle of a Reservoir

Use the Bernoulli equation and mass conservation to find a relation between the nozzle discharge velocity V_2 and tank free-surface height h as shown in Figure 4.

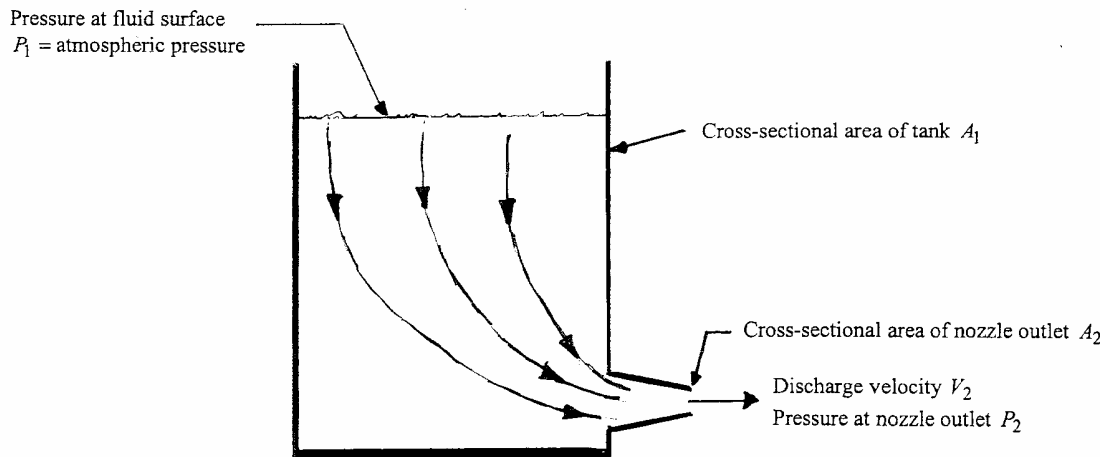


Figure 4: Discharge from Nozzle of a Reservoir

(Answer from: F.M. White, Fluid Mechanics 3^d edition, McGraw Hill, 1994, p. 161)

Choose point 1 upstream and point 2 downstream. Try to choose points 1 and 2 where maximum information is known or desired. Here we select point 1 as the tank free surface, where elevation and pressure are known, and point 2 as the nozzle exit, where again pressure and elevation are known. The two unknowns are V_1 and V_2 .

Mass conservation is usually a vital part of Bernoulli analyses. If A_1 is the tank cross section and A_2 the nozzle area, this is approximately a one-dimensional flow with constant density:

$$A_1 V_1 = A_2 V_2 \quad (1)$$

Bernoulli's equation gives :

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

But since sections 1 and 2 are both exposed to atmospheric pressure $P_1 = P_2$ pressure terms cancel, leaving:

$$V_2^2 - V_1^2 = 2g(z_1 - z_2) = 2gh \quad (2)$$

Eliminating V_1 , between Eqs. (1) and (2), we obtain the desired result:

$$V_2^2 = \frac{2gh}{1 - A_2^2/A_1^2} \quad (3)$$

Generally the nozzle area A_2 is very much smaller than the tank area A_1 so that the ratio A_2/A_1 is doubly negligible, and an accurate approximation for the outlet velocity is

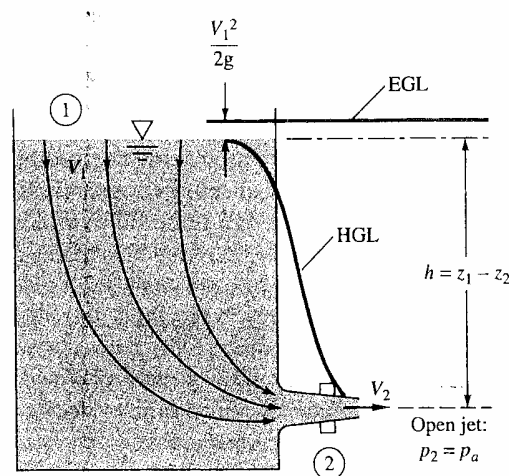
$$V_2 \approx \sqrt{2gh} \quad (4)$$

This formula, discovered by Evangelista Torricelli in 1644, states that the discharge velocity equals the speed which a frictionless particle would attain if it fell freely from point 1 to point 2. In other words, the potential energy of the surface fluid is entirely converted to kinetic energy of efflux, which is consistent with the neglect of friction and the fact that no net pressure work is done. Note that Eq. (4) is independent of the fluid density, a characteristic of gravity-driven flows.

Except for the wall boundary layers, the streamlines from 1 to 2 all behave in the same way, and we can assume that the Bernoulli constant h_0 is the same for all the core flow. However, the outlet flow is likely to be nonuniform, not one-dimensional, so that the average velocity is only approximately equal to Torricelli's result. The engineer will then adjust the formula to include a dimensionless discharge coefficient C_d

$$(V_2)_{av} \approx C_d \sqrt{2gh} \quad (5)$$

The discharge coefficient of a nozzle varies from about 0.6 to 1.0 as a function of (dimensionless) flow conditions and nozzle shape.



Question 6: Transition of Pipe Flow

A Reynolds number for flow in a circular pipe is defined as $Re = Vd/\nu$ (V : flow velocity averaged across the cross section of the pipe d : pipe diameter, ν : kinematic viscosity of the fluid flowing inside the pipe). The accepted critical Reynolds number for laminar-turbulent transition for the flow is $Re \approx 2300$. For flow through a 60 mm-diameter pipe, at what velocity will this occur for (a) water with $\nu = 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$ and (b) air with a dynamic viscosity of $\mu = 1.82 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$ and a density $\rho = 1.205 \text{ kg m}^{-3}$.

$$\text{Re} = \frac{V d}{\nu} \Rightarrow V = \frac{\nu \text{Re}}{d}$$

$$\text{i) } V = \frac{0.1 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \cdot 2300}{0.06 \text{ m}} = 0.0383 \frac{\text{m}}{\text{s}}$$

$$\text{ii) } \text{As } \nu = \frac{\mu}{\rho} = \frac{1.82 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}}{1.205 \text{ kg m}^{-3}} = 1.51 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$$

$$V = \frac{1.51 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \cdot 2300}{0.06 \text{ m}} = 0.579 \frac{\text{m}}{\text{s}}$$

Question 7: Minimum Flight Speed of Plane

Calculate the minimum flight speed U of an aircraft flying at constant altitude. Consider an aircraft with maximum take-off mass $m = 30,740 \text{ kg}$, a lift coefficient $C_L = 1.2$ and a total lifting surface of $A = 140 \text{ m}^2$. Assume that the cruising altitude is 10 km where the density of air is $\rho = 0.0414 \text{ kg.m}^{-3}$ and the acceleration due to gravity is $g = 9.776 \text{ m.s}^{-2}$. (Hint: For horizontal flight the lift force must balance the gravitational force)

For horizontal flight the lift force F_L must balance the gravitational force F_g which means:

$$F_g = F_L$$

Since

$$F_g = mg$$

And lift coefficient is :

$$C_L = \frac{F_L}{\frac{1}{2} \rho U^2 A} \Rightarrow F_L = C_L \frac{1}{2} \rho U^2 A$$

One gets

$$mg = C_L \frac{1}{2} \rho U^2 A$$

and after rearranging

$$U = \sqrt{\frac{2mg}{C_L \rho A}} = \sqrt{\frac{2 \times 30,740 \text{ kg} \times 9.776 \text{ ms}^{-2}}{1.2 \times 0.414 \text{ kg m}^{-3} \times 140 \text{ m}^2}} = \sqrt{8641.4 \frac{\text{m}^2}{\text{s}^2}} = 92.96 \frac{\text{m}}{\text{s}}$$

Question 8: Sprinter

Consider a sprinter running with velocity $V_0=10 \text{ m}\cdot\text{s}^{-1}$ in still air (density: $\rho=1.2 \text{ kg}\cdot\text{m}^{-3}$) as shown in Figure 5. Luckily the sprinter measured the atmospheric pressure before he set off and phoned you up to tell you that it is equal to $P_A=101.3 \text{ kPa}$. What pressure will the sprinter experience while running?

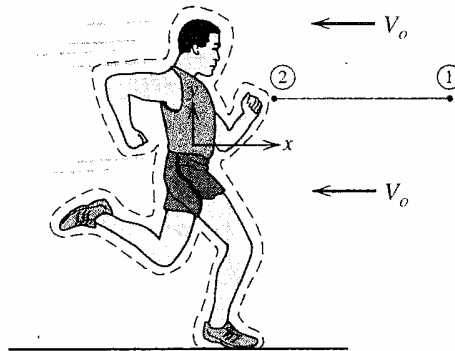
Select the sprinter as the control volume and prescribe a coordinate system attached to the sprinter as shown in the figure below. The problem is equivalent to having a stationary sprinter air flows past him with the velocity V_0 at atmospheric pressure P_A . The Bernoulli equation applied between points (1) and (2) along the selected streamline as shown in the figure gives

$$P_0 = P_A + \frac{\rho}{2} V^2$$

where at (1) $P = P_A$ and the velocity of Air is V , and at (2) the velocity of air is zero and the pressure is equal to $P = P_0$. This indicates that the sprinter experiences a pressure that increases with the square of the velocity.

For the given conditions, the stagnation pressure then is

$$P = 101.3 \text{ kPa} + 1.2 \frac{\text{kg}}{\text{m}^3} \times \frac{\left(10 \frac{\text{m}}{\text{s}}\right)^2}{2} = 101.36 \text{ kPa}$$



Relative to the sprinter