ES3C9

FLUID MECHANICS FOR MECHANICAL ENGINEERS

Example Sheet 3 - Pipe Flows

(1) Plane Poiseuille Flow: Consider the flow between two infinitely wide plates, driven by a pressure gradient, as shown in Fig. 1.

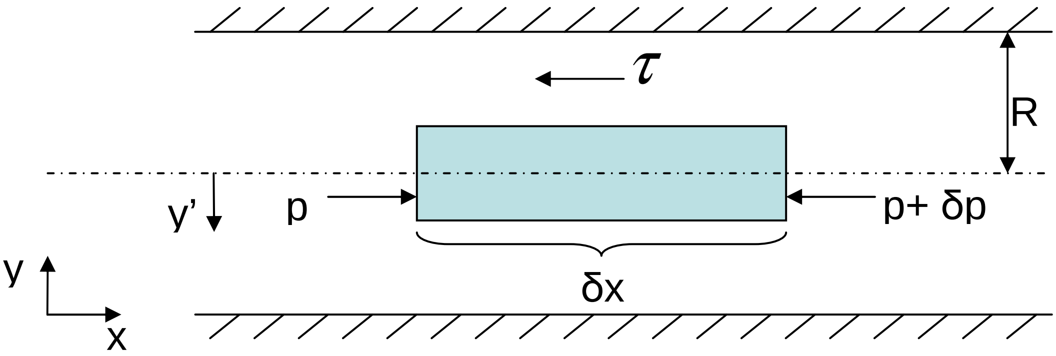


Fig. 1

(a) Use the suggested fluid element to show that the velocity profile for laminar flow is given by :

$$u(y) = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)\left(2Ry - y^2\right)$$

Write down the maximum velocity U_{max} in the pipe.

(b) By first evaluating the volumetric flow rate in the duct, show that the mean velocity is given by $u_m = 2u_{max}/3$. Compare the present results for U_m and U_{max} with those found for Poiseuille flow in a pipe with circular cross-section, ie. (from the notes):

$$u_m = \frac{R^2}{8\mu}\left(-\frac{dp}{dx}\right) \qquad u_{max} = \frac{R^2}{4\mu}\left(-\frac{dp}{dx}\right)$$

Is the difference reasonable ?

(c) Compute (i) the wall shear stress, (ii) the stream function, (iii) the vorticity, (iv) the velocity potential, and (v) the average velocity (Use y' instead of y).

(2) Laminar flow exists in the inclined pipe shown in Fig. 2a. The flow is "downhill" being driven by both the gravitational force and an applied pressure gradient, dp/dx, along the length of the pipe. z is an upward coordinate whilst x is a coordinate along the pipe centerline.

(a) By considering the suggested fluid element in Fig. 2a, show that the velocity profile is given by:

$$u(r) = \frac{1}{4\mu}\left(-\frac{d}{dx}(p+\rho g z)\right)\left(R^2 - r^2\right)$$

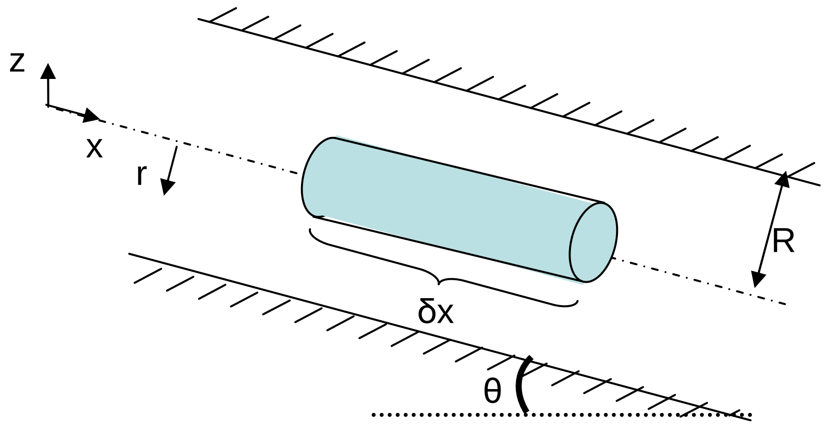


Fig. 2a

(b) A constant-head reservoir, of S.A.E. 30 oil (μ=0.290Ns/m^2, ρ=917kg/m^3) feeds the flow in an inclined pipe as shown in Fig. 2b. Given that h = 3.0 m, z_0=0.5m, x_o = 1.0m and R = 0.01m, find the maximum flow velocity U_{max} in the pipe.

[ANS. 2.71 m/s]

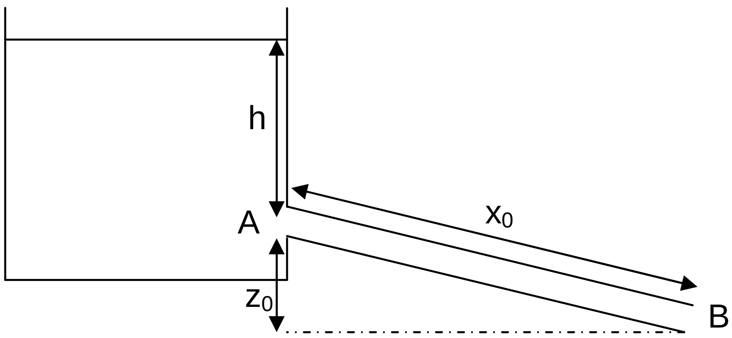


Fig. 2b

(d) Show that the volumetric flow rate through the pipe, Q, is given by:

$$Q = \frac{\pi R^4}{8\mu}\left(-\frac{d}{dx}(p+\rho g z)\right)$$

Hence determine the mean velocity and confirm that the flow is, indeed, laminar

[ANS. 1.36 m/s]

(3) Kerosine (μ=0.00192Ns/m^2, ρ =804kg/m^3) flows in a horizontal pipe of diameter 2.0cm and length 5.0m. The volumetric flow rate, Q is 0.754 liters/s

(a) Calculate the Reynolds number of the flow.
(b) Using the index laws for velocity profile, calculate the flow speed at the pipe centerline.
(c) Use the Blasius equation to calculate the coefficient of friction and hence the shear stress at the wall for this flow.
(d) Calculate the pressure head loss for the 5 m length of pipe.
(e) Calculate the (output) power of a pump needed to drive this flow.

[ANS. (a) 20,000, (b) 2.97m/s, (c) 3.322x10^{-3}, (d) 1.95m, (e) 11.6W]

(4) For the same flow as described in Question (3), use the logarithmic law to calculate the coefficient of friction. Compare this with the answer for C_f found using the index law in question (3c).

[ANS. 3.191×10^{-3}]

Calculate the friction velocity V* for the flow and hence the flow speed at the centerline using the logarithmic law for the velocity profile. Compare this with the answer to (3b).

[ANS. V*=2.841m/s, U_{max}=2.936m/s]

(5) The pipe used in Questions (3) and (4) is now discovered to have rough walls. The equivalent sand-grain roughness size, k_s, is 1.0 mm.

(a) Assuming the pipe wall to be completely rough calculate the new coefficient of friction.

(b) By first calculating the friction velocity, V*, verify that we are justified in making the completely rough assumption.

(c) Using the value for the coefficient of friction found in part (5a), calculate the pressure head loss for the 5m length of pipe. Compare this answer with that of Question (3d) pertaining to the hydraulically smooth pipe assumption.

[ANS. (a) 9.157×10^{-3}, (b) V*=0.230m/s, (c) 5.38m]

(6) Make use of the Moody chart (Fig. 7.11 in the notes) to determine the head loss due to friction when water flows through 300m of 150mm diameter galvanized steel pipe at 50 liters/s. Take the kinematic viscosity to be 1.14mm^2/s.

[ANS. 16.7m]

(7) Using the Moody chart, determine the diameter of galvanized steel pipe needed to carry water ($\mu = 10^{-3}$ Ns/m^2, $\rho = 10^3$ kg/m^3) a distance of 180 m at 85 liter/s with a head loss of 9m.

[ANS. 0.2 m]

(8) A simple hydraulic system is illustrated in Fig. 8a. It consists of a large reservoir and a horizontal pipeline 200 m long discharging to atmosphere through a valve. The first 120m of pipe connected to the reservoir has a diameter of 200 mm. The remainder of the pipeline has a diameter of 250 mm. Galvanized steel is used throughout as the pipe material. The pipeline is connected to the reservoir at a point 8 m below the water surface, Formulae for head losses due to sudden changes in pipe diameter are given in Fig. 8b.

If the valve is set to discharge 0.1 m^3/s of Water ($\mu=10^{-3}$ Ns/m^2, $\rho=10^3$ kg/m^3) plot, to scale, the hydraulic and energy gradelines (ie. the variations of piezometric and total heads) for the pipe-line.

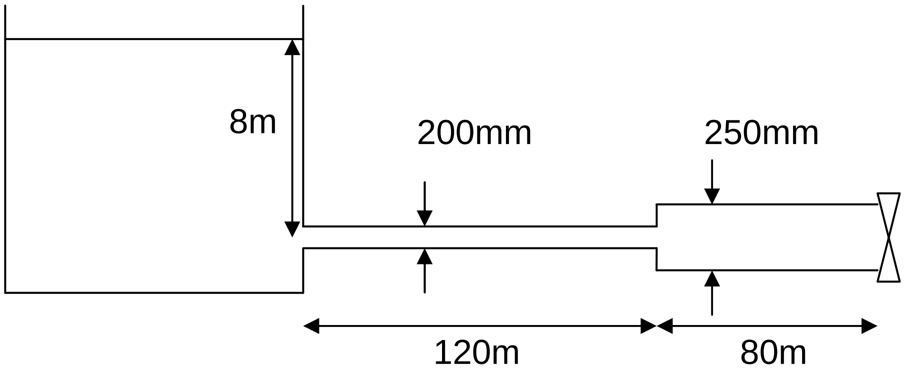
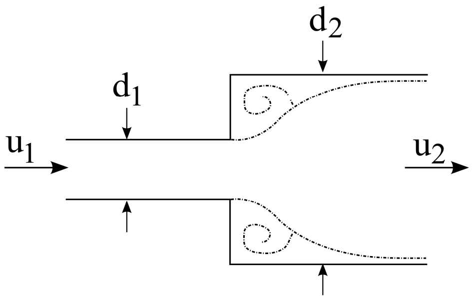


Fig. 8a

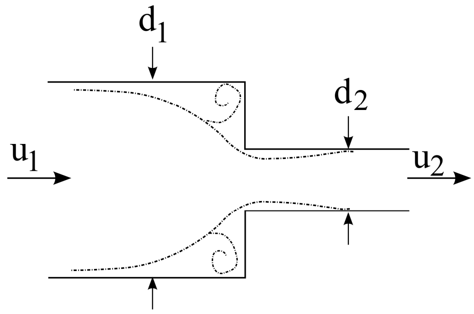
Fig. 8b (i) Head loss due to sudden enlargement : $\Delta h = \frac{u_1^2}{2g}\left(1-\left(\frac{d_1}{d_2}\right)^2\right)$

(ii) Head loss due to sudden contraction : $\Delta h = \frac{k u_2^2}{2g}$ where k verified :

d_2/d_1	0	0.2	0.4	0.6	0.8	1
k	0.5	0.45	0.38	0.28	0.14	0



(i)



(ii)

(9) A single uniform joins two reservoirs of fluid as shown in Fig. 9a. Calculate the percentage increase of flow rate obtainable if, from the mid-point of this pipe, another of the same diameter is added in parallel to it as shown in Fig 9b. Neglect all losses except pipe friction and assume a constant and equal f for both pipes.

[ANS. 26.5 %]

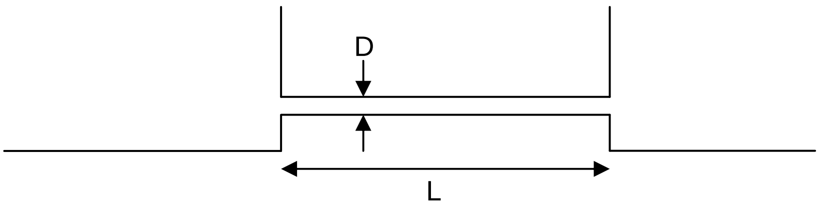


Fig. 9a

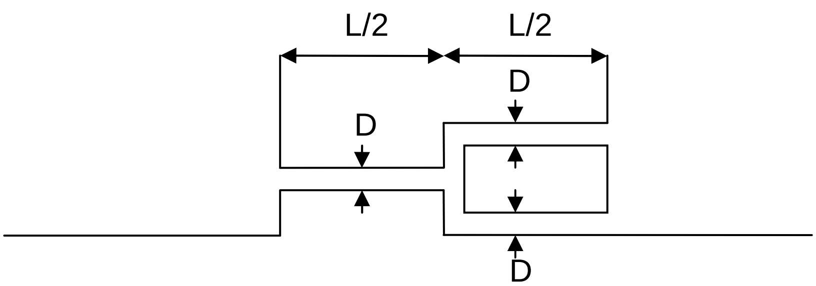


Fig. 9b