
MATHEMATICS PREPARATION WORKBOOK

ANSWERS

FP19 Engineering

UNIVERSITY OF WARWICK

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Surds and rationalising the denominator

Answers

- 1 a $3\sqrt{5}$ b $5\sqrt{5}$
c $4\sqrt{3}$ d $5\sqrt{7}$
- 2 a $15\sqrt{2}$ b $\sqrt{5}$
c $3\sqrt{2}$ d $\sqrt{3}$
- 3 a -1 b $9-\sqrt{3}$
c $10\sqrt{5}-7$ d $26-4\sqrt{2}$
- 4 a $\frac{2\sqrt{7}}{7}$ b $\frac{\sqrt{2}}{2}$
c $\frac{\sqrt{3}}{3}$ d $\frac{1}{3}$
- 5 a $\frac{3+\sqrt{5}}{4}$ b $\frac{2(4-\sqrt{3})}{13}$
- 6 $x-y$
- 7 a $3+2\sqrt{2}$ b $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Rules of indices

Answers

1 a 1

b 1

2 a 7

b 4

3 a 125

b 32

4 a $\frac{1}{25}$

b $\frac{1}{64}$

5 a $\frac{3x^3}{2}$

b $5x^2$

c $2x^6$

d x

6 a $\frac{1}{2}$

b $\frac{1}{9}$

c $\frac{8}{3}$

7 a x^{-1}

b x^{-7}

c $x^{\frac{1}{4}}$

8 a $\sqrt[5]{x^2}$

b $\frac{1}{\sqrt{x}}$

c $\frac{1}{\sqrt[4]{x^3}}$

9 a $5x^{\frac{1}{2}}$

b $2x^{-3}$

c $\frac{1}{3}x^{-4}$

10 a $x^3 + x^{-2}$

b $x^3 + x$

c $x^{-2} + x^{-7}$

Factorising expressions

Answers

- 1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$
 c $5x^2y^2(5 - 2x + 3y)$
- 2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$
 c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$
- 3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$
 c $2(3a - 10bc)(3a + 10bc)$
- 4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$
 c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$
- 5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$
 c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$
- 6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$
 c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$
- 7 $\frac{4(x+2)}{x-2}$

Completing the square

Answers

1 a $(x+2)^2 - 1$

b $(x-5)^2 - 28$

c $(x-4)^2 - 16$

d $(x+3)^2 - 9$

2 a $2(x-2)^2 - 24$

b $4(x-1)^2 - 20$

c $3(x+2)^2 - 21$

d $2\left(x+\frac{3}{2}\right)^2 - \frac{25}{2}$

3 a $2\left(x+\frac{3}{4}\right)^2 + \frac{39}{8}$

b $3\left(x-\frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x+\frac{3}{10}\right)^2 - \frac{9}{20}$

d $3\left(x+\frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x+3)^2 + 3$

Solving quadratic equations by factorisation

Answers

1 a $x = 0$ or $x = -\frac{2}{3}$

c $x = -5$ or $x = -2$

e $x = -1$ or $x = 4$

b $x = 0$ or $x = \frac{3}{4}$

d $x = 2$ or $x = 3$

f $x = -5$ or $x = 2$

2 a $x = -2$ or $x = 5$

c $x = -3$ or $x = 2\frac{1}{2}$

b $x = -1$ or $x = 3$

d $x = -\frac{1}{3}$ or $x = 2$

Solving quadratic equations by completing the square

1 a $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ b $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ d $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

2 a $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ b $x = \frac{-3 + \sqrt{23}}{2}$ or $x = \frac{-3 - \sqrt{23}}{2}$

c $x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$

Solving quadratic equations by using the formula

1 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

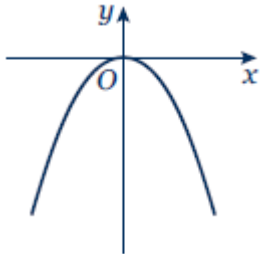
2 $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$

3 $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$

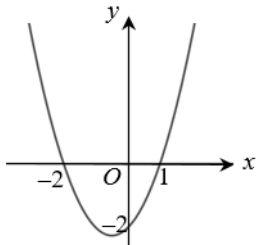
Sketching quadratic graphs

Answers

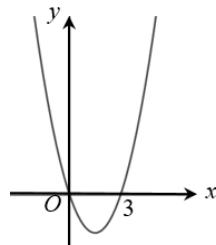
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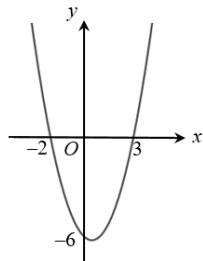
2 a



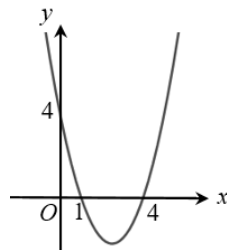
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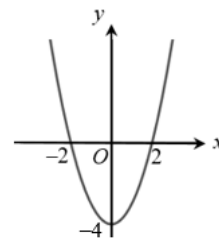
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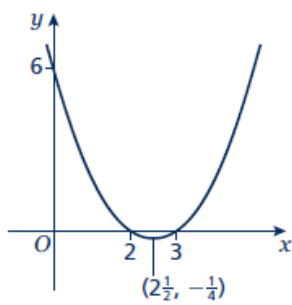
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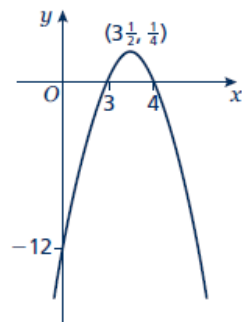
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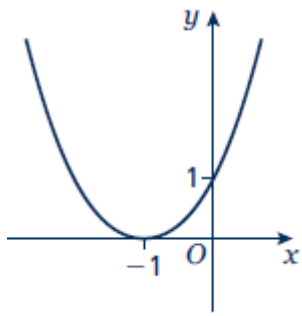
4 a



b



5



Line of symmetry at $x = -1$.

Solving linear simultaneous equations using the elimination method

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

Solving linear simultaneous equations using the substitution method

1 $x = 9, y = 5$

2 $x = -2, y = -7$

3 $x = \frac{1}{2}, y = 3\frac{1}{2}$

4 $x = \frac{1}{2}, y = 3$

5 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

Answers

1 $x = 1, y = 3$

$$x = -\frac{9}{5}, y = -\frac{13}{5}$$

2 $x = 2, y = 4$

$$x = 4, y = 2$$

3 $x = 3, y = 4$

$$x = 2, y = 1$$

4 $x = 7, y = 2$

$$x = -1, y = -6$$

5 $x = -2, y = -4$

$$x = 2, y = 4$$

6 $x = \frac{5}{2}, y = 6$

$$x = 3, y = 5$$

Solving simultaneous equations graphically

Answers

- 1** **a** $x = 2, y = 5$
 b $x = 2, y = -3$
- 2** **a** $x = -2, y = 2$
 b $x = 0.5, y = 0.5$
- 3** **a** $x = 1, y = 0$ and $x = 4, y = 3$
 b $x = -2, y = 7$ and $x = 2, y = -5$
- 4** $x = -3, y = 4$ and $x = 4, y = -3$

Linear inequalities

Answers

1 **a** $x > 4$ **b** $x \leq 2$ **c** $x \leq -1$

2 **a** $x < -20$ **b** $x \leq 3.5$ **c** $x < 4$

3 **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$

4 **a** $t < \frac{5}{2}$ **b** $n \geq \frac{7}{5}$

5 $x > 5$ (which also satisfies $x > 3$)

Quadratic inequalities

Answers

1 $-7 \leq x \leq 4$

2 $\frac{1}{2} < x < 3$

3 $-3 \leq x \leq 4$

4 $2 < x < 2\frac{1}{2}$

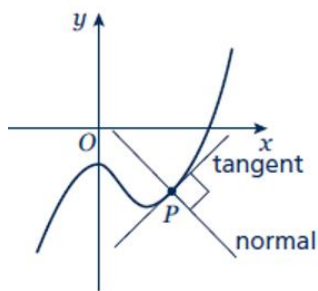
5 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

Sketching cubic and reciprocal graphs

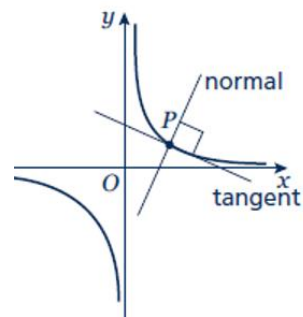
Answers

- 1 a i – C
 ii – E
 iii – B
 iv – A
 v – F
 vi – D

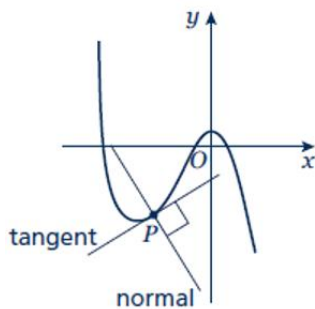
b ii



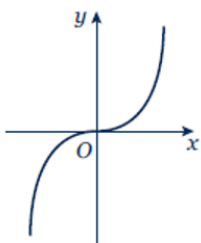
iv



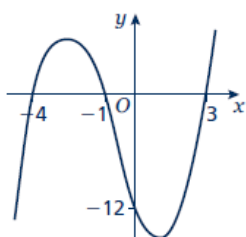
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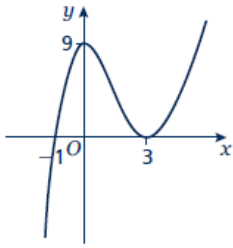
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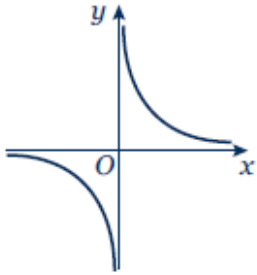
3



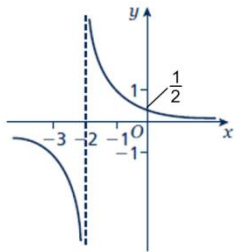
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5



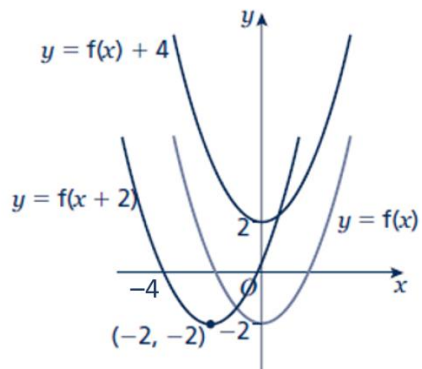
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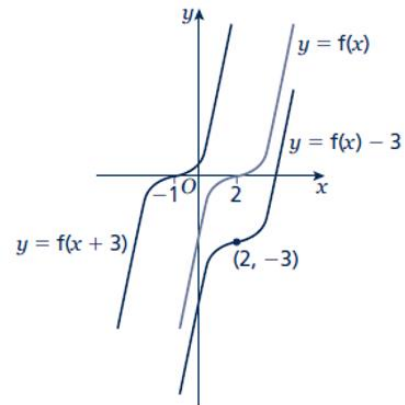
Translating graphs

Answers

1



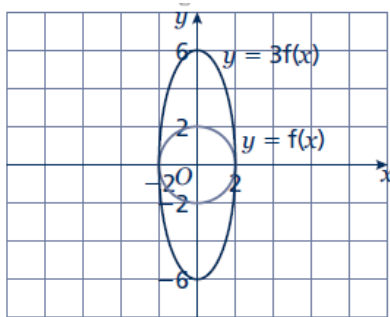
2



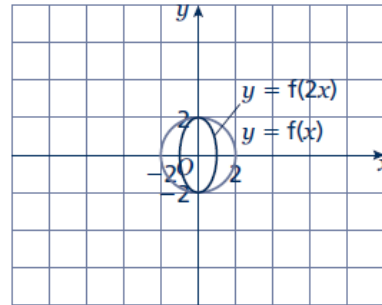
- 3 $C_1: y = f(x - 5)$
 $C_2: y = f(x) - 3$

Stretching graphs

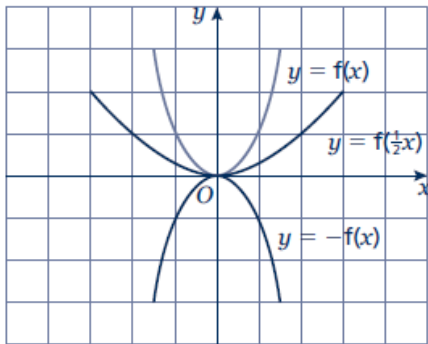
1 a



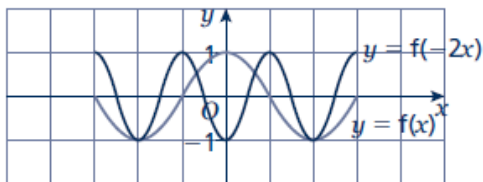
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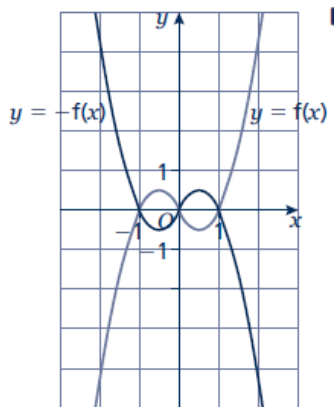
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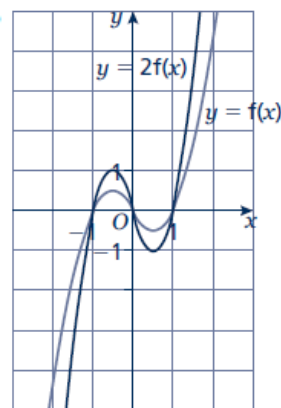
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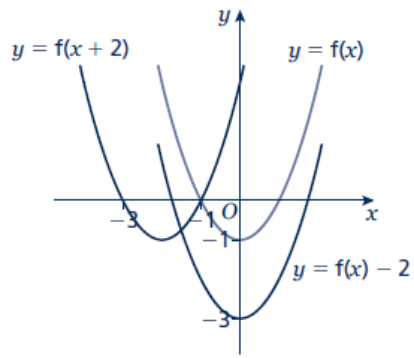
4 a



b



5



Straight line graphs

Answers

1 a $m = 3, c = 5$

b $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$

d $m = -1, c = 5$

2 a $x + 2y + 14 = 0$

b $2x - y = 0$

3 $y = -\frac{2}{3}x + 7$

4 a $y = 2x - 3$

b $y = -\frac{1}{2}x + 6$

Parallel and perpendicular lines

Answers

1 a $y = 3x - 7$

b $y = -2x + 5$

2 $y = -2x - 7$

3 a $y = -\frac{1}{2}x + 2$

b $y = 3x + 7$

4 a $y = -\frac{1}{2}x$

b $y = 2x$

5 a Parallel

b Neither

c Perpendicular

6 a $x + 2y - 4 = 0$

b $x + 2y + 2 = 0$

c $y = 2x$

Pythagoras' theorem

Answers

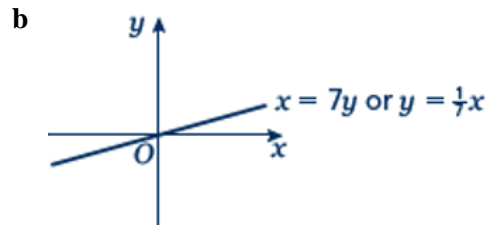
- 1 **a** 10.3 cm **b** 7.07 cm
- 2 **a** $4\sqrt{3}$ cm **b** $2\sqrt{21}$ cm
- 3 **a** $18\sqrt{13}$ mm **b** $2\sqrt{145}$ mm
- 4 95.3 mm
- 5 64.0 km

Proportion

Answers

1 £77

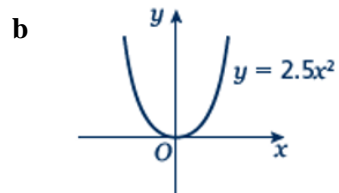
2 a $x = 7y$



c 91

d 9

3 a $y = 2.5x^2$



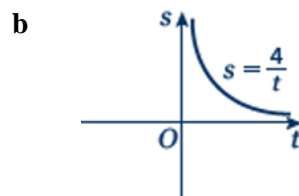
c ± 6

4 a 16

b 100

5 5

6 a $s = \frac{4}{t}$



c 4

7 1

8 1

Trigonometry

Answers

1 **a** 6.49 cm **b** 6.93 cm **c** 2.80 cm
 d 74.3 mm

2 **a** 36.9° **b** 57.1°

3 5.71 cm

4 20.4°

5 **a** 45° **b** 1 cm

The cosine rule

1 **a** 6.46 cm **b** 9.26 cm

2 **a** 22.2° **b** 52.9°

3 **a** 13.7 cm **b** 76.0°

The sine rule

1 **a** 4.33 cm **b** 15.0 cm

2 **a** 42.8° **b** 52.8°

3 **a** 8.13 cm **b** 32.3°

Areas of triangles

1 a 18.1 cm^2 b 18.7 cm^2

2 5.10 cm

3 a 6.29 cm b 84.3°

4 15.3 cm

Rearranging equations

Answers

1 $d = \frac{C}{\pi}$

2 $w = \frac{P - 2l}{2}$

3 $T = \frac{S}{D}$

4 $y = 2 + 3x$

5 $a = \frac{3x + 1}{x + 2}$

6 $d = \frac{b - c}{x}$

7 a $r = \sqrt{\frac{A}{\pi}}$

b $r = \sqrt[3]{\frac{3V}{4\pi}}$

8 a $x = \frac{abz}{cdy}$

b $x = \frac{3dz}{4\pi cpy^2}$

9 $\sin B = \frac{b \sin A}{a}$

10 a $x = \frac{q + pt}{q - ps}$

b $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$

Volume and surface area of 3D shapes

Answers

- 1
 - a $V = 396 \text{ cm}^3$
 - b $V = 402.5 \text{ cm}^3$
 - c $V = 1008\pi \text{ cm}^3$
 - d $V = 121.5\pi \text{ cm}^3$
 - e $V = 48\pi \text{ cm}^3$

- 2 17 cm

- 3 17 cm

- 4 21.4 cm

- 5 $r = \sqrt[3]{36x}$

Area under a graph

Answers

1 34 units^2

2 149 units^2

3 42 units^2

4 $26\frac{7}{8} \text{ units}^2$

5 35 units^2