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# MATHEMATICS PREPARATION WORKBOOK

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## ANSWERS

FP16 Mathematics & Statistics  
FP17 Economics  
FP18 Computer Science

UNIVERSITY OF WARWICK

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# Expanding brackets and simplifying expressions

## Answers

1 a  $6x - 3$

b  $-10pq - 8q^2$

2 a  $21x + 35 + 12x - 48 = 33x - 13$

b  $40p - 16 - 12p - 27 = 28p - 43$

3 a  $12x^2 + 24x$

b  $20k^3 - 48k$

4 a  $-y^2 - 4$

b  $5x^2 - 11x$

5 a  $-1 - 2m$

b  $5p^3 + 12p^2 + 27p$

6  $7x(3x - 5) = 21x^2 - 35x$

7 a  $x^2 + 9x + 20$

b  $x^2 + 10x + 21$

c  $x^2 + 5x - 14$

d  $x^2 - 25$

e  $10x^2 - 31x + 15$

f  $12x^2 + 13x - 14$

g  $4x^2 - 28x + 49$

h  $16x^2 - 24xy + 9y^2$

8 a  $x^2 - 1 - \frac{2}{x^2}$

b  $x^2 + 2 + \frac{1}{x^2}$

# Surds and rationalising the denominator

## Answers

1 a  $3\sqrt{5}$                       b  $5\sqrt{5}$   
c  $4\sqrt{3}$                          d  $5\sqrt{7}$

2 a  $15\sqrt{2}$                       b  $\sqrt{5}$   
c  $3\sqrt{2}$                          d  $\sqrt{3}$

3 a  $-1$                              b  $9-\sqrt{3}$   
c  $10\sqrt{5}-7$                     d  $26-4\sqrt{2}$

4 a  $\frac{2\sqrt{7}}{7}$                          b  $\frac{\sqrt{2}}{2}$   
c  $\frac{\sqrt{3}}{3}$                              d  $\frac{1}{3}$

5 a  $\frac{3+\sqrt{5}}{4}$                             b  $\frac{2(4-\sqrt{3})}{13}$

6  $x-y$

7 a  $3+2\sqrt{2}$                       b  $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

# Rules of indices

## Answers

1 a 1

b 1

2 a 7

b 4

3 a 125

b 32

4 a  $\frac{1}{25}$

b  $\frac{1}{64}$

5 a  $\frac{3x^3}{2}$

b  $5x^2$

c  $2x^6$

d  $x$

6 a  $\frac{1}{2}$

b  $\frac{1}{9}$

c  $\frac{8}{3}$

7 a  $x^{-1}$

b  $x^{-7}$

c  $x^{\frac{1}{4}}$

8 a  $\sqrt[5]{x^2}$

b  $\frac{1}{\sqrt{x}}$

c  $\frac{1}{\sqrt[4]{x^3}}$

9 a  $5x^{\frac{1}{2}}$

b  $2x^{-3}$

c  $\frac{1}{3}x^{-4}$

10 a  $x^3 + x^{-2}$

b  $x^3 + x$

c  $x^{-2} + x^{-7}$

# Factorising expressions

## Answers

- 1   **a**    $2x^3y^3(3x - 5y)$                       **b**    $7a^3b^2(3b^3 + 5a^2)$   
     **c**    $5x^2y^2(5 - 2x + 3y)$
- 2   **a**    $(x + 3)(x + 4)$                       **b**    $(x + 7)(x - 2)$   
     **c**    $(x - 5)(x - 6)$                       **d**    $(x - 8)(x + 3)$
- 3   **a**    $(6x - 7y)(6x + 7y)$                       **b**    $(2x - 9y)(2x + 9y)$   
     **c**    $2(3a - 10bc)(3a + 10bc)$
- 4   **a**    $(x - 1)(2x + 3)$                       **b**    $(3x + 1)(2x + 5)$   
     **c**    $(2x + 1)(x + 3)$                       **d**    $(3x - 1)(3x - 4)$
- 5   **a**    $\frac{2(x+2)}{x-1}$                                       **b**    $\frac{x}{x-1}$   
     **c**    $\frac{x+2}{x}$     **d**    $\frac{x}{x+5}$
- 6   **a**    $\frac{3x+4}{x+7}$                                       **b**    $\frac{2x+3}{3x-2}$   
     **c**    $\frac{2-5x}{2x-3}$                                       **d**    $\frac{3x+1}{x+4}$
- 7    $\frac{4(x+2)}{x-2}$

# Completing the square

## Answers

1 a  $(x+2)^2 - 1$

b  $(x-5)^2 - 28$

c  $(x-4)^2 - 16$

d  $(x+3)^2 - 9$

2 a  $2(x-2)^2 - 24$

b  $4(x-1)^2 - 20$

c  $3(x+2)^2 - 21$

d  $2\left(x+\frac{3}{2}\right)^2 - \frac{25}{2}$

3 a  $2\left(x+\frac{3}{4}\right)^2 + \frac{39}{8}$

b  $3\left(x-\frac{1}{3}\right)^2 - \frac{1}{3}$

c  $5\left(x+\frac{3}{10}\right)^2 - \frac{9}{20}$

d  $3\left(x+\frac{5}{6}\right)^2 + \frac{11}{12}$

4  $(5x+3)^2 + 3$

# Solving quadratic equations by factorisation

## Answers

1 a  $x = 0$  or  $x = -\frac{2}{3}$

c  $x = -5$  or  $x = -2$

e  $x = -1$  or  $x = 4$

b  $x = 0$  or  $x = \frac{3}{4}$

d  $x = 2$  or  $x = 3$

f  $x = -5$  or  $x = 2$

2 a  $x = -2$  or  $x = 5$

c  $x = -3$  or  $x = 2\frac{1}{2}$

b  $x = -1$  or  $x = 3$

d  $x = -\frac{1}{3}$  or  $x = 2$



# Solving quadratic equations by completing the square

1 a  $x = 2 + \sqrt{7}$  or  $x = 2 - \sqrt{7}$       b  $x = 5 + \sqrt{21}$  or  $x = 5 - \sqrt{21}$

c  $x = -4 + \sqrt{21}$  or  $x = -4 - \sqrt{21}$       d  $x = 1 + \sqrt{7}$  or  $x = 1 - \sqrt{7}$

2 a  $x = 1 + \sqrt{14}$  or  $x = 1 - \sqrt{14}$       b  $x = \frac{-3 + \sqrt{23}}{2}$  or  $x = \frac{-3 - \sqrt{23}}{2}$

c  $x = \frac{5 + \sqrt{13}}{2}$  or  $x = \frac{5 - \sqrt{13}}{2}$

# Solving quadratic equations by using the formula

1   **a**    $x = -1 + \frac{\sqrt{3}}{3}$  or  $x = -1 - \frac{\sqrt{3}}{3}$       **b**    $x = 1 + \frac{3\sqrt{2}}{2}$  or  $x = 1 - \frac{3\sqrt{2}}{2}$

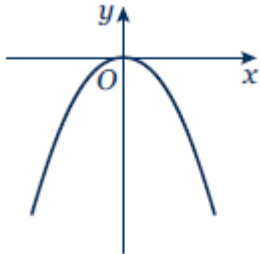
2    $x = \frac{7 + \sqrt{41}}{2}$  or  $x = \frac{7 - \sqrt{41}}{2}$

3    $x = \frac{-3 + \sqrt{89}}{20}$  or  $x = \frac{-3 - \sqrt{89}}{20}$

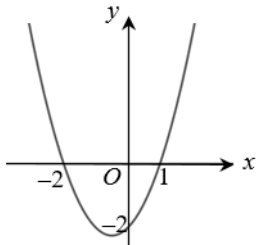
# Sketching quadratic graphs

## Answers

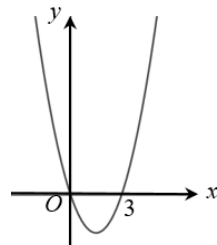
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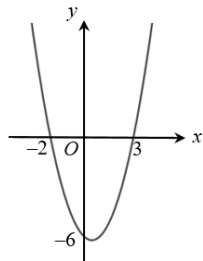
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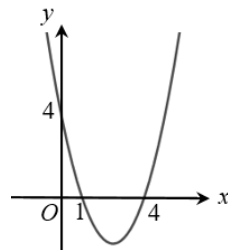
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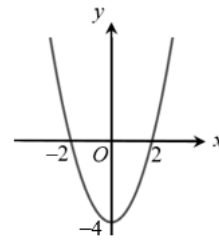
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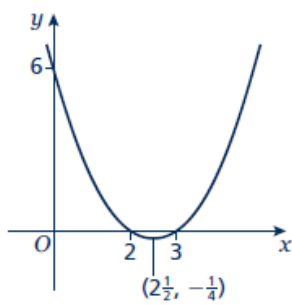
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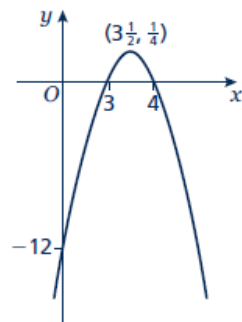
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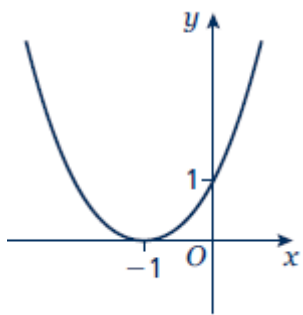
4 a



b



5



Line of symmetry at  $x = -1$ .

# Solving linear simultaneous equations using the elimination method

## Answers

1  $x = 1, y = 4$

2  $x = 3, y = -2$

3  $x = 2, y = -5$

4  $x = 3, y = -\frac{1}{2}$

# Solving linear simultaneous equations using the substitution method

1  $x = 9, y = 5$

2  $x = -2, y = -7$

3  $x = \frac{1}{2}, y = 3\frac{1}{2}$

4  $x = \frac{1}{2}, y = 3$

5  $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

# Solving linear and quadratic simultaneous equations

## Answers

**1**  $x = 1, y = 3$

$$x = -\frac{9}{5}, y = -\frac{13}{5}$$

**2**  $x = 2, y = 4$

$$x = 4, y = 2$$

**3**  $x = 3, y = 4$

$$x = 2, y = 1$$

**4**  $x = 7, y = 2$

$$x = -1, y = -6$$

**5**  $x = -2, y = -4$

$$x = 2, y = 4$$

**6**  $x = \frac{5}{2}, y = 6$

$$x = 3, y = 5$$

# Solving simultaneous equations graphically

## Answers

- 1**
  - a**  $x = 2, y = 5$
  - b**  $x = 2, y = -3$
  
- 2**
  - a**  $x = -2, y = 2$
  - b**  $x = 0.5, y = 0.5$
  
- 3**
  - a**  $x = 1, y = 0$  and  $x = 4, y = 3$
  - b**  $x = -2, y = 7$  and  $x = 2, y = -5$
  
- 4**  $x = -3, y = 4$  and  $x = 4, y = -3$



# Linear inequalities

## Answers

- 1   **a**    $x > 4$                       **b**    $x \leq 2$                       **c**    $x \leq -1$
- 2   **a**    $x < -20$                       **b**    $x \leq 3.5$                       **c**    $x < 4$
- 3   **a**    $x \leq -4$                       **b**    $-1 \leq x < 5$                       **c**    $x \leq 1$
- 4   **a**    $t < \frac{5}{2}$                       **b**    $n \geq \frac{7}{5}$
- 5    $x > 5$  (which also satisfies  $x > 3$ )

# Quadratic inequalities

## Answers

1  $-7 \leq x \leq 4$

2  $\frac{1}{2} < x < 3$

3  $-3 \leq x \leq 4$

4  $2 < x < 2\frac{1}{2}$

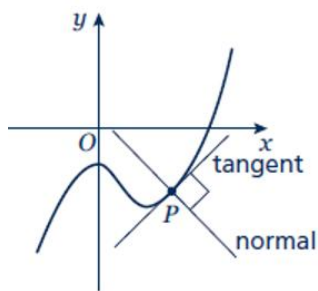
5  $x \leq -\frac{3}{2}$  or  $x \geq \frac{5}{3}$

# Sketching cubic and reciprocal graphs

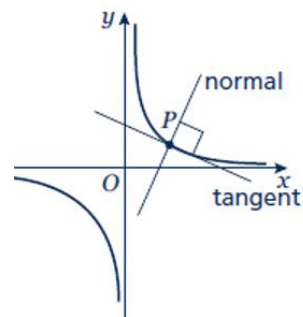
## Answers

- 1 a i - C  
ii - E  
iii - B  
iv - A  
v - F  
vi - D

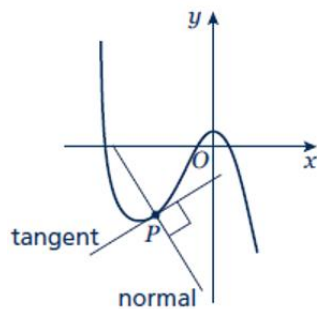
b ii



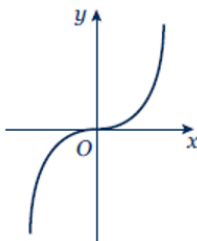
iv



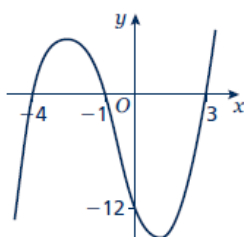
vi



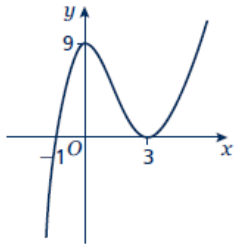
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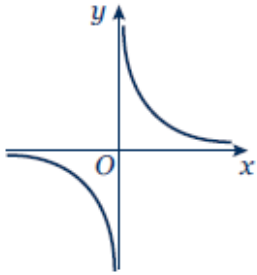
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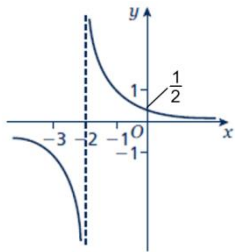
4



5



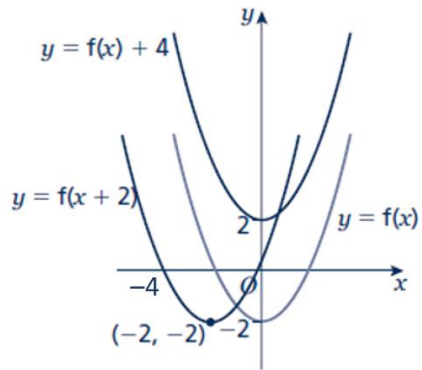
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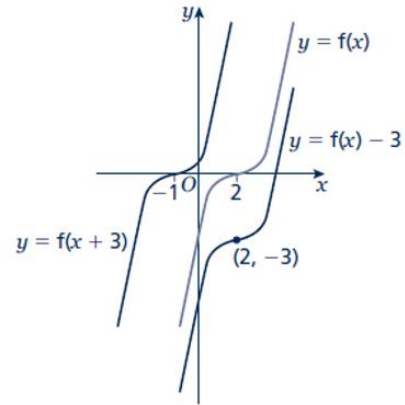
# Translating graphs

## Answers

1



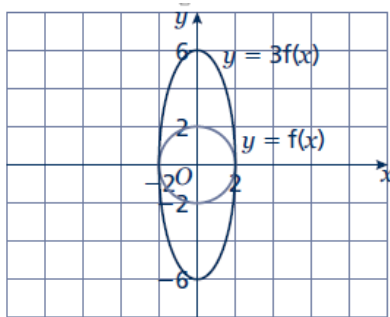
2



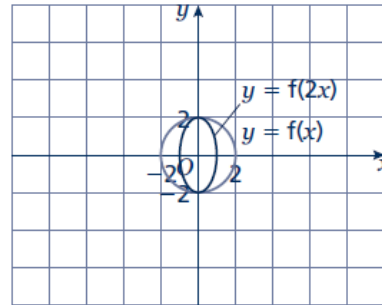
- 3  $C_1: y = f(x - 5)$   
 $C_2: y = f(x) - 3$

# Stretching graphs

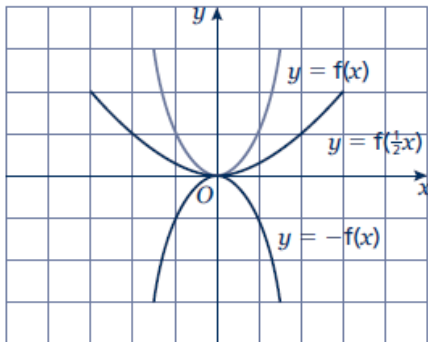
1 a



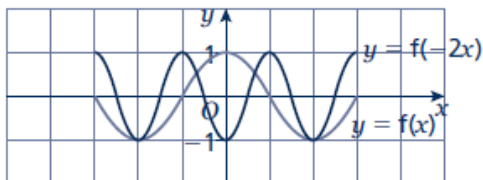
b



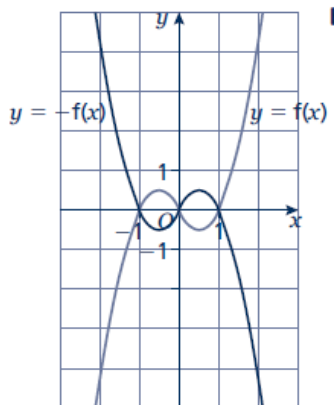
2



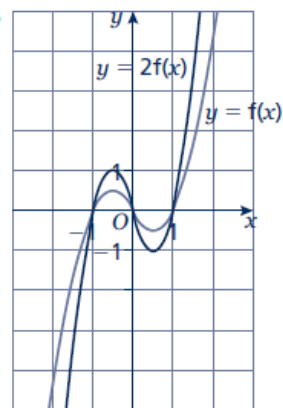
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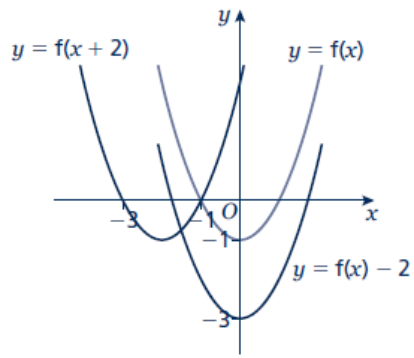
4 a



b



5



# Straight line graphs

## Answers

1 a  $m = 3, c = 5$

b  $m = -\frac{1}{2}, c = -7$

c  $m = 2, c = -\frac{3}{2}$

d  $m = -1, c = 5$

2 a  $x + 2y + 14 = 0$

b  $2x - y = 0$

3  $y = -\frac{2}{3}x + 7$

4 a  $y = 2x - 3$

b  $y = -\frac{1}{2}x + 6$



# Parallel and perpendicular lines

## Answers

1 a  $y = 3x - 7$

b  $y = -2x + 5$

2  $y = -2x - 7$

3 a  $y = -\frac{1}{2}x + 2$

b  $y = 3x + 7$

4 a  $y = -\frac{1}{2}x$

b  $y = 2x$

5 a Parallel

b Neither

c Perpendicular

6 a  $x + 2y - 4 = 0$

b  $x + 2y + 2 = 0$

c  $y = 2x$

# Pythagoras' theorem

## Answers

- 1   **a**   10.3 cm                      **b**   7.07 cm
- 2   **a**    $4\sqrt{3}$  cm                      **b**    $2\sqrt{21}$  cm
- 3   **a**    $18\sqrt{13}$  mm                      **b**    $2\sqrt{145}$  mm
- 4   95.3 mm
- 5   64.0 km

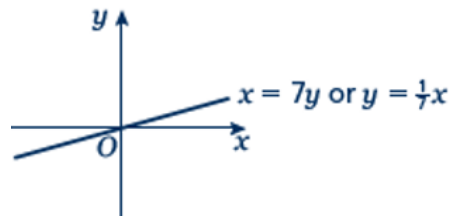
# Proportion

## Answers

1 £77

2 a  $x = 7y$

b

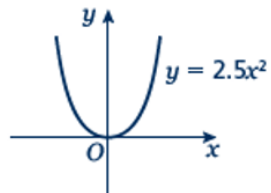


c 91

d 9

3 a  $y = 2.5x^2$

b



c  $\pm 6$

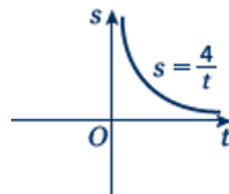
4 a 16

b 100

5 5

6 a  $s = \frac{4}{t}$

b



c 4

7 1

8 1

# Trigonometry

## Answers

1   **a**   6.49 cm                      **b**   6.93 cm                      **c**   2.80 cm  
     **d**   74.3 mm

2   **a**   36.9°                      **b**   57.1°

3   5.71 cm

4   20.4°

5   **a**   45°                      **b**   1 cm

# The cosine rule

1   **a**   6.46 cm                      **b**   9.26 cm

2   **a**   22.2°                            **b**   52.9°

3   **a**   13.7 cm                          **b**   76.0°

# The sine rule

1   **a**   4.33 cm                      **b**   15.0 cm

2   **a**   42.8°                              **b**   52.8°

3   **a**   8.13 cm                              **b**   32.3°

# Areas of triangles

1   **a**    $18.1 \text{ cm}^2$                       **b**    $18.7 \text{ cm}^2$

2    $5.10 \text{ cm}$

3   **a**    $6.29 \text{ cm}$                       **b**    $84.3^\circ$

4    $15.3 \text{ cm}$

# Rearranging equations

## Answers

1  $d = \frac{C}{\pi}$

2  $w = \frac{P - 2l}{2}$

3  $T = \frac{S}{D}$

4  $y = 2 + 3x$

5  $a = \frac{3x + 1}{x + 2}$

6  $d = \frac{b - c}{x}$

7 a  $r = \sqrt{\frac{A}{\pi}}$

b  $r = \sqrt[3]{\frac{3V}{4\pi}}$

8 a  $x = \frac{abz}{cdy}$

b  $x = \frac{3dz}{4\pi cpy^2}$

9  $\sin B = \frac{b \sin A}{a}$

10 a  $x = \frac{q + pt}{q - ps}$

b  $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$



# Volume and surface area of 3D shapes

## Answers

- 1
  - a  $V = 396 \text{ cm}^3$
  - b  $V = 402.5 \text{ cm}^3$
  - c  $V = 1008\pi \text{ cm}^3$
  - d  $V = 121.5\pi \text{ cm}^3$
  - e  $V = 48\pi \text{ cm}^3$
  
- 2 17 cm
  
- 3 17 cm
  
- 4 21.4 cm
  
- 5  $r = \sqrt[3]{36x}$

# Area under a graph

## Answers

1 34 units<sup>2</sup>

2 149 units<sup>2</sup>

3 42 units<sup>2</sup>

4  $26\frac{7}{8}$  units<sup>2</sup>

5 35 units<sup>2</sup>