MATHEMATICS PREPARATION WORKBOOK

FP14 BUSINESS MANAGEMENT

UNIVERSITY OF WARWICK

Contents

Order of operations

Key points

- Mathematics operations must be carried out in the correct order.
- BIDMAS are ways of remember this order.

B	**I**	**DM**	**AS**
Brackets	**I**ndices	**D**ivide and **M**ultiply – work from left to right	**A**dd and **S**ubtract – work from left to right

- To solve calcualtions involving brackets, always calculate the values inside the brackets first. If there are brackets inside other brackets, calculate the inside brackets first
- BIDMAS must also be used when using a calculator. Scientific calculators automatically apply the operations in the correct order, however extra brackets may be required.
- On a calculator, brackets must be used when raising negative numbers to a power. For example, to calculate "--2 squared" on a calculator, type in $(-2)^2$. This would give the correcr answer of 4. If brackets were not used BIDMAS would be applied and the answer given by the calualor would be -4.

Examples

Example 1 Calculate the value of $3 + 4^2 - 10 \div 2$

$3 + 16 - 10 \div 2$	1. There are no brackets (B) so calculate the **index number/power** (I) first: $4^2 = 16$
$3 + 16 - 5$	2. Do any divisions or multipications (DM), working from left to right: $10 \div 2 = 5$
$19 - 5$ 14	3. Do any additions or subtractions (AS), working from left to right: $3 + 16$ becomes 19 $19 - 5$ becomes 14

Example 2 Calculate the value of $[40-(2+4^2)]\times 2$

$[40-(\mathbf{2}+\mathbf{4^2})]\times 2$ $[40-(2+16)]\times 2$ $[40-(18)]\times 2$	1. Using the BIDMAS rule, first calculate the inside brackets (B) Work out the power/index in order to do this (I): $4^2=16$ Then the addition (A): $2+16=18$
22×2	2. Next, do the outer brackets (B) $40-18=22$
44	3. Once the brackets have been calculated, finish with the multipication (M): $22\times 2=44$

Example 3 Calculate the value of $\frac{2+4}{10-7}$

$(2+4)\div(10-7)$	1. Using the BIDMAS rule, the division (D) needs to be done first. The result of the top of the fraction needs to be divided by the bottom of the fraction. To do this rewrite with brackets (B).
$6\div 3$	2. Now do the bracket (B): $2+4=6$ $10-7=3$
2	3. Now do the division (D): $6\div 3=2$

Practice

1 Calculate $\mathbf{2^2 \times 5 - 6 \div 3}$

2 Calculate $[\mathbf{3} \times (\mathbf{6 - 4})^\mathbf{2}] + \mathbf{1}$

3 Use one pair of brackets to make the following statement correct

$$\mathbf{17 - 5 \times 2 + 4 = 28}$$

4 Calculate $\frac{\mathbf{3+2^3}}{\mathbf{15-2\times2}}$

5 Find the value of $\boldsymbol{x^2 + 7}$ when $\boldsymbol{x = -3}$

Rounding

Key points

- When **rounding using decimal places** (**d.p.**), the degree of accuracy that is required is usually given. However, there are certain calculations where the degree of accuracy may be more obvious. For example, calculations involving money should be given to two decimal places to represent the pence.

- To round to a **decimal place**:

  1 look at the first digit after the decimal point if rounding to one decimal place or the second digit for two decimal places, or the third for three decimal places etc.

  2 draw a vertical line to the right of the place value digit that is required

  3 look at the next digit

  4 **if it's 5 or more**, increase the previous digit by one

  5 **if it's 4 or less**, keep the previous digit the same

  6 remove any numbers to the **right** of the line

- The method of **rounding to a significant figure** is often used as it can be applied to any kind of number, regardless of how big or small it is. When a newspaper reports a lottery winner has won £3 million, this has been rounded to one significant figure. It rounds to the most important figure in the number.

- To round to a **significant figure**:

  1. look at the first non-zero digit if rounding to one significant figure
  2. look at the digit after the first non-zero digit if rounding to two significant figures or more
  3. draw a vertical line after the place value digit that is required
  4. look at the next digit
  5. if it's **5 or more**, increase the previous digit by one
  6. if it's **4 or less**, keep the previous digit the same
  7. fill any spaces to the **right** of the line with zeros, stopping at the decimal point if there is one

Examples

Example 1 Round 248.561 to 2 decimal places, then 1 decimal place.

- 248.5|**6**1 to 1 decimal place is 248.6
- 248.56|**1** to 2 decimal places is 248.56

For 2 decimal places:	
248.56|1	Count to the second digit after the decimal place and draw a vertical line to the right
248.56|**1**	Consider the digit to the right of the vertical line • **if it's 5 or more**, increase the previous digit by one • **if it's 4 or less**, keep the previous digit the same Since 1 is "4 or less" the digit "6" stays as a "6"
248.56 (2 d.p.)	
For 1 decimal place:	Count to the first digit after the decimal place and draw a vertical line to the right
248.5|61	
248.5|**6**1	Consider the digit to the right of the vertical line • **if it's 5 or more**, increase the previous digit by one • **if it's 4 or less**, keep the previous digit the same Since 6 is "5 or more" the digit "5" increases to "6"
248.6 (1 d.p.)	

Example 2: Round 53,879 to 2 significant figures, then 1 significant figure.

For 2 significant figures:	
53|879	Count to the second digit of the number and draw a vertical line to the right
53|**8**79	Consider the digit to the right of the vertical line • **if it's 5 or more**, increase the previous digit by one • **if it's 4 or less**, keep the previous digit the same
54,000 (2 s. f)	Since 8 is "5 or more" the digit "3" increases to "4" Notice that the number of significant figures in the question is the maximum number of non-zero digits in your answer.
For 1 significant figure:	
5|3,879	Count to the first digit of the number and draw a vertical line to the right
5|**3**,879	Consider the digit to the right of the vertical line • **if it's 5 or more**, increase the previous digit by one • **if it's 4 or less**, keep the previous digit the same
50,000 (1 s. f.)	Since 3 is "4 or less" the digit "5" stays as a "5"

Example 3: Round 0.005089 to 2 significant figures, then 1 significant figure.

For 2 significant figures: 0.0050|89	Count to the second non-zero digit of the number and draw a vertical line to the right
0.0050|**8**9	Consider the digit to the right of the vertical line • **if it's 5 or more**, increase the previous digit by one • **if it's 4 or less**, keep the previous digit the same
0.0051 (2 s.f)	Since 8 is "5 or more" the digit "0" increases to "1" Notice that the number of significant figures in the question is the maximum number of non-zero digits in your answer.
For 1 significant figure: 0.005|089	Count to the first non-zero digit of the number and draw a vertical line to the right
0.005|**0**89	Consider the digit to the right of the vertical line • **if it's 5 or more**, increase the previous digit by one • **if it's 4 or less**, keep the previous digit the same
0.005 (1 s.f.)	Since 0 is "4 or less" the digit "5" stays as a "5"

Practice

1 Round 249.5046 to

a) 1 decimal place

b) 2 decimal places

2 Round 0.9583 to

a) 3 decimal places

b) 1 decimal place

3 Round 598.093 to

a) 2 decimal places

b) 1 decimal place

4 Round 98,347 to

a) 3 significant figures

b) 2 significant figures

5 Round 3.5175

a) 1 significant figure

b) 2 significant figures

Rules of indices

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$

$= \frac{1}{16}$	**2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$	**1** Use the rule $a^m \times a^n = a^{m+n}$
$= x^{8-4} = x^4$	**2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
$= 4x^{-\frac{1}{2}}$	**2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 **b** 3^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3}

5 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$ **c** $\dfrac{\left(2x^2\right)^3}{4x^0}$ **d** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

7 Write the following as a single power of x.

a $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

8 Write the following without negative or fractional powers.

a $x^{\frac{2}{5}}$ **b** $x^{-\frac{1}{2}}$ **c** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{1}{3x^4}$

10 Write as sums of powers of x.

a $\dfrac{x^5 + 1}{x^2}$ **b** $x^2\left(x + \dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2 + \dfrac{1}{x^3}\right)$

Expanding brackets and simplifying expressions

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4
$= 3 - 5x$	**2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3
$= x^2 + 5x + 6$	**2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5
$= 2x^2 - 7x - 15$	**2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

Watch out!

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

5 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

6 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$

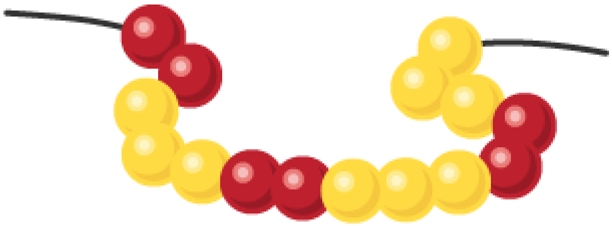
c $(x + 7)(x - 2)$ **d** $(x + 5)(x - 5)$

e $(5x - 3)(2x - 5)$ **f** $(3x - 2)(7 + 4x)$

Ratio and Proportion

Key Points

- **Ratios** are used to show how things are shared.
- For example, the necklace in the image has a pattern of two red beads for every three yellow beads.



The ratio of red beads to yellow beads is 2:3.

- Ratios can have more than two numbers for example, 3:4:2.
- Ratios can be fully simplified just like fractions. To simplify a ratio, divide all of the numbers in the ratio by the same number until they cannot be divided any more.
- **Proportion** is used to show how quantities and amounts are related to each other. The amount that quantities change in relation to each other is governed by proportion rules.
- For example, in the in the image of the necklace above the proportion of red beds is 6 out of the total of 15 which is equivalent to 40%

Examples

Example 1 Simplify 6 : 12

$6:12 = 1:2$	A quick way of doing this in just one step is to divide by the highest common factor of all the numbers in the ratio. In this example, the highest common factor of 6 and 12 is 6.

Example 2 Simplify 6 : 1.5

$6 : 1.5 = 12 : 3$	First multiply the numbers to make them whole numbers. We can multiply both numbers by 2
$12 : 3 = 4 : 1$	Divide both numbers by 3

Example 3 Simplify $\frac{1}{2} : \frac{3}{4}$

$\frac{1}{2} : \frac{3}{4} \rightarrow \frac{2}{4} : \frac{3}{4}$	Convert the fractions so they have a common denominator
$\frac{2}{4} : \frac{3}{4} = 2 : 3$	Multiply both fractions by the common denominator

Example 4 James and Helen get pocket money in the ratio 3 : 5. The total amount of pocket money they are given is £24. How much money do they each get?

	Since $3 + 5 = 8$, the amount is divided into 8 equal parts
	Since James' name us first he gets three of the parts as the 3 is the first number in the ratio. Helen gets 5 parts, since her name is second.
	Each "part" is $\frac{£24}{8} = £3$
James : Helen £9 : £15	So James gets $3 \times £3$ And Helen gets $5 \times £3$

Practice

1 Simplify the following ratios

a) 4 : 6

b) 9 : 12

c) 5 : 10 : 15

2 Simplify the ratio 2 : 3.6 so that it does not involve decimals.

3 Simplify the ratio $\frac{2}{3} : \frac{3}{4}$

4 Purple paint is made by mixing red and blue paint in the ratio **2 : 3**. If 15 litres of blue paint is used, how much red paint will be needed?

5 Two sisters share pocket money in the ratio of their ages. The first girl is 5 and the second girl is 7. The girls are given £36 between them. How much does the younger girl receive?