Pre-University Summer School

Game Theory (part 2)

Strategies and Equilibrium

‘Dominant strategies’ are *stable* strategies.

- My optimal action does not change, no matter what you will do.

However, ‘dominant strategies’ do not always exist, so they cannot be used to predict the outcome in every game.

We must look beyond stable strategies and focus on stable <u>outcomes</u>.

This is the essence of the concept of *equilibrium*.

The Studying Game – Which Outcomes are Stable?

Suppose we start here and allow players to take turns in reconsidering their action.

		Student B		
		No Effort	Some Effort	Very High Effort
Student A	No Effort	0, 0	-100, 400	-100, 150
	Some Effort	400, -100	100, 100	0, 150
	Very High Effort	150, -100	150, 0	50, 50

The Studying Game – Which Outcomes are Stable?

Let Student A have the first turn in revising their strategy

		Student B		
		No Effort	Some Effort	Very High Effort
Student A	No Effort	0, 0	-100, 400	-100, 150
	Some Effort	400, -100	100, 100	0, 150
	Very High Effort	150, -100	150, 0	50, 50

The Studying Game – Which Outcomes are Stable?

Now let Student B have a chance to revise their strategy.

			Student B	
		No Effort	Some Effort	Very High Effort
Student A	No Effort	0, 0	-100, 400	-100, 150
	Some Effort	400, -100	100, 100	0, 150
	Very High Effort	150, -100	150, 0	50, 50

The Studying Game – Which Outcomes are Stable?

A’s turn again

		Student B		
		No Effort	Some Effort	Very High Effort
Student A	No Effort	0, 0	-100, 400	-100, 150
	Some Effort	400, -100	100, 100	0, 150
	Very High Effort	150, -100	150, 0	50, 50

The Studying Game – Which Outcomes are Stable?

Does either player wish to change their action?

This kind of stable outcome is called a *Nash equilibrium*.

		Student B		
		No Effort	Some Effort	Very High Effort
Student A	No Effort	0, 0	-100, 400	-100, 150
	Some Effort	400, -100	100, 100	0, 150
	Very High Effort	150, -100	150, 0	50, 50

Other Examples of Nash Equilibrium

		Prisoner B	
		Confess	Stay Quiet
Prisoner A	Confess	-5, -5	0, -10
	Stay Quiet	-10, 0	-1, -1

		Player B	
		Split	Steal
Player A	Split	500, 500	0, 1000
	Steal	1000, 0	0, 0

Applying Nash Equilibrium

When is the Nash equilibrium of a game likely to be a useful prediction?

To illustrate some problems, we will study and play two games:

1. Keynes' beauty contest game
2. Schelling's mass coordination game

Game #1

Game #1

Rules

1. You will pick an integer number between 1 and 100.
2. I will take the average of these submissions.
3. The guess which is closest to 2/3rds of the average wins.

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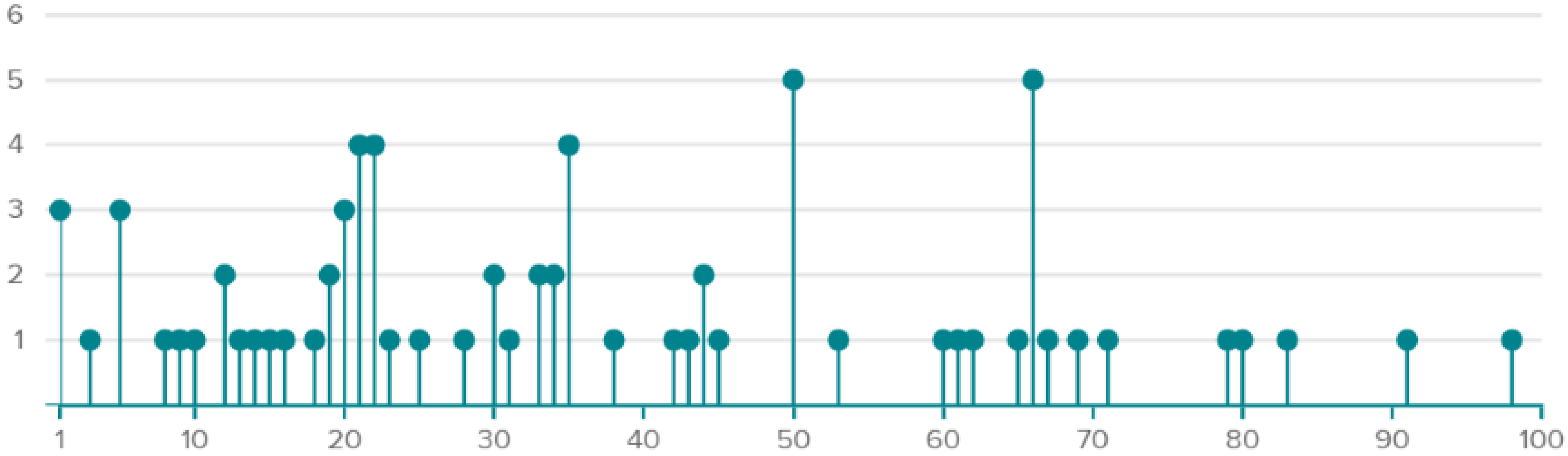
1 Pick a number between 1-100. Your aim is to pick a number which is 2/3rds of the average guess in the room.

73

Average response: 36.08



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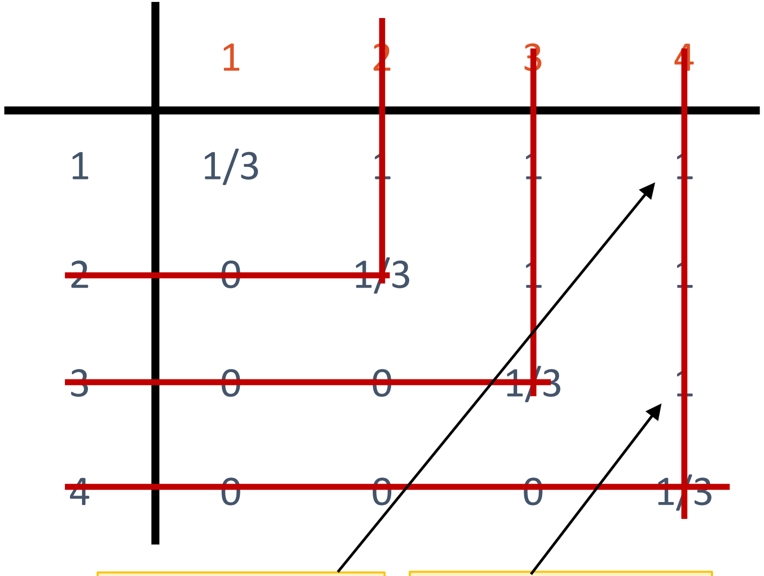
A Simplified Version (3 total players, guess 1-4)

Red number is the guess of the other players. Blue is our choice/our payoff.



	1	~~2~~	~~3~~	~~4~~
1	1/3	~~1~~	~~1~~	~~1~~
~~2~~	~~0~~	~~1/3~~	~~1~~	~~1~~
~~3~~	~~0~~	~~0~~	~~1/3~~	~~1~~
~~4~~	~~0~~	~~0~~	~~0~~	~~1/3~~

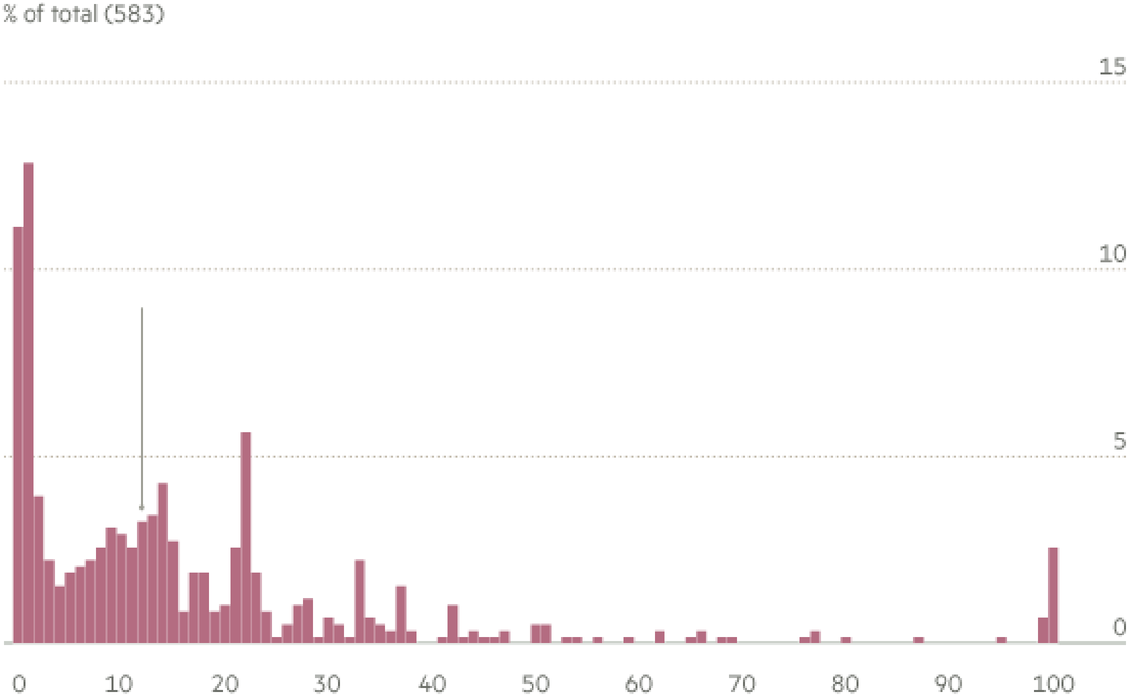
If they guess 4 and I guess 1. Average is 3, (2/3)*3=2

If they guess 4 and I guess 3. Average is 11/3, (2/3)*11/3=2.44

<u>Solving by iterated elimination:</u>

1. 4 can never be optimal, even if everyone else picks 4 (it is dominated by 3,2,1).
2. If 4 is never played then 3 can never be optimal (dominated by 2,1)
3. If 3 is never played then 2 can never be optimal (dominated by 1)
4. ‘1’ is left. This is the Nash equilibrium

Results from FT



The guesses to the FT challenge this year

Implications of ‘The Beauty Contest’

This illustrates three problems for players in a game:

1. Understanding the game.
2. Forming expectations about what the other players will do.
3. Finding the best response to what you anticipate them doing.

Is the game theoretic solution to the beauty contest useful?

- Does it provide a good prediction of how people actually play?
- Does it provide good advice about how you would optimally play?

Game #2

Game #2

The economist Thomas Schelling devised a version of the following game:

- You and a random person from the population are chosen to play a game.
- You both have the chance to win £10m.
- You must meet under Big Ben tomorrow but you don’t know which time.
- You must decide on the same time to meet, but you cannot communicate.

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Game #2

	10:00	11:00	12:00	1:00	2:00
10:00	10 , 10	0 , 0	0 , 0	0 , 0	0 , 0
11:00	0 , 0	10 , 10	0 , 0	0 , 0	0 , 0
12:00	0 , 0	0 , 0	10 , 10	0 , 0	0 , 0
1:00	0 , 0	0 , 0	0 , 0	10 , 10	0 , 0
2:00	0 , 0	0 , 0	0 , 0	0 , 0	10 , 10

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What time would you try to meet at?

60

Allowed selections: 1

Analysis

	10:00	11:00	12:00	1:00	2:00
10:00	10 , 10	0 , 0	0 , 0	0 , 0	0 , 0
11:00	0 , 0	10 , 10	0 , 0	0 , 0	0 , 0
12:00	0 , 0	0 , 0	10 , 10	0 , 0	0 , 0
1:00	0 , 0	0 , 0	0 , 0	10 , 10	0 , 0
2:00	0 , 0	0 , 0	0 , 0	0 , 0	10 , 10

Focal Points

Thomas Schelling’s theory of *focal points* give us some insight in to how we might select one of multiple Nash equilibria.

- In short: Certain NE are culturally more salient (or focal) than others.

Important feature is that focal points are common knowledge.

Examples of similar games:

- Both players have to name a mountain.
- Both players have to name a famous musician.
- Both players have to name a well-known dish.

Focal points work by being common knowledge, so culture dependent.

Game Theory (Part 2) - Summary

A Nash equilibrium is a prediction about what is likely to happen in a game.

▷ Nash equilibrium makes most sense when players have a chance to revise and adapt their strategy over time.

Two important practical issues when applying Nash equilibrium:

▷ A Nash equilibrium strategy is not always optimal for an individual to play (others may be acting irrationally!)

▷ We can have multiple Nash equilibria, are all equally likely?