Risk, Bonds, and the Determination of Interest Rates

Michael McMahon
To Cover

1. Examine how to calculate interest rates and yields;
2. The theory of bond prices;
3. The term structure of interest rates.
Present Value

Basic idea:

- If I offered you the choice between $9 now, and $10 at the end of the class, which would you rather?
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  • What about $9 now and $10 next week; which would you rather?

Money today is worth more than money tomorrow because if nothing else you could put it in the bank and earn interest.
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  • What about $9 now and $10 next week; which would you rather?
  • What about $9 now and $10 next year; which would you rather?

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  • What about $9 now and $10 next year; which would you rather?
  • How much could I lower the offer this week for you to reject the “bird in the hand”?
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Money today is worth more than money tomorrow because if nothing else you could put it in the bank and earn interest.
Present Value

Definition

The present value of a stream of money flows is the amount that these flows is worth today. The formula to use if the stream of money flows is $x_t$ (potentially different) in each period (from $t = 0$, to $t = N$) is:

$$PV = x_0 + \frac{x_1}{1 + r} + \frac{x_2}{(1 + r)^2} + \cdots + \frac{x_N}{(1 + r)^N}$$

where $r$ is the discount factor (interest rate, probability of death?).

You will use this concept for pricing of:

- Bonds (today);
- Equities (next week);
- any financial asset or investment project (life!).
Bonds

Definition

The bond market is a financial market where participants buy and sell debt securities usually in the form of bonds. It is also known as debt, credit, or fixed income market. It is a large and important market in modern economies - the size of outstanding U.S. bond market debt is $38.6 trillion (Q1 2013, SIFMA).

We learned that:

- Contractually fixed return
  - per period interest
  - principal at maturity date
- Preferential debtor
- No voting right
- Coupon bonds vs. Discount bonds (no coupon).
Calculating Bond Prices and Yields

The closest measure of the interest rate for bonds is the yield to maturity.

**Definition**

Yield to maturity is the interest rate that equates the present value of cash flow payments received from a debt instrument with its value today.

Prices and yields are inversely related.

The yield is different to the “return” which would take account of the selling price also, and express it all as a % of the purchase price.
Calculating Bond Prices and Yields

Valuation stream for a fixed interest coupon each coupon period \((c_t = c)\), with a final payment worth \(M\):

\[
P_0 = \sum_{t=1}^{N} \frac{c_t}{(1 + i)^t} + \frac{M}{(1 + i)^N}
\]

where \(t\) is the period between coupon payments and \(i\) is the yield to maturity.
Simple example to illustrate inverse relationship between price and yield:

\[ P_0 = \frac{c_1}{1 + i} + \frac{M}{1 + i} \]

where \( i \) is the yield to maturity.
Calculating Bond Prices and Yields III

We also know that we can value the coupon part separately as:

\[ P_{0}^{\text{annuity}} = c \cdot \frac{1 - (1 + i)^{-N}}{i} \]

And then the zero-coupon bit is simply:

\[ P_{0}^{\text{zero-coupon}} = \frac{M}{(1 + i)^{N}} \]

Aside the final price of the annuity can be worked out from:

\[ P_{N} = P_{0} \cdot (1 + i)^{N} \]
Calculating Bond Prices and Yields - Some Issues

1. Dirty price adds in the value of accrued interest:

\[ P_0 = \sum_{t=1}^{N} \frac{c_t}{(1 + i)^t} + \frac{M}{(1 + i)^N} + \frac{c \cdot \# \text{ days since last coupon}}{\# \text{ days in each coupon period}} \]
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\[ P_0^{\text{pure annuity}} = \frac{c}{i} \]
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4. The effective annual rate takes into account compounding:

\[ (1 + r) = (1 + i)^F \]

\( F \) is the number of coupon payments per year and periodic interest rate \( (i) \) is annual interest rate \( (R) \) divided by \( F \)
Calculating Bond Prices and Yields - Some More Issues

1. Simple Yield to Maturity

2. Holding Period Yield
   Assumes that you don’t hold the bond to maturity (sell early) and that the coupons are reinvested at different rates.

3. Real versus nominal interest rates - Fisher equation

\[ \text{nominal} = \text{real} + \pi^e \]
Calculating Bond Prices and Yields - Some More Issues

1. Simple Yield to Maturity

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3. Real versus nominal interest rates - Fisher equation

   \[ \text{nominal} = \text{real} + \pi^e \]

   Or, allowing for risks:

   \[ \text{nominal} = \text{real} + \pi^e + \text{risk} + \text{liquidity} \]
Theories of Interest Rate Determination
Supply and demand in the bond market

Demand factors:

- Wealth (+)
- Coupons (+), expected inflation (-)
- Risk (-)
- Liquidity (+)

Supply factors:

- Expected profitability (+)
- Expected inflation (-)
- Government budget deficit (+)
Theories of Interest Rate Determination

Business Cycle Expansions

\[ PB \]

\[ DB \]

\[ SB \]
Theories of Interest Rate Determination

Business Cycle Expansions

$P_B$ vs $B$

$D_B$ and $S_B$
Theories of Interest Rate Determination

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Theories of Interest Rate Determination

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$P_B$

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Theories of Interest Rate Determination

Business Cycle Expansions

$P_B$ vs $B$

- $S_B$
- $D_B$
Theories of Interest Rate Determination

Business Cycle Expansions

Diagram showing supply (S_B) and demand (D_B) curves for interest rates (P_B) and bonds (B).
A Simple Representative Investor Model I

The representative investor has endowment $y_0$ today & $y_1$ tomorrow. He maximizes the following objective function:

$$\max u(C_0) + \rho u(C_1)$$

subject to

$$C_0 = y_0 - b$$

$$C_1 = y_1 + (1 + r)b$$

where $b$ is the bond holdings.
A Simple Representative Investor Model I

The representative investor has endowment $y_0$ today & $y_1$ tomorrow.

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$$C_1 = y_1 + (1 + r)b$$

where $b$ is the bond holdings.

The investor’s utility function is defined by

$$u(C) = -\exp^{-\alpha C}$$

where $\alpha$ is some positive constant.
A Simple Representative Investor Model II

We will:

• Solve for the equilibrium interest rate $r$?
• Examine how $r$ is affected by:
  • increases in the representative investor’s next period endowment, $y_1$;
  • changes in investor time-patience $\rho$. 
Eliminating $b$ in the two constraints, we have

$$C_1 = y_1 + (1 + r) (y_0 - C_0)$$
Eliminating $b$ in the two constraints, we have

$$C_1 = y_1 + (1 + r)(y_0 - C_0)$$

The investor chooses consumption level to maximize his utility,

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s.t. $C_1 = y_1 + (1 + r)(y_0 - C_0)$
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s.t $C_1 = y_1 + (1 + r)(y_0 - C_0)$

$$\max -\exp^{-\alpha C_0} + \rho. - \exp^{-\alpha[y_1+(1+r)(y_0-C_0)]}$$
We then get the FOC:

\[
\frac{dU()}{dc_0} = 0
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\[-\exp^{-\alpha C_0} (-\alpha) + -\rho \exp^{-\alpha \left[ y_1 + (1+r)(y_0 - C_0) \right]} (-\alpha) \left( -(1+r) \right) = 0 \]
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\[
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\]

\[
\alpha \exp^{-\alpha C_0} = (\alpha) (1 + r) \rho \exp^{-\alpha C_1}
\]
We then get the FOC:

\[ \frac{dU()}{dc_0} = 0 \]

\[ -\exp^{-\alpha C_0} (-\alpha) - \rho \exp^{-\alpha[y_1+(1+r)(y_0-C_0)]} (-\alpha) (-1 + r) = 0 \]

\[ \alpha \exp^{-\alpha C_0} = (\alpha)(1 + r) \rho \exp^{-\alpha C_1} \]

\[ (1 + r) = \frac{1}{\rho} \exp^{\alpha(C_1-C_0)} \]
A Simple Representative Investor Model V

In equilibrium: \( C_0 = y_0 \) and \( C_1 = y_1 \), \( \implies \)

\[
r = \frac{1}{\rho} \cdot \exp^{\alpha(y_1 - y_0)} - 1
\]

Therefore, \( r \) will:

- Increase if the representative investor's next period endowment, \( y_1 \), increases. This is because the demand for saving has declined;
- Decrease if changes in investor time-patience \( \rho \) increases. The investor is now more patient and so values future consumption more - so demand for savings increases.
In equilibrium: $C_0 = y_0$ and $C_1 = y_1$, \[\implies \]

\[r = \frac{1}{\rho}.\exp^{\alpha.(y_1 - y_0) - 1}\]

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A Simple Representative Investor Model V

In equilibrium: \( C_0 = y_0 \) and \( C_1 = y_1 \), \( \implies \)

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Theories of Interest Rate Determination
Liquidity Preference Approach

An alternative theory of how interest rates are determined - due to Keynes.

\[ M_s + BS = BD + MD \]
\[ M_s - MD = BD - BS \]

Therefore it is the same to think about money equilibrium as it is to think about bond market equilibrium.

Key idea

The interest rate is the reward for giving up liquidity (not just saving). Liquidity effect: \( M_s \uparrow \) leads to interest rates \( \downarrow \).
The effect of MS on nominal interest rates

nominal = real + \pi^e

• Liquidity effect
• Fisher effect - higher future inflation
Theories of Interest Rate Determination

Central Bank Control of Interest Rates

In reality, the Central Bank actually announces the short-term interest rate (and then engineers it in the Bank lending market).

Therefore, we need to think about the effect that this has on how we think about interest rate setting.
Theories of Interest Rate Determination

Central Bank Control of Interest Rates

In reality, the Central Bank actually announces the short-term interest rate (and then engineers it in the Bank lending market).

Therefore, we need to think about the effect that this has on how we think about interest rate setting.

Actually - it is a combination of the two effects. The central bank is constrained by what the market thinks, but also the CB only controls the very short rate, while the longer rates are more important for economic decisions.
Bonds are Risky

Bonds are not perfectly safe (fixed payments and then fixed sum paid):

- Default risk (even some governments!);
- Liquidity risk;
- Reinvestment and Capital Risk (interest rate risk);
- Inflation risk - $ worth less when you get it;
- Tax considerations.

Affect the nature of the bond’s riskiness and so are reflected in the price (and yield which moves in the opposite direction).
Credit Ratings I

Credit Rating agencies are private companies who assess the credit worthiness of bond-issuers and, in announcing these ratings, affect the price of the bonds.

⇒ higher rating (more creditworthy) means a higher price and so a lower yield is required by investors.
## Credit Ratings II

<table>
<thead>
<tr>
<th>Description</th>
<th>Moody’s</th>
<th>S&amp;P</th>
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</thead>
<tbody>
<tr>
<td><strong>Investment grade</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Best quality companies, reliable and stable</td>
<td>Aaa</td>
<td>AAA</td>
</tr>
<tr>
<td>Quality companies, a bit riskier than above</td>
<td>Aa1, Aa2, Aa3</td>
<td>AA</td>
</tr>
<tr>
<td>Economic situation can affect finance</td>
<td>A1, A2, A3</td>
<td>A</td>
</tr>
<tr>
<td>Medium class companies</td>
<td>Baa1, Baa2, Baa3</td>
<td>BBB</td>
</tr>
<tr>
<td><strong>Speculative grade</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>More prone to changes in the economy</td>
<td>Ba1, Ba2, Ba3</td>
<td>BB</td>
</tr>
<tr>
<td>Financial situation varies noticeably</td>
<td>B1, B2, B3</td>
<td>B</td>
</tr>
<tr>
<td>Currently vulnerable</td>
<td>Caa1, Caa2, Caa3</td>
<td>CCC</td>
</tr>
<tr>
<td>Highly vulnerable, very speculative bonds</td>
<td>Ca</td>
<td>CC</td>
</tr>
<tr>
<td>Highly vulnerable (perhaps in arrears)</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Has defaulted on obligations</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

Downgrades of companies can have a large effect on their bond price.
The Term Structure of Interest Rates

Definition

“The term structure refers to the pattern of interest rates available on assets differentiated solely by their term to maturity”
(Howells and Bain, p. 322)
The Term Structure of Interest Rates

It’s a movie not a picture!

YTM

Time to Maturity
The Term Structure of Interest Rates

Mishkin highlights 3 key facts about the term structure of interest rates (Mishkin, p. 135):

1. Interest rates of different maturities tend to move together;
2. Yield curve slopes up if interest rates are historically low, and vice versa;
3. Long-term yields tend to be higher than short-term yields.

We have 3 or 4 different models to explain:

- Expectations theory
- Segmented markets theory
- Liquidity premium theory
- Preferred habitat theory
The Term Structure of Interest Rates

Expectations theory

Consider the simple example of a 2 year investment strategy. You can:

1. Invest in a 2 year bond paying $i_{02}$ each year;

\[
(1 + i_{01}) \cdot (1 + i_{12}) = (1 + i_{02})^2
\]

This is called the forward rate - implied future spot rate.
Consider the simple example of a 2 year investment strategy. You can:

1. Invest in a 2 year bond paying $i_{02}$ each year;

2. Invest in a 1 year bond paying $i_{01}$ in year 1; then reinvest in a 1 year bond which you expect to pay $i_{12}$.

In equilibrium, we should be indifferent between these two options - Why?

\[(1 + i_{01})(1 + i_{12}) = (1 + i_{02})^2 - 1\]

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The Term Structure of Interest Rates

Expectations theory

Consider the simple example of a 2 year investment strategy. You can:

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In equilibrium, we should be indifferent between these two options - Why?

$$(1 + i_{01})(1 + i_{12}) = (1 + i_{02})^2$$

$$i_{12} = \frac{(1 + i_{02})^2}{(1 + i_{01})} - 1$$

This is called the forward rate - implied future spot rate.
The Term Structure of Interest Rates

Expectations theory

\[(1 + i_{01})(1 + i_{12}) = (1 + i_{02})^2\]

\[1 + i_{12} + i_{01} + i_{01}i_{12} = 1 + 2i_{02} + (i_{02})^2\]

\[i_{12} + i_{01} \approx 2i_{02} \text{ if } i_{jk} \text{ is small}\]

\[i_{02} \approx \frac{(i_{01} + i_{12})}{2}\]
The Term Structure of Interest Rates

Expectations theory

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\]

\[
i_{02} \approx \frac{(i_{01} + i_{12})}{2}
\]

\[
i_{0T} \approx \frac{(i_{01} + i_{12} + \cdots + i(T-1)T)}{T}
\]
The Term Structure of Interest Rates

Expectations theory

\[(1 + i_{01})(1 + i_{12}) = (1 + i_{02})^2\]

\[1 + i_{12} + i_{01} + i_{01}i_{12} = 1 + 2i_{02} + (i_{02})^2\]

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\[i_{0T} \approx \frac{(i_{01} + i_{12} + \cdots + i(T-1)T)}{T}\]

- Can explain fact 1 and fact 2, but not fact 3 (unless consistent errors)
- With Fisher effect, can work out inflation expectations
  - Mankiw and Miron (1986) suggest little evidence that term structure predicts future interest rates
Imagine each market for the bonds is completely separate - 2 year bond investors don’t buy 1 year or 3 year bonds.

Now we can think about how demand and supply issues within each market separately determine the price and yield.

If people prefer shorter maturity bonds, then demand is less in the longer term markets so the price is lower (and yield higher).
The Term Structure of Interest Rates

Segmented markets theory

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If people prefer shorter maturity bonds, then demand is less in the longer term markets so the price is lower (and yield higher).

Explains only point 3, and in fact contradicts point 1 directly.
The Term Structure of Interest Rates

Liquidity premium theory

\[ i_{0T} = \left( \frac{i_0 + i_1 + \ldots + i_{(T-1)T}}{T} \right) + \theta_L^T \]

where \( \theta_L^T \) is the liquidity premium for a bond of maturity \( T \).

Expectations hypothesis + a premium for holding longer term bonds:

This is a risk premium (not necessarily representing liquidity):

- Default risk;
- Interest rate risk.

What about an inverted yield curve (downward sloping)?
The Term Structure of Interest Rates
Preferred habitat theory

Very similar to liquidity premium story.

Investors have a preferred maturity (habitat);

Other bonds must offer a premium to induce these investors away from their habit maturities - relies on preference for shorter rates.
The Term Structure of Interest Rates

Interpreting the yield curve is model dependent
The Term Structure of Interest Rates

Interpreting the yield curve is model dependent

YTM

Time to Maturity
The Term Structure of Interest Rates

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Time to Maturity
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Interpreting the yield curve is model dependent
The Term Structure of Interest Rates

Interpreting the yield curve is model dependent

![Graph showing the term structure of interest rates with YTM on the y-axis and time to maturity on the x-axis. The graph illustrates a positive slope, indicating an upward yielding curve.]
The Term Structure of Interest Rates

Interpreting the yield curve is model dependent

YTM

Time to Maturity
Questions?