The Economics of Risky decision making (1)

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An outline of the lectures

- Lecture 1: An introduction to economic risky decision making. Expected value and Expected Utility.
- Lecture 2: Beyond Expected Utility: Prospect theory, non-linear probability weighting, loss aversion, WTA/WTP.
- Lecture 3: The economics of time discounting. A brief history of time preferences and the models used to describe time preferences.
- Lecture 4: Applications of time discounting in fields of economics, with a particular focus on environmental economics.

Today's lecture

- In the lecture today we will outline a basic roadmap about decision making under risk.
- States and outcomes in decision theory. Models of Expected Utility theory and Expected Value.
- We will discuss some important economic concepts like utility, risk aversion, utility curvature and how they are related.
- Finally we will see some applications from financial markets (CAPM and market risk premium), agricultural economics and health economics.

States of the world, acts and outcomes

| | Rain Sunshine | | | |
|------------------------|-----------------|------------------------|--|--|
| Take umbrella | 0 ₁₁ | <i>O</i> ₁₂ | | |
| Leave umbrella at home | 021 | <i>O</i> ₂₂ | | |

Imagine you are facing the problem of the above matrix: you have to decide whether or not to get an umbrella with you. But this decision could depend on whether or not is going to rain.

The two possible states of the world are shown in the two columns (raining or not).

The two available acts are shown in the two rows (take or leave the umbrella).

States of the world, acts and outcomes

- The cells of the matrix represent the outcomes (the combination of acts and states of the worlds). Of course, the number of outcomes is equal to the product of states times acts; in our case, this means 2 * 2 = 4.
- The aforementioned is a generic representation of a decision problem an agent could face. We can assume that for an agent the outcomes have specific utility values and that specific probabilities are assigned to each state. Then, one can use decision theories to help guide himself about the dilemmas.

States of the world, acts and outcomes

- For example, assume that the following utilities are assigned to the four outcomes: $O_{11} = 0$, $O_{12} = -2$, $O_{21} = -8$, $O_{22} = 1$. Assume also that your belief about raining is 30% (0.3) and your belief abut sunshine is 70% (0.7).
- What is the expected utility for each act?

$$EU(take) = 0 * 0.3 + (-2) * 0.7 = -1.4$$
$$EU(leave) = (-8) * 0.3 + 1 * 0.7 = -2.4 + 0.7 = -1.7$$

So, taking the umbrella has a higher expected utility, so you might think that this the rationale thing to do.

Risk analysis in economics

Risk is an omnipresent factor in everyday life and practically all economic decisions are intertwined with risk.

Risk could depend very much on the context (business risk, environmental risk, health risk, safety risk etc.).

A question: But how to define risk? It is not always clear and there are no strict definitions.



A rough definition

In this lectures series, as risk in economics we will define event(s), outcomes and situations which are not known with certainty ahead of time.

Plenty of examples in economics: decisions about future investments, buying or trading stocks or bonds, purchasing insurance, volatility in the market for a commodity, change of economic policies etc.

Ultimately, risk could affect profits, losses, debt viability of an agent or a company.



Decision under risk

In this lecture we will consider situations where the decision maker is fully aware of the probabilities and the outcomes of the decisions examined.

The knowledge of these concepts will enable us through specific decision theories to determine if some actions are preferable or not over other actions.

Just being informed of the probability of an outcome does not mean that one can determine what should do.

It is the use of specific decision theories that should guide us.



A warning

- However, a couple of warnings must be offered here:
- In some situations, it might be very difficult to calculate probabilities accurately, for example, it might only be possible to make some rough estimates (confidence intervals, likelihood assessments).
- This means that randomness of some phenomena is very likely (e.g., atmospheric noise) and this makes very difficult to achieve accurate predictions for future action.
- An implication? It is very unlikely we will ever face risk-free economic decisions.

These are the confidence levels and likelihoods from the 4th National Climate Assessment, US Global Change Research Program (2017).

The link for the report is below.

https://science2017.globa lchange.gov/downloads/ CSSR2017_FullReport.pdf

Confidence Level

Very High

Strong evidence (established theory, multiple sources, consistent results, well documented and accepted methods, etc.), high consensus

High

Moderate evidence (several sources, some consistency, methods vary and/or documentation limited, etc.), medium consensus

Medium

Suggestive evidence (a few sources, limited consistency, models incomplete, methods emerging, etc.), competing schools of thought

Low

Inconclusive evidence (limited sources, extrapolations, inconsistent findings, poor documentation and/or methods not tested, etc.), disagreement or lack of opinions among experts

Likelihood

Virtually Certain

99%-100%

Extremely Likely

95%–100%

Very Likely

90%-100%

Likely

66%-100%

About as Likely as Not

33%-66%

Unlikely

0%-33%

Very Unlikely

0%-10%

Extremely Unlikely

0%–5%

Exceptionally Unlikely

0%–1%

Another issue is our limited ability to process all the available information.

Practically, in such cases people face cognitive constraints which limits the capacity of rational decision making. Then, one could ignore information and could use rules of thumb when making a decision. This fact has been called bounded rationality.

Even for supercomputers there are limitations in their functionalities and such limitations could impact the research on climate change.

https://newsupdate.uk/climatescientists-encounter-limits-of-computermodels-bedeviling-policy/



Blue Waters supercomputer University of Illinois at Urbana-Champaign.

The simplest example

You flip a coin: obviously the outcome could be either heads or tails.

But one cannot be sure beforehand for the exact outcome (assume that it is a fair coin).

This inability in predicting what is going to happen in this very simple problem has led to the generation of theories that can inform us in risky situations.

Heads or Tails?



Moving ahead

- The first concept we will use is the mathematical concept of probability. This is a particularly important concept and is at the centre of risk analysis.
- Another concept that will be introduced and used is utility. Again, utility is of fundamental importance to economics.
- These two concepts are used together in the Expected Utility Theory.

WHAT NEXT?



Putting things together

It is common to use lotteries (gambles, prospects) to describe risky situations.

A common format is below:

 $L = (x_1, p_1; x_2, p_2)$

where for this lottery L, $x_i(x_1, x_2)$ denotes the outcome and $p_i(p_1, p_2)$ the corresponding probability.

A large number of outcomes could be included in a lottery, not just two.



For example: you are asked to decide between the following lotteries which denote different situations when making an investment:

> $L_1 = (0.5, \pm 1000; 0.5, \pm 1200)$ $L_2 = (0.2, \pm 2500; 0.8, \pm 800)$

Would you prefer L_1 or L_2 ? And why?



How the probabilities are assigned in the above image?

Which would you choose?

- It's quite common in economic experiments for the elicitation of risk preferences to present lotteries in the form of pie-charts.
- For the adjacent examples: what would be your choice in each pair of lotteries?

A or B?







Expected Value

A simple principle to help you decide what to choose is the Expected Value.

The Expected Value (EV) of each lottery is computed by the following formula:

$$EV = p_1 * x_1 + p_2 * x_2$$

Or more generally:

$$EV = \sum_{1}^{n} p_i * x_i, i = 1, ... n$$

The lottery with the largest expected value could be your preferred option.





Let's make some calculations for the previous lotteries:

 $L_1 = 0.5 * 1000 + 0.5 * 1200 = 1100$ $L_2 = 0.2 * 2500 + 0.8 * 800 = 1140$

Since $EV(L_2) > EV(L_1)$, some of you could choose lottery L_2 .

Would you all agree with this finding? Some of you might disagree.



Is Expected Value always suitable?

- Expected Value implies that we consider monetary rewards at their face value. This is not necessarily correct.
- To explain situations where the Expected Value cannot accurately describe preferences, economists employ the concept of utility and of marginal utility.
- But when such situations can arise?



St. Petersburg paradox

Consider a simple game: flip a fair coin multiple times.

Assume that you get £2 if head appears in the 1^{st} toss and generally you receive $\pm 2^n$ if head appears after n times.

A simple question for you: what is the maximum amount of money you would pay for this game?



Hermitage Museum, St. Petersburg, Russia The expected value is

$$EV = \frac{1}{2} * 2 + \frac{1}{4} * 2^2 + \frac{1}{8} * 2^3 + \dots = \sum_{n=1}^{\infty} (\frac{1}{2})^n * 2^n$$
$$= \infty$$

Hence, the willingness to play the game would be infinite, a finding with which you might not agree.

The above question is called the St. Petersburg paradox.

Questions like this indicated that people do not behave so that to maximize the expected value of monetary rewards. Concepts like marginal utility could solve this paradox.



¹⁷⁰⁰⁻¹⁷⁸²

The concept of utility

- Utility is an abstract concept, not directly observable. It describes how valuable is an outcome/quantity/money to a person.
- Economists believe that money has a decreasing marginal utility. The more money you already have, the less additional (marginal) utility you will get by acquiring one more pound.
- At the end, one might reach a point of satiation where one is not willing to consume any more.



The importance of utility in economics

- The concept of utility has been extensively applied in all fields of economics in order to model preferences.
- Its origin can be traced back to moral philosophy and thinkers like Jeremy Bentham and John Stuart Mill.



Jeremy Bentham (1748-1832)

 Bentham's focus (he was a utilitarian philosopher) was on estimating pains and pleasures (hedonic calculus).



John Stuart Mill (1806-1873)

A note on utility

- Utility is a cornerstone concept of utilitarianism, a philosophical theory that advocates happiness and pleasure for most people in a society.
- Remember that Adam Smith, the author of "The Wealth of Nations" (1776), considered himself to be a moral philosopher.
- In modern economics though, utility is mostly associated with the classification and ordering of preferences.

UTILITARIANISM

JOHN STUART MILL.

REPRINTED FROM "FRASSA" MAGATTER.

LONDON : PARKER, SON, AND BOURN, WEST STRAND. 1863. [Be defler reverse the right of Translation.]

PRINCIPLES

POLITICAL ECONOMY

WHO BOAR OF THEIR APPLICATIONS TO SOCIAL FULLOBORRY.

JOHN STUART MILL

IN TWO VOLUMES, VOL 1.

LONDON. JOBN W. PARKER, WEST STRAND.

The Expected Utility Theory (EUT) model

- The next step was to formulate a formal decision theory for choice under risk by exploiting the concept of utility.
- This was achieved by Neumann and Morgenstern who introduced the Expected Utility theory ("Theory of Games and Economic Behavior", 1947).
- This theory employs a utility function and utilizes a set of axioms of preferences to describe decision preferences.



John von Neumann (1903-1957)



Oskar Morgenstern (1903-1977)

Expected Utility calculations

• The formula for estimating the Expected Utility is

> $EU = p_1 * u(x_1) + p_2 * u(x_2)$ $EU = \sum_{i=1}^{n} p_i * u(x_i), i = 1, ... n$

- The lottery with the highest Expected Utility is preferred (Expected Utility maximization).
- The (subjective) value of an outcome now becomes $u(x_i)$ and is dependent on a utility functional form.

Expected Utility formula

Another look at the previous example

Let's examine how choices are affected when EUT is used instead of EV.

Consider a utility function $u(x) = x^{0.7}$ and the lotteries mentioned earlier:

 $L_1 = (0.5, \pm 1000; 0.5, \pm 1200)$ $L_2 = (0.2, \pm 2500; 0.8, \pm 800)$

 $EU(L_1) = 0.5 * 1000^{0.7} + 0.5 * 1200^{0.7} = 134.46$ $EU(L_2) = 0.2 * 2500^{0.7} + 0.8 * 800^{0.7} = 133.96$

Thus, $EU(L_1) > EU(L_2)$, the lottery L_1 now is the preferred choice.





Axioms of EUT

• Transitivity:

If $x \ge y$ and $y \ge z$, then $x \ge z$

• Completeness:

 $x \ge y \text{ or } y \ge x$

• Independence axiom:

If $x \ge y$ and $0 , then <math>px + (1-p)z \ge py + (1-p)z$

• Continuity axiom:

If x > y and y > z, there are numbers 0 and <math>0 < q < 1 such that

$$px + (1-p)z \succ y \succ qx + (1-q)z$$

Utility curvature and risk attitudes

- Assume that the expected utility of a gamble (0.5, £10; 0.5, £30) is 14. An individual has £20 and he/she contemplates whether to keep the money or to take the gamble which also returns £20.
- An individual who chooses to keep the £20 over the gamble is a risk averse individual; that individual opts for the certain £20 over the gamble.
- Such an individual is risk averse and the representing utility function is concave.



The expected utility of wealth at £20 (point D) is above (larger) than the expected utility of the gamble (point F). This shows the concavity of the utility function. The adjacent graph illustrates this point more generally: the straight line depicts the expected utility of a gamble consisted of m_1, m_2 and with the corresponding probabilities. But this line is below the utility of the expected value. U(EV(L)) > EU(L)

This is also the definition of convexity:

f(px + (1 - p)y)> pf(x) + (1 - p)f(y)



- Of course, there is also the other possibility that an individual could prefer the gamble over the certain income.
- In other words, the utility of the certain income £20 is lower than the expected utility of the gamble. U(EV(L)) < EU(L)
- In such a case, the individual is risk seeking (risk-loving) and the corresponding utility function is convex.



- A final possibility is the individual to be risk neutral.
- The individual is then indifferent between the gamble and the certain income.
- Then, the utility function graph is a straight line, that is, u(x) = x.



Some popular utility functions

Below are some popular utility functional forms used in economics:

 $U(m) = m^r, r > 0$ (power function)

 $U(m) = 1 - e^{-rm}, r > 0$

$$U(m) = \frac{m^{1-r}}{1-r}, r > 0, r \neq 1$$

How to determine risk attitudes mathematically

- The standard conditions for a utility function are: U' > 0, U'' < 0 for all levels of wealth. This is translated to a monotonic function (graph that rises) but it increases at a decreasing rate; these two conditions imply concavity.
- What about risk neutrality? $U' = \rho$, U'' = 0.
- What about risk loving behaviour? U' > 0, U'' > 0 (convex function).
- So, by taking the first and second derivatives of a utility function with respect to monetary rewards (wealth), we can determine the nature of risk attitudes of an agent.

Arrow-Pratt coefficients

A question: Is there any way we can measure the utility curvature? We introduce the Arrow-Pratt measure of absolute risk aversion:

$$A(m) = -\frac{U''(m)}{U'(m)}$$

Note that this measure is local since it can vary with the argument m. This can be a problem if units of measurement for wealth change.

Example: Find the Arrow-Pratt measure of absolute risk aversion for the function $U(m) = m^{0.5}$.

It is $U' = 0.5m^{-0.5}$, $U'' = -0.5^2m^{-1.5}$

$$A(m) = -\frac{-0.25m^{-1.5}}{0.5m^{-0.5}} = 0.5m^{-1} = \frac{1}{2m}$$

Arrow-Pratt coefficients

• What might be a problem for A(m)? For the previous utility function, assume a wealth of £10. Then,

$$A(m) = \frac{1}{2m} = \frac{1}{2 * 10} = \frac{1}{20} = 0.05$$

What if the unit of wealth was in pence?

Say, 1,000 pence (which equals £10)?

Then,
$$A(m) = \frac{1}{2m} = \frac{1}{2*1000} = \frac{1}{2000} = 0.0005$$

Hence, you can see how sensitive A(m) is to units of measurements even though the wealth is the same (£10).

This paves the way for another measure of risk preferences.

Arrow-Pratt coefficients

Another risk aversion measure, is the Arrow-Pratt measure of relative risk aversion:

$$R = -m \frac{U^{\prime\prime}(m)}{U^{\prime}(m)} \,.$$

In essence, this measure captures how marginal utility changes when wealth changes as well.

Example: Find the Arrow-Pratt measure of relative risk aversion for the function $U(m) = \frac{m^{1-r}}{1-r}$ It is $U' = \frac{(1-r)m^{-r}}{1-r} = m^{-r}, U'' = -rm^{-r-1}$ $R(m) = -m\frac{-rm^{-r-1}}{m^{-r}} = mrm^{-1} = r$

$$\sim$$

A classic example-CAPM

A simple example (utilizing the expected value concept) is the construction of an investment portfolio based on the popular model CAPM (Capital Asset Pricing Model).

Assume that you invest your money in a portfolio with two assets: a risk-free asset (R_f) and a risky asset (R_m) .

The expected return, R of the portfolio would b

$$R_p = E(bR_m) + E((1-b)R_f)$$
$$R_p = bR_m + (1-b)R_f$$
$$R_p = R_f + b(R_m - R_f)$$





- The last equation can be further manipulated and written as $R_p = R_f + \beta \ (R_m - R_f)$
- The parameter $\beta = \frac{\sigma_p}{\sigma_m}$ (beta) represents practically the risk of the portfolio (*R*), the standard deviation of the portfolio over the s.d. of the risky asset.
- In essence, the expected return of a portfolio is equal to the risk-free asset and a risk adjustment.
- Note that CAPM can be modelled through various utility functions (e.g. quadratic utility functions).





One thing to remember

- In investment decisions it is important to remember one word: diversification. This is the rationale behind investing in a risk-free asset and a risky asset with the goal to reduce the risk of the investment.
- If you had put all your money in the risky asset then there is the danger you could have suffered losses. Investing in negative correlated assets could be a solution.
- Essentially, diversification implies what an old well-known proverb dictates: "Don't put all your eggs in one basket".

DON'T PUT ALL YOUR EGGS IN ONE BASKET

A health economics example

- The Expected Utility theory is also used in health care evaluations.
- The standard gamble is a popular method for measuring utility for health states based on Expected Utility theory.
- In this approach two different hypothetical health states, one certain and one risky (a gamble) are juxtaposed.





Typically, one chooses between a particular health state for the rest his/her life problem with certainty and a gamble with two possible outcomes: a state with good healt (under probability p) and a state assuming immediate death (probability 1-p).

 P
 Healthy for the rest of life

 Alternative 1
 1-P

 Immediate death

 Alternative 2

 Chronic health state for the rest of life

Source: Hamilton et al. (2014)

By varying the probability p, one can find a point of indifference between the two alternatives so, an evaluation for a health state can be identified. The mathematical formula implied is

p * U(good health) + (1 - p) * U(death) = U(health outcome)

 $\neg \land$

- Generally, utility of good health is set equal to 1 and utility of death equal to 0.
- One can continue this approach with more questions and calculate preferences for various health states. These results can be averaged to produce an index for health utility.
- In turn, this will help to estimate QALYs (Quality Life Adjusted Years), a measure for assessing quantitatively medical interventions.



FIGURE 1. Setups for rating states better (left) and worse (right) than death.

Source: Patrick et al. (1994)

Imagine that you have an illness from which, without treatment, you will die almost immediately. Your doctor tells you that there are two alternative available treatments for your illness. The two pies drawn below represent the chances of certain outcomes from the two treatments, treatment (a) and treatment (b):



(b)



Source: Oliver (2003)



Ultimately, this might seem like playing a game with the Death.

Determinants of Expected Utility

- A question that could be put forward here is what are the factors which could affect risk attitudes? Do such factors even exist?
- This is a question that the economic profession has attempted to examine in the last decades.
- There is not always a clear answer here since the results could be affected by the sample and characteristics of the sample.
- Socio-economic and political parameters could play a role: in the next page, you can see that risk aversion could be affected by factors like gender, age, race.

| | | | | Numbe | er of obs | s = | 14718 |
|----------------|--------------|-----------|---------|----------|-----------|-------|------------|
| | | | | Wald | chi2(6) | = | 53.91 |
| Log pseudolike | elihood = -9 | 311.596 | | Prob | > chi2 | = | 0.0000 |
| 2 | | (St | d. Err. | adjusted | for 240 | clust | ers in id) |
| | | | | | | | |
| | | Robust | | | | | |
| | Coef. | Std. Err. | Z | P> z | [95% | Conf. | Interval] |
| + | 0040650 | | | 0.010 | 1 5 0 5 | | 01 00 4 0 |
| Female | 0843659 | .0348/88 | -2.42 | 0.016 | 152 | /2/1 | 0160046 |
| Black | 1492568 | .0678122 | -2.20 | 0.028 | 2821 | L663 | 0163473 |
| Hispanic | 1948952 | .0765465 | -2.55 | 0.011 | 3449 | 9235 | 0448669 |
| Age | .0193893 | .0035911 | 5.40 | 0.000 | .0123 | 3508 | .0264277 |
| Business | 0126035 | .0330866 | -0.38 | 0.703 | 077 | 7452 | .0522451 |
| GPAlow | .0337201 | .0309452 | 1.09 | 0.276 | 0269 | 9314 | .0943715 |
| cons | 4536359 | 0811465 | 5.59 | 0.000 | 2945 | 5916 | 6126802 |
| | | | | | | | |

Estimating risk aversion coefficients, an econometric approach (Harrison, 2008). You can see that risk aversion could be negatively affected by gender (females are more risk averse) and race. Age has a slight positive impact.

In a field experiment parameters like education, income, marital status, number of children might also affect risk attitudes.

Criticisms

- A number of criticisms have been levelled against the Expected Utility and Expected Value model.
- Some of them include the consideration of non-objective probabilities, the insufficiency of utility curvature alone (Rabin, 2000), preference reversals depending on the size of probabilities (Allais paradox) etc.
- Look at Starmer (2000) for more details and a comprehensive survey. We will discuss the basic criticisms more analytically in the next lecture.

Wide applicability of EUT

- Expected Utility theory is by far the most dominant model used in economics to model risk and uncertainty.
- It has found applicability in practically every field of economics: health economics, environmental and agricultural economics, financial economics, insurance, decision modelling, estimation of risk attitudes etc.
- Why is this happening?





- Expected Utility Theory (EUT) has been advocated by many economists for normative purposes (i.e., how people should behave) and it has become the hallmark of rationality (Wakker, 2010).
- It has a simple and normative interpretation which is appealing to many economists.



Of course, rationality in economics has been very much debated.

A distinction to remember

- At this point we have to make a distinction between two terms, risk and uncertainty.
- Quite often people face situations where they do not know some of the outcomes and/or the probabilities.
- In such cases the tools mentioned before cannot be applied; this is an example of the Ellsberg paradox (Ellsberg, 1961).



- Frank Knight (1921) was the first who stressed the difference between risk and uncertainty.
- The latter arises in cases of non-quantifiable risk (quantity that cannot be measured) and it implies non-knowledge and ignorance.
- Different terminologies to describe such situations: ambiguity aversion, Knightian uncertainty, black swan events.
- Such situations could cause serious concerns about statistical modelling (e.g., about climate disasters). In such cases, typical valuation techniques (CBA) might not work.



Frank Knight (1885-1972)



Daniel Ellsberg (1931-)

An interesting quote

"... there are known knowns; there are things we know we know. We also know there are known unknowns. That is to say, we know there are some things we do not know. But there are also unknown unknowns—the ones we don't know we don't know." (February, 2002).

Known unknowns: things that are expected but have not been measured, it might be a hypothesis to be tested experimentally (Logan, 2009). Perhaps an economic example: the trade deal between UK and EU; we understand it will change but we don't know its exact future form.

Unknown unknowns: things that are not anticipated. An example could be the Fukushima nuclear plant disaster (2011) due to a massive earthquake (very difficult to predict earthquakes).



Donald Rumsfeld, US Secretary of Defense (2001-2006) Eventually each one of us must take decisions in our everyday life. The tools presented here can help and guide us through risky situations. This is how these tools should be viewed.



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