Business Strategy - Use of Game Theory

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Learning Objectives

- Pure strategies, dominance, and Nash equilibrium
- Simultaneous games including the prisoner's dilemma
- Mixed strategies
- Sequential games: threats, credibility, and commitment
- Repeated games

Suggested Reading

Textbooks

- Chapter 9 John Lipczynski, John Goddard, and John O.S. Wilson, *Industrial Organization: Competition Strategy and Policy*, 5th Edition, Pearson Education, 2017.
- Or any standard microeconomics textbook that covers game theory.

What is Game Theory?

- Game theory is a tool for studying strategic behavior which considers the expected behavior of others and the mutual recognition of interdependence.
- It shows how situations can arise where firms take decisions that may appear rational individually but lead to suboptimal outcomes for all combined.
- It is an approach to decision-making where two or more decision-makers or players face choices between a number of possible actions at any stage of the game.

What is Game Theory?

- Developed by the mathematicians John von Neumann and Oskar Morgenstern (1944).
- Game theory has many applications throughout the social, behavioural and physical sciences; and accordingly, its remit is much wider than just economics.
- Its focus on uncertainty, interdependence, conflict and strategy makes it ideally suited for the analysis of decision-making in oligopoly.

Applications of Game Theory

- Oligopoly
- Sports
- Chess
- Military strategy and nuclear deterrence
- International Business
- Economics
- Rock Paper Scissors Game
- Bidding at Auction
- Poker
- Market Shares and Stockholders

Common Features of Games

- **Rules**
- **Strategies**
- **Payoffs**
- **Outcome**

Player's Strategy

- A player's **strategy** is a set of rules telling him which action to choose under each possible set of circumstances that might exist at any stage in the game.
- Each player aims to select the strategy (or mix of strategies) that will maximise his own **payoff**.
- The players face a situation of **interdependence**.
- Each player is aware that the actions of other players can affect his payoff but at the time the player chooses his own action he may not know which actions are being chosen by the other players.

Types of Games

- **Constant-sum Game:** The sum of the payoffs to all players is always the same whatever strategies are chosen.
- **Non-constant-sum Game:** The sum of the payoffs depends on the strategies chosen.
- **Zero-sum Game:** A constant-sum game in which the sum of the gains and losses of all players is always zero. For example, a game of poker is a zero-sum game: one player's winnings are exactly matched by the losses of rival players.
- **The outcome** of a game is the set of strategies and actions that are actually chosen, and the resulting payoffs.
- **An equilibrium** is a combination of strategies, actions and payoffs that is optimal (in some sense) for all players.

Simultaneous vs. Sequential Games

- A game in which all players choose their actions simultaneously before knowing the actions chosen by other players is called a **simultaneous game**.
- **Dominant strategies** are those that are either unambiguously superior to or at least as good as all other strategies no matter which strategy the other player selects.
- A game in which the players choose their actions in turn so that a player who moves later knows the actions that were chosen by players who moved earlier is called a **sequential game**.
  - In a sequential game, sometimes the player who moves first is able to gain an advantage by influencing the future direction of the game in their favour.
- Finally, a game that is played more than once is called a multiple-period or **repeated game**. A multiple-period game can be repeated either indefinitely or a finite number of times.

Dominant Strategies

Figure 1. Production game with strictly dominant strategies

Firm B's strategies

Firm A's strategies		*Low*	*High*
	Low	**4** *4*	**2** *3*
	High	**3** *2*	**1** *1*

Production Game with Strictly Dominant Strategies

- The figure shows the payoff matrix for two firms A and B that have to decide simultaneously whether to produce *Low* or *High* levels of output.
- Firm A's strategies are denoted *Low* and *High* and similarly firm B's strategies are denoted *Low* and *High*.
- The elements in the matrix represent the payoffs (for example profit) to the two firms.
- Both firms' payoffs depend on their own output level and on the output level of the other firm since market price is a function of the combined output levels of both firms.
- Within each cell of the figure the first figure is A's payoff and the second figure is B's payoff. For example, if A selects *High* and B selects *Low*, A's payoff (profit) is 3 and B's payoff is 2.

Production Game Analysis:

A's Perspective

- If B selects *Low*, *Low* yields a payoff of 4 for A while *High* yields a payoff of 3. Therefore, if B selects *Low*, A's best response is *Low*.
- If B selects *High*, *Low* yields a payoff of 2 for A while *High* yields a payoff of 1. Therefore, if B selects *High*, A's best response is *Low*.

B's Perspective

- If A selects *Low*, *Low* yields a payoff of 4 for B while *High* yields a payoff of 3. Therefore, if A selects *Low*, B's best response is *Low*.
- If A selects *High*, *Low* yields a payoff of 2 for B while *High* yields a payoff of 1. Therefore, if A selects *High*, B's best response is *Low*.

Dominant Strategy Equilibrium

- Accordingly, no matter what strategy A selects, it is best for B to select *Low* rather than *High*.
- Therefore, *Low* is B's strictly dominant strategy.
- Following this approach, it appears A should select *Low* and B should select *Low*, so that both firms earn a payoff of 4.
- This outcome, denoted (*Low*, *Low*), is known as a Dominant Strategy Equilibrium.

Entry Game with a Weakly Dominant Strategy

- Another dominance concept is **weak dominance**, which refers to the case where a player can identify a strategy that is at least as good as any other strategy for all strategies the other player may choose and better than any other strategy for at least one strategy the other player may choose.
- The figure illustrates a game in which firm A has a weakly dominant strategy while firm B has neither a strictly dominant strategy nor a weakly dominant strategy.
- Firm A is an incumbent and firm B is a potential entrant. In the simultaneous version of the entry game, Firm B chooses whether or not to enter and Firm A plans whether to accommodate B's arrival in the event that B does enter or initiate a price war.
- To create the capability to fight a price war, A will have to take certain steps that will be irreversible before A knows B's decision. However, these steps will not impose any additional costs upon A in the event that B decides not to enter and A keeps the entire market to itself.

Figure 2. Entry game with a weakly dominant strategy

		Firm B's strategies	
		Entry	*No entry*
Firm A's strategies	**Accommodate**	**2**, *4*	**3**, *3*
	Fight	**1**, *1*	**3**, *3*

Entry game with a weakly dominant strategy

From A perspective

- If B selects No *Entry*, A does not need to execute the threatened price war. A's payoff of 3 is the same, regardless whether A had decided to *Accommodate* or *Fight*.
- If B selects *Entry*, *Accommodate* yields a payoff of 2 for A, while *Fight* yields a payoff of 1. If B selects *Entry*, A's best response is *Accommodate*.

From B perspective

- If A selects *Accommodate*, Entry yields a payoff of 4 for B, while No *Entry* yields a payoff of 3. If A selects *Accommodate*, B's best response is *Entry*.
- If A decides to *Fight*, *Entry* yields a payoff of 1 for B, while No *Entry* yields a payoff of 3. If A selects *Fight*, B's best response is No *Entry*.

From A perspective

- Firm A has no strictly dominant strategy, but *Accommodate* is weakly dominant: A prefers *Accommodate* if B selects *Entry*, but if B selects No *Entry* A is indifferent between *Accommodate* and *Fight*.

From B perspective

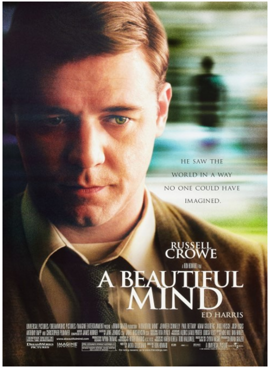
- Firm B, therefore, has neither any strictly dominant strategy nor any weakly dominant strategy.
- A might opt for the weakly dominant strategy of *Accommodate*, in which case B prefers *Entry*. Alternatively, A might decide to *Fight*, in which case B prefers No *Entry*.

It is hard to be certain which way this game will turn out.

Nash Equilibrium

Nash Equilibrium

- In a Nash Equilibrium, neither firm can improve its payoff by switching to a different strategy, assuming the strategy chosen by the other firm does not change.
- Nobel Prize winner in 1994 for the applications of game theory in Economics.
- Won 4 Oscars in 2001 with Best Actress in a Supporting Role Jennifer Connelly.



To read: https://www.economist.com/schools-brief/2016/08/20/prison-breakthrough

Nash Equilibrium in Figure 2

The game in Figure 2, has two Pure Strategy Nash Equilibria.

First (*Accommodate*, *Entry*) is a Nash Equilibrium:

- If B selects *Entry*, A's payoff would drop from 2 to 1 if A switches to *Fight*;
- If A selects *Accommodate*, B's payoff would drop from 4 to 3 if B switches to No *Entry*.

Likewise (*Fight*, No *Entry*) is a Nash Equilibrium:

- If B selects No *Entry*, A's payoff would remain at 3 if A switches to *Accommodate*;
- If A selects *Fight*, B's payoff would drop from 3 to 1 if B switches to *Entry*.

Both of the solutions (*Accommodate*, *Entry*) and (*Fight*, No *Entry*) satisfy the requirement for a Nash Equilbrium; that is, neither firm can improve its payoff, assuming the current strategy of the other firm is fixed.

Relationship between Dominant Strategy Equilibrium and Nash Equilibrium

- In the game shown in **Figure 1**, there is an exact correspondence between the Dominant Strategy Equilibrium and the (Pure Strategy) Nash Equilibrium.
- It can be shown that a Dominant Strategy Equilibrium in any game is always a Nash Equilibrium.
- If both players select their strictly dominant strategies, it is impossible for either to improve its own payoff by changing its strategy, given the current strategy of the other player.
- In games such as the one shown in **Figure 2**, however, there may exist one or more Nash Equilibria, although there are no strictly dominant strategies and no Dominant Strategy Equilibrium.

Figure 3. Payoff matrix for the advertising budgets of firms A and B

- In **Figure 3**, firms A and B must decide simultaneously their advertising expenditures.
- They have a choice between three levels of expenditure: *Low*, *Medium* or *High*.
- Both firms' payoffs from the advertising campaign depend on their own expenditure and on the expenditure of the other firm.

		Firm B's budget		
		Low	*Medium*	*High*
Firm A's budget	**Low**	**40** *40*	**35** *45*	**10** *25*
	Medium	**35** *35*	**45** *30*	**15** *20*
	High	**30** *25*	**25** *15*	**20** *30*

Payoff matrix for the advertising budgets of firms A and B

As before, consider firm A's choices:

- If B chooses *Low*, A's best response is *Low*.
- If B chooses *Medium*, A's best response is *Medium*.
- If B chooses *High*, A's best response is *High*.

Similarly, consider firm B's choices:

- If A chooses *Low*, B's best response is *Medium*.
- If A chooses *Medium*, B's best response is *Low*.
- If A chooses *High*, B's best response is *High*.

Relationship between Dominant Strategy Equilibrium and Nash Equilibrium

- There are no strictly dominant strategies, and no weakly dominant strategies, for either firm A or firm B. By inspection, however, it can be confirmed that (*High*, *High*) is a **Nash Equilibrium**.
- If B chooses *High*, then *High* is also A's best response; and if A chooses *High*, then *High* is also B's best response.
- Unfortunately, in **the absence of strictly dominant strategies, there is no simple decision-making procedure that will enable the two firms to reach the Nash Equilibrium easily**.
- If this solution is achieved by some means, however, it is stable in the sense that there is no incentive for either firm to depart from it, given the zero conjectural variation assumption.

Relationship between Dominant Strategy Equilibrium and Nash Equilibrium

- It is important to notice that firms A and B could both be better off by cooperating or agreeing to choose (*Low*, *Low*) in **Figure 3**, rather than remaining at the Nash Equilibrium of (High, High). In contrast to the Nash Equilibrium, however, this cooperative solution is unstable.
- If A chooses *Low*, B has an incentive to 'cheat' and choose *Medium* instead of *Low*. But if B chooses *Medium*, A would also prefer *Medium*; and then if A chooses *Medium*, B would prefer *Low*; and so on.
- The cooperative solution is vulnerable to defection by one or both of the firms and is likely to break down.

The Original Prisoner's Dilemma

The Prisoner's Dilemma

Situation:

- In the prisoner's dilemma game, two prisoners (Alan and Brian) have been caught committing a petty crime.

Rules:

- Each is held in a separate cell and cannot communicate with the other.
- Each is told that both are suspected of committing a more serious crime.

Options:

- If one of them confesses, he will get a 1-year sentence for cooperating while his accomplice will get a 10-year sentence for both crimes.
- If both confess to the more serious crime, each receives a 5-year sentence for both crimes.
- If neither confesses, each receives a 2-year sentence for the minor crime only.

Payoff Matrix for Alan and Brian

- **Strategies: Alan and Brian each have two possible actions**
  - Confess to the larger crime.
  - Deny having committed the larger crime.
- **Possible Outcomes:**
  - Both confess.
  - Both deny.
  - Alan confesses and Brian denies.
  - Brian confesses and Alan denies.
- **Payoffs:**
  - Each prisoner can work out what happens to him—can work out his payoff—in each of the four possible outcomes shown in **Figure 4**.

Figure 4. Payoff matrix for Alan and Brian: original prisoner's dilemma

		Brian's strategies	
		Not confess	*Confess*
Alan's strategies	**Not confess**	**–2** *–2*	**–10** *–1*
	Confess	**–1** *–10*	**–5** *–5*

Nash Equilibrium in Prisoner's Dilemma

- Alan's reasoning might be as follows: if Brian confesses I should confess because five years is better than ten years; and if Brian does not confess I should confess because one year is better than two years. Therefore, I will confess.
- Brian's reasoning is the same because the payoffs are symmetric between the two prisoners. Therefore both confess and both receive sentences of five years.
- But if they had been able to cooperate, they could have agreed not to confess and both would have received sentences of two years.
- Even acting independently, they might be able to reach a cooperative solution. Alan knows that if he does not confess, he receives a two-year sentence as long as Brian does the same.

Prisoner’s Dilemma

- However, Alan is worried because he knows there is a big incentive for Brian to ‘cheat’ on Alan by confessing. By doing so, Brian can earn the one-year sentence and leave Alan with a ten-year sentence!
- Brian is in a similar position: if he does not confess, he receives the two-year sentence as long as Alan also does not confess. However, Brian also knows there is a big incentive for Alan to cheat.
- The cooperative solution might be achievable, especially if Alan and Brian can trust one another not to cheat, but it is also unstable and liable to break down.

The Dilemma

- The dilemma arises as each prisoner contemplates the consequences of his decision and puts himself in the place of his accomplice.
- Each knows that it would be best if both denied.
- But each also knows that if he denies it is in the best interest of the other to confess.
- **The dilemma leads to the equilibrium of the game.**

A Bad Outcome

- For the prisoners, the equilibrium of the game is not the best outcome.
- If neither confesses, each gets a 2-year sentence.
- Can this better outcome be achieved?
- No, it can't because each prisoner can figure out that there is a best strategy for each of them.
- Each knows that it is not in his best interest to deny.

Figure 5: Production game with prisoner's dilemma structure

Firm B's strategies

Firm A's strategies		*Low*	*High*
	Low	**3** *3*	**1** *4*
	High	**4** *1*	**2** *2*

Production game with prisoner's dilemma structure

- *High* is a strictly dominant strategy for A and *High* is a strictly dominant strategy for B. Accordingly, it seems that A should select *High*and B should select *High*, in which case both firms earn a payoff of 2.
- As before, the Dominant Strategy Equilibrium (*High*, *High*) is also a Nash Equilibrium.
- Given that B selects *High*, if A switches from *High* to *Low*, A's payoff falls from 2 to 1; and given that A selects *High*, if B switches from *High* to *Low*, B's payoff also falls from 2 to 1. However, this time something appears to be wrong.
- If both firms had selected the other strategy (*Low*, *Low*), either by cooperating or perhaps by acting independently, both firms would have earned a superior payoff of 3 each, rather than their actual payoff of 2 each.

Conflict vs. Cooperation in Prisoner's Dilemma

Not all prisoner's dilemma games generate suboptimal outcomes, especially when the assumptions are relaxed.

- **First**, the optimal (cooperative) outcome might be achieved if there is good communication between the players. If firms meet frequently, they can exchange information and monitor each other's actions.
- **Second**, in practice, an important characteristic of any game is the length of the reaction lag: the time it takes for a player who has been deceived to retaliate. The longer the reaction lags, the greater the temptation for either player to act as an aggressor.
- **Third**, the dynamics of rivalry may also be relevant. Is the rivalry continuous or 'one-off'? If rivalry is continuous in a repeated game, players learn over time that cooperation is preferable to aggression.

Let's look at another example

Figure 6: Advertising game with no Pure Strategy Nash Equilibrium

		Firm B's strategies	
		Low	*High*
Firm A's strategies	**Low**	**1** *4*	**3** *2*
	High	**4** *1*	**0** *5*

Advertising Game with No Pure Strategy Nash Equilibrium

- Consider first the case of two firms that need to decide simultaneously their advertising budgets (**low or high**).
- As before, both firms' payoffs from the advertising campaign depend both on their own expenditure and on the other firm's expenditure.
- This is a **constant-sum game**. Whatever combination of strategies is chosen, the sum of the payoffs to both firms is 5.
- There is no strictly dominant strategy for either firm.

Figure 6: Advertising game with no Pure Strategy Nash Equilibrium

And from A's perspective:

- If B chooses Low, A's best response is **High**.
- If B chooses High, A's best response is **Low**.

And from B's perspective:

- If A chooses **Low**, B's best response is Low.
- If A chooses **High**, B's best response is High.

There is also no Pure Strategy Nash Equilibrium, because there is no pair of strategies from which neither firm would wish to defect if the other firm continues to follow the same strategy. A is in a difficult position.

- If A selects **Low**, B might select Low and A only earns a profit of 1. But, on the other hand, if A selects **High** and B selects High, A earns a profit of 0. Of course, B also faces a similar dilemma.

Mixed Strategies

Mixed Strategies

- In some games, **there is neither any Dominant Strategy Equilibrium nor any Pure Strategy Nash Equilibrium**.
- In others, there may be no Dominant Strategy Equilibrium but more than one Pure Strategy Nash Equilibrium.
- In such cases, it may be beneficial for the firms (or other players) to adopt what are known as **mixed strategies**.
- **A mixed strategy involves randomizing the choice between two or more options** with probabilities defined for each option.

Advertising game with no Pure Strategy Nash Equilibrium

- A possible solution lies in the concept of a mixed strategy, developed by von Neumann and Morgenstern (1944).
- A player follows a mixed strategy by choosing his action randomly, assigning fixed probabilities to the selection of each action.
- In contrast, previous examples have resulted in the choice of a **pure strategy** by both players.
- According to **the non-cooperative solution to the prisoner's dilemma game shown in Figure 5**, for example, A should only ever select **High** and B should only ever select **High**, because **High** and **High** are strictly dominant strategies.
- At the Mixed Strategy Nash Equilibrium, each player has the same expected payoff from either action and from a mixed strategy that assigns specific probabilities to both actions; and this expected payoff is unaffected by the mixed strategy selected by the other player.
- Therefore **neither player has any incentive to depart from his current mixed strategy, assuming the other player continues with his current mixed strategy.**

Battle of the Sexes Game

- Consider next the game represented by the payoff matrix shown in Figure 7, known as the battle of the sexes game.
- Arthur and Barbara are a couple with widely divergent preferences for an evening's live entertainment: Arthur prefers to watch football and Barbara prefers ballet.
- Despite their differences in taste, they both prefer each other's company to attending either event alone and both will go straight home if the other does not show up.
- Owing to a temporary mobile phone outage, they are unable to communicate with each other prior to the start of either event. Both must decide which event to attend without knowing the other's decision.

Figure 7: Battle of the Sexes strategies

		Barbara's strategies	
		Football	*Ballet*
Arthur's strategies	**Football**	**4**, *2*	**1**, *1*
	Ballet	**1**, *1*	**2**, *4*

Battle of the Sexes Payoff Matrix

- There are no strictly dominant strategies but both (Football, Football) and (Ballet, Ballet) are **Pure Strategy Nash Equilibria**.
- In respect of (Football, Football), if Barbara selects Football, Arthur would not wish to switch from Football (because his payoff would drop from 4 to 1); and if Arthur selects Football, Barbara would not wish to switch from Football (because her payoff would drop from 2 to 1).
- The same reasoning applies to (Ballet, Ballet).
- **Without communication, the couple faces a coordination problem in reaching either of these solutions.**
- The **selection of mixed strategies by both partners might resolve the dilemma**, but the Mixed Strategy Nash Equilibrium in this game is somewhat flawed.

Battle of the sexes game

Let Arthur assign a probability of x to the choice of Football, and a probability of $(1-x)$ to the choice of Ballet.

Barbara's expected payoffs (in terms of x) are as follows:

- If Barbara chooses Football, Barbara's possible payoffs are 2 (with a probability of x) and 1 (with a probability of $1-x$). Barbara's expected payoff is $2x + 1(1-x) = 1 + x$.
- If Barbara chooses Ballet, Barbara's possible payoffs are 1 (with a probability of x) and 4 (with a probability of $1-x$). Barbara's expected payoff is $1x + 4(1-x) = 4 - 3x$.

Let Barbara assign a probability of y to the choice of Football, and a probability of $(1-y)$ to the choice of Ballet. Arthur's expected payoffs (in terms of y) are as follows:

- If Arthur chooses Football, Arthur's possible payoffs are 4 (with a probability of y) and 1 (with a probability of $1-y$). Arthur's expected payoff is $4y + 1(1-y) = 1 + 3y$.
- If Arthur chooses Ballet, Arthur's possible payoffs are 1 (with a probability of y) and 2 (with a probability of $1-y$). Arthur's expected payoff is $1y + 2(1-y) = 2 - y$.

Battle of the sexes game

- Proceeding in the same way as before, **Arthur should choose x so as to minimise Barbara's expected payoff**. Arthur does so by selecting x such that Barbara's expected payoff is the same regardless of whether Barbara selects Football or Ballet, or any mixed strategy that combines Football and Ballet.
- Solving $1 + x = 4 - 3x$ yields $x = \frac{3}{4}$: Arthur should select Football with a probability of $x = \frac{3}{4}$, and Ballet with a probability of $(1 - x) = \frac{1}{4}$.
- Since the payoff matrix is symmetric, it is straightforward to verify that Barbara should select Football with a probability of $y = \frac{1}{4}$ and Ballet with a probability of $(1 - y) = \frac{3}{4}$.
- A troublesome feature of the Mixed Strategy Nash Equilibrium in the Battle of the sexes game is that the expected payoffs to both players are lower than the payoffs at either of the two Pure Strategy Nash Equilibria.

Battle of the sexes game: Arthur's expected payoff is

Arthur's expected payoff is calculated as follows:

$$\begin{aligned}
&P(\text{Arthur chooses Football}) \times P(\text{Barbara chooses Football}) \times 4 \\
&+ P(\text{Arthur chooses Football}) \times P(\text{Barbara chooses Ballet}) \times 1 \\
&+ P(\text{Arthur chooses Ballet}) \times P(\text{Barbara chooses Football}) \times 1 \\
&+ P(\text{Arthur chooses Ballet}) \times P(\text{Barbara chooses Ballet}) \times 2 \\
&= 4xy + x(1-y) + (1-x)y + 2(1-x)(1-y) = 1.75 \text{ when } x = \frac{3}{4} \text{ and } y = \frac{1}{4}.
\end{aligned}$$

- Likewise Barbara's expected payoff is 1.75; so the expected payoffs at the Mixed Strategy Nash Equilibrium are (1.75, 1.75).
- Both players would be better off at either of the two Pure Strategy Nash Equilibria, with payoffs of (4, 2) at (Football, Football) or (2, 4) at (Ballet, Ballet).
- As we have seen, however, the players encounter what is known **as a coordination problem in reaching either of these solutions**.
- **Example for firms**: The battle of the sexes game is relevant in describing the situation faced by two firms in deciding which of two alternative technological standards to adopt.

Sequential Games

Sequential Games

- In the games examined so far, the players act simultaneously and decide their strategies and actions before they know which strategies and actions have been chosen by their rivals.
- However, there are other games in which the players' decisions **follow a sequence**. One player makes a decision and the other player observes this decision before making a response.
- For a sequential game, it is convenient to map the choices facing the players in the form of **a game tree**.
- Now, suppose two breakfast cereal producers are both considering a new product launch. They each have a choice of launching one of two products: one product's appeal is '**crunchiness**' and the other's appeal is '**fruitiness**'. Assume the crunchy cereal is more popular with consumers than the fruity cereal.

Figure 8: Breakfast cereals game: strategic form representation

Firm B's strategies

Firm A's strategies		*Crunchy*	*Fruity*
	Crunchy	**3** *3*	**5** *4*
	Fruity	**4** *5*	**2** *2*

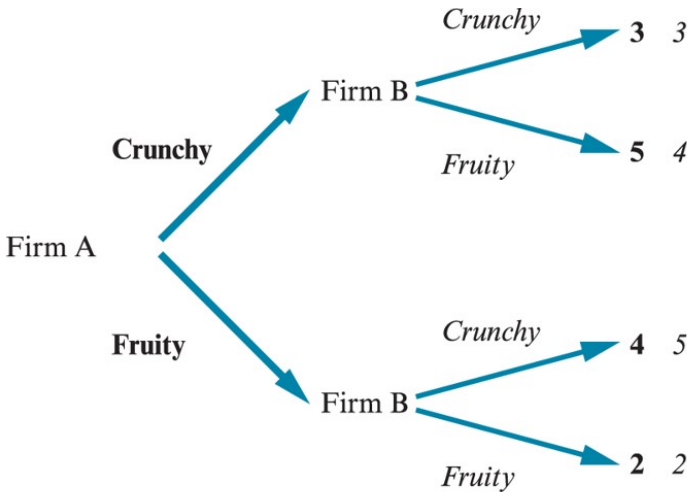
- The payoff structure is similar to the battle of the sexes game, with the exception that making different choices, rather than the same choices, is preferred by both firms.
- There **are no strictly dominant strategies**: if B produces Crunchy it is better for A to produce Fruity, but if B produces Fruity it is better for A to produce Crunchy. However, (Fruity, Crunchy) and (Crunchy, Fruity) are both Pure Strategy Nash Equilibria.
- In a sequential game, however, if A is the first to launch its new product and B then responds after having observed A's action, the outcome is different.

Figure 9 shows the game tree representation of the payoffs of the breakfast cereal game, also known as the extensive form representation. (The equivalent terminology for the payoff matrix used previously is the strategic form representation.) Consider A's decision:

- If A produces Crunchy, B's Fruity payoff of 4 exceeds B's Crunchy payoff of 3, so B will produce Fruity and A earns a payoff of 5.
- If A produces Fruity, B's Crunchy payoff of 5 exceeds B's Fruity payoff of 2, so B will produce Crunchy and A earns a payoff of 4.

A realises that whatever product A launches, the rational response of B is to launch the alternative product. A's best action is to produce Crunchy, and A earns the higher payoff of 5. B produces Fruity and earns the lower payoff of 4. At (Crunchy, Fruity), A ends up with the higher payoff, because A benefits from a first-mover advantage.

Figure 9: Breakfast cereals game: extensive form representation



Sequential games

- In many sequential games, the player who moves first gains an advantage, by influencing the shape of the game and forcing the other player to react to the first-mover's decision, rather than act in a way that is independent of the first-mover's presence.
- In the breakfast cereals game, from the symmetry of the payoff matrix it is obvious that if B were the first mover, the solution would be (Fruity, Crunchy), A would end up with the lower payoff of 4, and B would end up with the higher payoff of 5.
- Assuming the first mover is A, can B take any steps that might deliver the other outcome, in which the firms produce different products (Fruity, Crunchy) yielding payoffs of 4 to A and 5 to B?
- **Perhaps, prior to A's action, B could threaten to produce Crunchy regardless of A's decision. If A views B's threat as credible, A will calculate that by producing Crunchy, A will end up with a payoff of 3, but by producing Fruity A will achieve a payoff of 4.**
- According to this calculation, B's threat should steer A towards the (Fruity, Crunchy) outcome that B prefers. However, is B's threat to produce Crunchy truly credible?

Subgame Perfect Equilibrium (SPE)

- **A might calculate that if A produces Crunchy regardless, then B has no incentive to execute the threat.**
- Once A has taken the decision to produce Crunchy, the only payoffs relevant to B are those at the top of the game tree: 3 if B executes the threat and produces Crunchy, and 4 if B reneges on the threat and produces Fruity. Faced with these alternatives, **B reneges on the threat, and A's favoured outcome, (Crunchy, Fruity), is achieved.**
- In the breakfast cereals game with A as first mover, (Crunchy, Fruity) is a Subgame Perfect Equilibrium (SPE). Any SPE is a Nash Equilibrium in the strategic form representation, but not all Nash Equilibria are SPEs in the extensive form representation.
- **SPEs exclude any Nash Equilibrium, such as (Fruity, Crunchy), whose attainment would require either player to make non-credible threats that they would not execute if/when the time comes to do so.**
- In a sequential game the **classification of an SPE depends upon the order of play**: it is obvious that if B held the first-mover advantage and the payoffs were the same, (Fruity, Crunchy) would be an SPE, but (Crunchy, Fruity) would not be an SPE.

Repeated Games

Repeated Games

- In the previous discussion of single-period prisoner's dilemma and other games, it is assumed that the game is played only once.
- However, some games may be played repeatedly by the same players. In this case, it is more likely that cooperative behaviour will evolve as the two firms observe and learn from each other's behaviour.
- In a **repeated or multiple-period game**, each firm may attempt to influence its rival's behaviour by sending signals that **promise to reward cooperative behaviour and threaten to punish non-cooperative behaviour**.

Let's look again at Figure 5: prisoner's dilemma example

		Firm B's strategies	
		Low	*High*
Firm A's strategies	**Low**	**3** *3*	**1** *4*
	High	**4** *1*	**2** *2*

Tit-for-Tat Strategy in Repeated Games

- (**High, High**) is **the (suboptimal) non-cooperative solution**, which produces payoffs of 2 for firms A and B; and (**Low, Low**) is **the (optimal) cooperative choice, which produces payoffs of 3 for both firms.**
- In a single-period game, in which the firms act independently, **High** and **High** are the dominant strategies, and the **non-cooperative outcome is likely to occur**.

However, suppose the game is to be repeated over an indefinite number of periods.

- Firm A could adopt the following strategy, known as **tit-for-tat**, in an attempt to encourage firm B to always **select the cooperative choice**.

In this case, we would have:

- In period 1, A chooses Low.
- If B chose Low in period $t-1$, in period t $(t>1)$, A chooses Low.
- If B chose High in period $t-1$, in period t $(t>1)$, A chooses High.

Now, it's time to play and test!

1. To login, go to https://classEx.uni-passau.de
2. Select your institution: Warwick University (United Kingdom)
3. Choose your account name: Game theory
4. Participant
5. Enter your password: warwick2022

Practise Exercise

- Firms A and B are simultaneously planning an advertising campaign for the launch of a new product that will have similar characteristics when produced by either firm. Firm A is an established company, and is well known to consumers.
- Firm B is a relatively unknown recent entrant. Both firms have to decide whether to focus their advertising campaigns primarily on traditional media (TV and print), or on social media.
- If both firms decide to advertise using the same media, A's reputational advantage will tend to dominate the campaign, and B will struggle to establish a foothold in the market.
- If both firms decide to advertise using different media, B's campaign will succeed in enabling B to establish a foothold. The payoff matrix, expressed in terms of expected market share after the campaign, is as follows:

Practise Exercise

		B's campaign focused on:	
		Traditional media	*Social media*
A's campaign focused on:	**Traditional media**	**90**, *10*	**60**, *40*
	Social media	**50**, *50*	**80**, *20*

- Does this game have any Pure Strategy Nash Equilibria?
- Identify the Mixed Strategy Nash Equilibrium in the simultaneous game.
- Suppose B can observe A's decision before B decides how to focus its own campaign. Write down the extensive form representation of the sequential game.
- What is the likely outcome of the sequential game? Is there a first-mover advantage or a second-mover advantage?

1. Pure Strategy Nash Equilibria

To determine pure strategy Nash equilibria, analyze the payoff matrix:

	B: Traditional Media	**B: Social Media**
A: Traditional Media	(90, 10)	(60, 40)
A: Social Media	(50, 50)	(80, 20)

Best responses:

- Firm B: Traditional if A: Social; Social if A: Traditional
- Firm A: Traditional if B: Traditional; Social if B: Social

No pure strategy Nash equilibria.

- If Firm A chooses Traditional Media:

$$E_A(T) = 90q + 60(1-q) = 60 + 30q$$

- If Firm A chooses Social Media:

$$E_A(S) = 50q + 80(1-q) = 80 - 30q$$

For indifference:

$$60 + 30q = 80 - 30q \implies q = \frac{2}{9}$$

- If Firm B chooses Traditional Media:

$$E_B(T) = 10p + 50(1 - p) = 50 - 40p$$

- If Firm B chooses Social Media:

$$E_B(S) = 40p + 20(1 - p) = 20 + 20p$$

For indifference:

$$50 - 40p = 20 + 20p \implies p = \frac{1}{2}$$

Firm A's Expected Payoff

Using the mixed strategy probabilities:

- $p = \frac{1}{2}$ for Traditional Media
- $1 - p = \frac{1}{2}$ for Social Media

Firm A's expected payoff when Firm B uses $q = \frac{2}{9}$ for Traditional Media and $1 - q = \frac{7}{9}$ for Social Media:

$$E_A = p \cdot (90q + 60(1 - q)) + (1 - p) \cdot (50q + 80(1 - q))$$

Substituting the values:

$$E_A = \frac{1}{2} \cdot (90 \cdot \frac{2}{9} + 60 \cdot \frac{7}{9}) + \frac{1}{2} \cdot (50 \cdot \frac{2}{9} + 80 \cdot \frac{7}{9})$$

Simplifying each term:

$$E_A = \frac{1}{2} \cdot (20 + 46.67) + \frac{1}{2} \cdot (11.11 + 62.22)$$

$$E_A = \frac{1}{2} \cdot 66.67 + \frac{1}{2} \cdot 73.33$$

$$E_A = 33.33 + 36.67 = 70$$

Firm B's Expected Payoff

Using the mixed strategy probabilities:

- $q = \frac{2}{9}$ for Traditional Media
- $1 - q = \frac{7}{9}$ for Social Media

Firm B's expected payoff when Firm A uses $p = \frac{1}{2}$ for Traditional Media and $1 - p = \frac{1}{2}$ for Social Media:

$$E_B = q \cdot (10p + 50(1 - p)) + (1 - q) \cdot (40p + 20(1 - p))$$

Substituting the values:

$$E_B = \frac{2}{9} \cdot (10 \cdot \frac{1}{2} + 50 \cdot \frac{1}{2}) + \frac{7}{9} \cdot (40 \cdot \frac{1}{2} + 20 \cdot \frac{1}{2})$$

Simplifying each term:

$$E_B = \frac{2}{9} \cdot 30 + \frac{7}{9} \cdot 30$$

$$E_B = \frac{30}{9} \cdot 9$$

$$E_B = 30$$

- Firm A's mixed strategy:

$$p = \frac{1}{2}$$

- Firm B's mixed strategy:

$$q = \frac{2}{9}$$

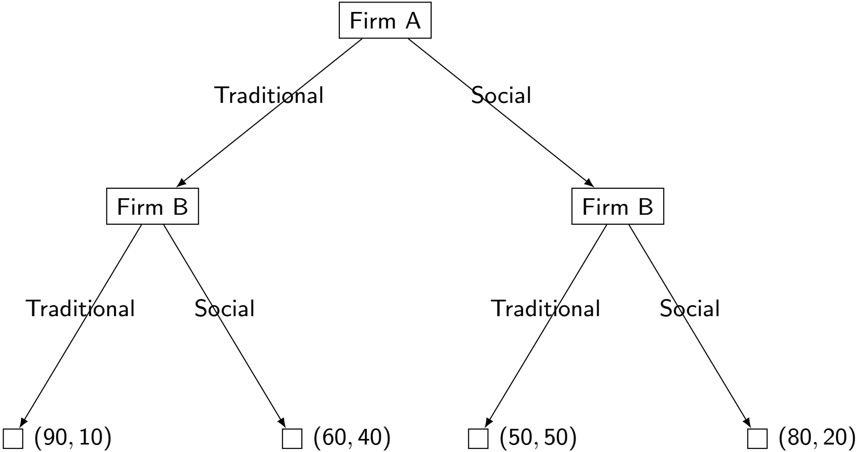
The mixed strategy Nash equilibrium is:

- Firm A: Traditional Media with probability $\frac{1}{2}$, Social Media with probability $\frac{1}{2}$.
- Firm B: Traditional Media with probability $\frac{2}{9}$, Social Media with probability $\frac{7}{9}$.

- The Mixed Strategy Nash Equilibrium expected payoffs are $(E_A; E_B) = (70; 30)$.

3. Extensive Form Representation

If Firm B observes Firm A's decision, the game tree is:



4. The outcome of the Sequential Game

Backward induction:

- If Firm A chooses Traditional Media, Firm B chooses Social Media ($40 > 10$).
- If Firm A chooses Social Media, Firm B chooses Traditional Media ($50 > 20$).

Firm A's best response is Traditional Media, leading to (60, 40).

First-mover advantage lies with Firm A.