International Business and Finance Seminar 3

Cholwoo Kim

Road map

- In the first two seminars, we've mainly covered
  - Bank essentials: balance sheet; how banks make money; measurements of banks' performance
  - Bond essentials: interest rate; compounding; present value and future value

- Today's seminar will be a bridge between the first two seminars and the next seminar when we will talk extensively about derivatives.

Geometric series

- *Geometric sequence*: A sequence of numbers where each term (apart from the first term) can be obtained by multiplying the previous term by a fixed non-zero number, *r*.

$$a, ar, ar^2, ar^3, \dots, ar^{n-1}, \dots$$

where *r* is the common ratio.

- *Geometric series*: The sum of the terms (entirely or partially) of a geometric sequence

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{i=0}^{n-1} ar^i$$

where S_n is the geometric series for *n* term.

How to derive S_n

- Begin with S_n

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{i=0}^{n-1} ar^i \qquad (1)$$

- Multiplying S_n by the common ratio r, we have:

$$S_n \times r = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^n \qquad (2)$$

- Subtracting (2) from (1), we have:

$$S_n - S_n \times r = a - ar^n$$

Or

$$S_n(1-r) = a(1-r^n) \overset{r\neq 1}{\Longrightarrow} S_n = \frac{a(1-r^n)}{(1-r)}$$

- If $|r| < 1$ and n is increasing infinitely, then we have:

$$\lim_{n\to\infty} S_n = \frac{a}{1-r}$$

▸ The PV of a perpetuity

- An asset with a stream of cash flow each year with no end
- Case 1 : The PV of a perpetuity with a constant stream of cash flow

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t} = \frac{C}{r}$$

where C is a fixed payment.

- Case 2: The PV of a perpetuity with a stream of cash flow growing at a constant rate g

$$PV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots$$

$$= \frac{C_1}{(1+r)} + \frac{C_1(1+g)}{(1+r)^2} + \frac{C_1(1+g)^2}{(1+r)^3} + \cdots$$

$$= \sum_{i=1}^{\infty} \frac{C_1(1+g)^{i-1}}{(1+r)^i} = \frac{C_1}{r-g}$$

The PV of a coupon bond

- Case 1: Finite maturity date

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^t} + \frac{Face\ Value}{(1+r)^t}$$

$$= \sum_{i=1}^{t} \frac{C}{(1+r)^i} + \frac{Face\ Value}{(1+r)^t} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^t}\right] + \frac{Face\ Value}{(1+r)^t}$$

- Case 2: No finite maturity date

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots = \sum_{i=1}^{\infty} \frac{C}{(1+r)^i} = \frac{C}{r}$$

Applications

Net present value

- A measure of "is it a good deal?"
- NPV is essentially all your future earnings discounted to today, minus all costs discounted to today
- Positive NPV usually indicates good investment, because — well, why not?

- Case 1: Just one investment (I_0) in the beginning:

$$NPV = \sum_{i=1}^{t} \frac{C_i}{(1+r)^i} - I_0 > 0$$

- Case 2: Series of investment during the life-time of the project:

$$NPV = \sum_{i=1}^{t} \frac{C_i}{(1+r)^i} - \sum_{j=0}^{m} \frac{I_j}{(1+r)^j} > 0 \quad (m \leq t)$$

Questions

Question 1: It costs a risk neutral firm £800 to set up a factory (fixed cost). The factory can produce one unit of output per year forever. The current price of a unit of output is £100.

The product price is currently uncertain. In the next period the price will increase or decrease by 50% (with equal probability) and remain fixed forever at the level it has reached. The interest rate is 10%.

According to NPV, should the firm invest now?

Question 2: In 2019, Daimler Chrysler had just paid a dividend of €2 per share on its equity. The dividends are expected to grow at a constant rate of 5% per year indefinitely. If investors require an 11% return on the company's equity,

a. What is the current price?
b. What will be the price in 2022?
c. What will be the price in 2034?

Questions

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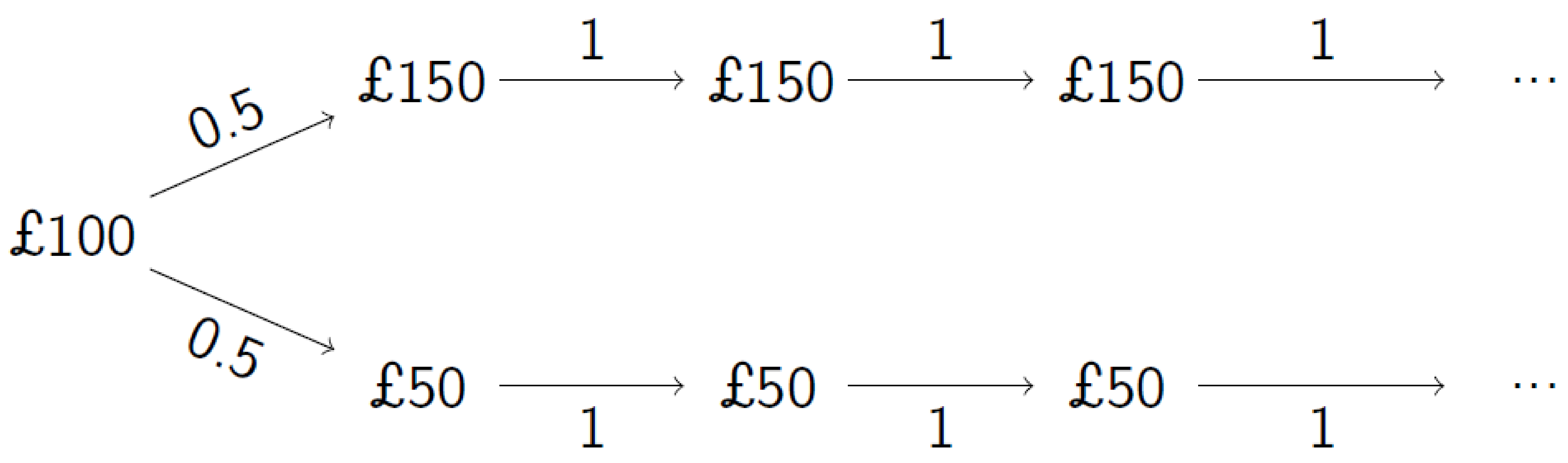
The expected cash flow:



The expected future price of output at any time is:

$$0.5 \times £150 + 0.5 \times £50 = £100$$

Questions

Now let us compute the NPV:

$$NPV = \overbrace{£100}^{\text{today's income}} + \overbrace{\sum_{t=1}^{\infty} \frac{£100}{(1+0.1)^t}}^{\text{expected discounted future cash flow}} - \overbrace{£800}^{\text{investment cost}}$$

$$= \underbrace{\sum_{t=0}^{\infty} \frac{£100}{(1+0.1)^t}}_{\text{infinite geometric series}} - £800$$

We can use the formula for infinite geometric series

$$NPV = \frac{1+0.1}{0.1} \times £100 - £800$$

$$= £1100 - £800$$

$$= £300 > 0$$

Thus, the firm should invest now.

Questions

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Derivation of the constant growth rate DDM

$$P_0 = \frac{D_1}{1+\overline{r}} + \frac{D_1(1+g)}{(1+\overline{r})^2} + \frac{D_1(1+g)^2}{(1+\overline{r})^3} + \cdots$$

$$= \frac{D_1}{1+\overline{r}}\left[1 + \frac{1+g}{1+\overline{r}} + \left(\frac{1+g}{1+\overline{r}}\right)^2 + \cdots\right]$$

$$= \frac{D_1}{1+\overline{r}}\left[\frac{1}{1-\left(\frac{1+g}{1+\overline{r}}\right)}\right]$$

(Infinite geometric series)

$$= \frac{D_1}{1+\overline{r}}\left[\frac{1+\overline{r}}{\overline{r}-g}\right]$$

$$= \frac{D_1}{\overline{r}-g}$$

Questions

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a. Current price at a constant growth of 5 per cent per year indefinitely

$P = Div_1 / (R - g)$, whereas $Div_1 = D_0 (1+g)$ i.e. $Div_1 = €2 (1.05) = €2.1$
$P = €2.1 / (0.11 - 0.05) = €35$

b. Price in 3 years: $P = P_0(1+g)^3 = €35(1.05)^3 = €40.52$

c. Price in 15 years: $P = P_0(1+g)^{15} = €35(1.05)^{15} = €72.76$