International Business and Finance
Seminar 4

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Exercise on NPV

- You can decide today to invest in *a machine that costs £1600*, paid regardless of the state of nature *at the end of a year*
- At the end of year, the machine produces *one unit of product*, which is worth £300 or £100, with *50-50 probability*
- The interest rate is 10%
- What is the NPV today if the machine can be only used for one period?
- What is the NPV today if the machine continues to produce forever and the price level stays the same forever?

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$$\text{NPV(one period)} = \frac{(-1600 + 0.5^{*}300 + 0.5^{*}100)}{1.1} = -1272.7$$

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$$NPV_1 = -1,600 + [0.5(300) + 0.5(100)] + \sum_{t=1}^{\infty} \frac{0.5(300) + 0.5(100)}{(1+0.1)^t}$$

$$= -1,600 + 200 + \frac{200}{0.1} = 600$$

$$NPV_0 = 600/1.1 = 545.5.$$

Terminology Recap

- Option contracts: the right, not the obligation, to buy or sell something at some price.
  - Call option: the right to **buy** at the strike price
  - Put option: the right to **sell** at the strike price
  - Strike price: the price that you can buy or sell the stock
  - American options: can be exercised at any time between the date of purchase and the expiration date.
  - European options: can only be exercised at the end of their lives on their expiration date.

Options

- **How do option holders decide to exercise their options?**
- The payoff of a call option

$$C = \max(S - K, 0)$$

where S : Stock price, K : Strike price, C: Call option payoff

- The payoff of a put option

$$P = \max(K - S, 0)$$

where P: Put option payoff

- Suppose John owns a **put option** on Dell stock with a strike price of $12 that expires today. If a Dell stock is trading at $13 currently, does he exercise the option?

- Suppose Jane owns a **call option** on Dell stock with a strike price of $12 that expires today. If a Dell stock is trading at $13 currently, does she exercise the option?

Options

Put-call parity

- The Put-Call Parity defines **a relationship between the price of a European call option and a European put option** with an identical strike price and expiration date, and the underlying asset.

- The relationship is given by:

$$\underbrace{\text{Stock Price} + \text{Put Price}}_{\text{Buy a stock and a put option with a strike price, K}} = \underbrace{\text{Call Price} + \text{Present Value of the strike price (K)}}_{\text{Buy a call option and a risk-free asset that pays K at expiration}}$$

$$S + P = C + PV(K)$$

❑ Suppose Nokia shares are selling for £10.95. A three-month call option with a £10.95 strike price goes for £0.35. The risk-free rate is 0.5% per month. What is the value of a 3-month put option with a £10.95 strike price?

Answer: Using the put-call parity equation we have

$$P = -S + C + PV(K) \Longrightarrow P = -£10.95 + £0.35 + \frac{£10.95}{1.005^3} = £0.1873$$

Questions

Question 1: Answer the following questions.

a) Suppose that the stock price and the prices of **at-the-money put and call options** are given by S = $100, P = $10, and C =$15. What must be the **one-year interest rate**?

b) If the one-year, risk-free interest rate is **4%**, is there an arbitrage opportunity? If so, what would you do to obtain risk-less profit?

Question 2: In mid-February **2016**, European-style options on the **S&P 100** index (OEX) expiring in December **2017** were priced as follows:

Dec 2017 OEX Index Options

Strike Price	Call Price	Put Price
840	**88.00**	
860	**76.30**	**102.21**
880		**111.56**

Given an interest rate of **0.40%** for a December **2017** maturity (**22** months in the future), use put-call parity (with dividends) to determine:

a) The price of a December **2017** OEX put option with a strike price of **840**.
b) The price of a December **2017** OEX call option with a strike price of **880**.

Questions

Question 1: Answer the following questions.

a) Suppose that the stock price and the prices of **at-the-money put and call options** are given by S = $100, P = $10, and C =$15. What must be the **one-year interest rate**?

What we have:

$$S = 100, P = 10, C = 15, K = 100 \text{ (at-the-money)}$$

The Put-Call Parity implies:

$$\begin{aligned} S - PV(K) &= C - P \\ 100 - \frac{100}{1+r} &= 15 - 10 \\ r &= 0.053 = 5.3\% \end{aligned}$$

Questions

Question 1: Answer the following questions.

b) If the one-year, risk-free interest rate is **4%**, is there an arbitrage opportunity? If so, what would you do to obtain risk-less profit?

There is an arbitrage opportunity as the actual risk-free interest rate is different from the one-year interest rate that would prevent an arbitrage.

If $r = 4\%$, then an investor could make arbitrage profits by:

1. Borrowing funds at 4% risk-free interest rate
2. Investing the borrowed funds in a synthetic bond with a 5.3% rate of return:
   - From put-call parity $(PV(K) = S + P - C)$, we know that the synthetic bond consists of 1) a share of stock 2) a European put option 3) a short position in a European call option.
   - The synthetic bond would cost \$95 $(PV(K) = S + P - C = 100 + 10 - 15 = 95)$ and would pay off \$100 at maturity in one year.
   - The investor would repay the principal and the interest on \$95, which is equal to $95 \times (1 + 0.04) = \$98.8$.
   - The investor would make an arbitrage profit of \$1.2 $(= \$100 - \$98.8)$ per bond a year.

Questions

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a) From put-call parity:

$$S - PV(Div) = C - P + PV(K)$$

From the 860 calls and puts:

$$S - PV(Div) = 76.30 - 102.21 + \frac{860}{1.004^{22/12}} = 827.82$$

Therefore, for the 840 put:

$$P = C + PV(K) - (S - PV(Div)) = 88.00 + \frac{840}{1.004^{22/12}} - 827.82 = 94.05$$

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a) The price of a December **2017** OEX put option with a strike price of **840**.
b) The price of a December **2017** OEX call option with a strike price of **880**.

b) And for the 880 call:

$$C = P - PV(K) + (S - PV(Div)) = 111.56 - \frac{880}{1.004^{22/12}} + 827.82 = 65.80$$