International Business and Finance
Week 2 Seminar 1

Cholwoo Kim

Pricing an option

- When pricing options or derivatives we rely on the closely related principle: law of one price.

- **Law of One Price (LOP)**: LOP means that any two portfolios that generate the same payoff must have the same price.

- We will derive the price of an option based on LOP in two ways:

- **Replicating portfolio approach**: The replicating portfolio approach involves replicating the payoff of an option using a portfolio of stocks and **bonds (risk-free assets)**.

- **Risk-neutral probability approach** (Considering an equilibrium with risk-neutral agents):
  - **Find the probability** that will make the stock's expected return the same as the risk-free rate.
  - **The value of the option is the expected payoff of the option** (based on the above probabilities) discounted back using the risk-free rate.

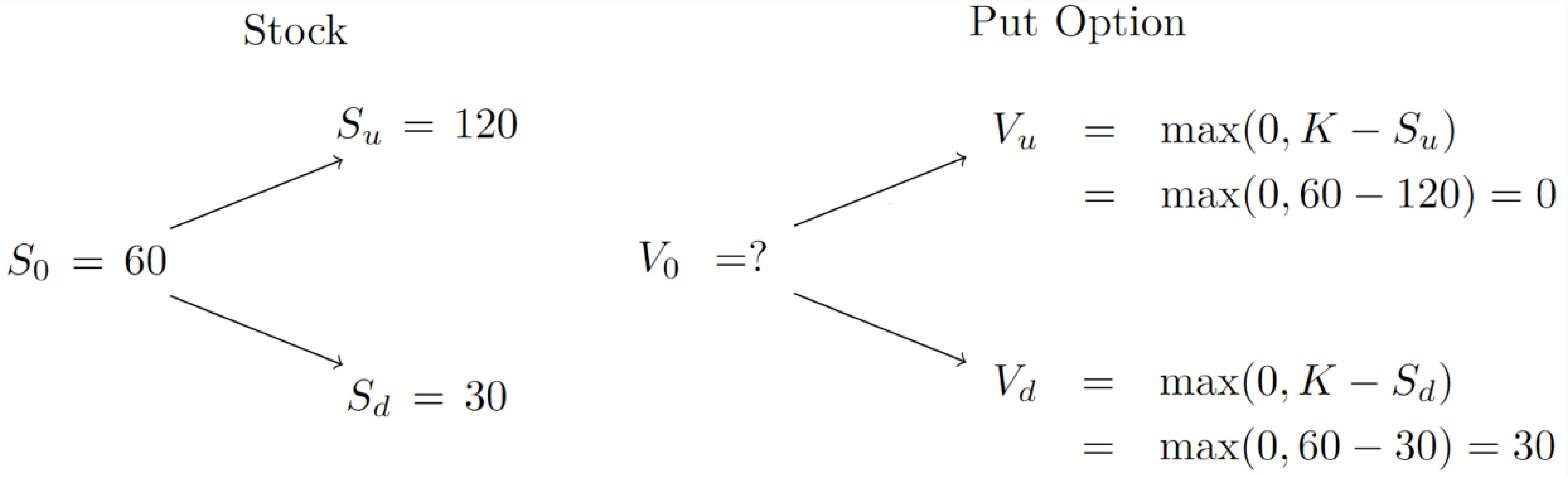
Example

This question asks you to use the binomial model to compute the price of a European put option on Meta stock with a strike price of $60. The current price of Meta stock is $60 per share, and the price will either increase by 100% or decrease by 50% on the exercise date of the option. The interest rate is 10%. Meta stock does not pay any dividends.

Using the replicating portfolio approach, what portfolio of Meta stock and borrowing or lending will replicate the change in the value of the option? What is the theoretical value of the put option described above?

- What we know:
  - Current stock price: S_0 = 60
  - Strike price: K = 60
  - S_u = 120
  - S_d = 30
  - Risk-free rate: r_f = 0.1
  - Put options values: V_u = 0 and V_d = 30

Example



Construct a portfolio of Δ shares and B:

$$\Delta S_u + (1+r)\, B = V_u$$

$$\Delta S_d + (1+r)\, B = V_d$$

By subtracting the second equation from the first one, we obtain Δ (the share of the Meta stock):

$$\Delta S_u - \Delta S_d = V_u - V_d$$

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{0 - 30}{120 - 30} = -\frac{1}{3}$$

Example

We can now retrieve the quantity of bonds B (or borrowing/lending):

$$\begin{aligned} \Delta S_u + (1+r)\,B &= V_u \\ \left(-\frac{1}{3}\right) \times 120 + 1.1 \times B &= 0 \\ B &= \frac{40}{1.1} = 36.36 \end{aligned}$$

The value of the option is the cost of this position at t=0:

$$\begin{aligned} V_0 &= \Delta S_0 + B \\ &= \left(-\frac{1}{3}\right) \times 60 + 36.36 = 16.36 \end{aligned}$$

Questions

A current stock price is **£100**. In the next period, it will be either **£120 or £80** with equal probabilities. The risk-free rate is **5%**.

1) Find the fair price of **a call option** with a strike of **K=£100** based on
   i) Replicating portfolio approach
   ii) Risk-neutral probability approach

For the remainder of this question suppose that the borrowing and lending rates differ, and that the **borrowing rate** is **6%** and the **lending rate** is **4%**.

2) If no options currently trade on this stock, is there a way to create a synthetic call option with identical payoffs to the call option described? If there is a way, how would you do it?

3) Redo 2) for a short position in the call option.

Questions

A current stock price is **£100**. In the next period, it will be either **£120 or £80** with equal probabilities. The risk-free rate is **5%**.

1) Find the fair price of **a call option** with a strike of **K=£100** based on
    i) Replicating portfolio approach

Stock

$S_0 = 100$ → $S_u = 120$

$S_0 = 100$ → $S_d = 80$

Option

$V_0 = ?$ → $V_u = \max(0, S_u - \text{K}) = \max(0, 120 - 100) = 20$

$V_0 = ?$ → $V_d = \max(0, S_d - \text{K}) = \max(0, 80 - 100) = 0$

Construct a portfolio of Δ shares and *B*:

$$\Delta S_u + (1 + r)B = V_u$$

$$\Delta S_d + (1 + r)B = V_d$$

By subtracting the second equation from the first one, we obtain Δ (the share of the stock):

$$\Delta S_u - \Delta S_d = V_u - V_d$$

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{20 - 0}{120 - 80} = \frac{1}{2}$$

Questions

A current stock price is **£100**. In the next period, it will be either **£120 or £80** with equal probabilities. The risk-free rate is **5%**.

1) Find the fair price of **a call option** with a strike of **K=£100** based on
   i) Replicating portfolio approach

We can now retrieve the quantity of the risk-free asset, *B*

$$\Delta S_u + (1+r)B = 20$$

$$\frac{1}{2} \times 120 + (1+0.05)B = 20$$

$$B = -\frac{40}{1.05} = -38.095$$

The value of the option is then given by:

$$V_0 = \Delta S_0 + B = \frac{1}{2} \times 100 - \frac{40}{1.05} = 11.905$$

Questions

A current stock price is **£100**. In the next period, it will be either **£120 or £80** with equal probabilities. The risk-free rate is **5%**.

1) Find the fair price of **a call option** with a strike of **K=£100** based on

ii) Risk-neutral probability approach

$$\underbrace{(1+r)S_0}_{\text{Return of a risk-free asset}} = \underbrace{pS_u + (1-p)\,S_d}_{\text{Stock's expected return}}$$

$$S_0 = \frac{pS_u + (1-p)\,S_d}{1+r} = \frac{puS_0 + (1-p)\,dS_0}{1+r}$$

Canceling out S_0 on both sides, we have:

$$1 = \frac{pu + (1-p)\,d}{1+r}$$

$$p = \frac{(1+r)-d}{u-d} = \frac{(1+0.05)-0.8}{1.2-0.8} = 0.625$$

We can now find the value of the put option:

$$V_0 = \frac{pV_u + (1-p)\,V_d}{1+r} = \frac{0.625 \times 20 + 0.375 \times 0}{1+0.05} = 11.905$$

Questions

For the remainder of this question suppose that the borrowing and lending rates differ, and that the **borrowing rate** is **6%** and the **lending rate** is **4%**.

2) If no options currently trade on this stock, is there a way to create **a synthetic call option** with identical payoffs to the call option described? If there is a way, how would you do it?

The replicating portfolio entails borrowing *B* at 6%,

$$120\Delta + (1 + 0.06)B = 20$$

$$80\Delta + (1 + 0.06)B = 0$$

Then, we have:

$$\Delta = \frac{1}{2}, B = -\frac{40}{1.06}$$

Note that since B is negative this is consistent with the borrowing at 6%.

The cost is then given by

$$V_0 = \Delta S_0 + B = \frac{1}{2} \times 100 - \frac{40}{1.06} = 12.264$$

Questions

For the remainder of this question suppose that the borrowing and lending rates differ, and that the **borrowing rate** is **6%** and the **lending rate** is **4%**.

3) Redo 2) for a short position in the call option.

For the short position, we consider lending *B* at 4%.

$$120\Delta + (1 + 0.04)B = -20$$

$$80\Delta + (1 + 0.04)B = 0$$

Then, we have:

$$\Delta = -\frac{1}{2}, B = \frac{40}{1.04}$$

Note that since B is positive this is consistent with the 4% lending rate.

The payoff from synthetic short is

$$-[\Delta S_0 + B] = -\left[-\frac{1}{2} \times 100 + \frac{40}{1.04}\right] = -[-11.538] = 11.538$$