International Business and Finance
Week 2 Seminar 1

Junxi Liu

2024

Terminology Recap

- Option contracts: the right, not the obligation, to buy or sell something at some price.
  - Call option: the right to buy at the strike price
  - Put option: the right to sell at the strike price
  - American options: can be exercised at any time between the date of purchase and the expiration date.
  - European options: can only be exercised at the end of their lives on their expiration date.
  - In-the-money: Describes an option whose value, if immediately exercised, would be positive

Options

- Put-call parity

Current Stock Price = Call Option Contract Price - Put Option Contract Price + Present value of strike price

$$S = C - P + PV(K)$$

- If the stock pays a dividend, put-call parity becomes

$$S = C - P + PV(K) + PV(Div)$$

Arbitrage

Question 8: Answer the following questions:

a) Suppose that $\mathbf{S=£100}$, $\mathbf{P=£10}$, and $\mathbf{C=£15}$. What must be the one-year interest rate?

b) If the one-year, risk-free interest rate is 4%, is there an arbitrage opportunity? If so, what would you do to obtain risk-less profit?

Question 8:

a) Assuming the option (put or call) is at-the-money (i.e. its exercise price is equal to the current stock price), then K=£100.
Using the put-call parity:

$$S - PV(K) = C - P \Longrightarrow 100 - \frac{100}{1+r} = 15 - 10 \Longrightarrow r = 0.053 = 5.3\%$$

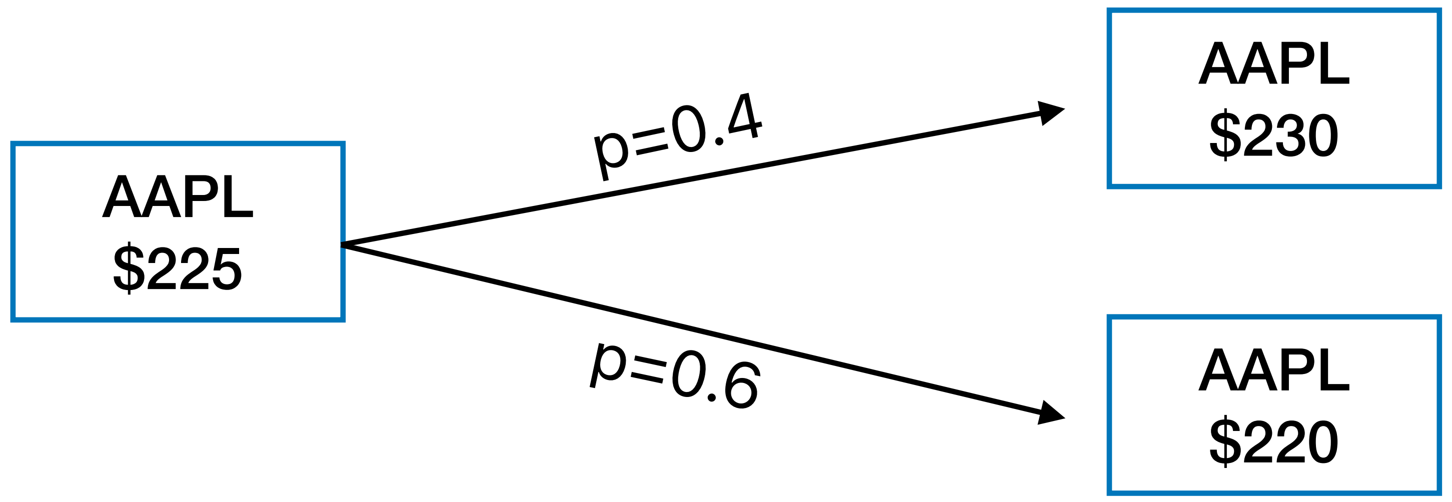
b) If r=4%, then one could make risk-less arbitrage profit by i) borrowing at 4% and ii) investing in a synthetic one-year bond with 5.3% return.

Using the put-call parity we know that $PV(K) = S + P - C$. Therefore, a synthetic one-year bond consists of a share of stock, a European put option, and a short position in a European call option. This synthetic bond would cost $PV(K) = S + P - C = 100 + 10 - 15 = £95$, and its payoff is £100 at maturity in one-year.

The principal and interest on the £95 synthetic bond would be $£95 \times 1.04 = £98.8$. So, there would be a pure risk-less profit of £1.2=£100-£98.8 per bond a year from now.

Pricing an Option

- More often, we are essentially interested in what's the "value added" for an option, given risks
- Each possible scenario could be seen as a "possibility". For simplicity, let's assume that there are only 2 possibilities: in a week, going up to 230 or going down to 220 from 225.
  - Using calculus and integration, this could actually easily be generalized
- What should be the value of a call option with a strike price of 228 that expires in a week?



Expected payoff: (230-228)*0.4+0*0.6 = 0.8

Pricing an Option

- In reality, rarely do we know the objective probability of any move. Rather, we might be able to observe some correlating securities or other already-priced financial instruments.
- If we know the risk-free interest rate, we can create a risk-neutral portfolio, by leveraging two assets, to make a two-equation-two-unknowns problem
- More specifically,
  - by holding a certain amount of the two assets, you can replicate an option's return in both the good state and the bad state;
  - after knowing the number for the risky project and riskless asset you need, the option is just a linear combination of the two assets you know – the risky asset and the risk-free asset.
  - therefore, you will be able to price the option.
- If the created portfolio is risk-neutral, then we can confidently use the risk-free interest rate to discount between different periods

Replicating Portfolios

- Essentially a mathematical problem
  - With two variables, you can solve a system of equations with two unknowns
  - With two risk products, you can create a portfolio

- Stock price today: $100
- Future stock prices:
  - Up-move: $110
  - Down-move: $95
- Call option strike: $102
- Risk-free rate: 5%

1. **Equations**:
   - Up-move: $\Delta \times \$110 + 1.05B = \$8$
   - Down-move: $\Delta \times \$95 + 1.05B = \$0$

   Option's payoff

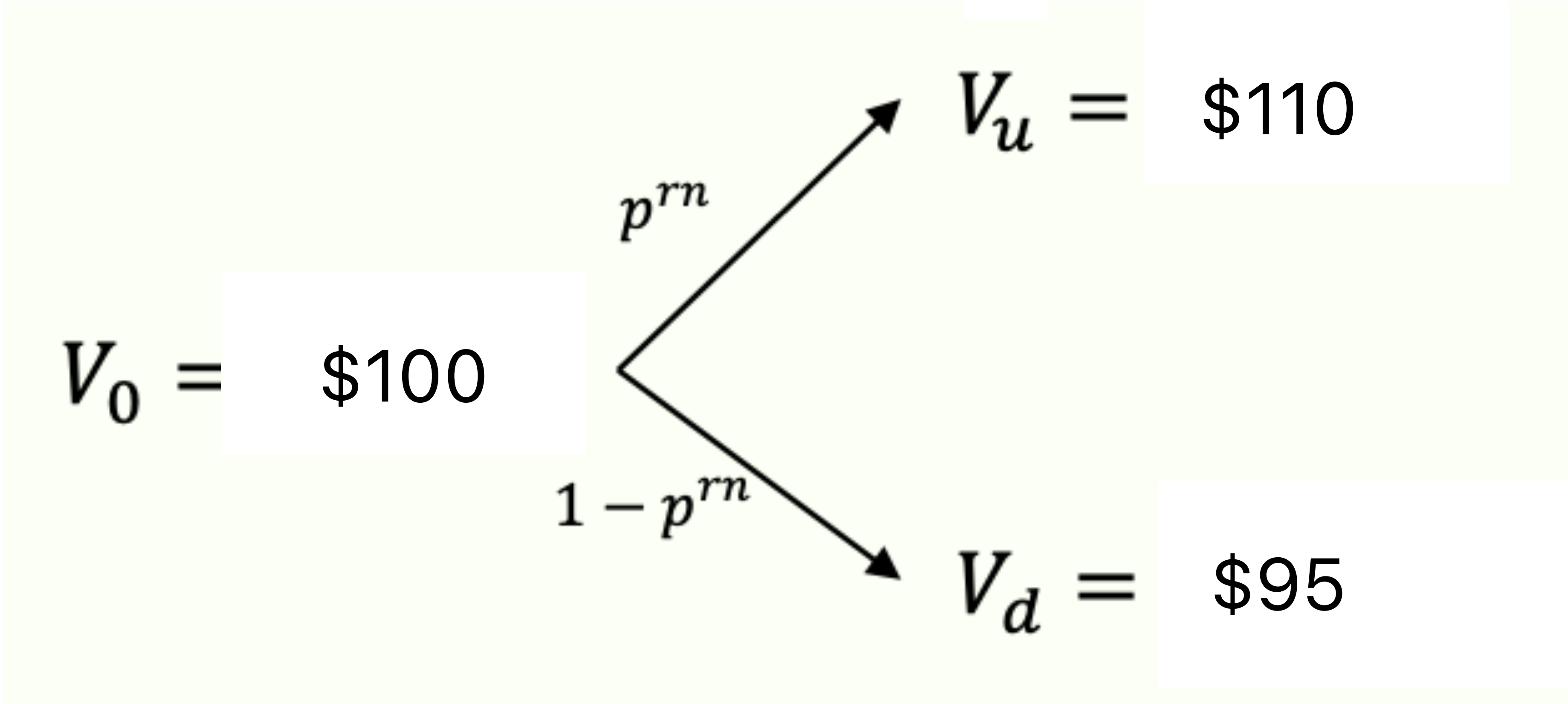
2. **Solutions**:
   - Δ (Shares): $\$\ \frac{8}{15}$
   - B (Borrow): $\$\ \frac{\frac{8}{15} \times 95}{1.05}$

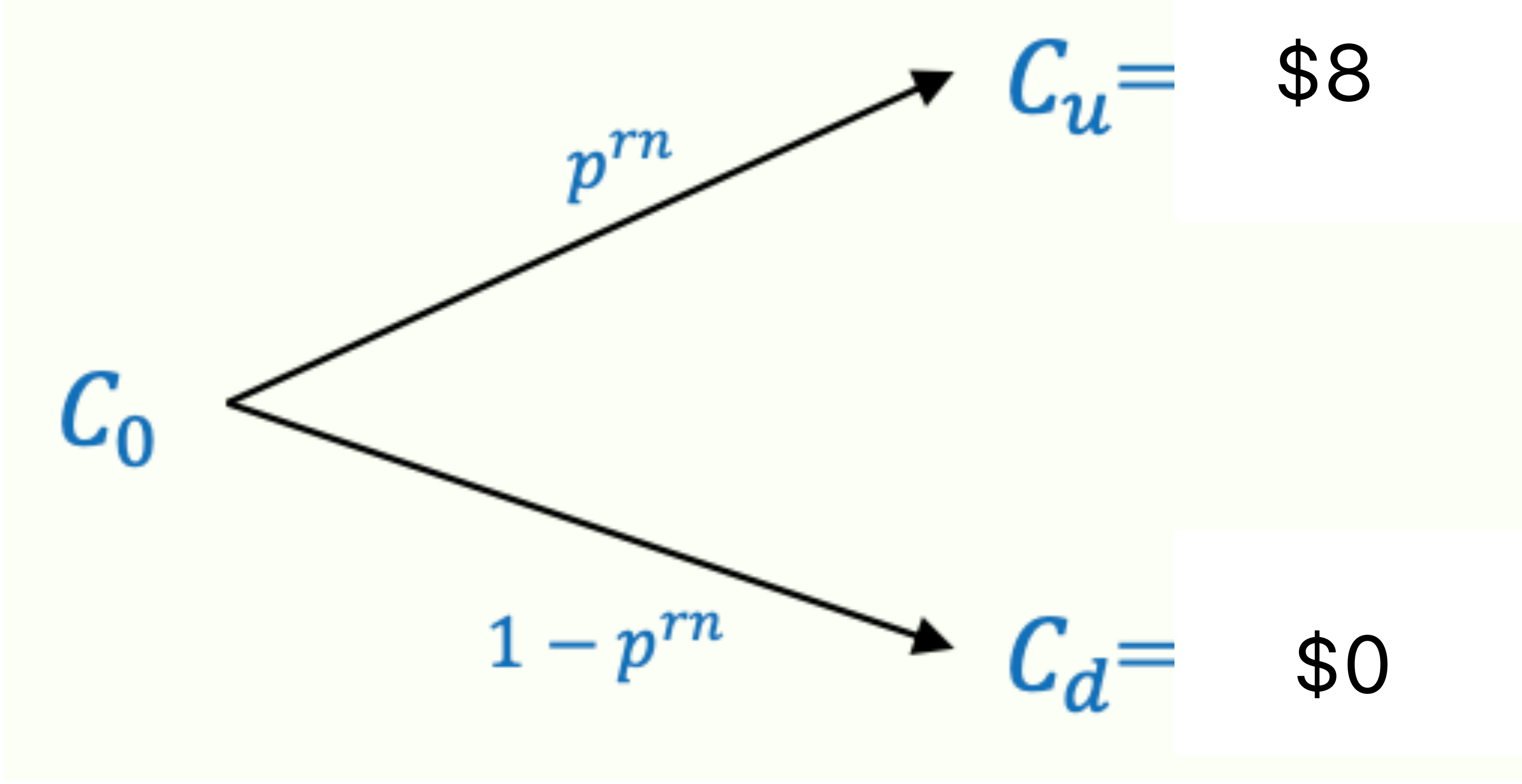
- Cost to set up the replicating portfolio = $\Delta \times StockPrice + B$
- Theoretical call option price: $5.08

Risk-Neutral Probability

- Q: At what probabilities, the option is riskless (i.e., have the same expected return as a risk-free asset)?

$$\frac{p^{RN}R_{+} + (1 - p^{RN})R_{-}}{(1 + r_f)} = R_0 \qquad p^{RN} = \frac{(1 + r_f)R_0 - R_{-}}{R_{+} - R_{-}}$$





Solution: $C = \frac{p^{rn}C_u + (1 - p^{rn})C_d}{1 + r_f} = \frac{p^{rn}\ \$8 + (1 - p^{rn})\$0}{1.05} = \$5.08$

Risk-Neutral Probability

Question 10: This question asks you to use the binomial model to compute the price of a European put option on Meta stock with a strike price of **$60**. The current price of Meta stock is **$60** per share, and the price will either increase by **100%** or decrease by **50%** on the exercise date of the option (that is, **u = 2** and **d = 0.5**). The interest rate is **10%**. Meta stock does not pay any dividends.

a) Using the replicating portfolio approach, what portfolio of Meta stock and borrowing or lending will replicate the change in the value of the option? What is the theoretical value of the put option described above?
b) Using the risk-neutral probability approach, what is the theoretical value of the put option described above?

Now, there are two periods and in each period the price will either increase by **100%** or decrease by **50%** (that is, **u = 2** and **d = 0.5**). The interest rate is **10%** per period.

If there are two periods then there are **3** dates; call the three dates times **0**, **1**, and **2**. The put option expires at time **2**, and the payoff of the put option is based on the stock price at time **2**. The strike price of the option is **$60**.

c) Using the replicating portfolio approach What is the theoretical value of the put option at each time?
d) Check your answers in part c) by using the risk-neutral probability approach.

Risk-Neutral Probability

According to the information we have:

$$S_0 = 60, K = 60, U = 2, d = 0.5, \text{ and } r = 0.1$$

Stock Price

$S_0 = 60$ → $S_u = 120$

$S_0 = 60$ → $S_d = 30$

Put Payoff

$V_0 = ?$ → $V_u = Max(0, 60 - 120) = 0$

$V_0 = ?$ → $V_d = Max(0, 60 - 30) = 30$

a) The replicating portfolio approach:

$$\begin{cases} \Delta S_u + (1+r)B = V_u \\ \Delta S_d + (1+r)B = V_d \end{cases} \Rightarrow \begin{cases} 120\Delta + 1.1B = 0 \\ 30\Delta + 1.1B = 30 \end{cases} \Rightarrow \Delta = -\frac{1}{3}, B = 36.36$$

So, the value of the put option is:

$$V_0 = \Delta S_0 + B = -\frac{1}{3}(60) + 36.36 = 16.36$$

Risk-Neutral Probability

b) The risk-neutral probability approach:

$$S_0 = \frac{p.S_u + (1-p).S_d}{1+r} = \frac{p.u.S_0 + (1-p).d.S_0}{1+r}$$

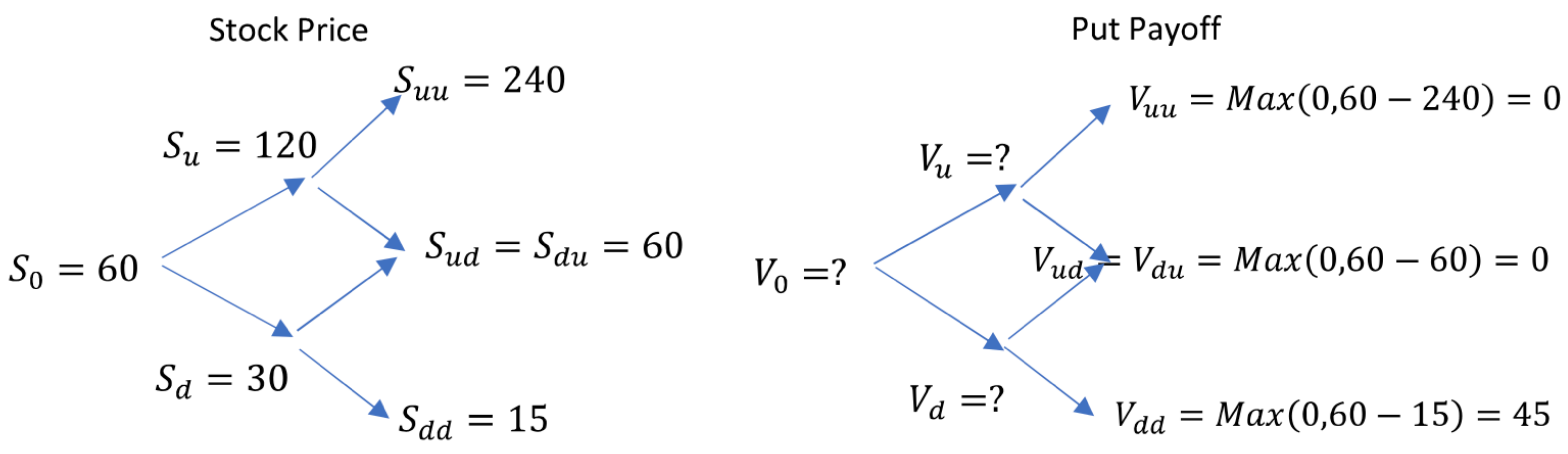
By factorising S_0 from the right-hand-side and eliminating it from both sides and re-arranging everything with respect to p, we will have~:

$$p = \frac{(1+r)-d}{u-d} = \frac{1.1-0.5}{2-0.5} = 0.4 \Longrightarrow 1-p = 0.6$$

$$\therefore The\ value\ of\ the\ option\ V_0 = \frac{p.V_u + (1-p).V_d}{1+r} = \frac{0.4(0)+0.6(30)}{1.1} = 16.36$$

This is the same answer we had from the previous approach.

Risk-Neutral Probability



$V_u = 0$, as there is no positive payoff in the upper branch and lower branch. But for V_d we need to calculate as following:

$$\begin{cases} \Delta S_{du} + (1+r)B = V_{du} \\ \Delta S_{dd} + (1+r)B = V_{dd} \end{cases} \Longrightarrow \begin{cases} 60\Delta + 1.1B = 0 \\ 15\Delta + 1.1B = 45 \end{cases} \Longrightarrow \Delta = -1\,, B = 54.55$$

$$\therefore \text{So, the value of } V_d = \Delta S_d + B = -1 \times (30) + 54.55 = 24.55$$

And for V_0 we need to go through the following steps:

$$\begin{cases} \Delta S_u + (1+r)B = V_u \\ \Delta S_d + (1+r)B = V_d \end{cases} \Longrightarrow \begin{cases} 120\Delta + 1.1B = 0 \\ 30\Delta + 1.1B = 24.55 \end{cases} \Longrightarrow \Delta = -0.273, B = 29.75$$

$$\therefore \text{So, the value of } V_0 = \Delta S_0 + B = -0.273 \times (60) + 29.75 = 13.388$$

Risk-Neutral Probability

d) From part b) we know that $p = 0.4$ and $(1 - p) = 0.6$, so:

$$V_u = \frac{p.V_{uu} + (1-p).V_{ud}}{1+r} = \frac{0.4(0) + 0.6(0)}{1.1} = 0$$

$$V_d = \frac{p.V_{du} + (1-p).V_{dd}}{1+r} = \frac{0.4(0) + 0.6(45)}{1.1} = 24.55$$

$$V_0 = \frac{p.V_u + (1-p).V_d}{1+r} = \frac{0.4(0) + 0.6(24.55)}{1.1} = 13.388$$

These are similar answers we reached through the replication method.