International Business and Finance
Week 2 Seminar 4

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Game Theory

- You had a quarrel with your boyfriend/girlfriend/partner; the relationship is tense right now
  - You can admit it's your fault, though you might not think so. So if you do this, you feel not that good, but still the relationship is back on track
  - You can wait for your partner to admit it's their fault, and if they do that you will feel really good, and the relationship is back on track
  - But, there is also a possibility that neither of you break the ice, and you two break up. You feel so bad.

Game Theory

- Normal form game: three key aspects
  - Payoffs
  - Players
  - Actions
- In the previous example
  - Payoffs: you feel good or not
  - Players: you and your partner
  - Actions: admit or wait

(your payoff, his/her payoff)		Your partner: Admit	Your partner: Wait
You	Admit	2,2	1,3
You	Wait	3,1	-100,-100

Definition 3.3 A **normal-form game** includes three components as follows:

1. A finite **set of players,** $N = \{1, 2, \ldots, n\}$.
2. A **collection of sets of pure strategies,** $\{S_1, S_2, \ldots, S_n\}$.
3. A **set of payoff functions,** $\{v_1, v_2, \ldots, v_n\}$, each assigning a payoff value to each combination of chosen strategies, that is, a set of functions $v_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ for each $i \in N$.

Dominance: It's Always Better To Do So

		Player 2	
		M	F
Player 1	M	$-2, -2$	$-5, -1$
	F	$-1, -5$	$-4, -4$

Definition 4.2 $s_i \in S_i$ is a **strictly dominant strategy** for i if every other strategy of i is strictly dominated by it, that is,

$$v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}) \quad \text{for all } s_i' \in S_i, \quad s_i' \neq s_i, \quad \text{and all } s_{-i} \in S_{-i}.$$

Best Response: It's Good Enough Given Your Information

		Chris O	Chris F
Alex	O	2, 1	0, 0
Alex	F	0, 0	1, 2

As the matrix demonstrates, the best choice of Alex depends on what Chris will do. If Chris goes to the opera then Alex would rather go to the opera instead of going to the football game. If, however, Chris goes to the football game then Alex's optimal action is switched around.

Definition 4.5 The strategy $s_i \in S_i$ is player i's **best response** to his opponents' strategies $s_{-i} \in S_{-i}$ if

$$v_i(s_i, s_{-i}) \geq v_i(s_i', s_{-i}) \quad \forall s_i' \in S_i.$$

Solution: Dominant Strategy Equilibrium

- Every one chooses dominate strategy

		Firm B's strategies	
		Low	*High*
Firm A's strategies	**Low**	**4** *4*	**2** *3*
	High	**3** *2*	**1** *1*

Solution: Nash Equilibrium

- Nash Equilibrium is when all plays' best responses meet
- Best response is a function of the other person's choice

		Player 2	
		action C	action D
Player 1	action A	5, 3	1, 0
	action B	0, 1	2, 4

Solution: Nash Equilibrium

Pure-Strategy Nash Equilibrium in a Matrix

This short section presents a simple method to find all the pure-strategy Nash equilibria in a matrix game if at least one exists. Consider the following two-person finite game in matrix form:

		Player 2		
		L	C	R
	U	7, 7	4, 2	1, 8
Player 1	M	2, 4	5, 5	2, 3
	D	8, 1	3, 2	0, 0

Solution: Nash Equilibrium

Step 1: For every *column,* which is a strategy of player 2, find the highest payoff entry for player 1. By definition this entry must be in the row that is a best response for the particular column being considered. Underline the pair of payoffs in this row under this column:

		Player 2		
		L	C	R
	U	7, 7	4, 2	1, 8
Player 1	M	2, 4	<u>5, 5</u>	<u>2, 3</u>
	D	<u>8, 1</u>	3, 2	0, 0

Step 1 identifies the best response of player 1 *for each of the pure strategies* (columns) of player 2. For instance, if player 2 is playing L, then player 1's best response is D, and we underline the payoffs associated with this row in column 1. After performing this step we see that there are three pairs of pure strategies at which player 1 is playing a best response: (D, L), (M, C), and (M, R).

Solution: Nash Equilibrium

Step 2: For every *row*, which is a strategy of player 1, find the highest payoff entry for player 2. By definition this entry must be in the column that is a best response for the particular row being considered. Overline the pair of payoffs in this entry:

		Player 2		
		L	*C*	*R*
	U	7, 7	4, 2	$\overline{1, 8}$
Player 1	*M*	2, 4	$\overline{\underline{5, 5}}$	$\underline{2, 3}$
	D	$\underline{8, 1}$	$\overline{3, 2}$	0, 0

Step 2 similarly identifies the pairs of strategies at which player 2 is playing a best response. For instance, if player 1 is playing D, then player 2's best response is C, and we overline the payoffs associated with this column in row 3. We can continue to conclude that player 2 is playing a best response at three strategy pairs: (D, C), (M, C), and (U, R).

Solution: Nash Equilibrium

Step 3: If any matrix entry has both an under- and an overline, it is the outcome of a Nash equilibrium in pure strategies.

This follows immediately from the fact that both players are playing a best response at any such pair of strategies. In this example we find that (M, C) is the unique pure-strategy Nash equilibrium—it is the only pair of pure strategies for which both players are playing a best response. If you apply

this approach to the Battle of the Sexes, for example, you will find both pure-strategy Nash equilibria, (O, O) and (F, F). For the Prisoner's Dilemma only (F, F) will be identified.

		Chris	
		O	*F*
Alex	*O*	2, 1	0, 0
	F	0, 0	1, 2

		Player 2	
		M	*F*
Player 1	*M*	−2, −2	−5, −1
	F	−1, −5	−4, −4

Mixed Strategy Nash Equilibrium

- Sometimes there is no discrete Nash Equilibrium, but there could be a probability-based mixed strategy Nash Equilibrium

Example: Matching Pennies

Consider the Matching Pennies game,

		Player 2: H	Player 2: T
Player 1	H	$1, -1$	$-1, 1$
	T	$-1, 1$	$1, -1$

and recall that we showed that this game does not have a pure-strategy Nash equilibrium. We now ask, does it have a mixed-strategy Nash equilibrium? To answer this, we have to find mixed strategies for both players that are mutual best responses.

Mixed Strategy Nash Equilibrium

- Consider the matching pennies.
- Suppose player 1 plays the mixed strategy σ_1 with $\sigma_1(H) = p$ and $\sigma_1(T) = 1 - p$. We also write $p \circ H + (1 - p) \circ T$ to denote this strategy.
- Suppose player 2 plays the mixed strategy $\sigma_2 = q \circ H + (1 - q) \circ T$.
- We always assume that players' mixtures are independent.
- What is the outcome of this strategy profile?

	H	T
H	$p \times q$	$p \times (1 - q)$
T	$(1 - p) \times q$	$(1 - p) \times (1 - q)$

Figure 2.20: The outcome is a distribution over the strategy profiles

- Outcome HH will occur with probability pq. Outcome HT will occur with probability $p(1 - q)$, and so on.

$$v_i(\sigma_1, \sigma_2) = pqv_i(H, H) + p(1 - q)v_i(H, T) + (1 - p)qv_i(T, H) + (1 - p)(1 - q)v_i(T, T).$$

Mixed Strategy Nash Equilibrium

To simplify the notation, define mixed strategies for players 1 and 2 as follows: Let p be the probability that player 1 plays H and $1 - p$ the probability that he plays T. Similarly let q be the probability that player 2 plays H and $1 - q$ the probability that he plays T.

Using the formulas for expected payoffs in this game, we can write player 1's expected payoff from each of his two pure actions as follows:

$$v_1(H, q) = q \times 1 + (1 - q) \times (-1) = 2q - 1 \qquad (6.1)$$

$$v_1(T, q) = q \times (-1) + (1 - q) \times 1 = 1 - 2q. \qquad (6.2)$$

With these equations in hand, we can calculate the best response of player 1 for any choice q of player 2. In particular player 1 will prefer to play H over playing T if and only if $v_1(H, q) > v_1(T, q)$. Using (6.1) and (6.2), this will be true if and only if

$$2q - 1 > 1 - 2q,$$

which is equivalent to $q > \frac{1}{2}$. Similarly playing T will be strictly better than playing H for player 1 if and only if $q < \frac{1}{2}$. Finally, when $q = \frac{1}{2}$ player 1 will be indifferent between playing H or T.

Mixed Strategy Nash Equilibrium

$$BR_1(q) = \begin{cases} 0, & \text{if } q < 1/2, \\ [0,1], & \text{if } q = 1/2, \\ 1, & \text{if } q > 1/2. \end{cases}$$

$$BR_2(p) = \begin{cases} 1, & \text{if } p < 1/2, \\ [0,1], & \text{if } p = 1/2, \\ 0, & \text{if } p > 1/2. \end{cases}$$

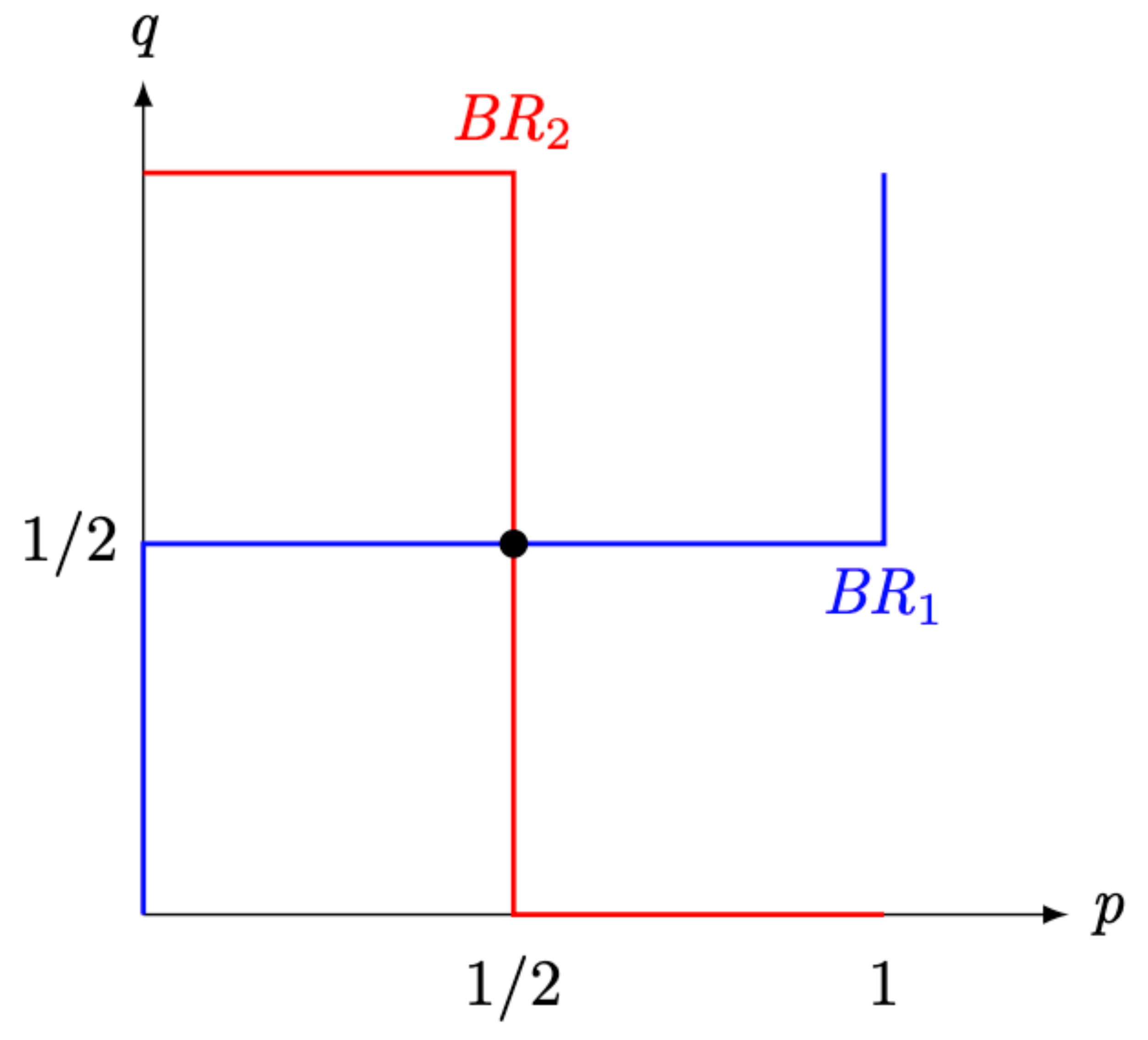


Figure 2.22: Best response correspondences in the matching pennies

Unique Nash equilibrium: $(\frac{1}{2} \circ H + \frac{1}{2} \circ T, \frac{1}{2} \circ H + \frac{1}{2} \circ T)$.

Different Types of Games

	Complete Information	Incomplete Information
Static	Players have all relevant information and move simultaneously	Not all players have all the information and move simultaneously
Dynamic	Players have all relevant information and move in turns	Not all players have all the information and move in turns

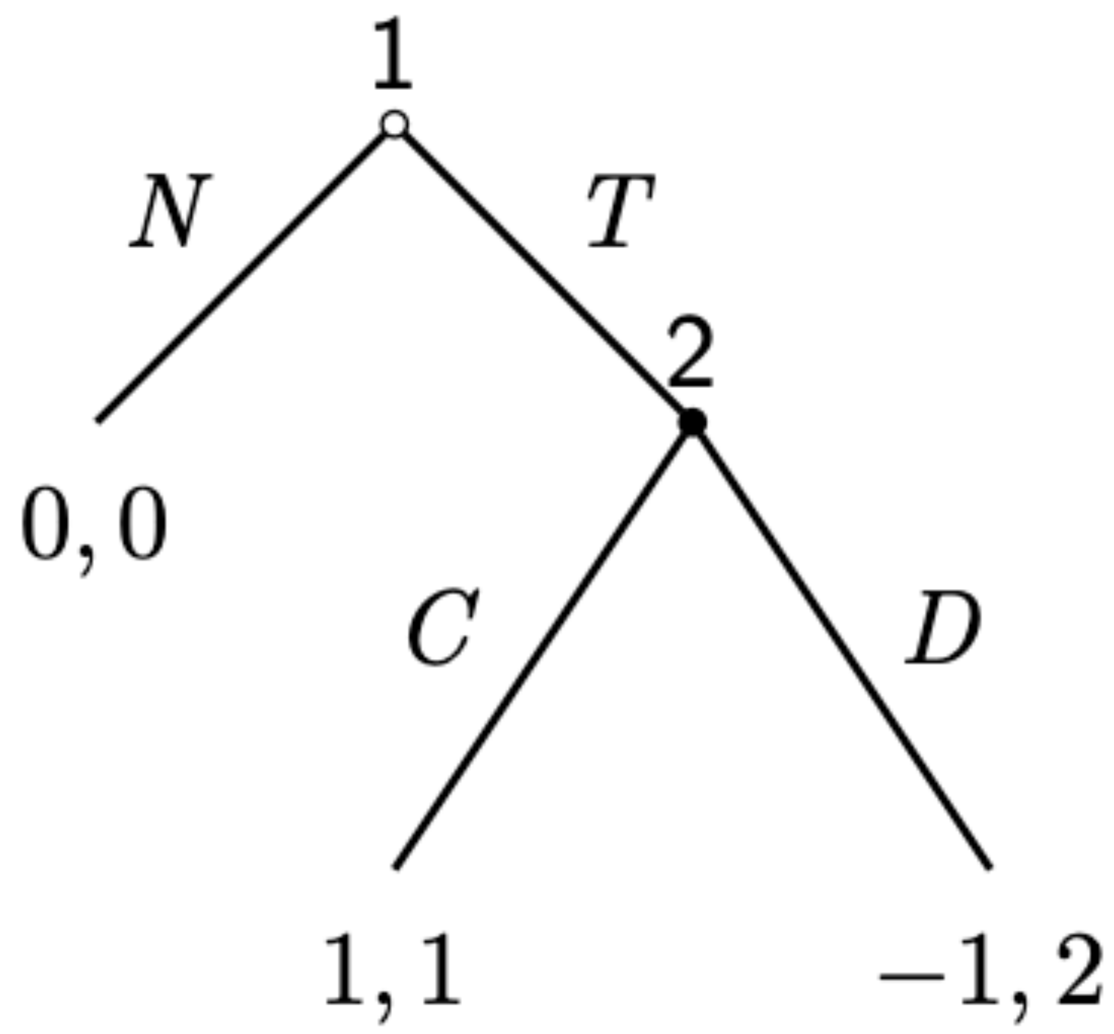


Figure 3.1: A trust game.

- Player 1 first chooses whether to ask for the services of player 2.
- He can trust player 2 (T) or not trust him (N).
- The latter choice gives both players a payoff of 0.
- If player 1 plays T, player 2 can choose to cooperate (C) or defect (D).
- If player 2 cooperates, both players get 1.
- If player 2 defects, player 1 gets -1 while player 2 gets 2.

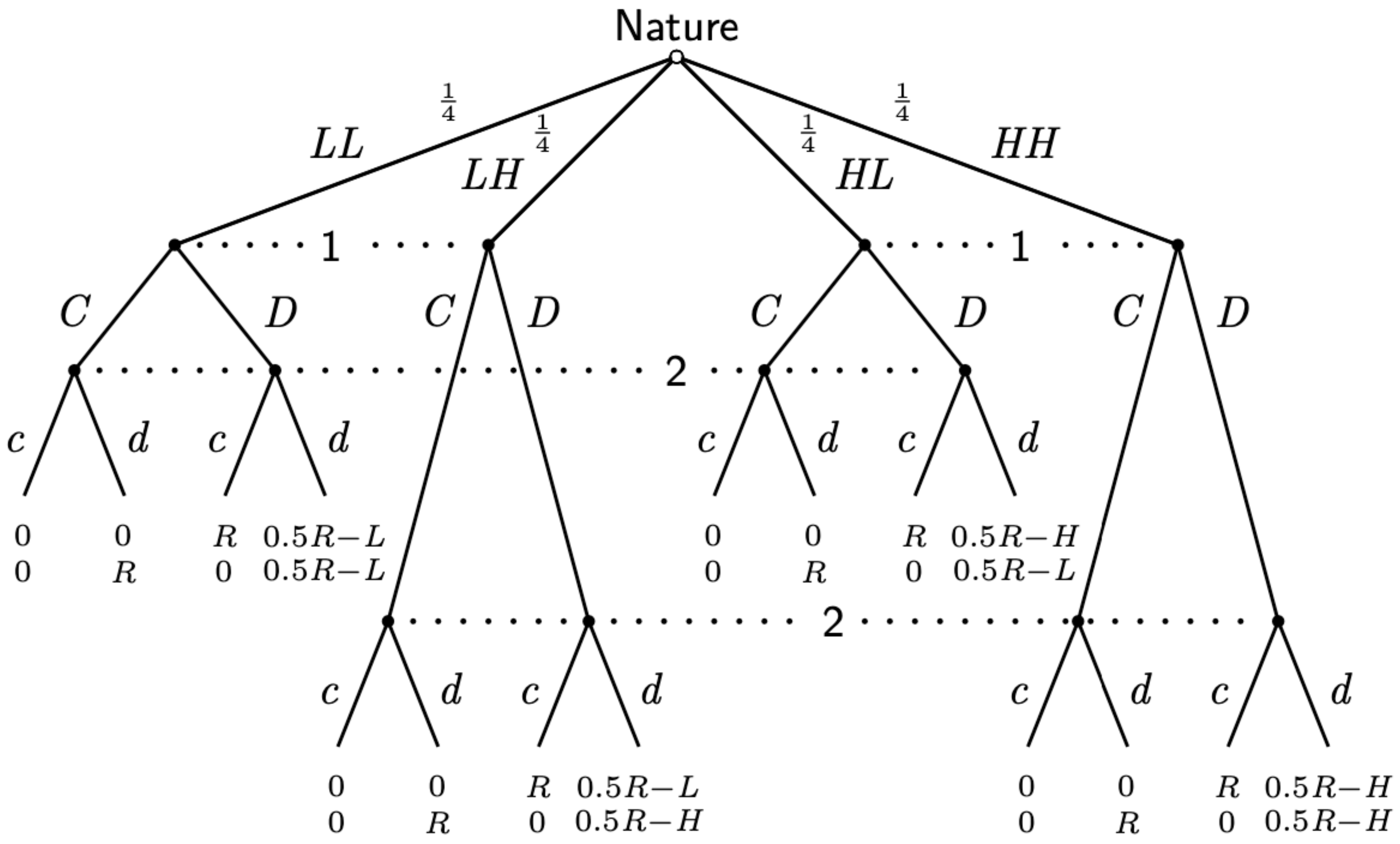
Static Games With Incomplete Information

- Two teenagers drive toward each other.
- Just before impact, they must simultaneously choose whether to be chicken and swerve to the right, or continue driving head on.
- They want to show they are brave and gain the respect from their friends.
- But they also suffer a loss if a collision occurs (e.g., parents reprimand, if seriously injured).
- If everything is common knowledge, it is a complete information game that we have learned.

	c	d
C	$0, 0$	$0, R$
D	$R, 0$	$\frac{R}{2} - k, \frac{R}{2} - k$

- Now, assume the punishment k is each player's *private information*.
- For instance, every teenage knows whether his own parents are harsh or lenient, but does not know others' parents.
- If i's parents are harsh, then $k_i = H$ is high.
- If i's parents are lenient, then $k_i = L$ is low.
- Assume every player's parents can be harsh or lenient with equal probabilities.

Game of chicken with incomplete information



Dynamic Games With Incomplete Information

Player 1 is a potential entrant to an industry that has a monopolistic incumbent, player 2. If player 1 stays out (O) then the incumbent earns a profit of 2, while the potential entrant gets 0. The entrant's other option is to enter (E), which gives the incumbent a chance to respond. If the incumbent chooses to accommodate entry (A), then both the entrant and the incumbent receive a payoff of 1. The incumbent's other option is to fight entry (F), in which case the payoff for each player is –1.

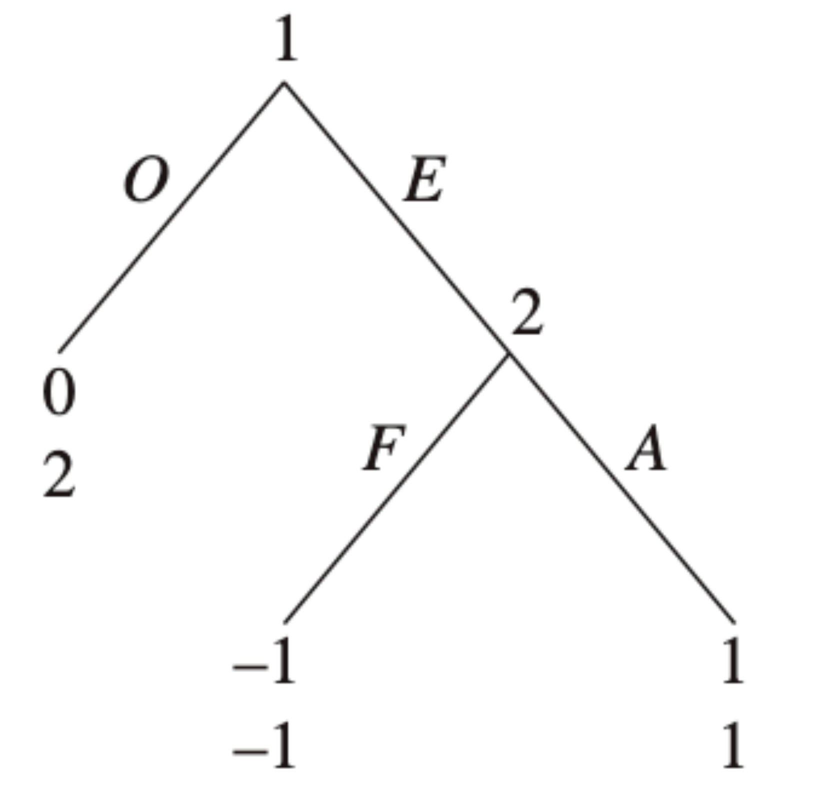


FIGURE 15.1 A simple entry game.

Now consider a straightforward variant of this game that includes incomplete information. In particular imagine that the entrant may have a technology that is as good as that of the incumbent, in which case the game above describes the payoffs. However, the entrant may also have an inferior technology, in which case he would not gain by entering and the incumbent would lose less if fighting occurred.

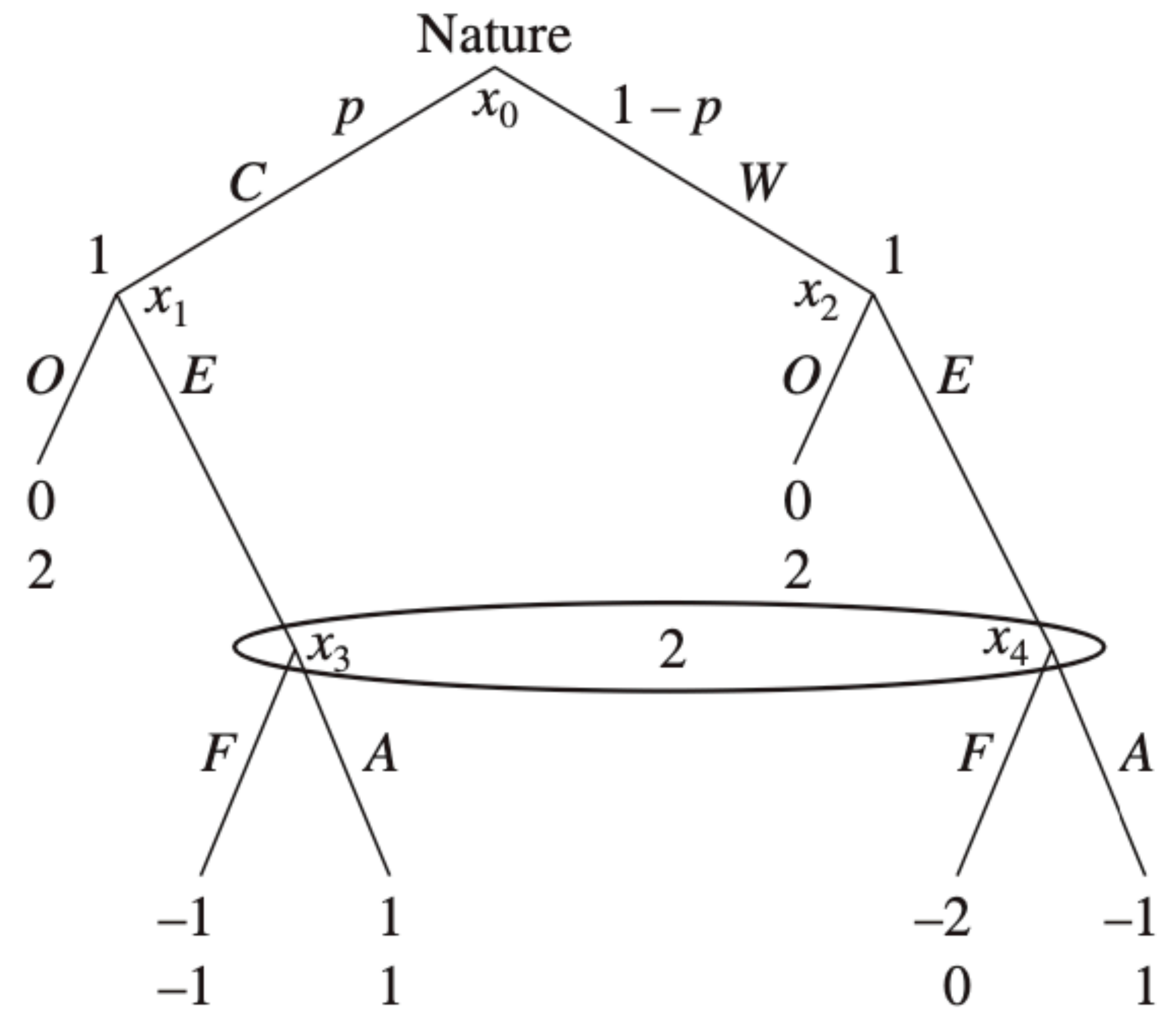


FIGURE 15.2 An entry game with incomplete information.