

# Quantum impurity problem in ultracold gases: from dark solitons to quantum ferromagnets

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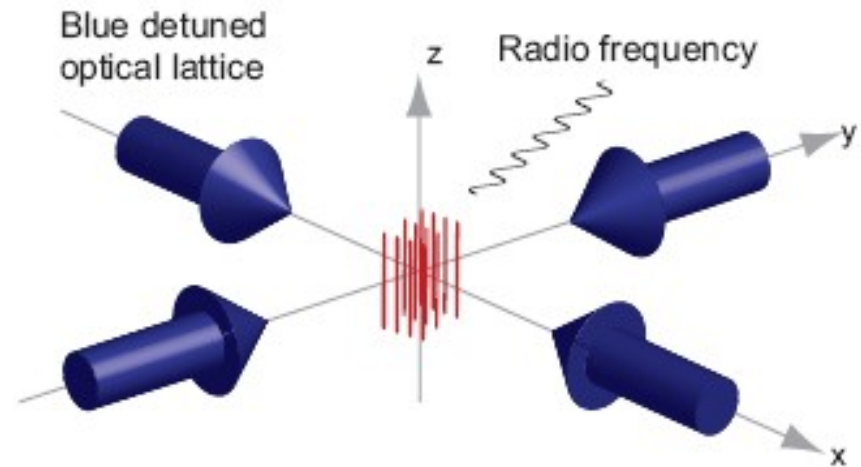
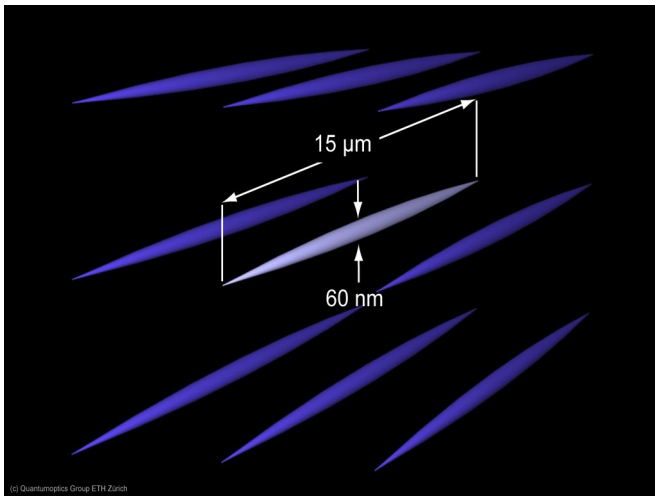
MUARC – Midlands Ultra Cold Atoms  
Research Centre

In collaboration with **Alex Kamenev**, University of Minnesota

Phys. Rev. Lett. 102, 070402 (2009), arXiv:0908.4513

**EPSRC**

# Experiments in 1D



$N \sim 100$  atoms in each tube

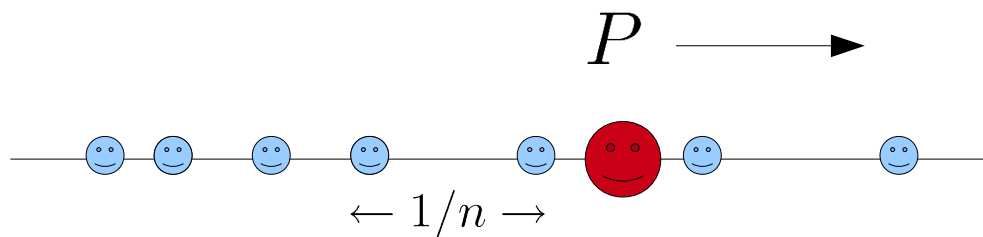
1 dimensional regime

$$\hbar\omega_{\perp} \gg \mu, T$$

Short-range interactions  $V_{\text{int}} = g\delta(x_i - x_j)$   $g = 2\hbar a\omega_{\perp}$

# Impurity in 1D

1 impurity of mass  $M$  making its way through the liquid....



Questions:

- How to characterise its interactions with background
- Dissipative processes
- Motion under slow external fields (harmonic trap)

# Bose spinor gas

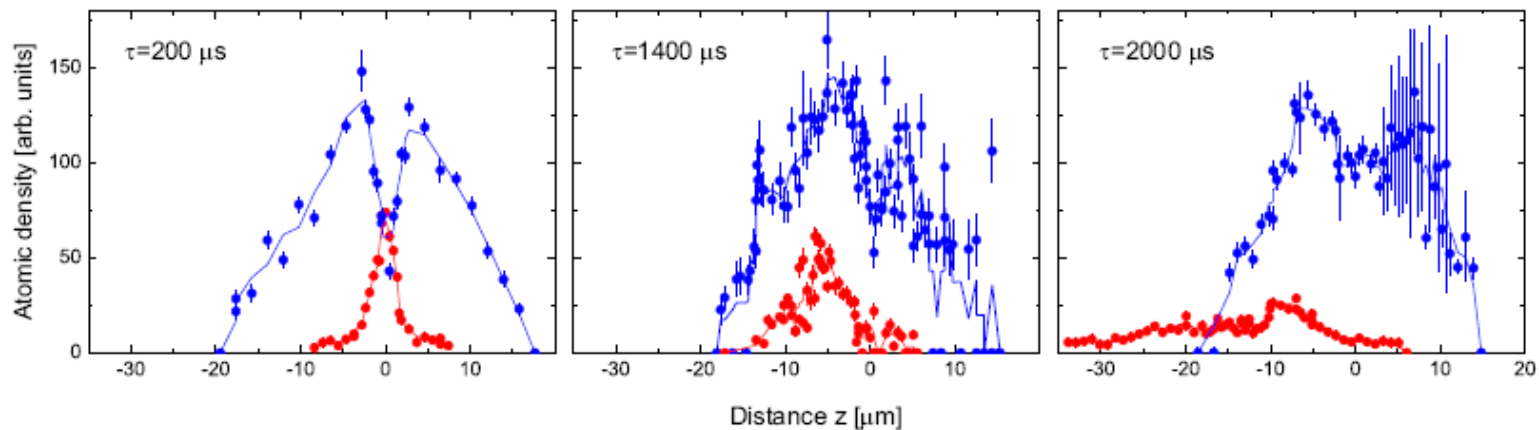
$\text{Rb}^{87}$

$$|F = 1, m_F = -1\rangle \rightarrow |\uparrow\rangle$$

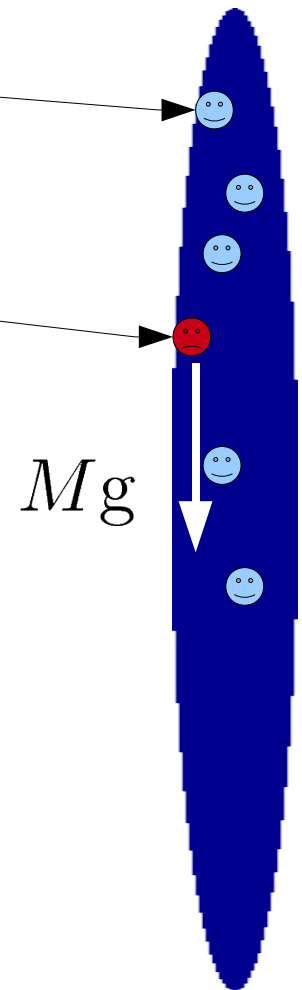
Magnetically trapped

$$|F = 2, m_F = 0\rangle \rightarrow |\downarrow\rangle$$

Untrapped



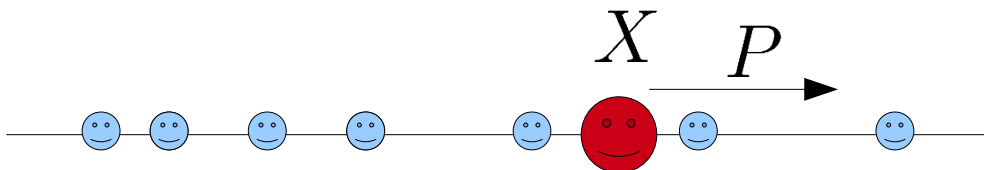
S. Palzer et al. PRL 2009



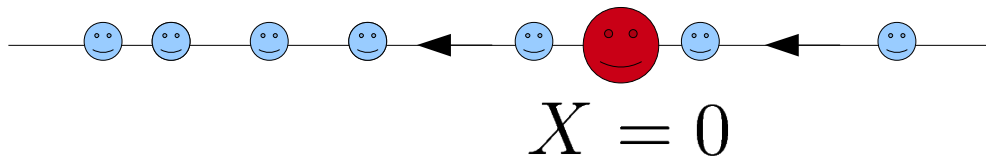
# Constructing effective action

Impurity  $P(t), X(t)$       Bose Liquid  $\bar{\psi}(x, t), \psi(x, t)$

$$L = P\dot{X} - \frac{P^2}{2M} - G|\psi(X, t)|^2 + \int dx \left( i\bar{\psi}\partial_t\psi - \frac{1}{2m}\partial_x\bar{\psi}\partial_x\psi - \frac{g}{2}\bar{\psi}\bar{\psi}\psi\psi \right)$$



# co-moving frame



$$L = P\dot{X} - \frac{1}{2M} (P - P(\bar{\psi}, \psi))^2 - G|\psi(0, t)|^2 + \int dx \left( i\bar{\psi}\partial_t\psi - \frac{1}{2m}\partial_x\bar{\psi}\partial_x\psi - \frac{g}{2}\bar{\psi}\bar{\psi}\psi\psi \right)$$

$P$  is TOTAL momentum

$$P(\bar{\psi}, \psi) = \frac{1}{i} \int dx \bar{\psi}\partial_x\psi$$

# Renormalised collective dispersion

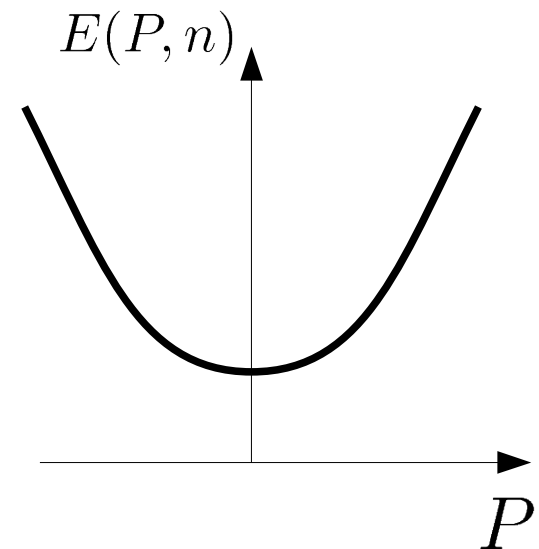
Minimise the total energy of impurity + liquid for given total momentum

ground state of  $E(\bar{\psi}, \psi) - VP(\bar{\psi}, \psi)$

Control parameters: velocity  $V$ , background chemical pot.  $\mu$

$$E(V, \mu) \longrightarrow E(P, n)$$

$$\simeq \varepsilon_0(n) + P^2/2M^*$$



# Dispersion periodicity

## 'superfluid' argument

Transfer momentum quantum  $\frac{2\pi\hbar}{L}$  to each particle  
No change to internal state!

Total momentum 
$$P = P_{\text{imp}} + N \times \frac{2\pi\hbar}{L} = P_{\text{imp}} + 2\pi\hbar n$$

Total energy 
$$E(P) = E(P_{\text{imp}}) + N \times \frac{1}{m} \left( \frac{2\pi\hbar}{L} \right)^2$$

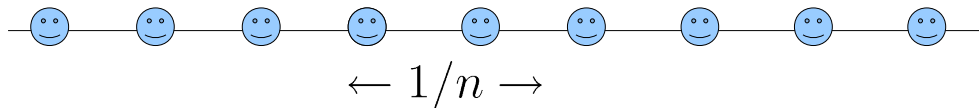
Periodic dispersion 
$$E(P) = E(P - 2\pi n\hbar)$$



# Dispersion periodicity

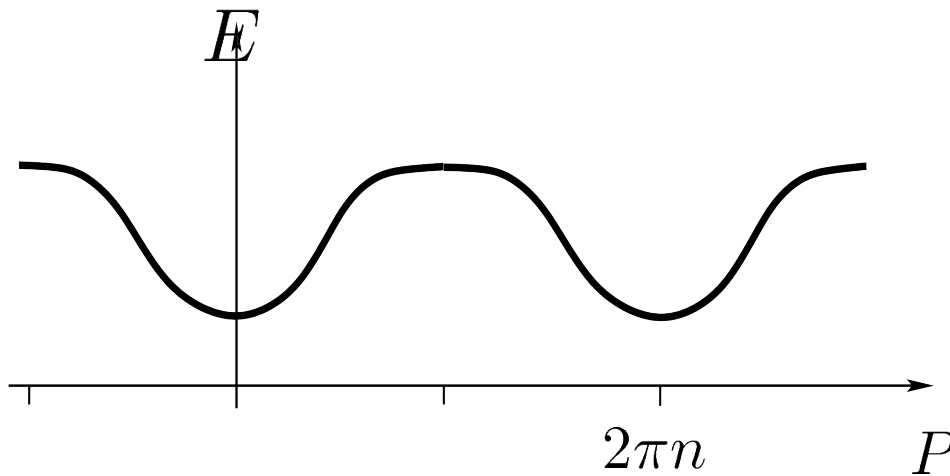
## 'umklapp' argument

Assume no (quantum, thermal) fluctuations: perfect lattice



Periodic zone with period

$$\frac{2\pi\hbar}{a} = 2\pi\hbar n$$



$E(P)$  contains contribution of all excitations with

$$k > k_c = \sqrt{m\mu}$$

# Phonons

Phase-density representation for slow fields

$$\psi(x, t) = \sqrt{n + \rho(x, t)} e^{i\phi(x, t)}$$

Close to equilibrium

$$L = P\dot{X} - E(P, n) + \int dx \left( -\rho\partial_t\phi - \frac{mc^2}{2n}\rho^2 - \frac{n}{2m}(\partial_x\phi)^2 \right)$$

+ interactions

# Real processes: dissipation and viscous friction

~~One phonon processes~~

$$\Delta E \sim c\Delta P - V(P)\Delta P$$

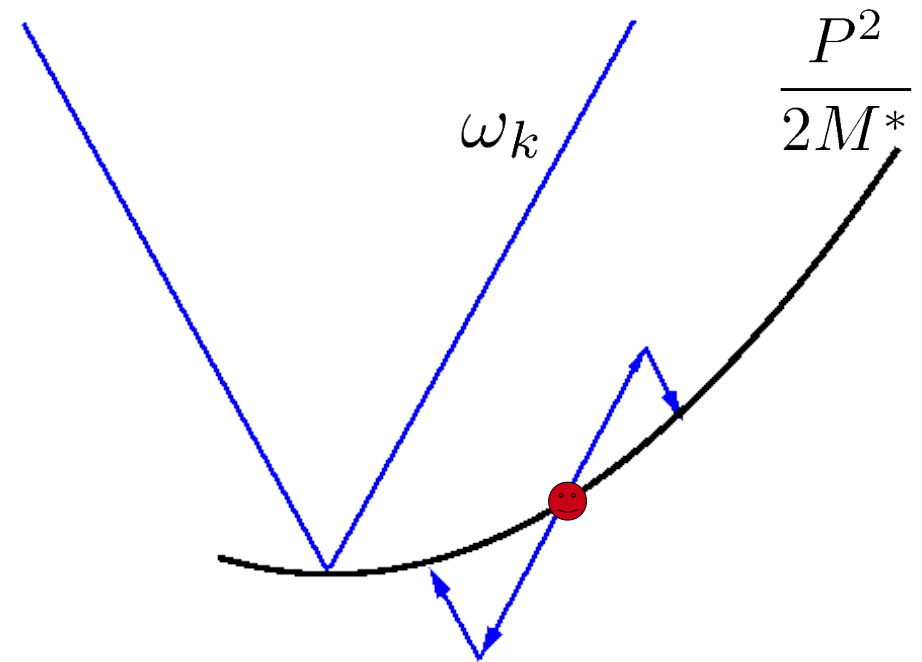
$$V(P) = P/M^* \ll c$$

- Landau criterion

Superfluid at  $T = 0$

$T \neq 0$

two phonon processes



# Viscosity coefficient

$$\dot{P} = -\kappa V$$

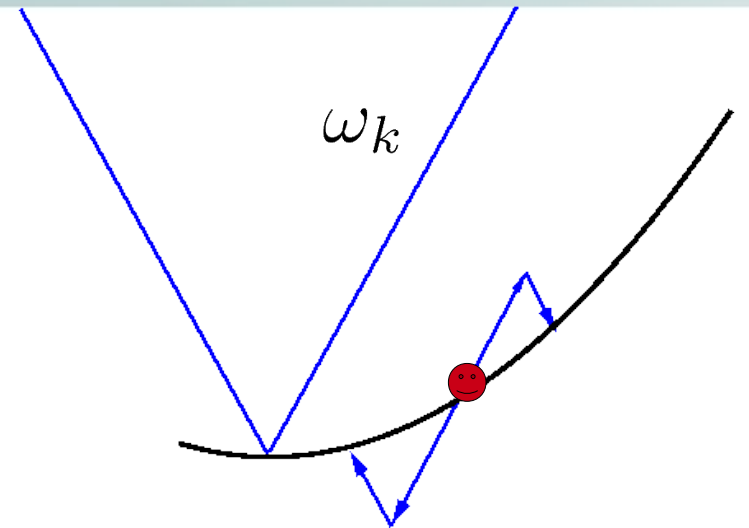
$$\omega_{\max} = cq_{\max} \sim T$$

$$\kappa \sim \int_0^T dq q q q^{2d-1} \sim T^{2d+2}$$

In 3 dimensions  $\kappa \sim T^8$

depends crucially on coupling with phonons

In 1 dimension  $\kappa \sim (\dots) \frac{T^4}{c^4 \hbar^3 n^2}$  Castro Neto & M.P.A Fisher, 1996



He<sup>3</sup> - He<sup>4</sup>

Landau & Khalatnikov, 1949  
Baym & Ebner, 1967

# Impurity – phonon coupling

Coupling to density

$$E(P, n + \rho(X, t)) = \varepsilon_0(n) + \frac{P^2}{2M} + \frac{\partial \varepsilon_0}{\partial n} \rho + \frac{1}{2} \frac{\partial^2 \varepsilon_0}{\partial n^2} \rho^2$$

Coupling to (super) velocity  $v = \frac{\hbar}{m} \partial_x \phi$

Galilean transformation:  $P' = P - Mv$

$$\begin{aligned} E' &= E(P, n) - Pv + \frac{Mv^2}{2} = \\ &= \varepsilon_0(n) + \frac{P'^2}{2M^*} - \left( \frac{M^* - M}{M^*} \right) P'v + \left( \frac{M^* - M}{M^*} \right) \frac{Mv^2}{2} \end{aligned}$$

# Gauge transformation

$$\rho(x, t) \rightarrow \rho(x, t) - R(t) \delta(x - X)$$

$$\phi(x, t) \rightarrow \phi(x, t) - F(t) \theta(x - X)$$

$$P \rightarrow P + mRu(X, t) + F\rho(X, t)$$

Choose  $R(t), F(t)$  to eliminate linear couplings:

$$H_{\text{imp-ph}} = \frac{1}{2} \Gamma_{\rho} [\rho(X)]^2 + \frac{1}{2} \Gamma_{\phi} [\partial_x \phi(X)]^2$$

# Integrating out phonons

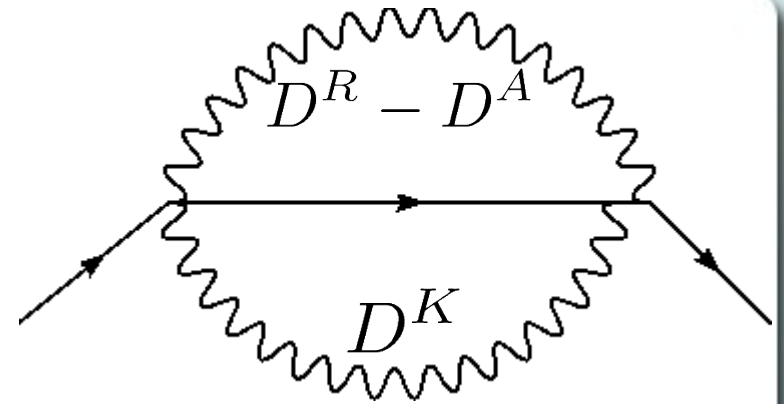
$$\dot{P} = -\kappa V$$

$$\kappa = \text{Tr} [\Gamma (D^A - D^R) \Gamma D^K]$$

$$= \frac{1}{4} \left( \Gamma_\rho - \frac{m^2 c^2}{n^2} \Gamma_\vartheta \right)^2 \int \frac{dq}{2\pi} q \Pi(q, qV)$$



Quantum interference – possible cancellation



$$\Pi(q, \omega) = \frac{n^2}{4m^2 c^3} \left( q^2 - \frac{\omega^2}{c^2} \right) \left( \coth \frac{cq - \omega}{4T} - \coth \frac{cq + \omega}{4T} \right)$$

# Effective coupling

$$\Gamma_\rho - \frac{m^2 c^2}{n^2} \Gamma_\phi \sim \left( N_s - \frac{1}{g} \frac{\partial E_s}{\partial n_0} \right) + \left( N_s - \frac{M_s^*}{m} \right)$$

Number of particles expelled by the impurity

$$R = N_s = - \int (|\psi|^2 - n_0) dx$$

Integrable cases  $\frac{\partial E_s}{\partial \mu} = N_s \quad M_s^* = m N_s$



# Integrability ?

Mechanism of relaxation?

~~3-body collisions~~

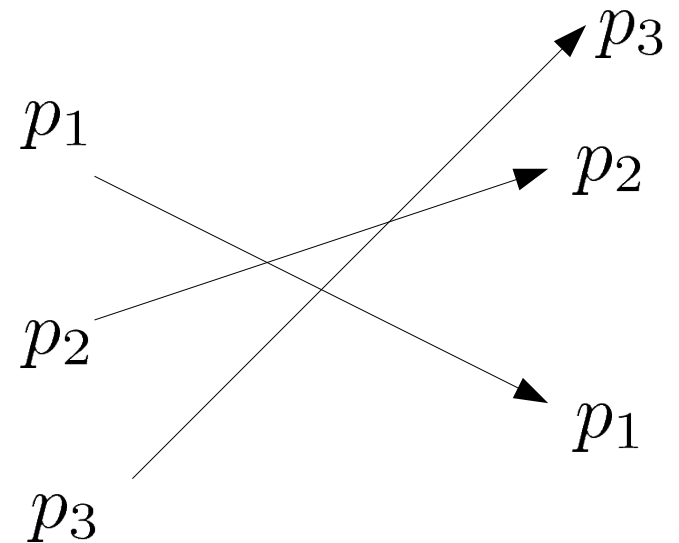
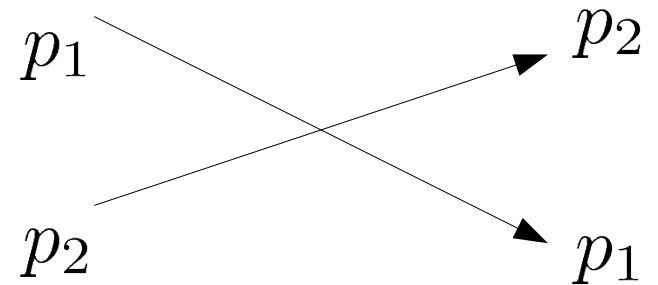
STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM  
Document LA-1940 (May 1955).

No dissipation

No decay of supercurrents

Even at  $T \neq 0$



# Dark Soliton

Phase drop  $\theta_s$ :

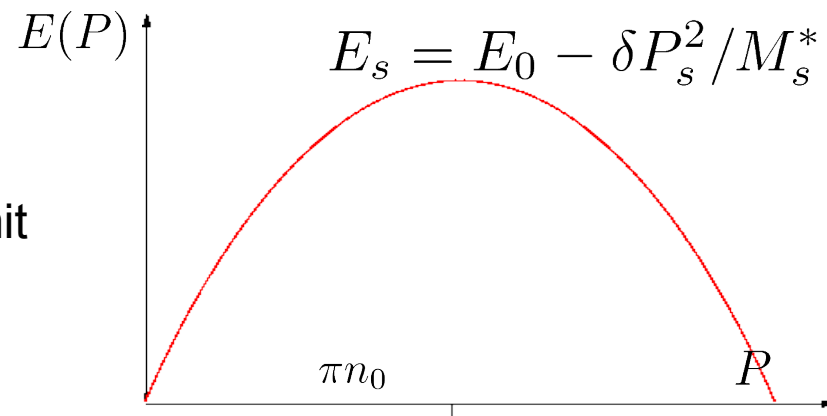
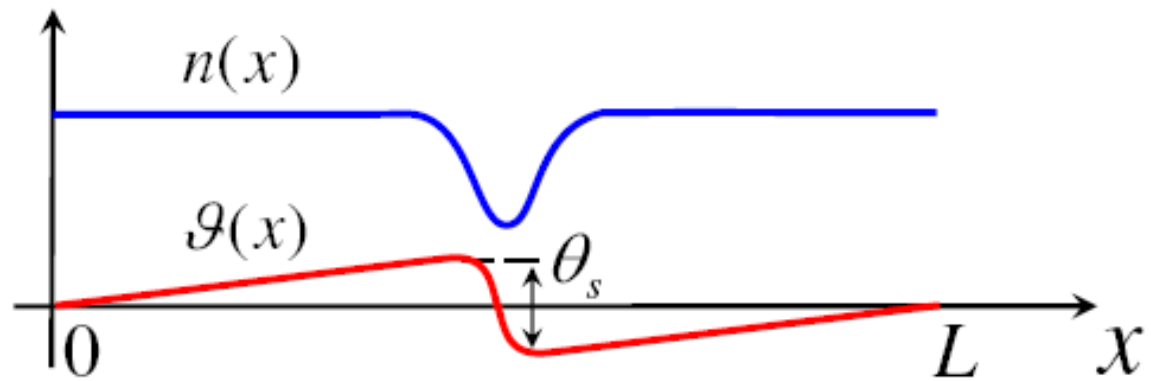
$$\cos \frac{\theta_s}{2} = \frac{V}{c}$$

Number of part. expelled

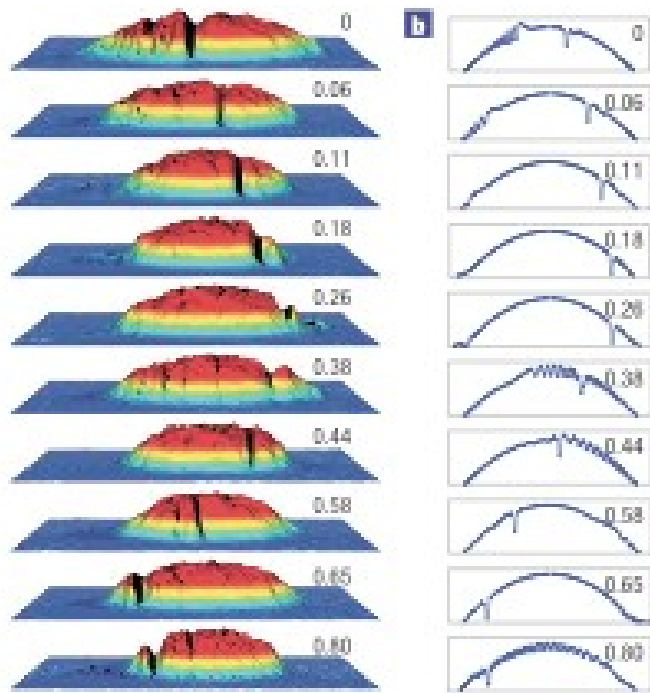
$$N_s = \frac{2n}{mc} \sin \frac{\theta_s}{2}$$

$E(P, n)$  can be obtained in the limit

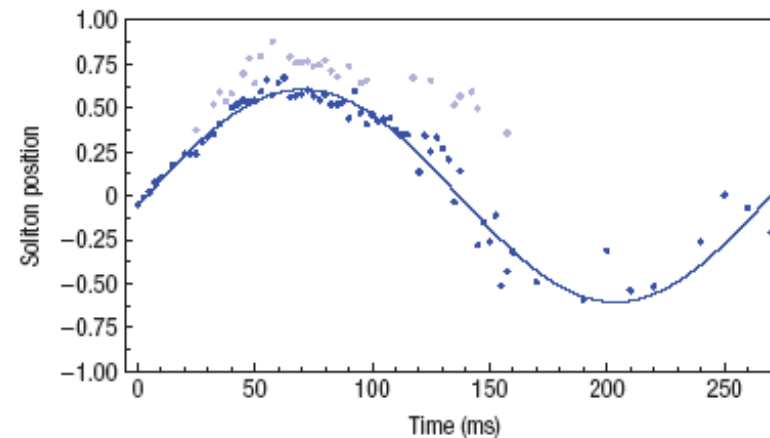
$$G \rightarrow 0, M \rightarrow 0$$



# Experiments. Life-Time



C. Becker *et al.*, Nature Physics **4**, 496 (2008)



Lifetime  $\sim 300$ ms

- Finite lifetime due to background fluctuations (phonons)
- Absent at  $T=0$  (Landau criterion)
- Absent for integrable case

# 1D fermions with spin $\frac{1}{2}$

(Gaudin '67, Young '67)

$$H_{\text{int}} = u \int dx \psi_{\uparrow}^{\dagger} \psi_{\downarrow}^{\dagger} \psi_{\downarrow} \psi_{\uparrow}$$

Spin -1 excitation above a (metastable) ferromagnetic state

$$\kappa = 0 \Rightarrow \frac{m}{m^*} = \frac{\pi p_F}{m} \frac{1}{\frac{\partial}{\partial n} (\mu - \mu_d)} + \frac{p_F^2}{m} \frac{\frac{\partial^2}{\partial n^2} (\mu - \mu_d)}{\left(\frac{\partial}{\partial n} (\mu - \mu_d)\right)^2}$$

$$\mu - \mu_d = \frac{p_F u}{2\pi} \left[ \left( \frac{2p_F}{mu} + \frac{mu}{2p_F} \right) \arctan \frac{2p_F}{mu} - 1 \right]$$

Castella, Zotos '93

$$\frac{m}{m^*} = \frac{\pi}{2 \arctan^2 \frac{2p_F}{mu}} \left( \arctan \frac{2p_F}{mu} - \frac{\frac{2p_F}{mu}}{1 + \left(\frac{2p_F}{mu}\right)^2} \right)$$

# Perturbation around integrability

Weakly interacting bosons  $\mu = gn$   $\mu_d = Gn$

$$\kappa = \frac{16\pi^3}{15} \frac{T^4}{c^4 \hbar^3 n^2} \left(\frac{G}{g}\right)^2 \left(\frac{mG}{Mg} - 1\right)^2 \quad M^* = M$$

Rb<sup>87</sup>  $G/g \sim 1.05$

Strong interactions  $m \neq M$   $g, G \rightarrow \infty$

$$\kappa = \frac{16\pi^3}{15} \frac{T^4}{c^4 \hbar^3 n^2} \left[ \left(\frac{m}{M}\right)^{1/3} - \frac{M}{m} \right]^2$$

$m = M$   $g, G \gg \hbar^2 n/m$

$$\kappa \sim \frac{\pi^3 T^4}{c^4 \hbar^3 n^2} \left(\frac{\hbar^2 n}{gm}\right)^4 \left(1 - \frac{g}{G}\right)^2$$

# Concluding remarks

- Impurity in 1D – model to study nonequilibrium many body quantum dynamics.
- Universal coupling to phonons
- Dissipation is important. Mechanism of dissipation
- Sensitive to parameters. Absence of dissipation for integrable systems even at nonzero temperature
- Can be applied to study excitation dynamics in nearly integrable systems. Lifetime of quasiparticles
- Experiments....