The role of excitations statistic and nonlinearity in energy harvesting from random impulsive excitations

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Design of an efficient energy harvester is now feasible as technology develops and a viable approach to solve this need is to exploit the concept and application of a nonlinear design. In this letter, we conducted a comparative analysis of linear and nonlinear piezoelectric energy harvesting from random impulsive excitations modelled by white Poisson noise. It is shown that the harvester performance depends on both nonlinearity and properties of ambient energy, and nonlinearity should be optimized for a given type of ambient vibration in order to achieve efficient harvesting.


Energy harvesting is a transformation of an external (ambient) energy of one type, for example, mechanical vibration or heat into an electrical form by using different transduction mechanisms for subsequent utilization. The main target application of harvesting is to provide power for small scale devices by effective replacement of the traditional power supply, i.e., battery. Typically, mechanical vibrations are sources of ambient energy for such devices. The primary design of the harvesters is based on the resonant principle, aiming to a harmonic (sinusoidal) form of vibration, when a linear oscillator with a large quality factor is employed for harvesting energy. Generally, harvesting efficiency should grow as $1/\omega^2$ ($\omega$ is frequency of the harmonic vibration) for a typical linear harvester. Consequently, an efficiency should reach its maximum as $\omega$ tends to zero. However, for practical applications, the resonant frequency is typically in the order (or more) of 100 Hz, whereas a wide range of energy sources for example, human motions have low frequency vibrations (1–10 Hz). In many cases, this energy has a wide bandwidth and a stochastic nature. Thus, the primary design requires modifications in order to address the spectral band limitations. Recently, introducing a nonlinearity has been suggested to overcome the limitations. The bistable regime of harvester operation for broadband random excitations in the form of white Gaussian noise has been discussed.

The use of nonlinear design poses other challenges. It is important to take into account the features of vibration with the aim of selecting the best form of nonlinearity. The question arises whether it is sufficient only to understand the spectral context of the vibration or whether a detailed description of the vibration is a prerequisite requirement? These issues are still open and have not been adequately addressed. In this letter, we employ a model of a piezoelectric harvesting device to analyze the role of a nonlinearity and the importance of a detailed description of vibration. We selected vibration in the form of random impulsive excitations which are described by two independent parameters (see below). The form of the impulsive excitations that can be represented by a sequence of pulses is often observed and refers to impact excitations. For the impacts from uncorrelated sources, the impulsive excitations form random pulse train corresponding to Poisson white noise. These factors are discussed here. Also, the efficiency of the harvester with different types of nonlinearities is compared for Poisson noise excitations.

A feasibility to create different nonlinearities in piezoelectric harvesting devices by using additional magnets has been recently demonstrated. Corresponding models of devices have been validated and such models have been used to understand the outcome of the harvesters relating to harmonic signals and white Gaussian noise. The model under consideration here describes a ferromagnetic cantilever beam with piezoceramic layers. It includes two parts: mechanical and electrical forming the following system:

$$\ddot{x} = -\alpha \dot{x} - \frac{dU(x)}{dx} + \chi z + f(t),$$

$$\dot{z} = -\lambda z - \kappa \dot{x}. \quad (1)$$

The mechanical part is described by the variables $x$ and $\dot{x}$ and this corresponds to a cantilever beam dynamics. Coordinate $z$ corresponds to the electrical part and this is proportional to the voltage arising from the interaction between piezoceramic layers and beam bending. The extracted power is defined as

$$P = \rho \lim_{T \to \infty} \frac{1}{T} \int_0^T \dot{z}^2(t) dt, \quad (2)$$

where $T$ is the averaging time, and $\rho$ corresponds to the load conductivity. For simplicity $\rho = 1$. The term $f(t)$ corresponds to an external mechanical energy source from the vibration of the beam support. We fix values of the parameters $x = 0.02$, $\chi = 0.05$, $\lambda = 0.05$, and $\kappa = 0.5$. The term $dU(x)/dx$ describes the non-linearity of the system (1), where $U(x)$ is the magnetoelastic potential formed by permanent magnets. By changing the magnets locations and/or using different numbers of magnets, $U(x)$ can be varied consequently changing the system nonlinearity. However, it is unclear as to how the potential shape and this system nonlinearity influence energy harvesting. In order to address this
question, let us consider several types of $U(x)$ with distinct shapes
\begin{align*}
U_L(x) &= x^2/2, \\
U_B(x) &= -x^2/4 + x^4/2, \\
U_F(x) &= x^4, \\
U_M(x) &= x^2/2 + x^4/2,
\end{align*}
where indices $L$, $B$, $F$, and $M$ correspond to linear, bistable, flat, and monostable potentials, respectively. Note that $U_L$ is realized in the absence of magnets. The system (1) with linear, bistable, and monostable potentials has the same resonant frequency $\omega_0 \approx 1.0124$ for weak vibration, whereas for the flat potential $\omega_0 = 0$.

To compare the performance of Eq. (1) for different potential shapes, we introduce an efficiency index in the following form:
\begin{equation}
\gamma = 10 \log_{10} \left( \frac{P_N}{P_L} \right),
\end{equation}
where $P_L$ is the power (2) for the linear case, and $P_N$ represents the power (2) for different nonlinear potentials. The results for $\gamma$ are given in dB units. For the input signals as considered below, the output power $P_L$ is a linear function of the input power, $P_{in}$, that results in a constant ratio $P_L/P_{in} = C$ ($C =$ constant). Numerical simulations of Eq. (1) were performed in order to obtain values of $\gamma$. The simulations used the Heun methods for stochastic differential equations with corresponding modifications for Poisson noise.

After a relaxation period $T_c = 10^5$, the power $P$ was calculated for a finite $T = 10^{10}$. The zero initial conditions $x = 0$, $\dot{x} = 0$, and $z = 0$ were used to mimic corresponding experimental setup.

The linear potential $U_L$ is an optimal shape for a harmonic signal $f(t) = A \sin(\omega_0 t)$, and a nonlinear design cannot significantly improve the performance. Indeed, for weak signal (A is small), the efficiency of the system (1) with bistable and monostable potentials is close to the linear case since they are characterized by the same resonant frequency $\omega_0$. The growth of $A$ induces a nonlinear response and the efficiency decreases. Since the resonant frequency is zero for the flat potential, significantly poorer efficiency for $U_F(x)$ is expected than for the case of $U_L(x)$. However, the scenario can change when a different form of ambient energy is involved.

Let us then consider the signal $f(t)$ in the form of Poisson white noise which represents a sequence of independent pulses
\begin{equation}
f(t) = \sum_{i=1}^{N} \lambda \delta(t - t_i),
\end{equation}
where $\lambda$ is the pulse amplitude, and $t_i$ are the independent random times of the pulse appearance. Time intervals $t_i = t_{i+1} - t_i$ between two subsequent pulses have exponential distribution, with the parameter $\Lambda$ and a number of pulses, $n$, for time interval $\delta t$, and obeys the Poisson distribution
\begin{equation}
p(n) = \frac{(\delta \Lambda)^n \exp(-\delta \Lambda)}{n!}.
\end{equation}

The parameter $\Lambda$ defines the frequency of the pulse appearance. Since pulses are independent, the resulting spectrum is white. The input power of Poisson noise is $P_{in} = \lambda^2 \Lambda$ and the noise induces a constant bias $I = \lambda \Lambda$. In the limit of $1/I \to 0$ (while $P_{in}$ is constant), Poisson noise can be substituted by Gaussian noise. By varying the bias $I$ for a fixed $P_{in}$, the shape of the Poisson distribution Eq. (5) changes from symmetrical (close to Gaussian, when $1/I \to 0$) to a strongly asymmetrical non-Gaussian shape ($1/I \to \infty$). The value of $1/I$ can, therefore, be considered as a parameter of the asymmetry of distribution Eq. (5).

Let us now consider $\gamma$ as a function of the input power, $P_{in} = \lambda^2 \Lambda$ for variation in $\lambda$ and the fixed value of $\Lambda = 1$ (Fig. 1). The flat potential demonstrates the best performance for weak input power. In this case, the system Eq. (1) performs oscillations near an equilibrium state and nonlinear potentials can be replaced by corresponding linear potentials. For flat potential $\omega_0 = 0$, this leads to an efficient transformation of lower frequency components of the Poisson noise with $1/\omega^2$ scaling. The response of Eq. (1) for $U_B(x)$ and $U_M(x)$ potentials coincides with the linear case since they are characterized by the same $\omega_0$. The growth of $P_{in}$ induces larger amplitudes of oscillations, and the response becomes correspondingly nonlinear. As a result, the efficiency $\gamma$ decreases for $U_M(x)$ and $U_F(x)$ with increase of $\lambda$. In the bistable case, there are noise-induced jumps between states of $U_B(x)$ starting with a given value of $\lambda$, leading to enhanced efficiency. Further increase of $\lambda$ leads to a decrease in $\gamma$ for $U_B(x)$, which indicates that $\gamma$ has an extremum. For both potentials $U_F(x)$ and $U_B(x)$, the efficiency is greatly improved than for the linear case, for a wide range of $P_{in}$, whereas this is not the case for $U_M(x)$.

For moderate and strong $P_{in}$, the bistable potential shows better performance than $U_F(x)$, although the flat potential shows the best performance for a weak input signal $f(t)$. For the latter, noise induced oscillations occur near an equilibrium state where the potential is maximally flat, i.e., $\partial^2 U/\partial x^2 = 0$. The extension of this maximally flat region should, therefore, lead to an improvement of the performance for higher $P_{in}$. This conclusion is validated by the consideration of $U_{FF} = x^2/4$ (marker × in Fig. 1), for which the
efficiency $\gamma$ is larger than for the bistable potential in the entire range of $P_{in}$.

The influence of the asymmetry of noise distribution has been investigated where the input power is fixed $P_{in} = k^2 \lambda = 0.001$, and the bias $I$ is varying as shown in Fig. (2). It is to be noted here that $P_L$ changes weakly with $I$ and is nearly a constant. The efficiency $\gamma$ is negative and decreases monotonically with $1/I$ for $U_{BF}(x)$. In contrast, $\gamma$ shows a non-monotonic behaviour for both $U_F(x)$ and $U_B(x)$ with a maximal efficiency $\gamma$ in between the two limits $1/I \to 0$ and $1/I \to \infty$. Thus, $\gamma$ depends non-linearly on statistical characteristics of Poisson noise. Although similar parametric dependences are observed for the flat and bistable potentials, underlying mechanisms are different. For the flat case, the performance is maximal when pulses are rare enough to be processed as a single pulse, having the low-frequency spectral content. Since the response for $U_F(x)$ is larger in the low-frequency region, harvesting is efficient. More frequent pulses shift an effective frequency content to the high frequency region leading to decrease of the power; additionally, the bias $I$ changes the flatness of the potential. The other limit ($1/I \to \infty$) of rare pulses with a large amplitude leads to the power decrease as well since the pulses induce deviations far away from the flat area of $U_F(x)$. In the bistable case, the performance is defined by a frequency of jumps between the states. For small values of $1/I$, the bias $I$ increases the barrier between the states and the output power decreases. In the opposite region of large values of $1/I$, the probability of jumps also decreases due to a nonlinear dependence of the jump probability on $I$.

As a result, there is a region of $1/I$ with a maximal performance.

In conclusion, the in-depth comparison of the linear and several nonlinear potentials and related analysis as presented here show that the efficiency of energy harvesting is strongly dependent on the properties of ambient vibrations. The efficiency of the harvester with the flat potential changes from an inferior state for a harmonic signal to the best (if $U_{BF}(x)$ is selected) for the random impulsive excitation. A good performance can be observed in the bistable case for harvesting impulsive excitations. The results (Fig. 2) demonstrate a non-trivial dependence of efficiency of the harvesters on statistical properties of random excitations, viz., whilst the input power remains constant, the output power can change over a wide range of values depending on the symmetry of the distribution of the amplitude of the excitations. This conclusion strongly suggests that in order to design an efficient harvester, it is vitally important to take into account a comprehensive description of properties of the ambient vibrations.

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16Qualitatively similar results were observed for varying $\Lambda$ and fixed $\lambda$.