Risk Sharing with Higher-Dimensional Security Markets

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In this talk, we discuss how the aggregated risk in an economy populated by n agents with varying attitudes towards risk can be reduced by redistributing the individual risks. Our model considers aggregated and individual losses in a one-period market economy populated by a finite number $n \geq 2$ of agents. Each of the agents has a quantitative attitude towards the risk an individual loss poses. Following the seminal paper by Artzner et al. [1] and the works of Farkas et al. [2] and Munari [3], we define the risk of a loss net of gains as the minimal price that has to be raised today and invested in a security portfolio such that, tomorrow, the hedged position falls in a range of losses that is deemed acceptable by the agent. Thus, our approach goes far beyond the commonly assumed cash-additivity property of risk measures.

We present a precise mathematical formulation of the studied risk sharing model and define notions of optimality for partitions of an aggregated risk, Pareto optimal and equilibrium allocations. Our main results concern the existence of optimal allocations in two practically relevant settings: (i) the aggregated losses form a Banach lattice and, for each agent, acceptability is determined by a finite number of linear constraints; (ii) aggregated losses are random variables over an atomless probability space, and each agent determines acceptability with a criterion that is probabilistically sophisticated with respect to the reference probability measure. These results will be complemented with observations on continuity properties of set-valued maps either mapping an aggregated initial loss to its Pareto optimal allocations or an initial loss endowment to its equilibrium allocations.

References

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